# Database Systems DATA 514 

## Lecture 5: <br> E/R Diagrams and Constraints

## Database Design

What it is:

- Starting from scratch, design the database schema: relation, attributes, keys, foreign keys, constraints etc
Why it's hard
- The database will be in operation for a very long time (years). Updating the schema while in production is very expensive (why?)


## Database Design

- Consider issues such as:
- What entities to model
- How entities are related
- What constraints exist in the domain
- Several formalisms exists
- We discuss E/R diagrams
- Reading: Sec. 4.1-4.6


## Database Design Process

Conceptual Model:


Relational Model:
Tables + constraints
And also functional dep.

## Normalization:

Eliminates anomalies

## Conceptual Schema

Physical storage details
Physical Schema


## Entity / Relationship Diagrams

- Entity set = a class
- An entity = an object
- Attribute

Product city

- Relationship makes



## Keys in E/R Diagrams

- Every entity set must have a key



## What is a Relation?

- A mathematical definition:
- if $A, B$ are sets, then a relation $R$ is a subset of $A \times B$
- $A=\{1,2,3\}, B=\{a, b, c, d\}$,

$$
\begin{aligned}
& A \times B=\{(1, a),(1, b), \ldots,(3, d)\} \\
& R=\{(1, a),(1, c),(3, b)\}
\end{aligned}
$$



- makes is a subset of Product $\times$ Company:


Company

## Multiplicity of E/R Relations

- one-one:
- many-one

- many-many




## Multi-way Relationships

How do we model a purchase relationship between buyers, products and stores?


Can still model as a mathematical set (Q. how ?)
A. As a set of triples $\subseteq$ Person $\times$ Product $\times$ Store

## Arrows in Multiway Relationships

Q: What does the arrow mean?


A: A given person buys a given product from at most one store
[Arrow pointing to E means that if we select one entity from each of the other entity sets in the relationship, those entities are related to at most one entity in E]

## Arrows in Multiway Relationships

Q: What does the arrow mean?


A: A given person buys a given product from at most one store AND every store sells to every person at most one product

## Converting Multi-way Relationships to Binary



## Converting Multi-way Relationships to Binary



## 3. Design Principles

## What's wrong?



Moral: be faithful to the specifications of the app!

## Design Principles: What's Wrong?



## Design Principles: What's Wrong?



# From E/R Diagrams to Relational Schema 

- Entity set $\rightarrow$ relation
- Relationship $\rightarrow$ relation


## Entity Set to Relation



Product(prod-ID, category, price)

| prod-ID | category | price |
| :--- | :--- | :--- |
| Gizmo55 | Camera | 99.99 |
| Pokemn19 | Toy | 29.99 |

## N-N Relationships to Relations



Represent this in relations

## N-N Relationships to Relations



## N-1 Relationships to Relations



Represent this in relations

## N-1 Relationships to Relations



Remember: no separate relations for many-one relationship

## Multi-way Relationships to Relations



Purchase(prod-ID, ssn, name)

## Modeling Subclasses

Some objects in a class may be special

- define a new class
- better: define a subclass


So --- we define subclasses in E/R

## Subclasses



## Subclasses to Relations



# Modeling Union Types with Subclasses 

## FurniturePiece

## Person

## Company

Say: each piece of furniture is owned either by a person or by a company

## Modeling Union Types with Subclasses

Say: each piece of furniture is owned either by a person or by a company
Solution 1. Acceptable but imperfect (What's wrong?)


## Modeling Union Types with Subclasses

## Solution 2: better, more laborious



## Weak Entity Sets

Entity sets are weak when their key comes from other classes to which they are related.


Team(sport, number, universityName)
University(name)

## What Are the Keys of R ?



## What makes good schemas?



## Integrity Constraints Motivation

An integrity constraint is a condition specified on a database schema that restricts the data that can be stored in an instance of the database.

- ICs help prevent entry of incorrect information
- How? DBMS enforces integrity constraints
- Allows only legal database instances (i.e., those that satisfy all constraints) to exist
- Ensures that all necessary checks are always performed and avoids duplicating the verification logic in each application


## Constraints in E/R Diagrams

Finding constraints is part of the modeling process.
Commonly used constraints:

Keys: social security number uniquely identifies a person.
Single-value constraints: a person can have only one father.
Referential integrity constraints: if you work for a company, it must exist in the database.

Other constraints: peoples' ages are between 0 and 150 .

## Keys in E/R Diagrams

## Underline:



# Single Value Constraints 



## Referential Integrity Constraints

## Product

## makes

Company

Each product made by at most one company. Some products made by no company

## Product

## makes

Company
Each product made by exactly one company.

