Lecture 2:
Image Classification

Administrative: Assignment 0

- Due 1/13 by 11:59pm
- Easy assignment
- Hardest part is learning how to use colab and how to submit on gradescope
- Worth 0% of your grade
- Used to evaluate how prepared you are to take this course

Administrative: Assignment 1

Due 1/23 11:59pm

- K-Nearest Neighbor
- Linear classifiers: SVM, Softmax

Administrative: Course Project

Project proposal due 4/29 11:59pm

Find your teammates on EdStem. We will help find teammates as well.

Collaboration: EdStem

“Is X a valid project for 493G1?”

- Anything related to **deep learning + pixels**
- Maximum of 3 students per team
- Make a EdStem private post or come to TA Office Hours

More info on the website

Administrative: Fridays

This Friday 9:30-10:30am and again 12:30-1:30pm

Broadcasting

Presenter: Tanush (Head TA)

Syllabus

Deep learning Fundamentals

- Data-driven approaches
- Linear classification & kNN
- Loss functions
- Optimization
- Backpropagation
- Multi-layer perceptrons
- Neural Networks
- Convolutions
- RNNs / LSTMs
- Transformers

Practical training skills

- Pytorch 1.4 / Tensorflow 2.0
- Activation functions
- Batch normalization
- Transfer learning
- Data augmentation
- Momentum / RMSProp / Adam
- Architecture design

Applications

- Image captioning
- Interpreting machine learning
- Generative AI
- Fairness & ethics
- Data-centric AI
- Deep reinforcement learning
- Self-supervised learning
- Diffusion
- LLMs

Image Classification

A Core Task in Computer Vision

Today:

- The image classification task
- Two basic data-driven approaches to image classification
    - K-nearest neighbor and linear classifier

Image Classification: A core task in Computer Vision

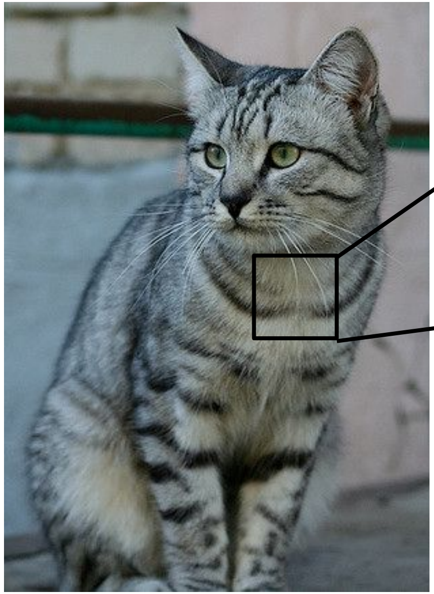
This image by Nikita is licensed under CC-BY 2.0

(assume given a set of possible labels)

→

Cat
Dog
Bird
Truck
Plane

The Problem: Semantic Gap



This image by Nikita is licensed under CC-BY 2.0

```
[[105 112 108 111 104  99 106  99  96 103 112 119 104  97  93  87]
 [ 91  98 102 106 104  79  98 103  99 105 123 136 110 105  94  85]
 [ 76  85  90 105 128 105  87  96  95  99 115 112 106 103  99  85]
 [ 99  81  81  93 120 131 127 100  95  98 102  99  96  93 101  94]
 [106  91  61  64  69  91  88  85 101 107 109  98  75  84  96  95]
 [114 108  85  55  55  69  64  54  64  87 112 129  98  74  84  91]
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 [115 114 109 123 150 148 131 118 113 109 100  92  74  65  72  78]
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 [ 63  65  75  88  89  71  62  81 120 138 135 105  81  98 110 118]
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 [122 121 102  80  82  86  94 117 145 148 153 102  58  78  92 107]
 [122 164 148 103  71  56  78  83  93 103 119 139 102  61  69  84]]
```
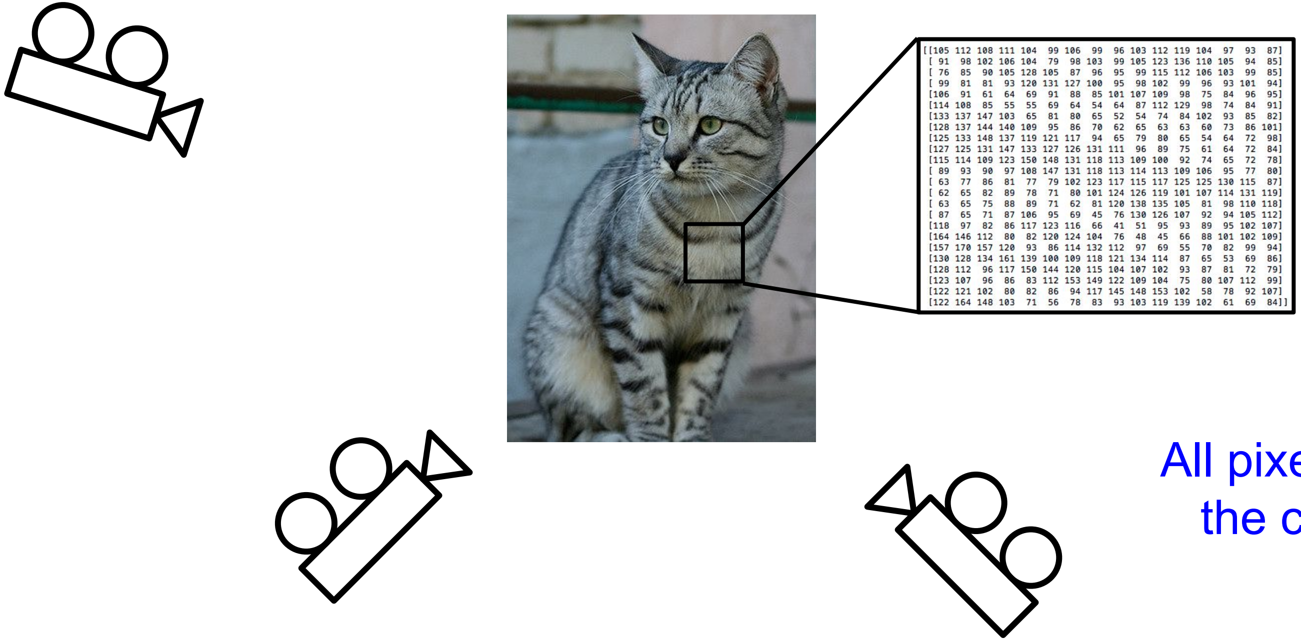

What the computer sees

An image is a tensor of integers between [0, 255]:

e.g. 800 x 600 x 3
(3 channels RGB)

Challenges: Viewpoint variation



```
[[105 112 108 111 104  99 106  99  96 103 112 119 104  97  93  87]
 [ 91  98 102 106 104  79  98 103  99 105 123 136 110 105  94  85]
 [ 76  85  90 105 128 105  87  96  95  99 115 112 106 103  99  85]
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 [122 121 102  80  82  86  94 117 145 148 153 102  58  78  92 107]
 [122 164 148 103  71  56  78  83  93 103 119 139 102  61  69  84]]
```
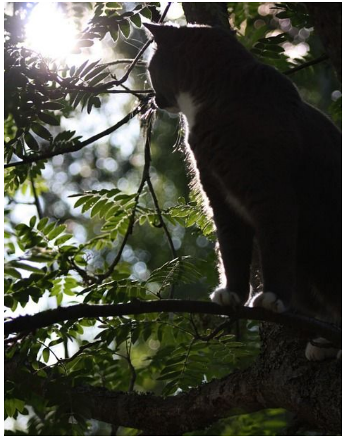

All pixels change when the camera moves!

Challenges: Illumination



RGB values are a function of surface materials, color, light source, etc.

Challenges: Background Clutter

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Challenges: Occlusion

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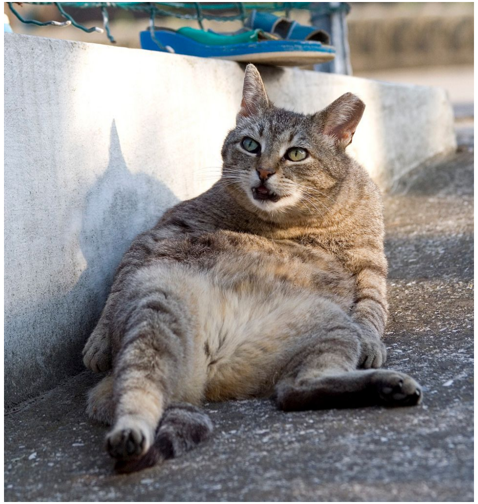
Challenges: Deformation



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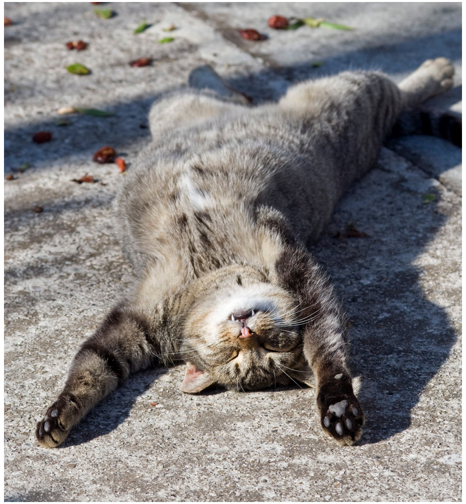

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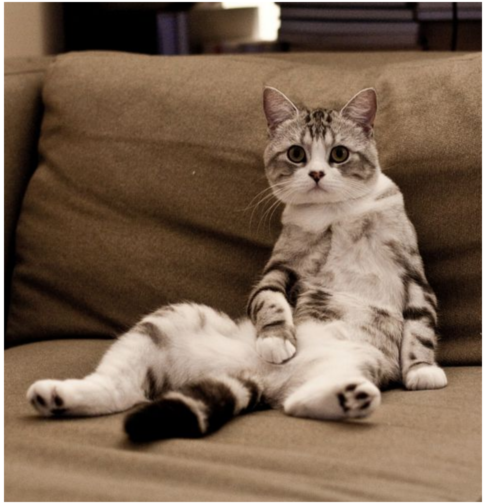
This image by sare bear is licensed under CC-BY 2.0



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Challenges: Intraclass variation

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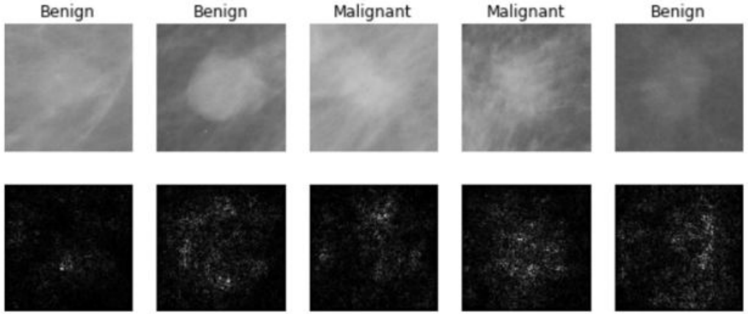
Image classification is a building block for other tasks

Medical Imaging



Levy et al, 2016 Figure reproduced with permission

Galaxy Classification

Dieleman et al, 2014

From left to right: public domain by NASA, usage permitted by ESA/Hubble, public domain by NASA, and public domain.

Whale recognition

Kaggle Challenge

This image by Christin Khan is in the public domain and originally came from the U.S. NOAA.

Image classification is a building block for other tasks

Image Captioning

Vinyals et al, 2015

Karpathy and Fei-Fei, 2015

A white teddy bear sitting in the grass

A man in a baseball uniform throwing a ball

A woman is holding a cat in her hand

A man riding a wave on top of a surfboard

A cat sitting on a suitcase on the floor

A woman standing on a beach holding a surfboard

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https://pixabay.com/en/luggage-antique-cat-1643010/
https://pixabay.com/en/teddy-plush-bears-cute-teddy-bear-1623436/
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https://pixabay.com/en/woman-female-model-portrait-adult-983967/
https://pixabay.com/en/handstand-lake-meditation-496008/
https://pixabay.com/en/baseball-player-shortstop-infield-1045263/

Captions generated by Justin Johnson using Neuraltalk2

Image classification is a building block for other tasks

Example: Playing Go

This image is CC0 public domain

→

(1, 1)

(1, 2)

...

(1, 19)

...

(19, 19)

Where to play next?

Modern computer vision algorithms

Classifiers today take 1ms to classify images. And can handle an unlimited (open-set) of categories.

This image is CC0 1.0 public domain

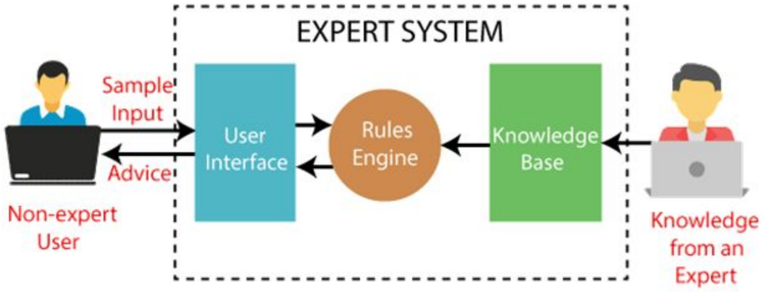
An image classifier: can we implement this as a normal software function?

```
def classify_image(image):
  # Some magic here?
  return class_label
```

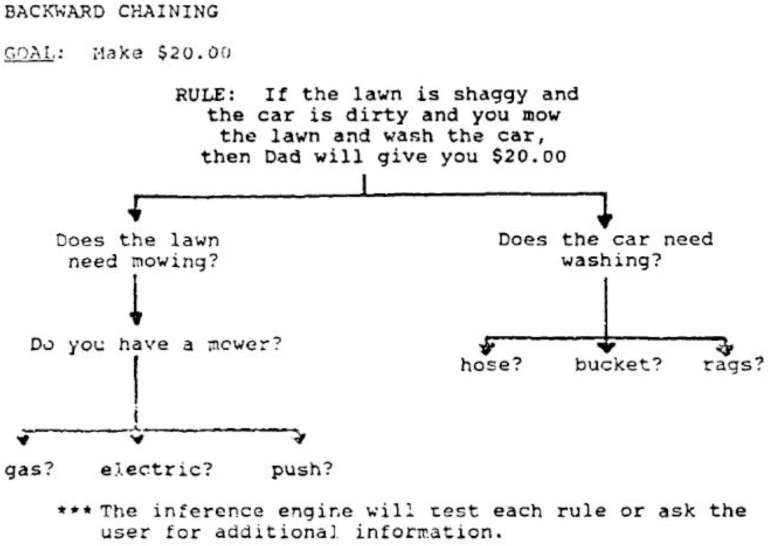
Unlike e.g. sorting a list of numbers,

no obvious way to hard-code the algorithm for recognizing a cat, or other classes.

This is why expert systems in the 80s led to the AI winter.



Originally called heuristic programming project.