## Other Constraints



Q: What does this mean?
A: A Company entity cannot be connected by relationship to more than 99 Product entities

## Constraints in SQL

Constraints in SQL:

- Keys, foreign keys


## simplest

- Attribute-level constraints
- Tuple-level constraints
- Global constraints: assertions Most
complex
- The more complex the constraint, the harder it is to check and to enforce


## Key Constraints

Product(name, category)

## CREATE TABLE Product ( name CHAR(30) PRIMARY KEY, category VARCHAR(20))

OR: CREATE TABLE Product ( name CHAR(30), category VARCHAR(20), PRIMARY KEY (name))

## Keys with Multiple Attributes

Product(name, category, price)

> | CREATE TABLE Product ( |
| :--- |
| name CHAR(30), |
| category VARCHAR(20), |
| price INT, |
| PRIMARY KEY (name, category)) |

| Name | Category | Price |
| :---: | :---: | :---: |
| Gizmo | Gadget | 10 |
| Camera | Photo | 20 |
| Gizmo | Photo | 30 |
| Gizano | Garetget | 40 |

## Other Keys

## CREATE TABLE Product ( productID CHAR(10), name CHAR(30), category VARCHAR(20), price INT, PRIMARY KEY (productID), UNIQUE (name, category))

There is at most one PRIMARY KEY; there can be many UNIQUE

## Foreign Key Constraints

CREATE TABLE Purchase ( prodName CHAR(30) REFERENCES Product(name), date DATETIME)
prodName is a foreign key to Product(name) name must be a key in Product

May write just Product if name is PK

## Foreign Key Constraints

- Example with multi-attribute primary key

```
CREATE TABLE Purchase (
    prodName CHAR(30),
    category VARCHAR(20),
    date DATETIME,
    FOREIGN KEY (prodName, category)
        REFERENCES Product(name, category)
```

- (name, category) must be a KEY in Product


## What happens when data changes?

Types of updates:

- In Purchase: insert/update
- In Product: delete/update



## What happens when data changes?

- SQL has three policies for maintaining referential integrity:
- NO ACTION reject violating modifications (default)
- CASCADE after delete/update do delete/update
- SET NULL set foreign-key field to NULL
- SET DEFAULT set foreign-key field to default value
- need to be declared with column, e.g., CREATE TABLE Product (pid INT DEFAULT 42)


## Maintaining Referential Integrity

## CREATE TABLE Purchase ( prodName CHAR(30), category VARCHAR(20), date DATETIME, <br> FOREIGN KEY (prodName, category) REFERENCES Product(name, category) ON UPDATE CASCADE ON DELETE SET NULL )

|  | Purchase |  |
| :--- | :---: | :---: |
| Name Category <br> Gizmo gadget <br> Camera Photo <br> OneClick Photo$\quad$ProdName Category <br> Gizmo Gizmo <br> Snap Camera <br> EasyShoot Camera |  |  |

## Constraints on Attributes and Tuples

- Constraints on attributes: NOT NULL CHECK condition
-- obvious meaning...
-- any condition!
- Constraints on tuples CHECK condition


## Constraints on <br> Attributes and Tuples

```
CREATE TABLE Product ( productID CHAR(10), name CHAR(30), category VARCHAR(20), price INT CHECK (price > 0), PRIMARY KEY (productID))
```


## Constraints on Attributes and Tuples

CREATE TABLE Product ( productID CHAR(10), name CHAR(30), category VARCHAR(20) CHECK (category in ('toy','gadget','apparel')), price INT CHECK (price > 0), PRIMARY KEY (productID))

## Constraints on Attributes and Tuples

CREATE TABLE Product ( productID CHAR(10), name CHAR(30) NOT NULL, category VARCHAR(20) CHECK (category in ('toy','gadget','apparel')), price INT CHECK (price > 0), PRIMARY KEY (productID))

## Constraints on Attributes and Tuples

```
CREATE TABLE R (
    A int NOT NULL,
    B int CHECK (B > 50 and B < 100),
    C varchar(20),
    D int,
    CHECK (C >= 'd' or D > 0))
```


## Constraints on Attributes and Tuples

What does this constraint do?

> CREATE TABLE Purchase ( prodName CHAR(30)

> CHECK (prodName IN
> (SELECT Product.name FROM Product),
> date DATETIME NOT NULL)

## General Assertions

CREATE ASSERTION myAssert CHECK (NOT EXISTS(

SELECT Product.name FROM Product, Purchase WHERE Product.name = Purchase.prodName GROUP BY Product.name HAVING count(*) > 200) )

But most DBMSs do not implement assertions Because it is hard to support them efficiently Instead, they provide triggers

## Database Design Process

Conceptual Model:


Relational Model:
Tables + constraints
And also functional dep.


Normalization:
Eliminates anomalies
Conceptual Schema


Physical storage details
Physical Schema


## Relational Schema Design

| Name | SSN | PhoneNumber | City |
| :--- | :--- | :--- | :--- |
| Fred | $123-45-6789$ | $206-555-1234$ | Seattle |
| Fred | $123-45-6789$ | $206-555-6543$ | Seattle |
| Joe | $987-65-4321$ | $908-555-2121$ | Westfield |

One person may have multiple phones, but lives in only one city
Primary key is thus (SSN, PhoneNumber)
What is the problem with this schema?

## Relational Schema Design

| Name | $\underline{\text { SSN }}$ | PhoneNumber | City |
| :--- | :--- | :--- | :--- |
| Fred | $123-45-6789$ | $206-555-1234$ | Seattle |
| Fred | $123-45-6789$ | $206-555-6543$ | Seattle |
| Joe | $987-65-4321$ | $908-555-2121$ | Westfield |

## Anomalies:

- Redundancy = repeat data
- Update anomalies = what if Fred moves to "Bellevue"?
- Deletion anomalies = what if Joe deletes his phone number?


## Relation Decomposition

## Break the relation into two:



- No more repeated data
- Easy to move Fred to "Bellevue" (how ?)
- Easy to delete all Joe's phone numbers (how ?)


## Relational Schema Design (or Logical Design)

How do we do this systematically?

- Start with some relational schema
- Find out its functional dependencies (FDs)
- Use FDs to normalize the relational schema


## Functional Dependencies (FDs)

## Definition

If two tuples agree on the attributes

$$
A_{1}, A_{2}, \ldots, A_{n}
$$

then they must also agree on the attributes

$$
B_{1}, B_{2}, \ldots, B_{m}
$$

Formally:

$$
A_{1}, A_{2}, \ldots, A_{n} \rightarrow B_{1}, B_{2}, \ldots, B_{m}
$$

## Functional Dependencies (FDs)

Definition $A_{1}, \ldots, A_{m} \rightarrow B_{1}, \ldots, B_{n}$ holds in $R$ if:
$\forall \mathrm{t}, \mathrm{t}^{\prime} \in \mathrm{R}$,
$\left(\mathrm{t} . \mathrm{A}_{1}=\mathrm{t}^{\prime} . \mathrm{A}_{1} \wedge \ldots \wedge \mathrm{t} . \mathrm{A}_{\mathrm{m}}=\mathrm{t}^{\prime} . \mathrm{A}_{\mathrm{m}} \rightarrow \mathrm{t} . \mathrm{B}_{1}=\mathrm{t}^{\prime} . \mathrm{B}_{1} \wedge \ldots \wedge \mathrm{t} . \mathrm{B}_{\mathrm{n}}=\mathrm{t}^{\prime} . \mathrm{B}_{\mathrm{n}}\right)$

if $t$, t' agree hereathenvin, el agree here

## Example

An FD holds, or does not hold on an instance:

| EmpID | Name | Phone | Position |
| :--- | :--- | :--- | :--- |
| E0045 | Smith | 1234 | Clerk |
| E3542 | Mike | 9876 | Salesrep |
| E1111 | Smith | 9876 | Salesrep |
| E9999 | Mary | 1234 | Lawyer |

EmpID $\rightarrow$ Name, Phone, Position
Position $\rightarrow$ Phone
but not Phone $\rightarrow$ Position

## Example

| EmpID | Name | Phone | Position |
| :--- | :--- | :--- | :--- |
| E0045 | Smith | 1234 | Clerk |
| E3542 | Mike | $9876 \leftarrow$ | Salesrep |
| E1111 | Smith | $9876 \leftarrow$ | Salesrep |
| E9999 | Mary | 1234 | Lawyer |

Position $\rightarrow$ Phone

## Example

| EmpID | Name | Phone | Position |
| :--- | :--- | :--- | :--- |
| E0045 | Smith | $1234 \rightarrow$ | Clerk |
| E3542 | Mike | 9876 | Salesrep |
| E1111 | Smith | 9876 | Salesrep |
| E9999 | Mary | $1234 \rightarrow$ | Lawyer |

But not Phone $\rightarrow$ Position

## Example

## name $\rightarrow$ color

category $\rightarrow$ department color, category $\rightarrow$ price

| name | category | color | department | price |
| :---: | :---: | :---: | :---: | :---: |
| Gizmo | Gadget | Green | Toys | 49 |
| Tweaker | Gadget | Green | Toys | 99 |

Do all the FDs hold on this instance?