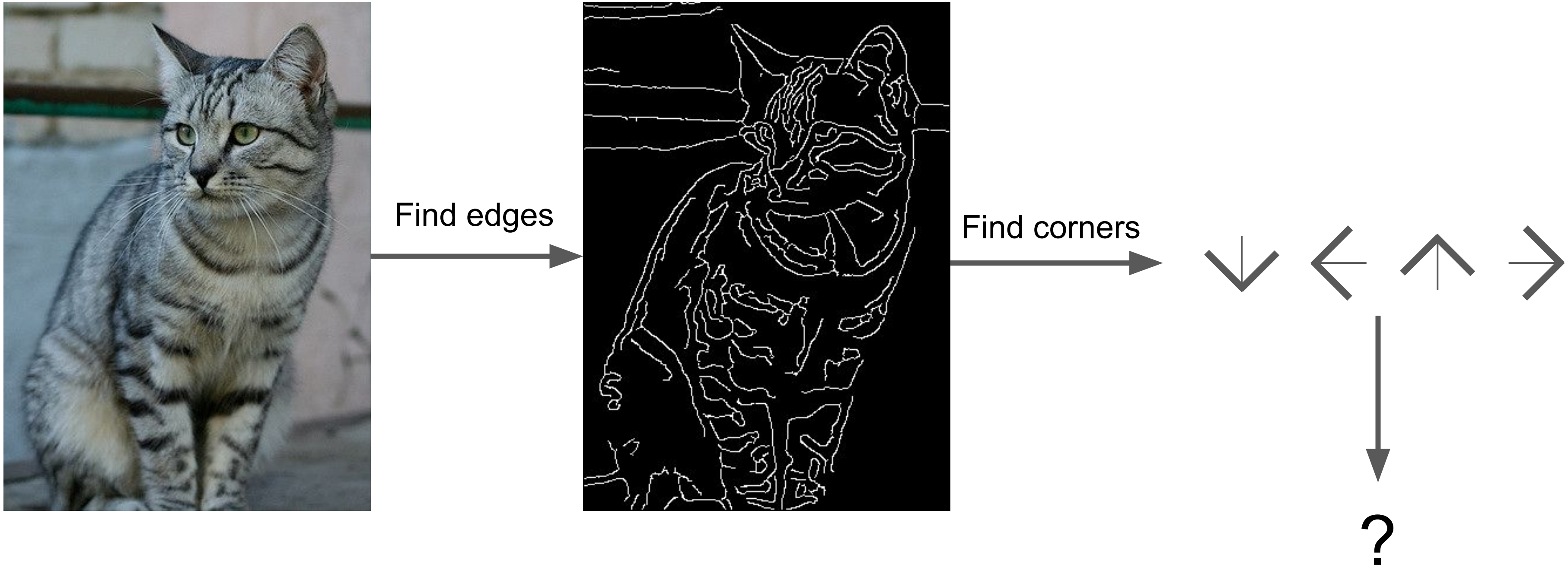

BACKWARD CHAINING

GOAL: Make $20.00

RULE: If the lawn is shaggy and the car is dirty and you mow the lawn and wash the car, then Dad will give you $20.00

*** The inference engine will test each rule or ask the user for additional information.

Attempts have been made



Find edges

Find corners

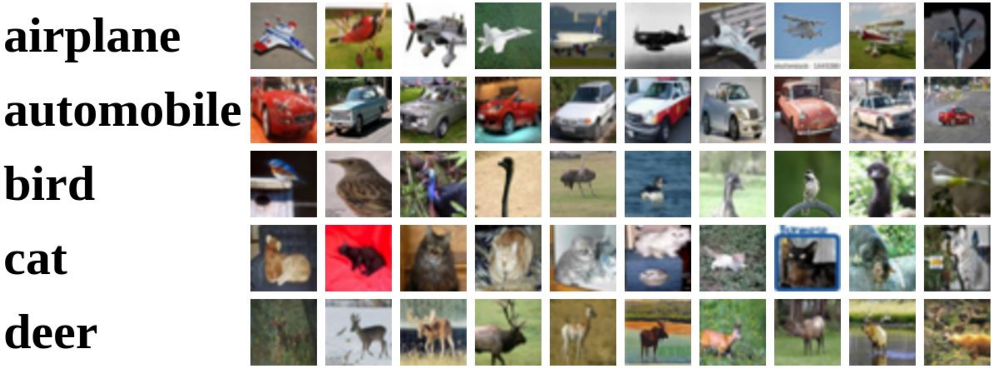
?

John Canny, "A Computational Approach to Edge Detection", IEEE TPAMI 1986

Machine Learning: Data-Driven Approach

1. Collect a dataset of images and labels

Example training set



Example dataset: MNIST

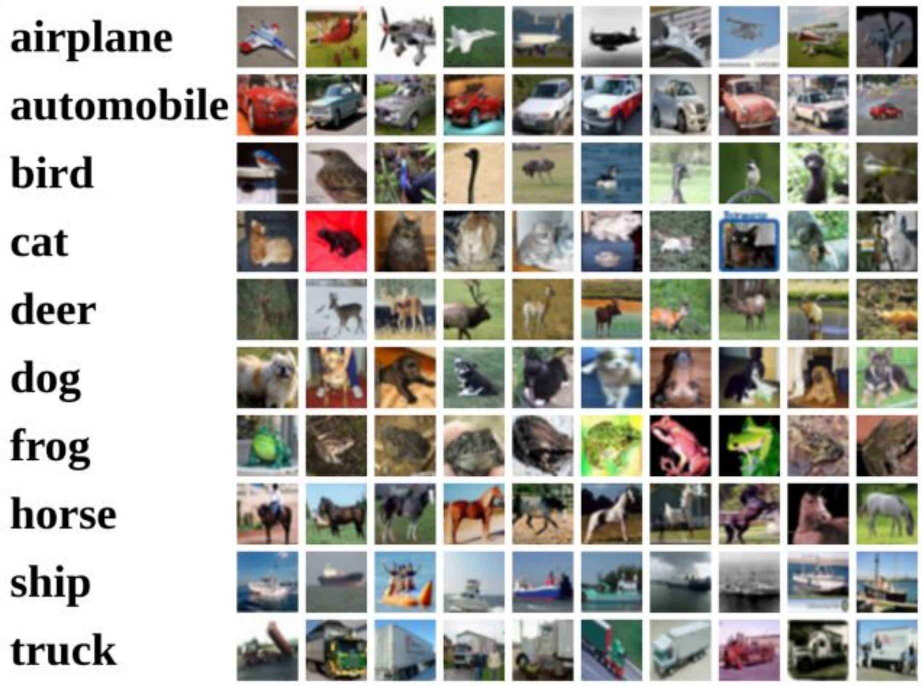
10 classes: Digits 0 to 9

28x28 grayscale images

50k training images

10k test images

Example dataset: CIFAR10



10 classes

50k training images (5k per class)

10k testing images (1k per class)

32x32 RGB images

We will use this dataset for homework assignments

Example dataset: CIFAR100

100 classes
50k training images (500 per class)
10k testing images (100 per class)
32x32 RGB images

20 superclasses with 5 classes each:

<u>Aquatic mammals</u>: beaver, dolphin, otter, seal, whale
<u>Trees</u>: Maple, oak, palm, pine, willow

Example dataset: ImageNet (ILSVRC challenge)

ILSVRC = ImageNet Large-Scale Visual Recognition Challenge

1000 classes

~1.3M training images (~1.3K per class)
50K validation images (50 per class)
100K test images (100 per class)

Performance metric: **Top 5 accuracy**
Algorithm predicts 5 labels for each image; one of them needs to be right

Example dataset: MIT Places

365 classes of different scene types

~8M training images
18.25K val images (50 per class)
328.5K test images (900 per class)

Images have variable size, often resize to **256x256** for training

Example dataset: Omniglot

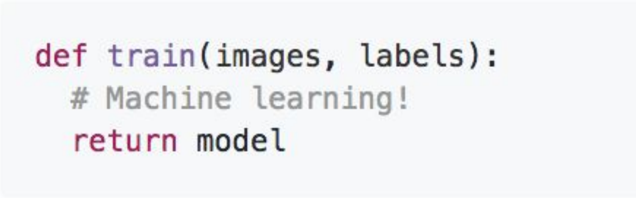
1623 categories: characters from 50 different alphabets

20 images per category

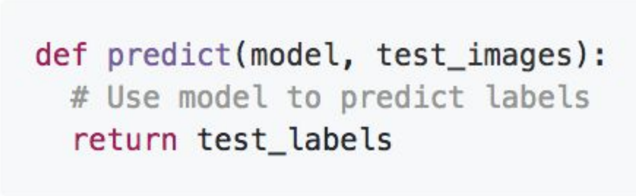
Meant to test **few shot learning**

Machine Learning: Data-Driven Approach

1. Collect a dataset of images and labels
2. Use Machine Learning algorithms to train a classifier
3. Evaluate the classifier on new images

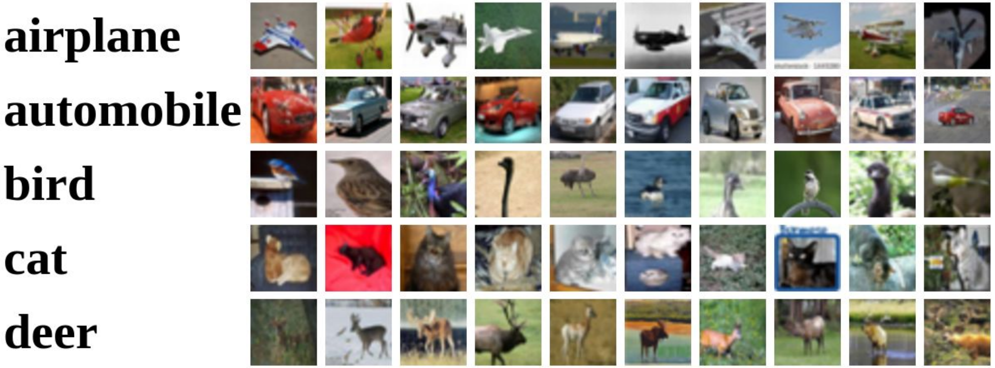


```
def train(images, labels):
  # Machine learning!
  return model
```



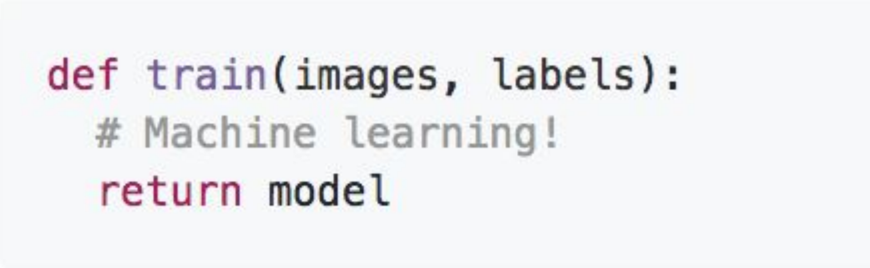
```
def predict(model, test_images):
  # Use model to predict labels
  return test_labels
```

Example training set



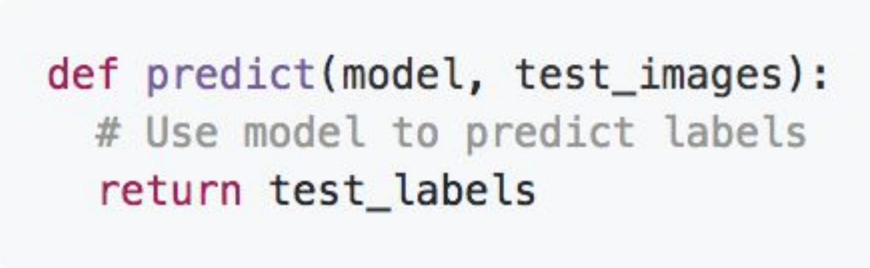
Nearest Neighbor Classifier

First classifier: **Nearest Neighbor**



```
def train(images, labels):
  # Machine learning!
  return model
```

→ Memorize all data and labels



```
def predict(model, test_images):
  # Use model to predict labels
  return test_labels
```

→ Predict the label of the most similar training image

First classifier: **Nearest Neighbor**

Training data with labels

query data

Distance Metric $\Big|$  $\Big| \to \mathbb{R}$

What is a good distance metric?

Distance Metric to compare images

L1 distance:

$$d_1(I_1, I_2) = \sum_p |I_1^p - I_2^p|$$

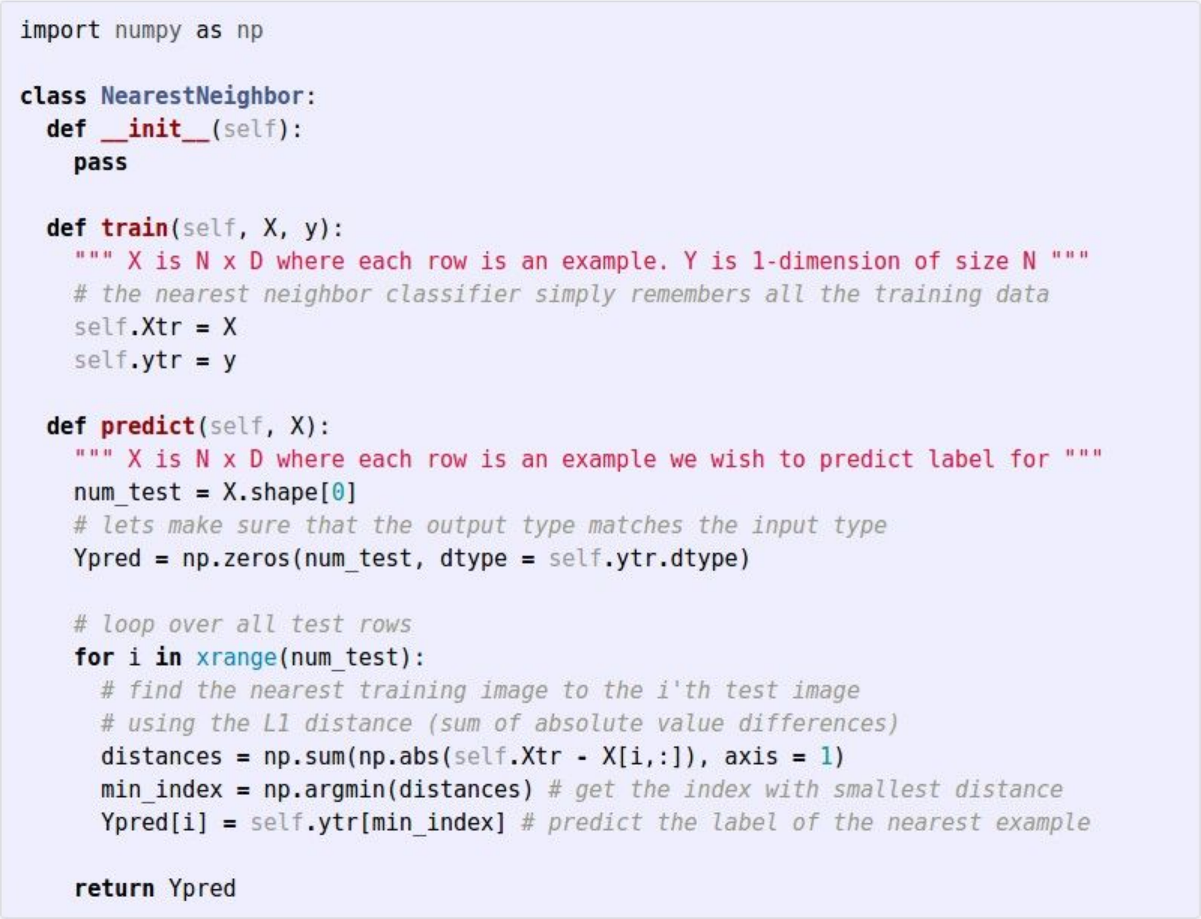
test image

training image

pixel-wise absolute value differences

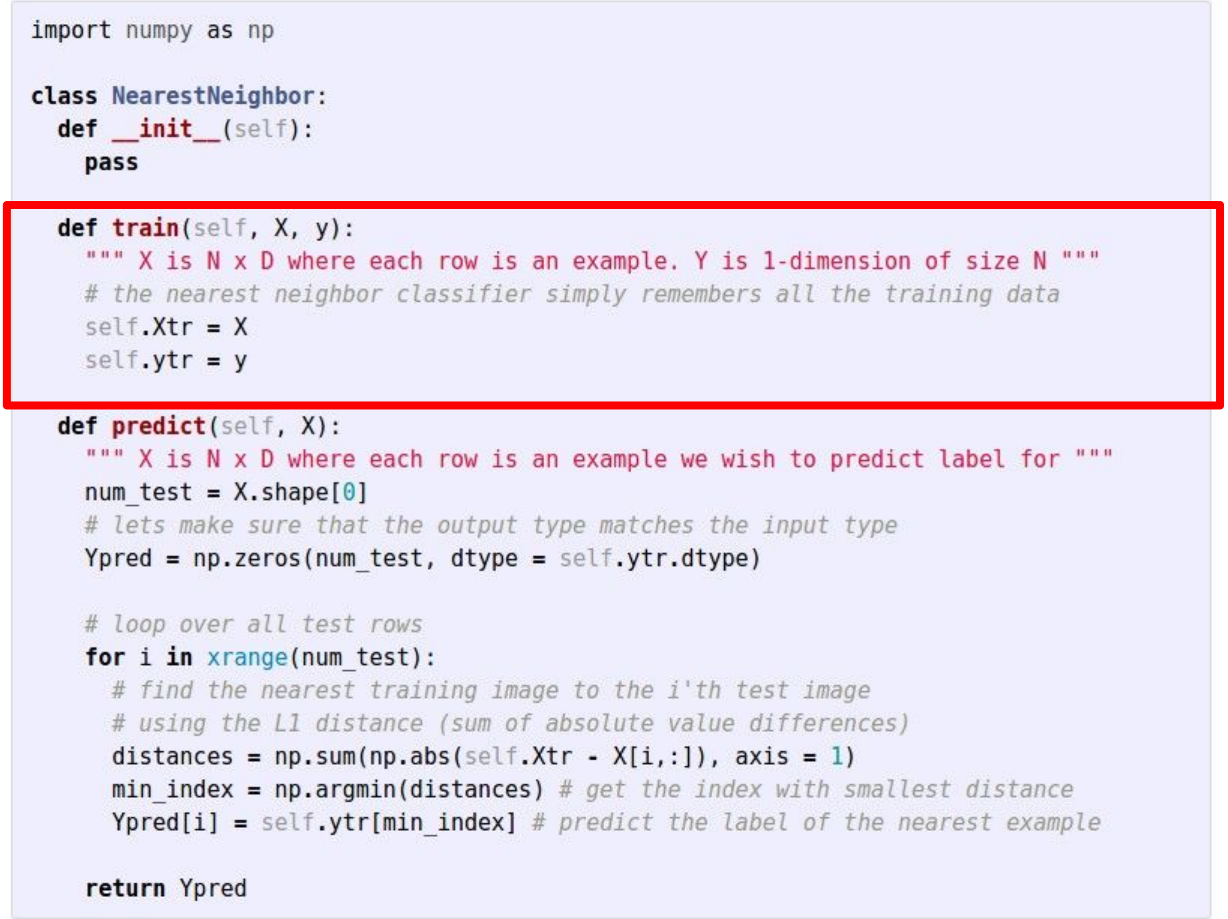
$$\left|\begin{array}{|c|c|c|c|}\hline 56 & 32 & 10 & 18 \\ \hline 90 & 23 & 128 & 133 \\ \hline 24 & 26 & 178 & 200 \\ \hline 2 & 0 & 255 & 220 \\ \hline \end{array} - \begin{array}{|c|c|c|c|}\hline 10 & 20 & 24 & 17 \\ \hline 8 & 10 & 89 & 100 \\ \hline 12 & 16 & 178 & 170 \\ \hline 4 & 32 & 233 & 112 \\ \hline \end{array}\right| = \begin{array}{|c|c|c|c|}\hline 46 & 12 & 14 & 1 \\ \hline 82 & 13 & 39 & 33 \\ \hline 12 & 10 & 0 & 30 \\ \hline 2 & 32 & 22 & 108 \\ \hline \end{array} \xrightarrow{\text{add}} 456$$

Nearest Neighbor classifier



```
import numpy as np

class NearestNeighbor:
  def __init__(self):
    pass

  def train(self, X, y):
    """ X is N x D where each row is an example. Y is 1-dimension of size N """
    # the nearest neighbor classifier simply remembers all the training data
    self.Xtr = X
    self.ytr = y

  def predict(self, X):
    """ X is N x D where each row is an example we wish to predict label for """
    num_test = X.shape[0]
    # lets make sure that the output type matches the input type
    Ypred = np.zeros(num_test, dtype = self.ytr.dtype)

    # loop over all test rows
    for i in xrange(num_test):
      # find the nearest training image to the i'th test image
      # using the L1 distance (sum of absolute value differences)
      distances = np.sum(np.abs(self.Xtr - X[i,:]), axis = 1)
      min_index = np.argmin(distances) # get the index with smallest distance
      Ypred[i] = self.ytr[min_index] # predict the label of the nearest example

    return Ypred
```

Nearest Neighbor classifier



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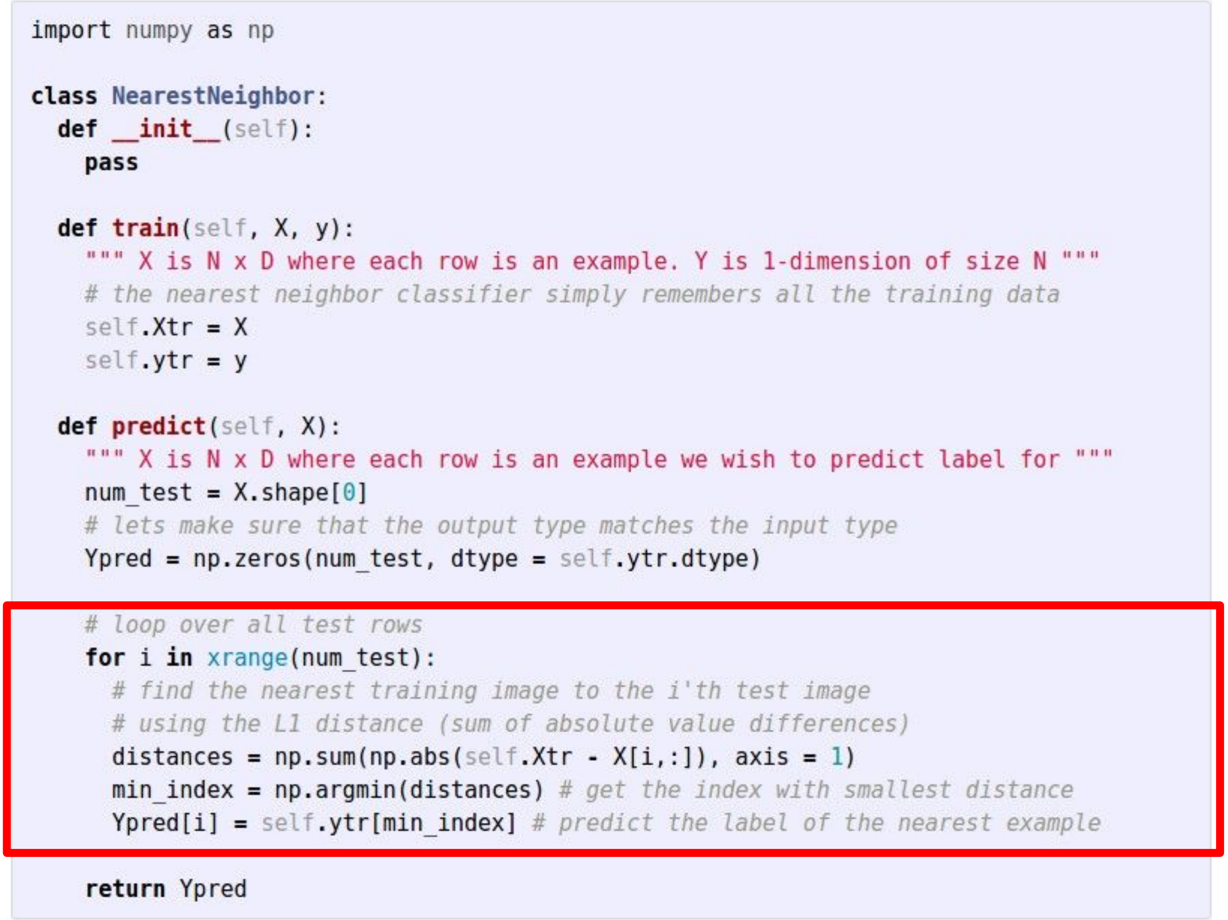
```

Memorize training data

Nearest Neighbor classifier



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      Ypred[i] = self.ytr[min_index] # predict the label of the nearest example

    return Ypred
```
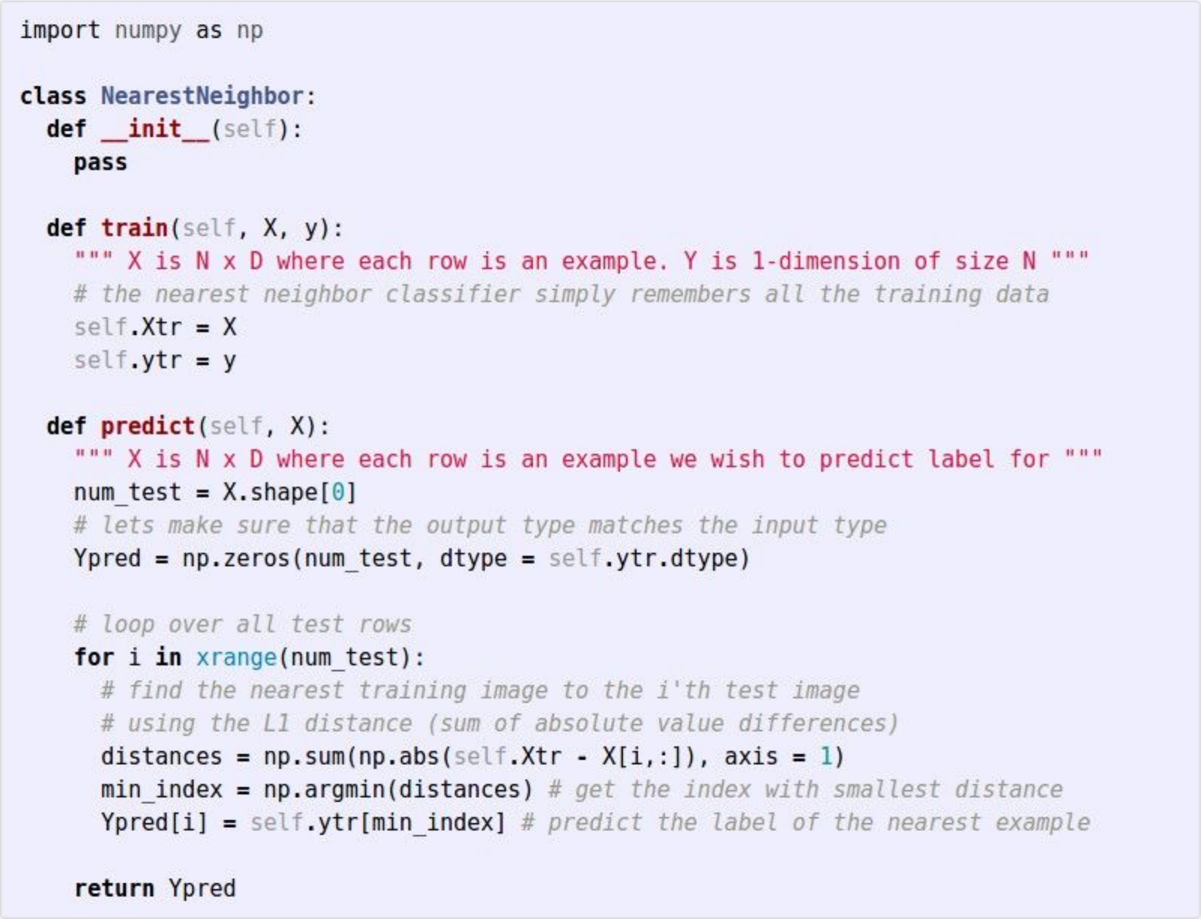

For each test image:
- Find closest train image
- Predict label of nearest image



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```

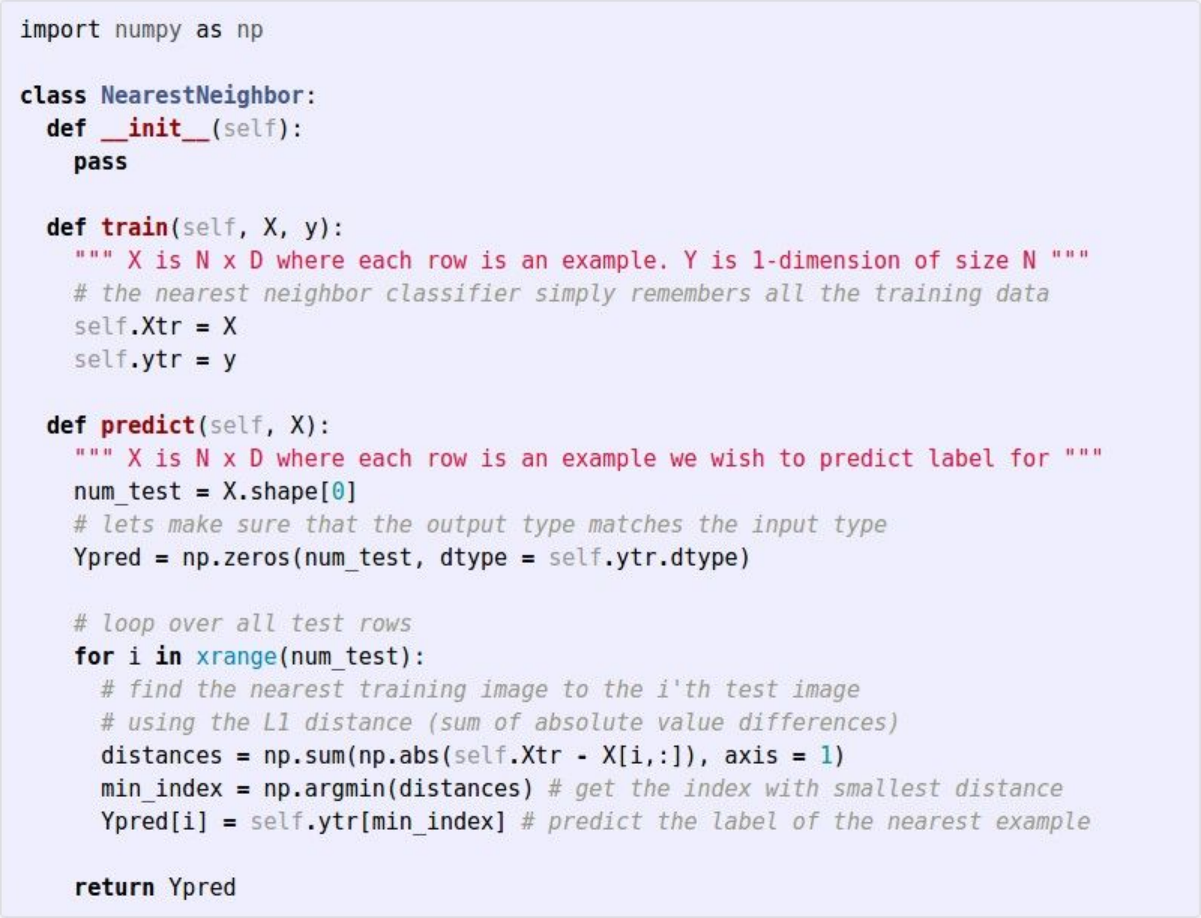
Nearest Neighbor classifier

Q: With N examples, how fast are training and prediction?

Ans: Train O(1),
predict O(N)

This is bad: we want classifiers that are **fast** at prediction; **slow** for training is ok

Nearest Neighbor classifier



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      distances = np.sum(np.abs(self.Xtr - X[i,:]), axis = 1)
      min_index = np.argmin(distances) # get the index with smallest distance
      Ypred[i] = self.ytr[min_index] # predict the label of the nearest example

    return Ypred
```
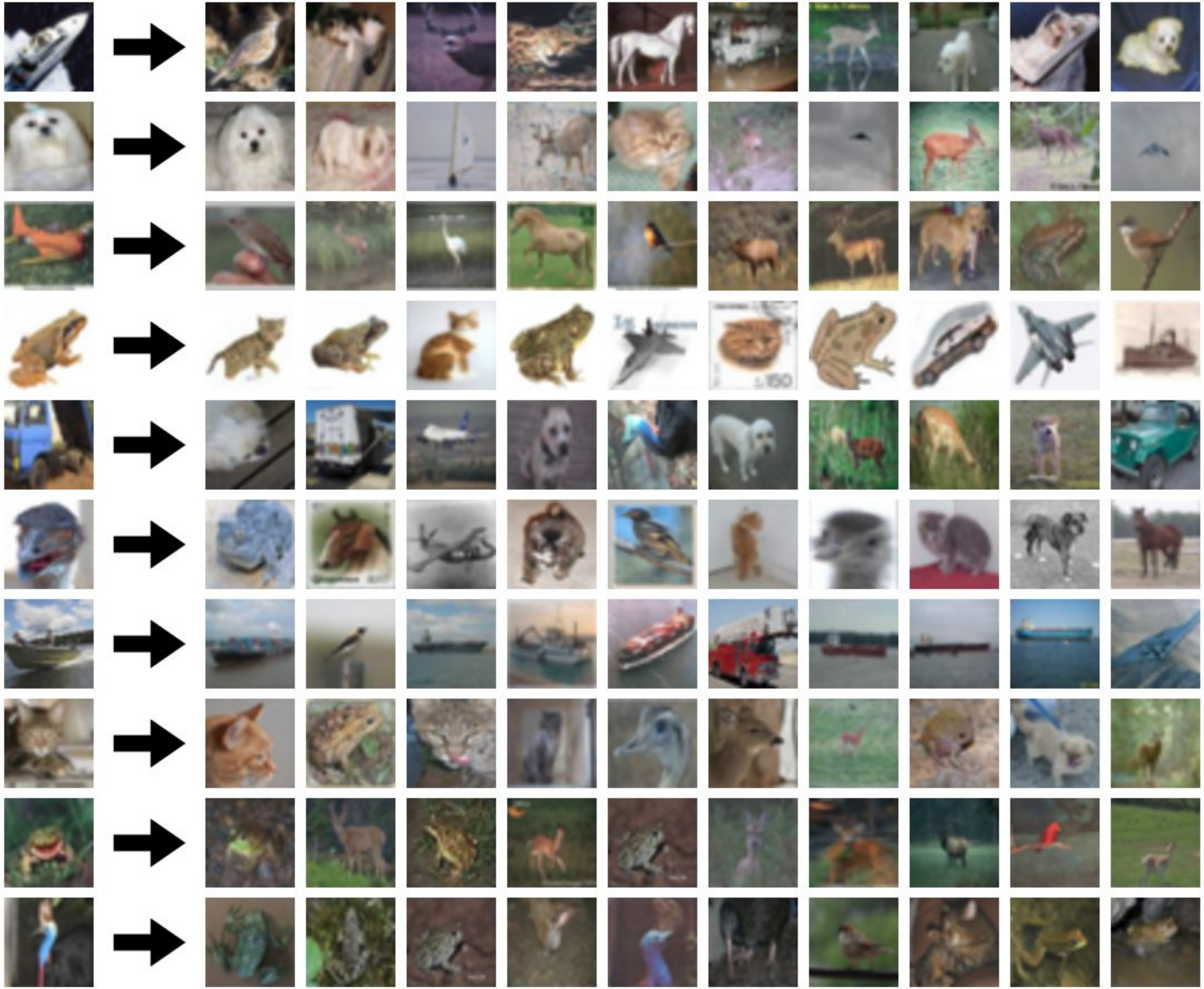

Many methods exist for fast / approximate nearest neighbor (beyond the scope of this course!)

A good implementation:
https://github.com/facebookresearch/faiss

Example outputs from a NN classifier on CIFAR:

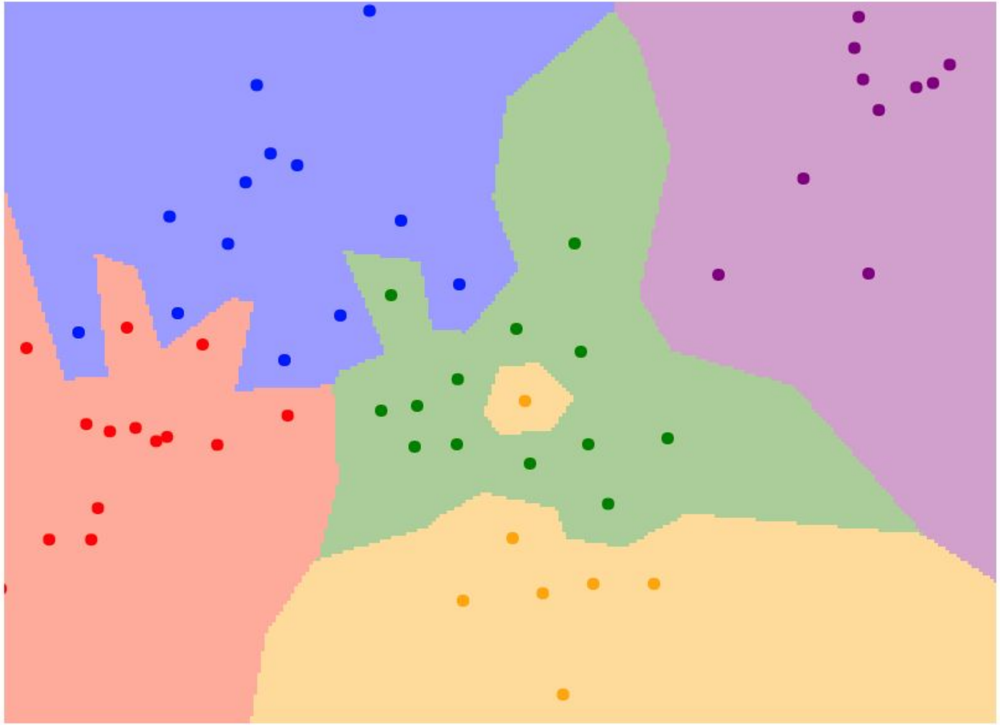


Example outputs from a NN classifier on CIFAR:

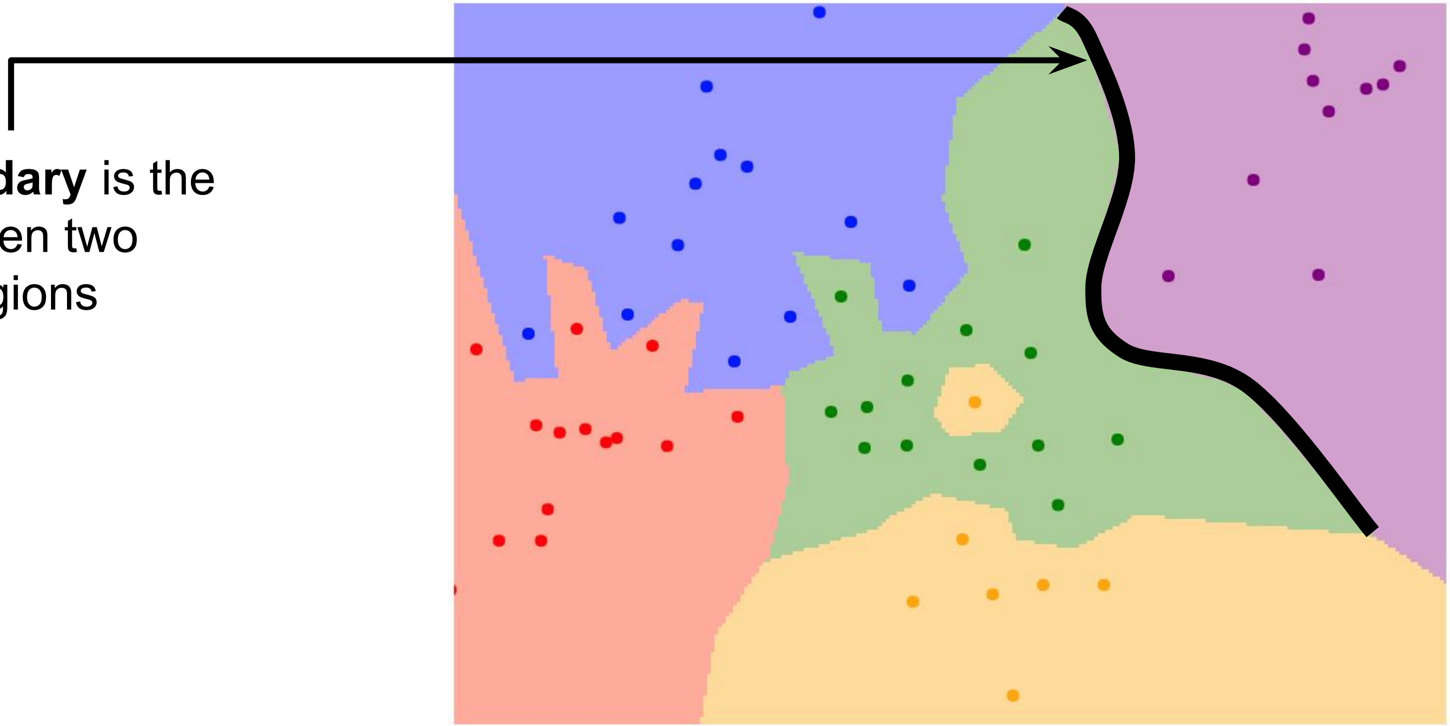
Assume each dot is a training image.

Assume all images are two dimensional.

What does this classifier look like?



1-nearest neighbor

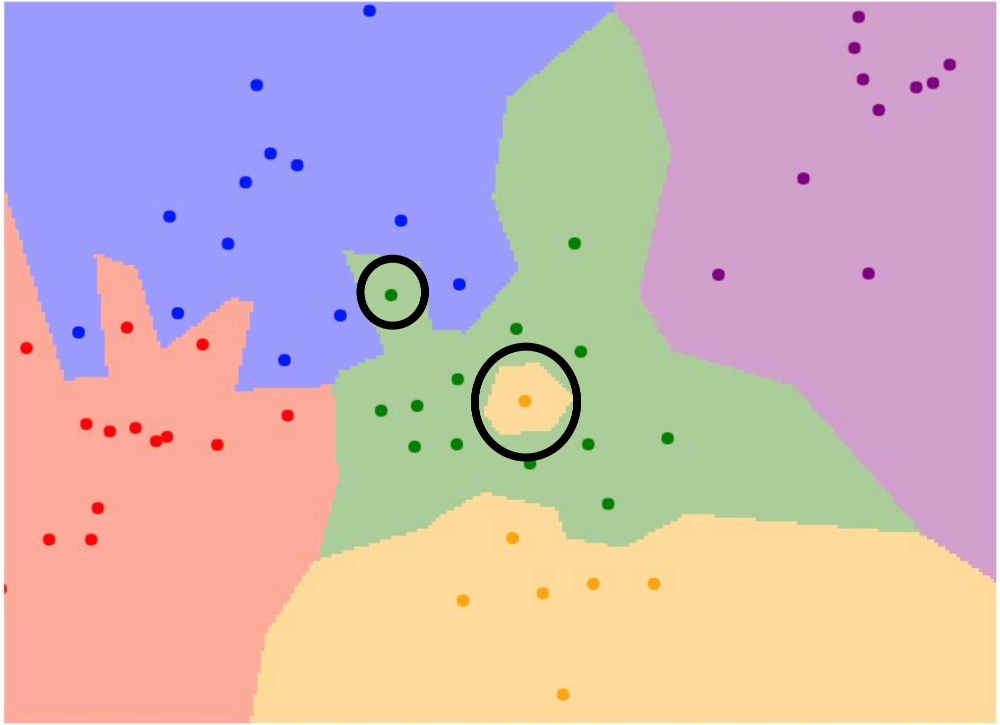


Decision boundary is the boundary between two classification regions

1-nearest neighbor

Yellow point in the middle of green might be mislabeled.

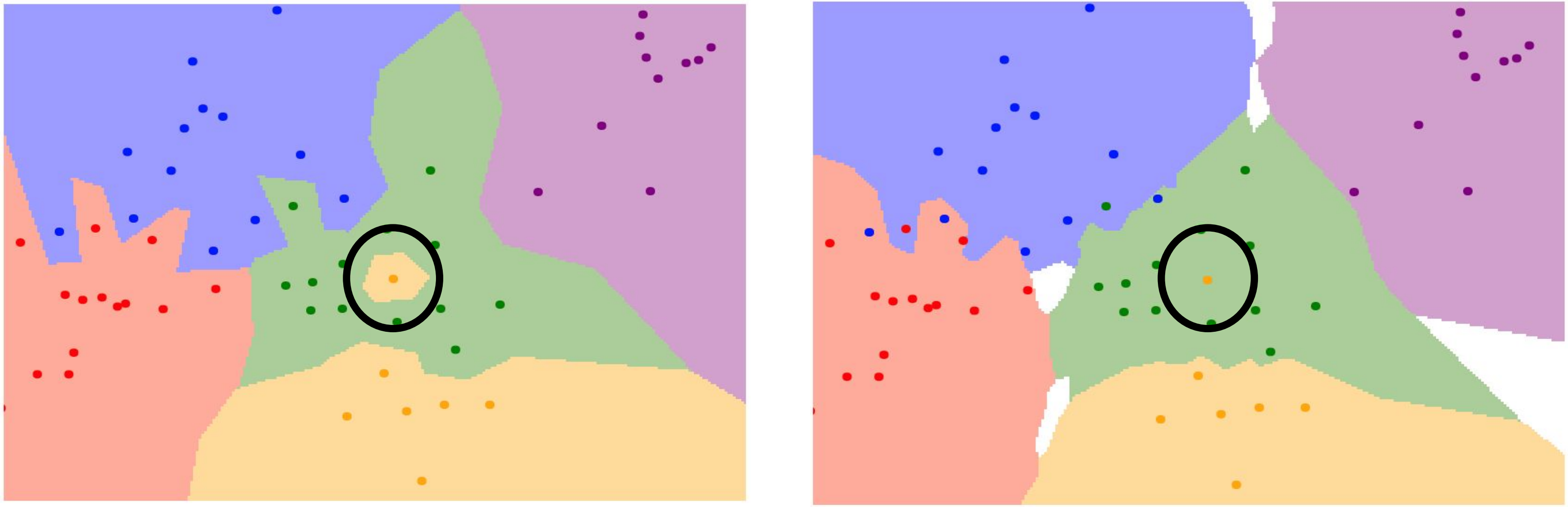
1-NN is not robust to label noise.



1-nearest neighbor

K-Nearest Neighbors

Instead of copying label from nearest neighbor,
take **majority vote** from K closest points

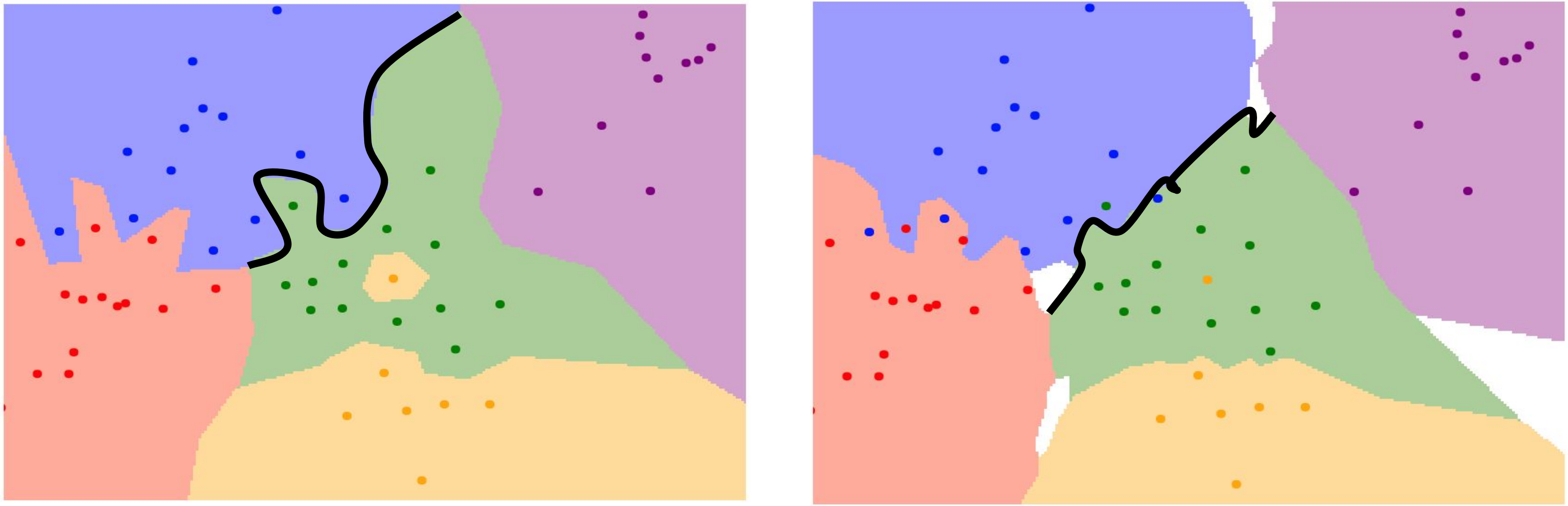


K = 1

K = 3

K-Nearest Neighbors

Using more neighbors helps smooth out rough decision boundaries

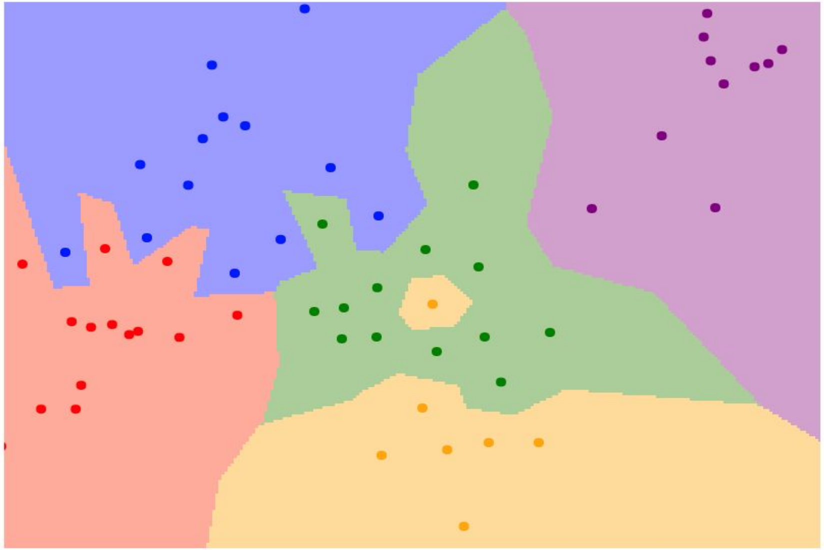


K = 1

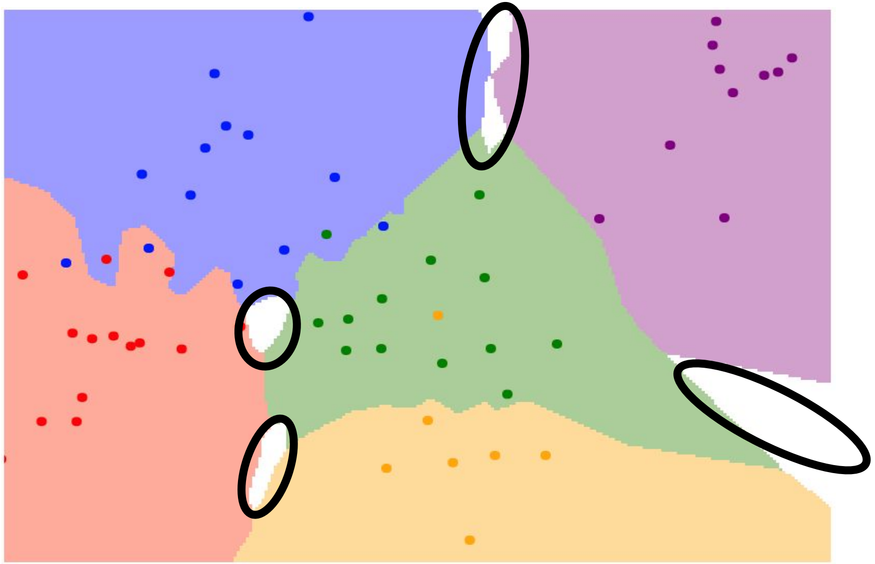
K = 3

K-Nearest Neighbors

Find more labels near uncertain white regions



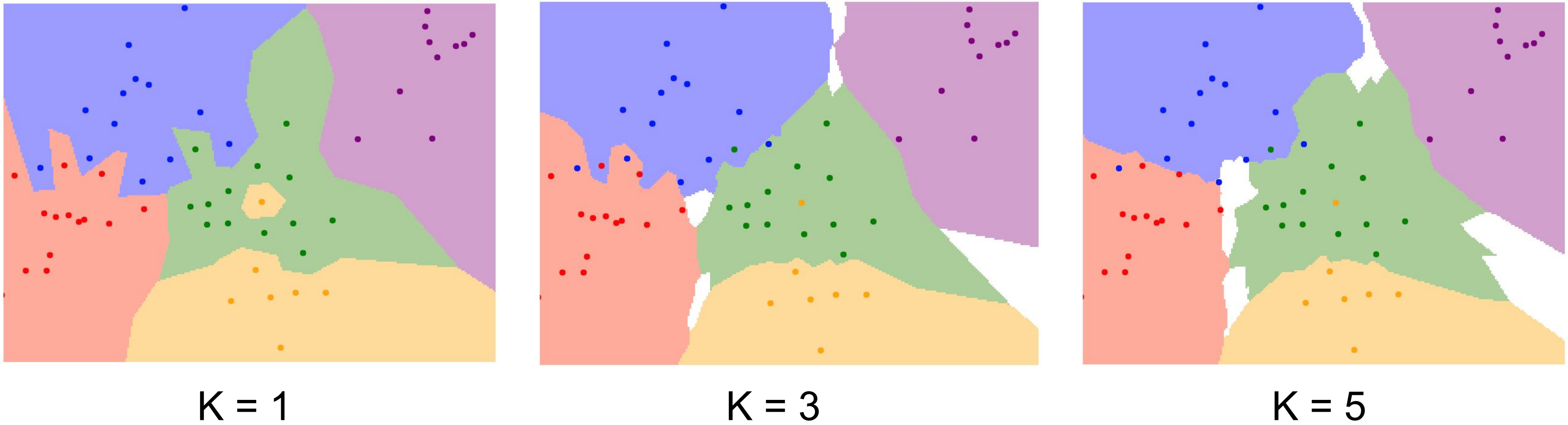
K = 1



K = 3

K-Nearest Neighbors

Larger K smooths boundaries more and leads to more uncertain regions



K = 1

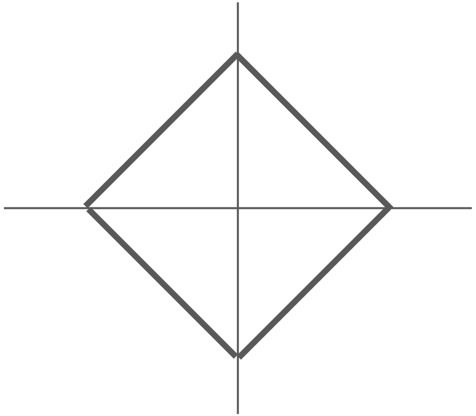
K = 3

K = 5

K-Nearest Neighbors: Distance Metric

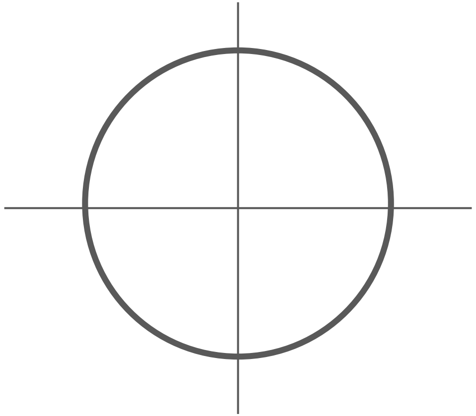
L1 (Manhattan) distance

$$d_1(I_1, I_2) = \sum_p |I_1^p - I_2^p|$$



L2 (Euclidean) distance

$$d_2(I_1, I_2) = \sqrt{\sum_p \left(I_1^p - I_2^p\right)^2}$$



K-Nearest Neighbors: Distance Metric

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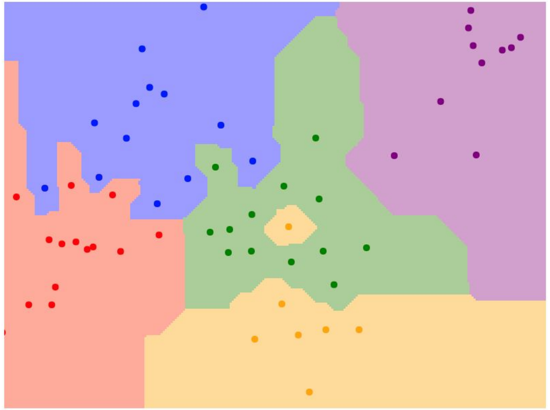
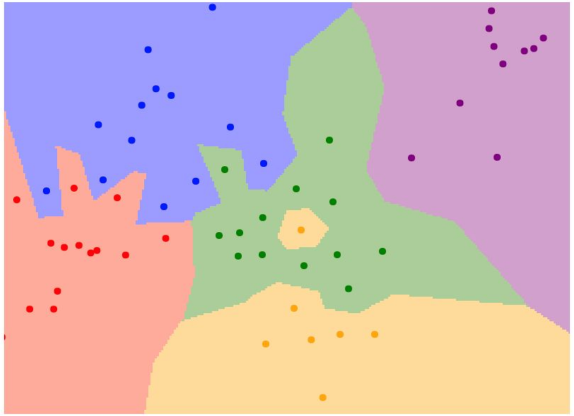


K = 1

L2 (Euclidean) distance

$$d_2(I_1, I_2) = \sqrt{\sum_p \left(I_1^p - I_2^p\right)^2}$$



K = 1