## Example

## name $\rightarrow$ color

category $\rightarrow$ department color, category $\rightarrow$ price

| name | category | color | department | price |
| :---: | :---: | :---: | :---: | :---: |
| Gizmo | Gadget | Green | Toys | 49 |
| Tweaker | Gadget | Green | Toys | 49 |
| Gizmo | Stationary | Green | Office-supp. | 59 |

## Terminology

- FD holds or does not hold on an instance
- If we can be sure that every instance of $R$ will be one in which a given FD is true, then we say that $R$ satisfies the FD
- If we say that $R$ satisfies an FD F, we are stating a constraint on $R$


## An Interesting Observation

If all these FDs are true:
name $\rightarrow$ color
category $\rightarrow$ department color, category $\rightarrow$ price

Then this FD also holds: name, category $\rightarrow$ price

If we find out from application domain that a relation satisfies some FDs, it doesn't mean that we found all the FDs that it satisfies! There could be more FDs implied by the ones we have.

## Closure of a set of Attributes

Given a set of attributes $A_{1}, \ldots, A_{n}$
The closure, $\left\{A_{1}, \ldots, A_{n}\right\}^{+}=$the set of attributes $B$ s.t. $A_{1}, \ldots, A_{n} \rightarrow B$

Example: 1. name $\rightarrow$ color
2. category $\rightarrow$ department
3. color, category $\rightarrow$ price

Closures:
name ${ }^{+}=$\{name, color\}
\{name, category $\}^{+}=\{$name, category, color, department, price\} color $^{+}=$\{color $\}$

## Closure Algorithm

$\mathrm{X}=\{\mathrm{A} 1, \ldots, \mathrm{An}\}$.
Example:
Repeat until $X$ doesn't change do: if $\quad B_{1}, \ldots, B_{n} \rightarrow C$ is a FD and $B_{1}, \ldots, B_{n}$ are all in $X$
then $\operatorname{add} \mathrm{C}$ to X .

1. name $\rightarrow$ color
2. category $\rightarrow$ department
3. color, category $\rightarrow$ price
\{name, category\} ${ }^{+}=$ \{ name, category, color, department, price \}
Hence: name, category $\rightarrow$ color, department, price

## Example

In class:
R(A,B,C,D,E,F)

$$
\begin{array}{lll}
\mathrm{A}, \mathrm{~B} & \rightarrow \mathrm{C} \\
\mathrm{~A}, \mathrm{D} & \rightarrow & \mathrm{E} \\
\mathrm{~B} & \rightarrow & \mathrm{D} \\
\mathrm{~A}, \mathrm{~F} & \rightarrow & \mathrm{~B}
\end{array}
$$

Compute $\{\mathrm{A}, \mathrm{B}\}^{+} \quad \mathrm{X}=\{\mathrm{A}, \mathrm{B}$,


Compute $\{\mathrm{A}, \mathrm{F}\}^{+} \quad \mathrm{X}=\{\mathrm{A}, \mathrm{F}$,

$$
\text { \} }
$$

## Example

In class:
R(A,B,C,D,E,F)

$$
\begin{array}{lll}
\mathrm{A}, \mathrm{~B} & \rightarrow \mathrm{C} \\
\mathrm{~A}, \mathrm{D} & \rightarrow & \mathrm{E} \\
\mathrm{~B} & \rightarrow & \mathrm{D} \\
\mathrm{~A}, \mathrm{~F} & \rightarrow & \mathrm{~B}
\end{array}
$$

Compute $\{\mathrm{A}, \mathrm{B}\}^{+} \quad \mathrm{X}=\{\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{D}, \mathrm{E}\}$
Compute $\{\mathrm{A}, \mathrm{F}\}^{+} \quad \mathrm{X}=\{\mathrm{A}, \mathrm{F}$,

$$
\}
$$

## Example

In class:
R(A,B,C,D,E,F)

$$
\begin{array}{lll}
\mathrm{A}, \mathrm{~B} & \rightarrow \mathrm{C} \\
\mathrm{~A}, \mathrm{D} & \rightarrow & \mathrm{E} \\
\mathrm{~B} & \rightarrow & \mathrm{D} \\
\mathrm{~A}, \mathrm{~F} & \rightarrow & \mathrm{~B}
\end{array}
$$

Compute $\{\mathrm{A}, \mathrm{B}\}^{+} \quad \mathrm{X}=\{\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{D}, \mathrm{E}\}$
Compute $\{\mathrm{A}, \mathrm{F}\}^{+} \quad \mathrm{X}=\{\mathrm{A}, \mathrm{F}, \mathrm{B}, \mathrm{C}, \mathrm{D}, \mathrm{E}\}$