k-Nearest Neighbor with pixel distance is **never used.**

- Distance metrics on pixels are not informative

Original image is CC0 public domain

Original

Occluded

Shifted (1 pixel)

Tinted

(All three images on the right have the same pixel distances to the one on the left)

Hyperparameters

What is the best value of **k** to use?
What is the best **distance** to use?

These are **hyperparameters**: choices about the algorithms themselves.

Very problem/dataset-dependent.
Must try them all out and see what works best.

Setting Hyperparameters

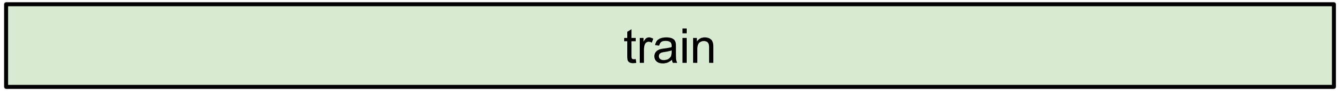
Idea #1: Choose hyperparameters that work best on the **training data**

train

Setting Hyperparameters

Idea #1: Choose hyperparameters that work best on the **training data**

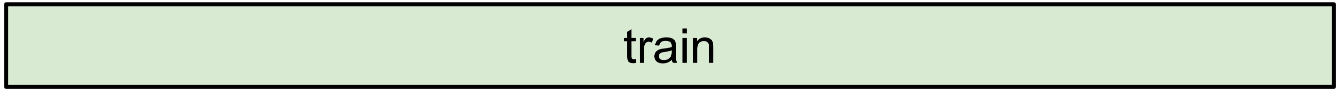
BAD: K = 1 always works perfectly on training data



Setting Hyperparameters

Idea #1: Choose hyperparameters that work best on the **training data**

BAD: K = 1 always works perfectly on training data

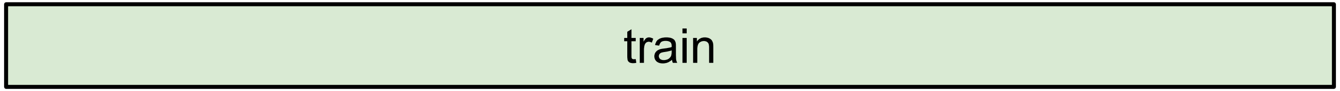


Idea #2: choose hyperparameters that work best on **test** data

Setting Hyperparameters

Idea #1: Choose hyperparameters that work best on the **training data**

BAD: K = 1 always works perfectly on training data



Idea #2: choose hyperparameters that work best on **test** data

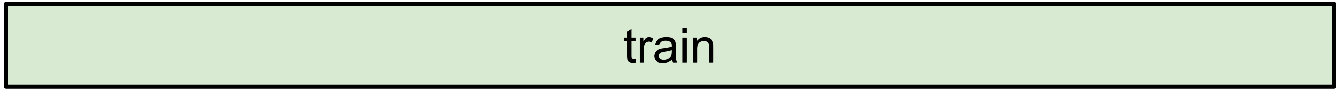
BAD: No idea how algorithm will perform on new data

Never do this!

Setting Hyperparameters

Idea #1: Choose hyperparameters that work best on the **training data**

BAD: K = 1 always works perfectly on training data



Idea #2: choose hyperparameters that work best on **test** data

BAD: No idea how algorithm will perform on new data

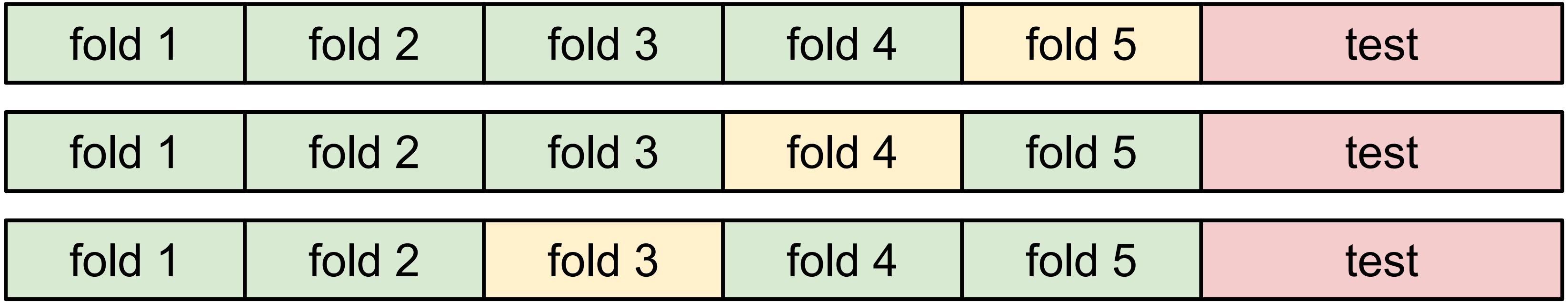
Idea #3: Split data into **train**, **val**; choose hyperparameters on val and evaluate on test

Better!

Setting Hyperparameters

train

Idea #4: **Cross-Validation**: Split data into **folds**, try each fold as validation and average the results



fold 1	fold 2	fold 3	fold 4	fold 5	test
fold 1	fold 2	fold 3	fold 4	fold 5	test
fold 1	fold 2	fold 3	fold 4	fold 5	test

Useful for small datasets, but not used too frequently in deep learning

Example Dataset: **CIFAR10**

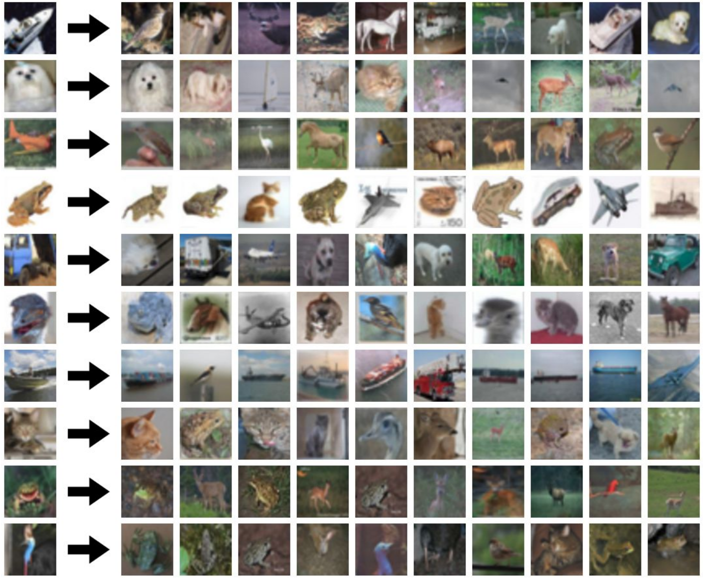
10 classes
50,000 training images
10,000 testing images

Alex Krizhevsky, “Learning Multiple Layers of Features from Tiny Images”, Technical Report, 2009.

Example Dataset: **CIFAR10**

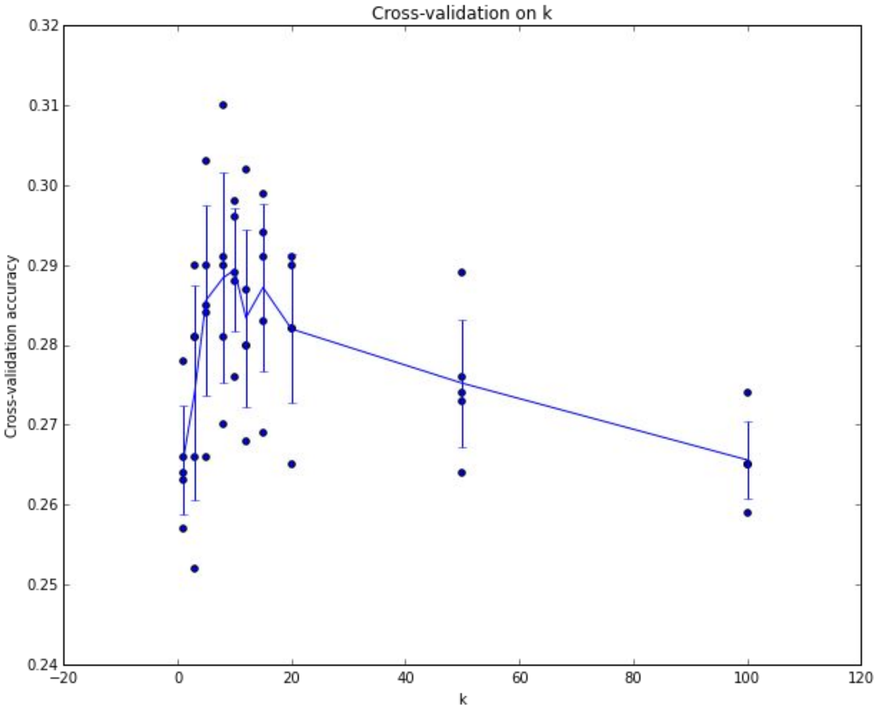
10 classes
50,000 training images
10,000 testing images

Test images and nearest neighbors



Alex Krizhevsky, “Learning Multiple Layers of Features from Tiny Images”, Technical Report, 2009.

Setting Hyperparameters



Example of
5-fold cross-validation
for the value of **k.**

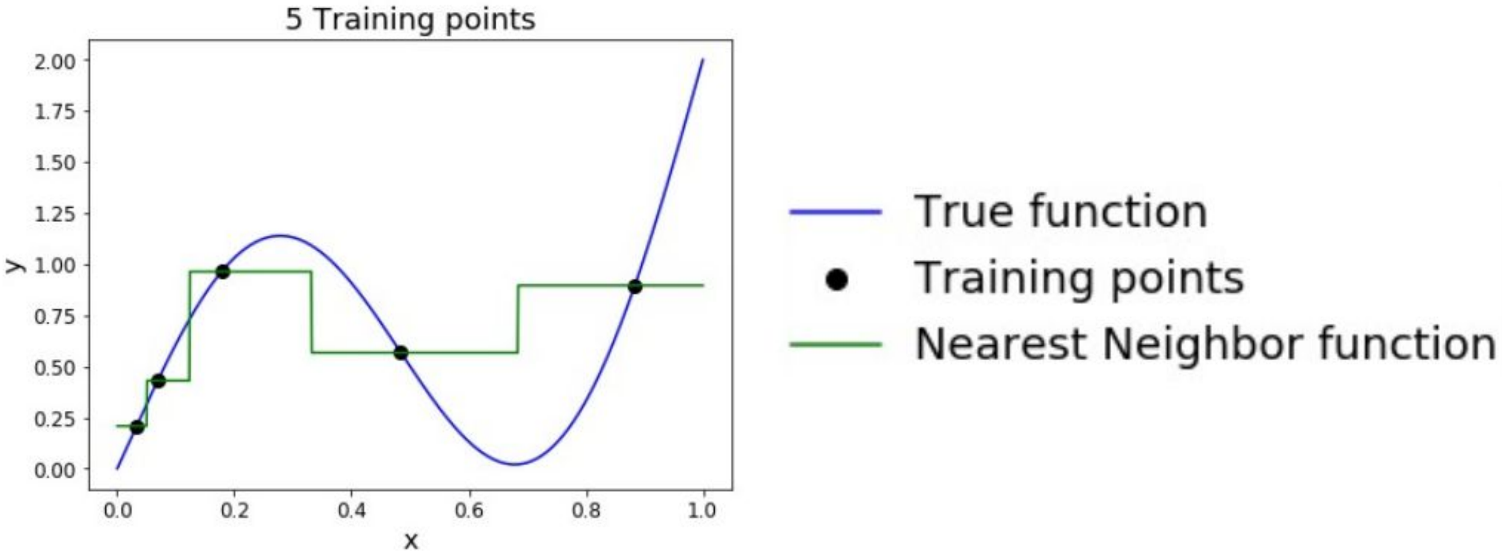
Each point: single
outcome.

The line goes
through the mean, bars
indicated standard
deviation

(Seems that k ~= 7 works best
for this data)

K-Nearest Neighbor: Universal Approximation

As the number of training samples goes to infinity, nearest neighbor can represent any(*) function!



K-Nearest Neighbor: Universal Approximation

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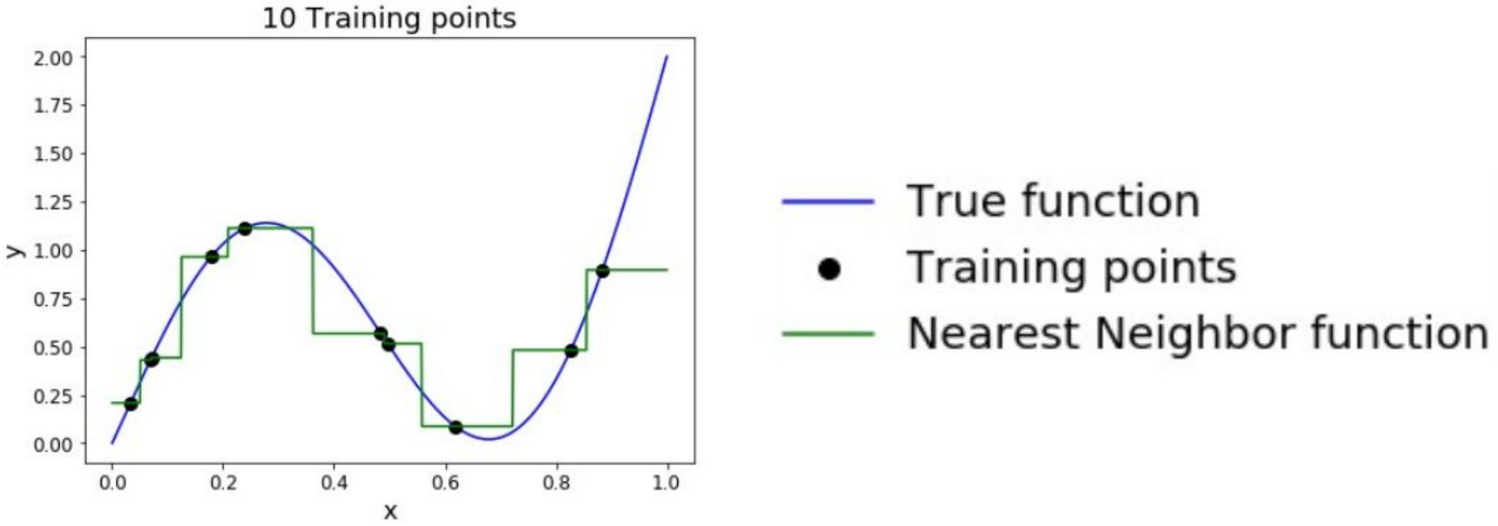

K-Nearest Neighbor: Universal Approximation

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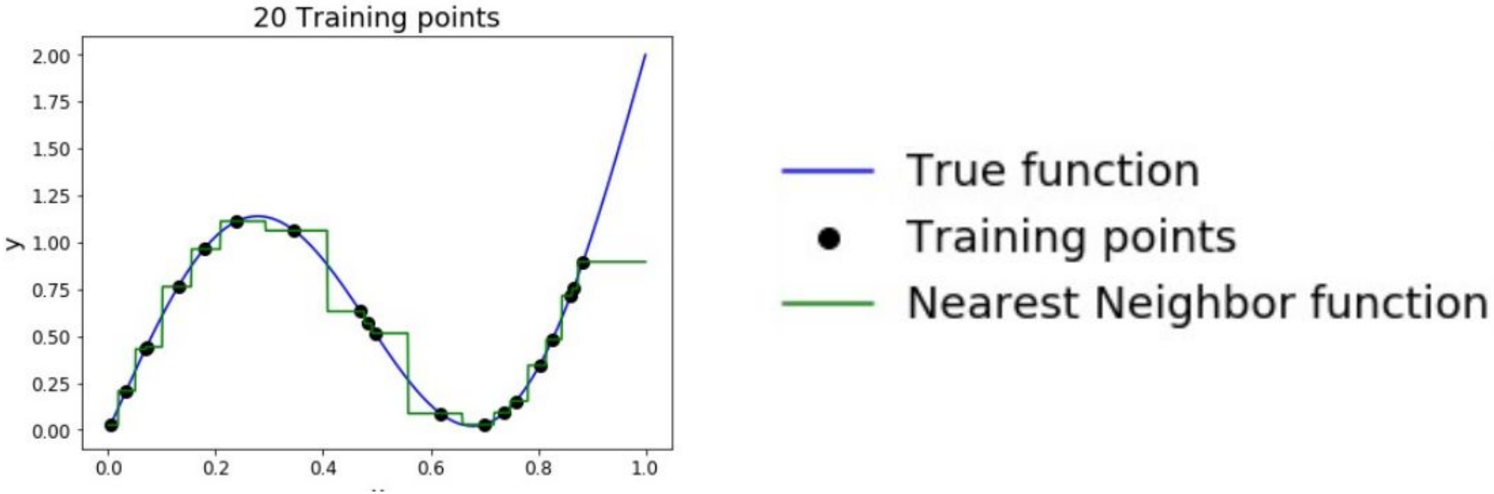
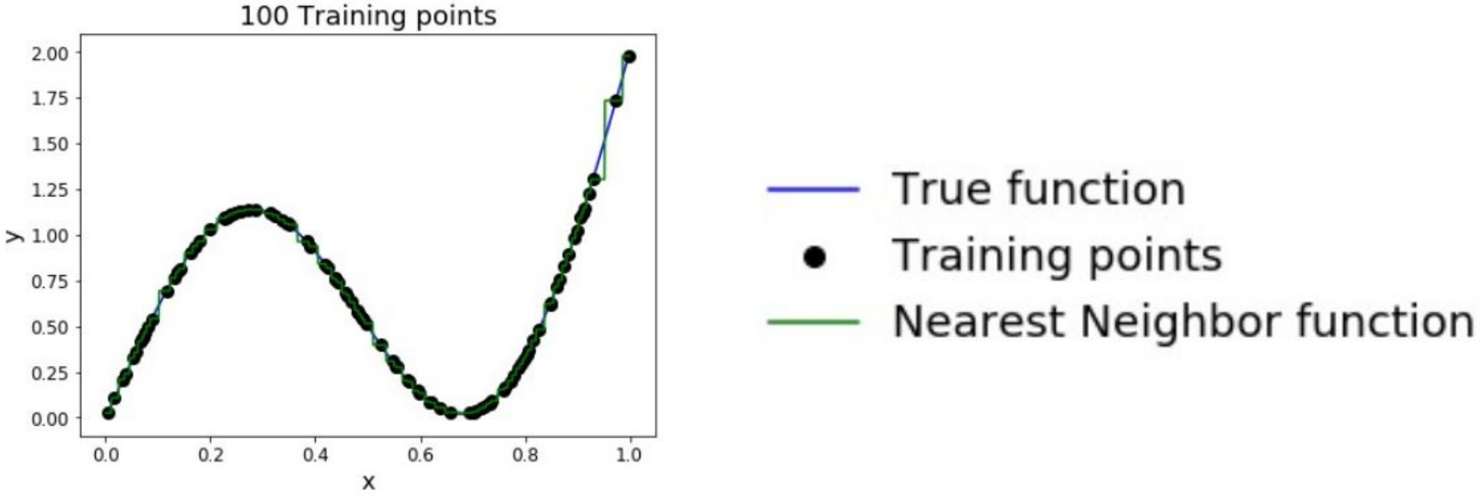


20 Training points

K-Nearest Neighbor: Universal Approximation

As the number of training samples goes to infinity, nearest neighbor can represent any(*) function!

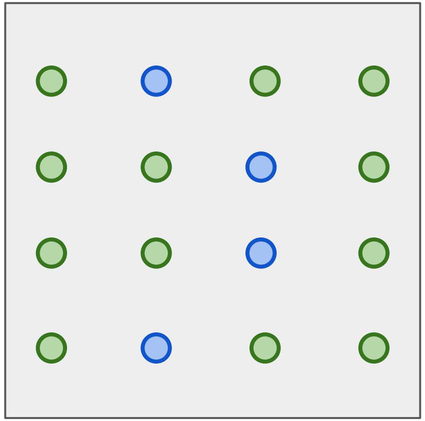


Problem: curse of dimensionality

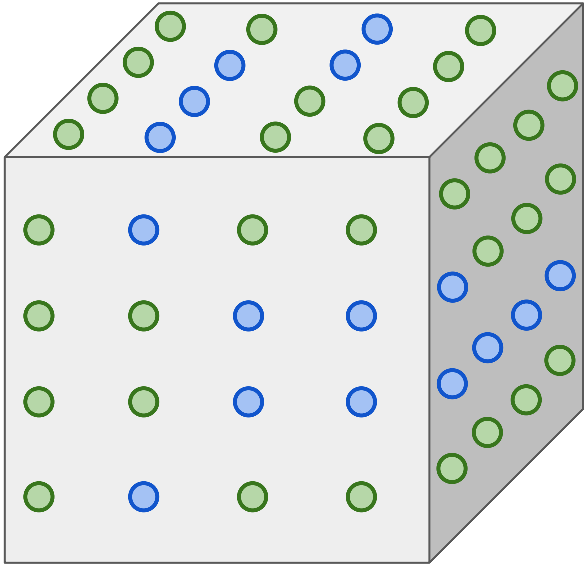
Curse of dimensionality: : For uniform coverage of space, number of training points needed grows exponentially with dimension

Dimensions = 1
Points = 4

Dimensions = 2
Points = 4^2



Dimensions = 3
Points = 4^3



Problem: curse of dimensionality

Curse of dimensionality: : For uniform coverage of space, number of training points needed grows exponentially with dimension

Number of possible 32x32 binary images:

$2^{32x32} = 10^{308}$

Number of elementary particles in the visible universe:
10^{97}

K-Nearest Neighbors: Summary

In **image classification** we start with a **training set** of images and labels, and must predict labels on the **test set**

The **K-Nearest Neighbors** classifier predicts labels based on the K nearest training examples

Distance metric and K are **hyperparameters**

Choose hyperparameters using the **validation set**;

Only run on the test set once at the very end!

Linear Classifier

Parametric Approach

Image

Array of **32x32x3** numbers
(3072 numbers total)

$f(\mathbf{x},\mathbf{W})$

10 numbers giving
class scores

W
parameters
or weights

Parametric Approach: Linear Classifier

$$f(x,W) = Wx$$

Image

Array of **32x32x3** numbers