## Example

In class:
R(A,B,C,D,E,F)

$$
\begin{array}{lll}
\mathrm{A}, \mathrm{~B} & \rightarrow \mathrm{C} \\
\mathrm{~A}, \mathrm{D} & \rightarrow & \mathrm{E} \\
\mathrm{~B} & \rightarrow & \mathrm{D} \\
\mathrm{~A}, \mathrm{~F} & \rightarrow & \mathrm{~B}
\end{array}
$$

Compute $\{\mathrm{A}, \mathrm{B}\}^{+} \quad \mathrm{X}=\{\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{D}, \mathrm{E}\}$
Compute $\{\mathrm{A}, \mathrm{F}\}^{+} \quad \mathrm{X}=\{\mathrm{A}, \mathrm{F}, \mathrm{B}, \mathrm{C}, \mathrm{D}, \mathrm{E}\}$

## Practice at Home

Find all FD's implied by:

| $A, B$ | $\rightarrow$ | $C$ |
| :--- | :--- | :--- |
| $A, D$ | $\rightarrow$ | $B$ |
| $B$ | $\rightarrow$ | $D$ |

## Practice at Home

Find all FD's implied by:

$$
\begin{array}{lll}
\mathrm{A}, \mathrm{~B} & \rightarrow \mathrm{C} \\
\mathrm{~A}, \mathrm{D} & \rightarrow & \mathrm{~B} \\
\mathrm{~B} & \rightarrow & \mathrm{D}
\end{array}
$$

Step 1: Compute $\mathrm{X}^{+}$, for every X :

$$
\begin{aligned}
& \mathrm{A}+=\mathrm{A}, \quad \mathrm{~B}+=\mathrm{BD}, \quad \mathrm{C}+=\mathrm{C}, \quad \mathrm{D}+=\mathrm{D} \\
& \mathrm{AB}+=\mathrm{ABCD}, \mathrm{AC}+=\mathrm{AC}, \mathrm{AD}+=\mathrm{ABCD}, \\
& \mathrm{BC}+=\mathrm{BCD}, \mathrm{BD}+=\mathrm{BD}, \mathrm{CD}+=\mathrm{CD}
\end{aligned}
$$

$A B C+=A B D+=A C D^{+}=A B C D$ (no need to compute- why ?) $B C D^{+}=B C D, \quad A B C D+=A B C D$
Step 2: Enumerate all FD's $X \rightarrow Y$, s.t. $Y \subseteq X^{+}$and $X \cap Y=\varnothing$ :


## Keys

- A superkey is a set of attributes $A_{1}, \ldots, A_{n}$ s.t. for any other attribute $B$, we have $A_{1}, \ldots, A_{n} \rightarrow B$
- A key is a minimal superkey
- A superkey and for which no subset is a superkey


## Computing (Super)Keys

- For all sets $X$, compute $X^{+}$
- If $X^{+}=$[all attributes], then $X$ is a superkey
- Try only the minimal X's to get the key


## Example

# Product(name, price, category, color) 

## name, category $\rightarrow$ price category $\rightarrow$ color

What is the key?

## Example

Product(name, price, category, color)

## name, category $\rightarrow$ price category $\rightarrow$ color

What is the key?
(name, category) + = \{ name, category, price, color \}
Hence (name, category) is a key

## Key or Keys ?

Can we have more than one key?

Given $R(A, B, C)$ define $F D$ 's s.t. there are two or more keys

| $A \rightarrow B$ <br> $B \rightarrow C$ <br> $C \rightarrow A$ |
| :--- |
| or $\quad$$A B \rightarrow C$ <br> $B C \rightarrow A$ |
| what are the keys here ? |$\quad$| $A \rightarrow B C$ |
| :--- |
| $B \rightarrow A C$ |

## Eliminating Anomalies

Main idea:

- $X \rightarrow A$ is OK if $X$ is a (super)key
- $\mathrm{X} \rightarrow \mathrm{A}$ is not OK otherwise
- Need to decompose the table, but how?


## Boyce-Codd Normal Form

## Boyce-Codd Normal Form

## Dr. Raymond F. Boyce

## Boyce-Codd Normal Form

There are no "bad" FDs:

Definition. A relation R is in BCNF if:
Whenever $X \rightarrow B$ is a non-trivial dependency, then $X$ is a superkey.