(3072 numbers total)

→ $f(\mathbf{x},\mathbf{W})$ →

10 numbers giving
class scores

↑

W

parameters
or weights

Parametric Approach: Linear Classifier

$$\underbrace{\boxed{f(x,W)}}_{\mathbf{10x1}} = \underbrace{\boxed{W}}_{\mathbf{10x3072}}\overset{\mathbf{3072x1}}{\boxed{x}}$$

Image

Array of **32x32x3** numbers
(3072 numbers total)

$$\xrightarrow{\quad} f(\mathbf{x},\mathbf{W}) \xrightarrow{\quad}$$

10 numbers giving
class scores

$$\uparrow$$

W

parameters
or weights

Parametric Approach: Linear Classifier

$$\underbrace{\boxed{f(x,W)}}_{10x1} = \underbrace{\boxed{W}}_{10x3072}\overset{3072x1}{\boxed{x}} + \boxed{b}\quad 10x1$$

Image

Array of **32x32x3** numbers
(3072 numbers total)

→ $f(\mathbf{x},\mathbf{W})$ →

10 numbers giving class scores

↑

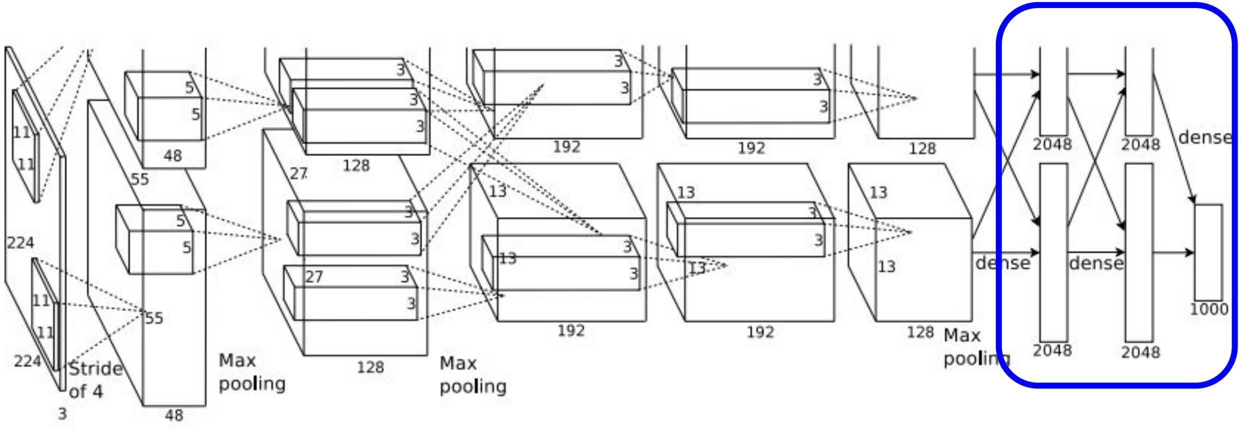
W

parameters
or weights

Neural Network

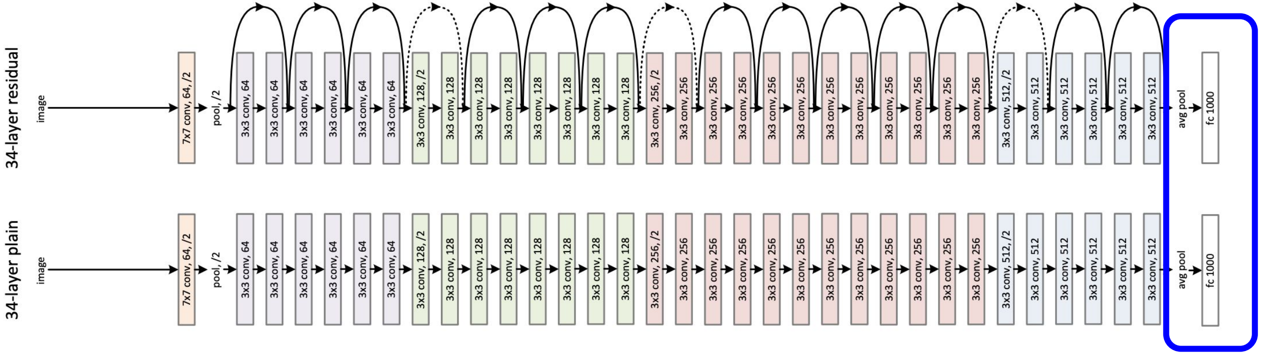
Linear classifiers

This image is CC0 1.0 public domain



[Krizhevsky et al. 2012]

Linear layers



[He et al. 2015]

Recall CIFAR10

50,000 training images
each image is **32x32x3**

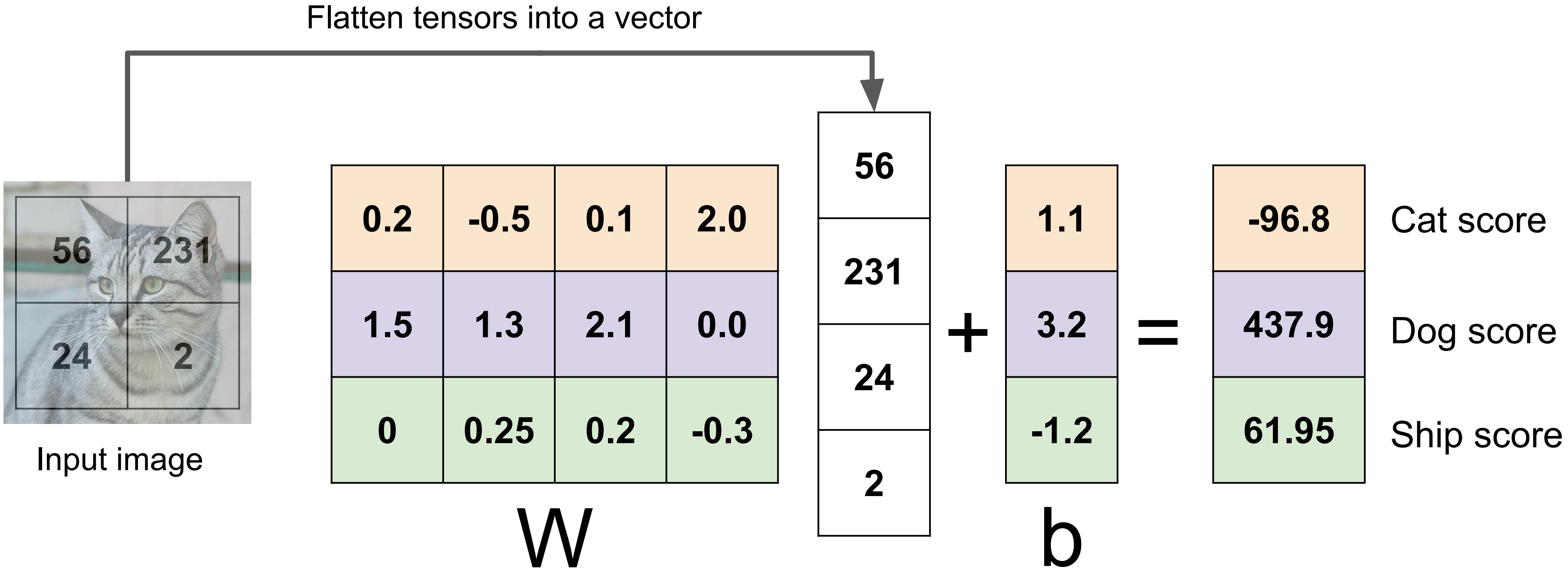
10,000 test images.

Algebraic viewpoint: Example with an image with 4 pixels, and 3 classes (cat/dog/ship)


Flatten tensors into a vector

Input image

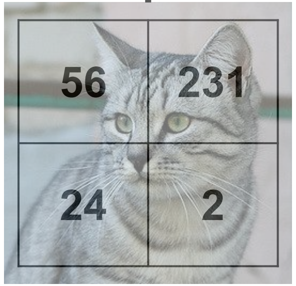
Algebraic viewpoint: Example with an image with 4 pixels, and 3 classes (cat/dog/ship)



$$\underbrace{\begin{bmatrix}0.2 & -0.5 & 0.1 & 2.0\\ 1.5 & 1.3 & 2.1 & 0.0\\ 0 & 0.25 & 0.2 & -0.3\end{bmatrix}}_{W}\begin{bmatrix}56\\ 231\\ 24\\ 2\end{bmatrix}+\underbrace{\begin{bmatrix}1.1\\ 3.2\\ -1.2\end{bmatrix}}_{b}=\begin{bmatrix}-96.8\\ 437.9\\ 61.95\end{bmatrix}\begin{matrix}\text{Cat score}\\ \text{Dog score}\\ \text{Ship score}\end{matrix}$$

Algebraic viewpoint: Example with an image with 4 pixels, and 3 classes (cat/dog/ship)

Flatten tensors into a vector

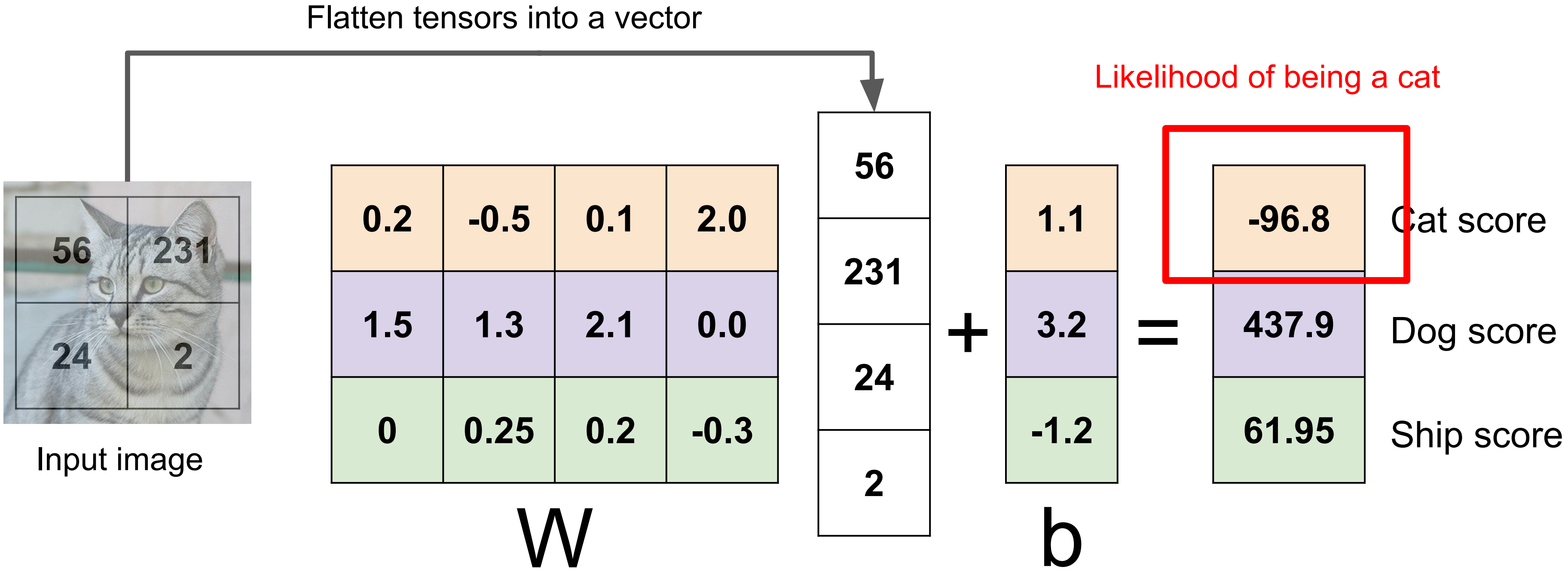


Input image

(2,2)

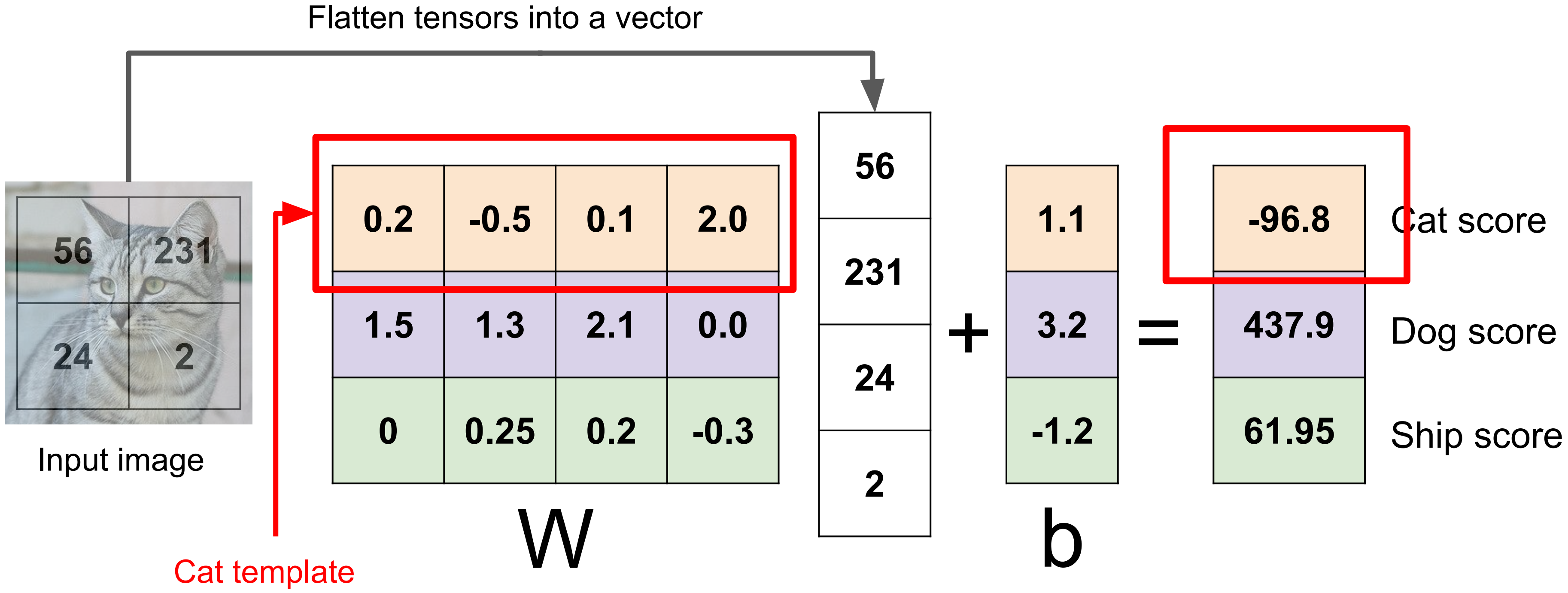
$$\underset{(3,4)}{\underset{W}{\begin{bmatrix}0.2 & -0.5 & 0.1 & 2.0\\ 1.5 & 1.3 & 2.1 & 0.0\\ 0 & 0.25 & 0.2 & -0.3\end{bmatrix}}}\underset{(4,)}{\begin{bmatrix}56\\ 231\\ 24\\ 2\end{bmatrix}}+\underset{(3,)}{\underset{b}{\begin{bmatrix}1.1\\ 3.2\\ -1.2\end{bmatrix}}}=\underset{(3,)}{\begin{bmatrix}-96.8\\ 437.9\\ 61.95\end{bmatrix}}\begin{array}{l}\text{Cat score}\\ \text{Dog score}\\ \text{Ship score}\end{array}$$