Equivalently: Definition. A relation R is in BCNF if: $\forall X$, either $\mathrm{X}^{+}=\mathrm{X}$ or $\mathrm{X}^{+}=[$all attributes $]$

## BCNF Decomposition Algorithm

Normalize(R)
find $X$ s.t.: $X \neq X^{+}$and $X^{+} \neq$[all attributes]
if (not found) then " $R$ is in BCNF" let $Y=X^{+}-X ; \quad Z=$ [all attributes $]-X^{+}$ decompose R into R1 $(X \cup Y)$ and R2 $(X \cup Z)$ Normalize(R1); Normalize(R2);


## Example

| Name | SSN | PhoneNumber | City |
| :--- | :--- | :--- | :--- |
| Fred | $123-45-6789$ | $206-555-1234$ | Seattle |
| Fred | $123-45-6789$ | $206-555-6543$ | Seattle |
| Joe | $987-65-4321$ | $908-555-2121$ | Westfield |
| Joe | $987-65-4321$ | $908-555-1234$ | Westfield |

## SSN $\rightarrow$ Name, City

The only key is: \{SSN, PhoneNumber\} Hence SSN $\rightarrow$ Name, City is a "bad" dependency


In other words:
SSN+ = SSN, Name, City ${ }^{\text {adAAFA4is }}$ N"ineriffier SSN nor All Attribule

## Example BCNF Decomposition

| Name | SSN | City |
| :--- | :--- | :--- |
| Fred | $123-45-6789$ | Seattle |
| Joe | $987-65-4321$ | Westfield |


| SSN | PhoneNumber |
| :--- | :--- |
| $123-45-6789$ | $206-555-1234$ |
| $123-45-6789$ | $206-555-6543$ |
| $987-65-4321$ | $908-555-2121$ |
| $987-65-4321$ | $908-555-1234$ |

Let's check anomalies:

- Redundancy?
- Update ?
- Delete?

Find $X$ s.t.: $X \neq X^{+}$and $X^{+} \neq[$all attributes $]$

## Example BCNF Decomposition

Person(name, SSN, age, hairColor, phoneNumber)
SSN $\rightarrow$ name, age
age $\rightarrow$ hairColor

Find X s.t.: $\mathrm{X} \neq \mathrm{X}^{+}$and $\mathrm{X}^{+} \neq$[all attributes]

## Example BCNF Decomposition

Person(name, SSN, age, hairColor, phoneNumber)
SSN $\rightarrow$ name, age age $\rightarrow$ hairColor
Iteration 1: Person: SSN+ = SSN, name, age, hairColor Decompose into: P(SSN, name, age, hairColor) Phone(SSN, phoneNumber)


Find X s.t.: $\mathrm{X} \neq \mathrm{X}^{+}$and $\mathrm{X}^{+} \neq$[all attributes]

## Example BCNF Decomposition

Person(name, SSN, age, hairColor, phoneNumber)

SSN $\rightarrow$ name, age age $\rightarrow$ hairColor

What are the keys?

Iteration 1: Person: SSN+ = SSN, name, age, hairColor Decompose into: P(SSN, name, age, hairColor) Phone(SSN, phoneNumber)

Iteration 2: P: age+ = age, hairColor
Decompose: People(SSN, name, age)
Hair(age, hairColor)
Phone(SSN, phoneNumber)

Find X s.t.: $\mathrm{X} \neq \mathrm{X}^{+}$and $\mathrm{X}^{+} \neq$[all attributes]

## Example BCNF Decomposition

Person(name, SSN, age, hairColor, phoneNumber)
SSN $\rightarrow$ name, age age $\rightarrow$ hairColor

## Note the keys!

Iteration 1: Person: SSN+ = SSN, name, age, hairColor Decompose into: P(SSN, name, age, hairColor) Phone(SSN, phoneNumber)

Iteration 2: P: age+ = age, hairColor
Decompose: People(SSN, name, age)
Hair(age, hairColor)
Phone(SSN, phoneNumber)

## Example: BCNF

## $A \rightarrow B$ <br> $B \rightarrow C$

R(A,B,C,D)
$R(A, B, C, D)$

## Example: BCNF

## $A \rightarrow B$ <br> $B \rightarrow C$

Recall: find X s.t. $X \subsetneq X^{+} \subsetneq$ [all-attrs] $\quad R(A, B, C, D)$

## Example: BCNF

## $A \rightarrow B$ <br> $B \rightarrow C$

## R(A,B,C,D) <br> $A^{+}=A B C \neq A B C D$

## Example: BCNF

$$
\begin{aligned}
& A \rightarrow B \\
& B \rightarrow C
\end{aligned}
$$


$R(A, B, C, D)$

## Example: BCNF

## $A \rightarrow B$ <br> $B \rightarrow C$



## R(A,B,C,D)

## Example: BCNF

$$
\begin{aligned}
& A \rightarrow B B \\
& B \rightarrow C
\end{aligned}
$$




## Decompositions in General


$S_{1}=$ projection of $R$ on $A_{1}, \ldots, A_{n}, B_{1}, \ldots, B_{m}$
$S_{2}=$ projection of $R$ on $A_{1}, \ldots, A_{n}, C_{1}, \ldots, C_{p}$

## Lossless Decomposition



## Lossy Decomposition

## What is lossy here?



| Name | Price | Category |
| :---: | :---: | :---: |
| Gizmo | 19.99 | Gadget |
| OneClick | 24.99 | Camera |
| Gizmo | 19.99 | Camera |


| Name | Category |
| :---: | :---: |
| Gizmo | Gadget |
| OneClick | Camera |
| Gizmo | Camera |


| Price | Category |
| :---: | :---: |
| 19.99 | Gadget |
| 24.99 | Camera |
| 19.99 | Camera |

## Lossy Decomposition



## Decomposition in General

$$
R\left(A_{1}, \ldots, A_{n}, B_{1}, \ldots, B_{m}, C_{1}, \ldots, C_{p}\right)
$$

$$
S_{1}\left(A_{1}, \ldots, A_{n}, B_{1}, \ldots, B_{m}\right) \quad S_{2}\left(A_{1}, \ldots, A_{n}, C_{1}, \ldots, C_{p}\right)
$$

Let: $\quad S_{1}=$ projection of $R$ on $A_{1}, \ldots, A_{n}, B_{1}, \ldots, B_{m}$
$S_{2}=$ projection of $R$ on $A_{1}, \ldots, A_{n}, C_{1}, \ldots, C_{p}$
The decomposition is called lossless if $R=S_{1} \bowtie S_{2}$
Fact: If $A_{1}, \ldots, A_{n} \rightarrow B_{1}, \ldots, B_{m}$ then the decomposition is lossless
It follows that every B6ata F $_{4}$ deramposition is lossless

Example from textbook Ch. 3.4.2

## The Chase Test for Lossless Join

$R(A, B, C, D)=S 1(A, D) \bowtie S 2(A, C) \bowtie S 3(B, C, D)$
$R$ satisfies: $A \rightarrow B, B \rightarrow C, C D \rightarrow A$
$S 1=\Pi_{A D}(R), S 2=\Pi_{A C}(R), S 3=\Pi_{B C D}(R)$,
hence $R \subseteq S 1 \bowtie S 2 \bowtie S 3$
Need to check: $\mathrm{R} \supseteq \mathrm{S} 1 \bowtie \mathrm{~S} 2 \bowtie \mathrm{~S} 3$

Example from textbook Ch. 3.4.2

## The Chase Test for Lossless Join

$$
R(A, B, C, D)=S 1(A, D) \bowtie S 2(A, C) \bowtie S 3(B, C, D)
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hence $R \subseteq S 1 \bowtie S 2 \bowtie S 3$
Need to check: $\mathrm{R} \supseteq \mathrm{S} 1 \bowtie \mathrm{~S} 2 \bowtie \mathrm{~S} 3$
Suppose (a,b,c,d) $\in S 1 \bowtie S 2 \bowtie S 3$ Is it also in $R$ ? $R$ must contain the following tuples:

| A | B | C | D |
| :---: | :---: | :---: | :---: |
| Why ? |  |  |  |
| a | b1 | c1 | d |
| $(a, d) \in S 1=\Pi_{A D}(R)$ |  |  |  |

Example from textbook Ch. 3.4.2

## The Chase Test for Lossless Join

$$
R(A, B, C, D)=S 1(A, D) \bowtie S 2(A, C) \bowtie S 3(B, C, D)
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Suppose (a,b,c,d) $\in S 1 \bowtie S 2 \bowtie S 3$ Is it also in $R$ ? $R$ must contain the following tuples:

| A | B | C | D | Why?$\begin{aligned} & (\mathrm{a}, \mathrm{~d}) \in \mathrm{S} 1=\Pi_{\mathrm{AD}}(\mathrm{R}) \\ & (\mathrm{a}, \mathrm{c}) \in \mathrm{S} 2=\Pi_{\mathrm{BD}}(\mathrm{R}) \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: |
| a | b1 | c1 | d |  |
| a | b2 | C | d2 |  |

Example from textbook Ch. 3.4.2

## The Chase Test for Lossless Join

$$
R(A, B, C, D)=S 1(A, D) \bowtie S 2(A, C) \bowtie S 3(B, C, D)
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Suppose (a,b,c,d) $\in S 1 \bowtie S 2 \bowtie S 3$ Is it also in $R$ ? $R$ must contain the following tuples:

| A | B | C | D | Why?$\begin{aligned} & (\mathrm{a}, \mathrm{~d}) \in \mathrm{S} 1=\Pi_{\mathrm{AD}}(\mathrm{R}) \\ & (\mathrm{a}, \mathrm{c}) \in \mathrm{S} 2=\Pi_{\mathrm{BD}}(\mathrm{R}) \\ & (\mathrm{b}, \mathrm{c}, \mathrm{~d}) \in \mathrm{S} 3=\Pi_{\mathrm{BCD}}(\mathrm{R}) \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: |
| a | b1 | c1 | d |  |
| a | b2 | c | d2 |  |
| a3 | b | c | d |  |

Example from textbook Ch. 3.4.2

## The Chase Test for Lossless Join

$$
R(A, B, C, D)=S 1(A, D) \bowtie S 2(A, C) \bowtie S 3(B, C, D)
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Need to check: $\mathrm{R} \supseteq \mathrm{S} 1 \bowtie \mathrm{~S} 2 \bowtie \mathrm{~S} 3$
Suppose (a,b,c,d) $\in S 1 \bowtie S 2 \bowtie S 3$ Is it also in $R$ ? $R$ must contain the following tuples:
"Chase" them (apply FDs):

| A | B | C | D | Why ?$\begin{aligned} & (\mathrm{a}, \mathrm{~d}) \in S 1=\Pi_{\mathrm{AD}}(\mathrm{R}) \\ & (\mathrm{a}, \mathrm{c}) \in S 2=\Pi_{\mathrm{BD}}(\mathrm{R}) \\ & (\mathrm{b}, \mathrm{c}, \mathrm{~d}) \in S 3=\Pi_{\mathrm{BCD}}(\mathrm{R}) \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: |
| a | b1 | c1 | d |  |
| a | b2 | C | d2 |  |
| a3 | b | C | d |  |

## $A \rightarrow B$

| A | B | C | D |
| :---: | :---: | :---: | :---: |
| a | b1 | c1 | d |
| a | b1 | c | d2 |
| a3 | b | c | d |

Example from textbook Ch. 3.4.2

## The Chase Test for Lossless Join

$$
R(A, B, C, D)=S 1(A, D) \bowtie S 2(A, C) \bowtie S 3(B, C, D)
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Need to check: R $\supseteq$ S1 $\bowtie$ S2 $\bowtie$ S3
Suppose (a,b,c,d) $\in S 1 \bowtie S 2 \bowtie S 3$ Is it also in $R$ ? $R$ must contain the following tuples:
"Chase" them (apply FDs):

| A | B | C | D | Why?$\begin{aligned} & (\mathrm{a}, \mathrm{~d}) \in \mathrm{S} 1=\Pi_{\mathrm{AD}}(\mathrm{R}) \\ & (\mathrm{a}, \mathrm{c}) \in \mathrm{S} 2=\Pi_{\mathrm{BD}}(\mathrm{R}) \\ & (\mathrm{b}, \mathrm{c}, \mathrm{~d}) \in \mathrm{S} 3=\Pi_{\mathrm{BCD}}(\mathrm{R}) \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: |
| a | b1 | c1 | d |  |
| a | b2 | C | d2 |  |
| a3 | b | C | d |  |



Example from textbook Ch. 3.4.2

## The Chase Test for Lossless Join

$$
R(A, B, C, D)=S 1(A, D) \bowtie S 2(A, C) \bowtie S 3(B, C, D)
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hence $R \subseteq S 1 \bowtie S 2 \bowtie S 3$
Need to check: R $\supseteq$ S1 $\bowtie$ S2 $\bowtie$ S3
Suppose (a,b,c,d) $\in S 1 \bowtie S 2 \bowtie S 3$ Is it also in $R$ ? $R$ must contain the following tuples:
"Chase" them (apply FDs):

$|$| $A \rightarrow B$ |  |  |  |
| :---: | :---: | :---: | :---: |
| $A$ | $B$ | $C$ | $D$ |
| $a$ | $b 1$ | $c 1$ | $d$ |
| $a$ | $b 1$ | $c$ | $d 2$ |
| $a 3$ | $b$ | $c$ | $d$ |



Hence R
contains (a,p,p,c,d)

## Schema Refinements = Normal Forms

- 1st Normal Form = all tables are flat
- 2nd Normal Form = obsolete
- Boyce Codd Normal Form = no bad FDs
- 3rd Normal Form = see book
- BCNF is lossless but can cause loss of ability to check some FDs (see book 3.4.4)
- 3NF fixes that (is lossless and dependencypreserving), but some tables might not be in BCNF - i.e., they may have redundancy anomalies