Algebraic viewpoint: Example with an image with 4 pixels, and 3 classes (cat/dog/ship)



$$\begin{bmatrix} 0.2 & -0.5 & 0.1 & 2.0 \\ 1.5 & 1.3 & 2.1 & 0.0 \\ 0 & 0.25 & 0.2 & -0.3 \end{bmatrix} \begin{bmatrix} 56 \\ 231 \\ 24 \\ 2 \end{bmatrix} + \begin{bmatrix} 1.1 \\ 3.2 \\ -1.2 \end{bmatrix} = \begin{bmatrix} -96.8 \\ 437.9 \\ 61.95 \end{bmatrix}$$

Algebraic viewpoint: Example with an image with 4 pixels, and 3 classes (cat/dog/ship)



Flatten tensors into a vector

56 231
24 2

Input image

Cat template

$$\begin{bmatrix} 0.2 & -0.5 & 0.1 & 2.0 \\ 1.5 & 1.3 & 2.1 & 0.0 \\ 0 & 0.25 & 0.2 & -0.3 \end{bmatrix} \begin{bmatrix} 56 \\ 231 \\ 24 \\ 2 \end{bmatrix} + \begin{bmatrix} 1.1 \\ 3.2 \\ -1.2 \end{bmatrix} = \begin{bmatrix} -96.8 \\ 437.9 \\ 61.95 \end{bmatrix}$$

W

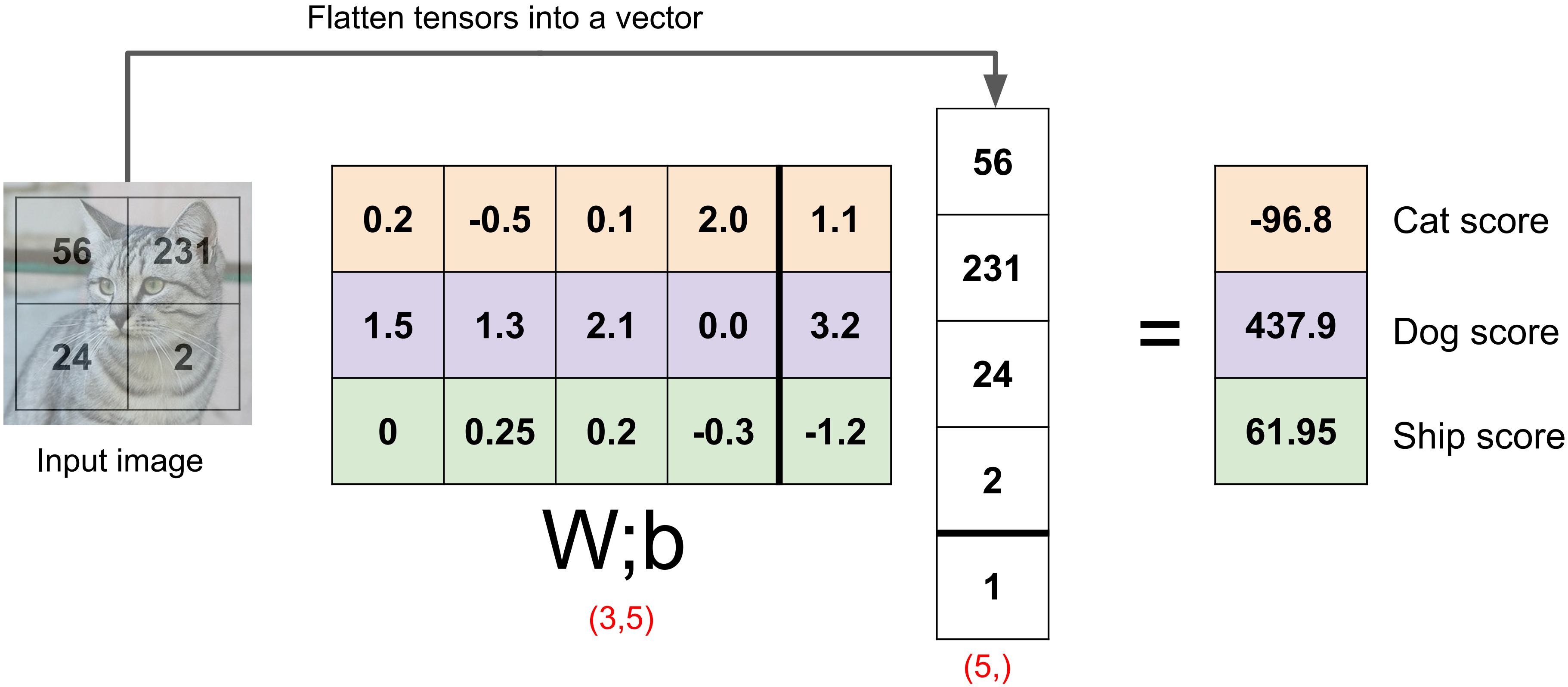
b

Cat score

Dog score

Ship score

Algebraic viewpoint: Bias trick to simplify computation



Flatten tensors into a vector

Input image

0.2	-0.5	0.1	2.0	1.1
1.5	1.3	2.1	0.0	3.2
0	0.25	0.2	-0.3	-1.2

W;b

(3,5)

56
231
24
2
1

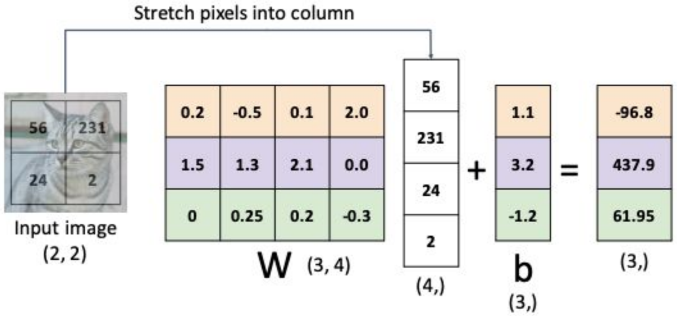
(5,)

=

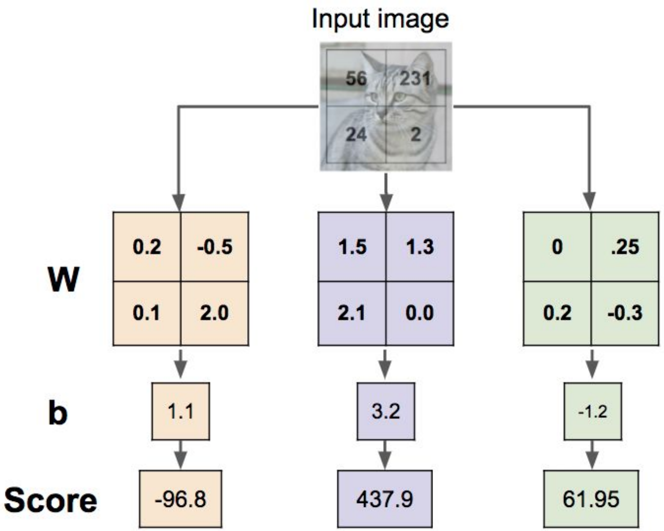
-96.8	Cat score
437.9	Dog score
61.95	Ship score

Visual Viewpoint: learning templates

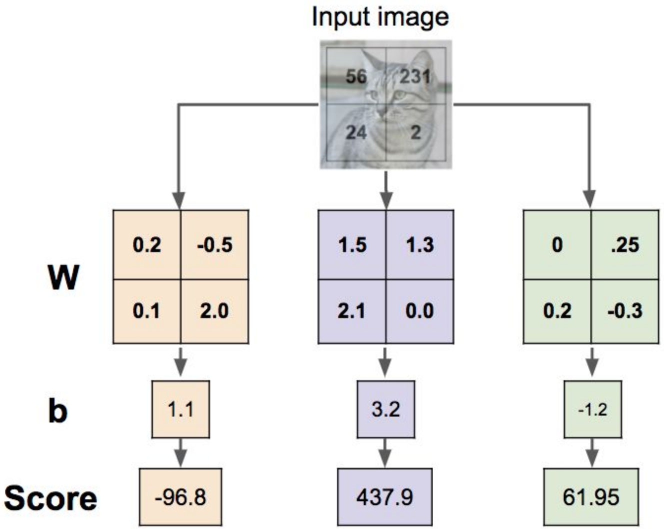
Algebraic viewpoint:



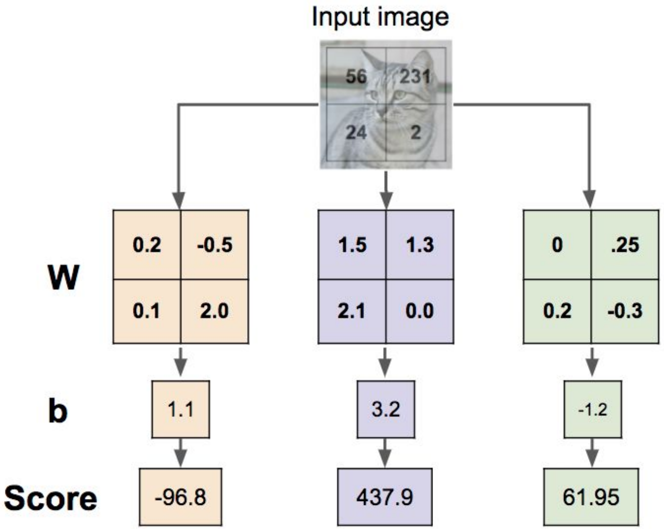
$$\begin{bmatrix} 0.2 & -0.5 & 0.1 & 2.0 \\ 1.5 & 1.3 & 2.1 & 0.0 \\ 0 & 0.25 & 0.2 & -0.3 \end{bmatrix} \begin{bmatrix} 56 \\ 231 \\ 24 \\ 2 \end{bmatrix} + \begin{bmatrix} 1.1 \\ 3.2 \\ -1.2 \end{bmatrix} = \begin{bmatrix} -96.8 \\ 437.9 \\ 61.95 \end{bmatrix}$$



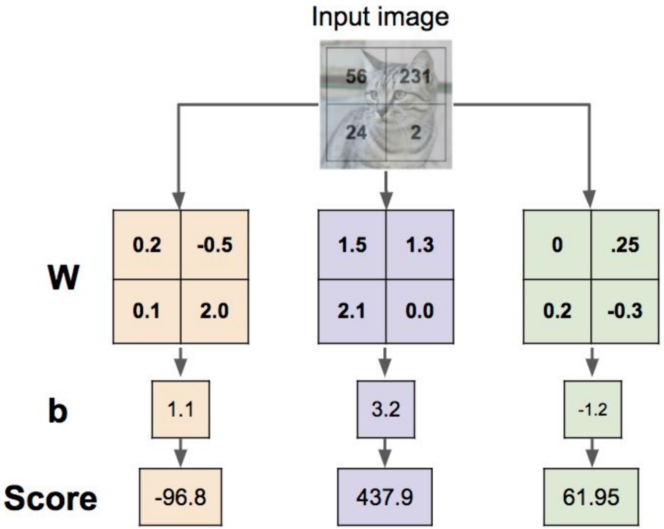
Visual Viewpoint: learning templates

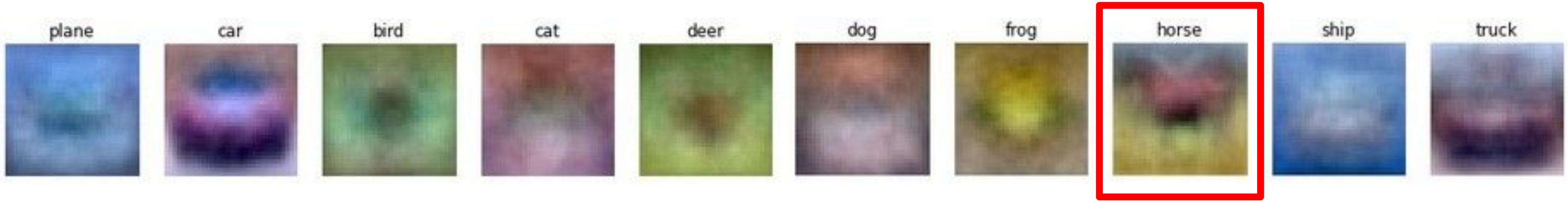


Visual Viewpoint: learning templates

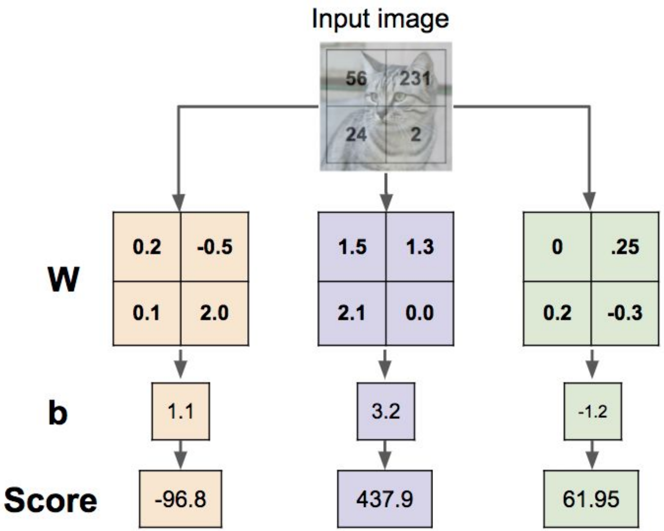


Visual Viewpoint: learning templates





Visual Viewpoint: learning templates


Input image

56 231
24 2

W

0.2	-0.5
0.1	2.0

1.5	1.3
2.1	0.0

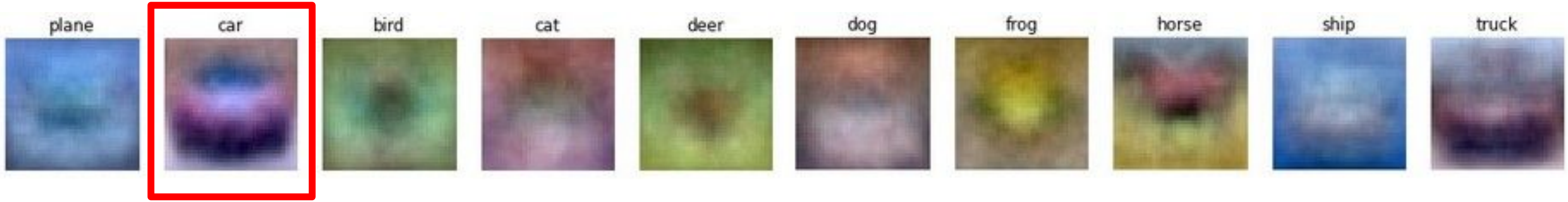
0	.25
0.2	-0.3

b

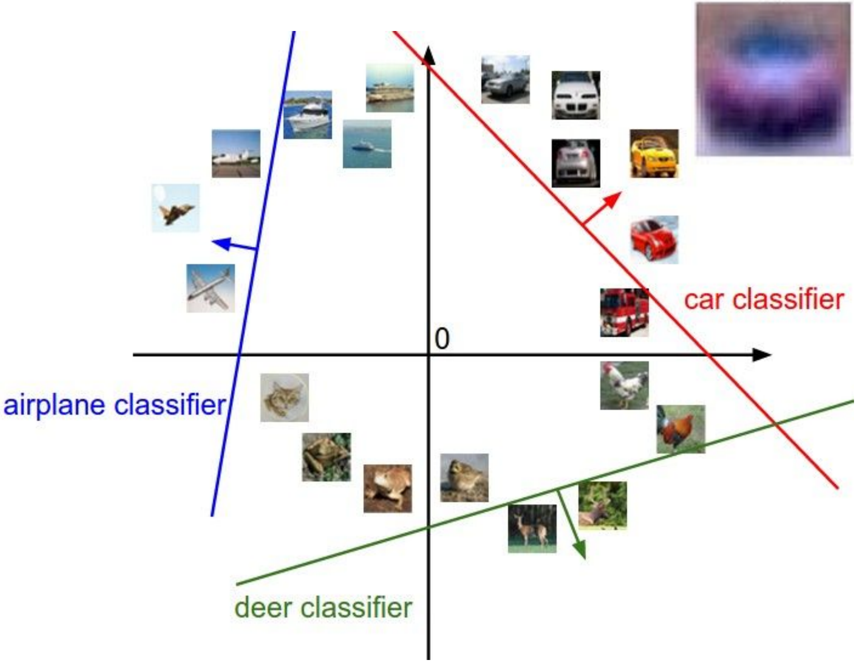
1.1
3.2
-1.2

Score

-96.8
437.9
61.95



Geometric Viewpoint: linear decision boundaries

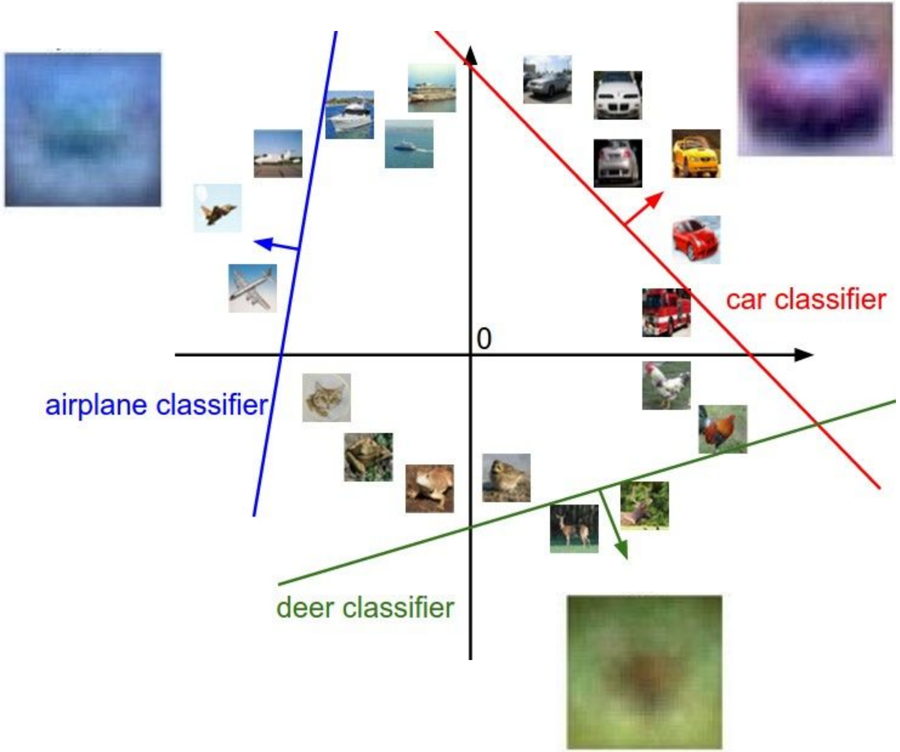


$$f(x,W) = Wx + b$$

Array of **32x32x3** numbers
(3072 numbers total)

Plot created using Wolfram Cloud

Geometric Viewpoint: linear decision boundaries



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Array of **32x32x3** numbers
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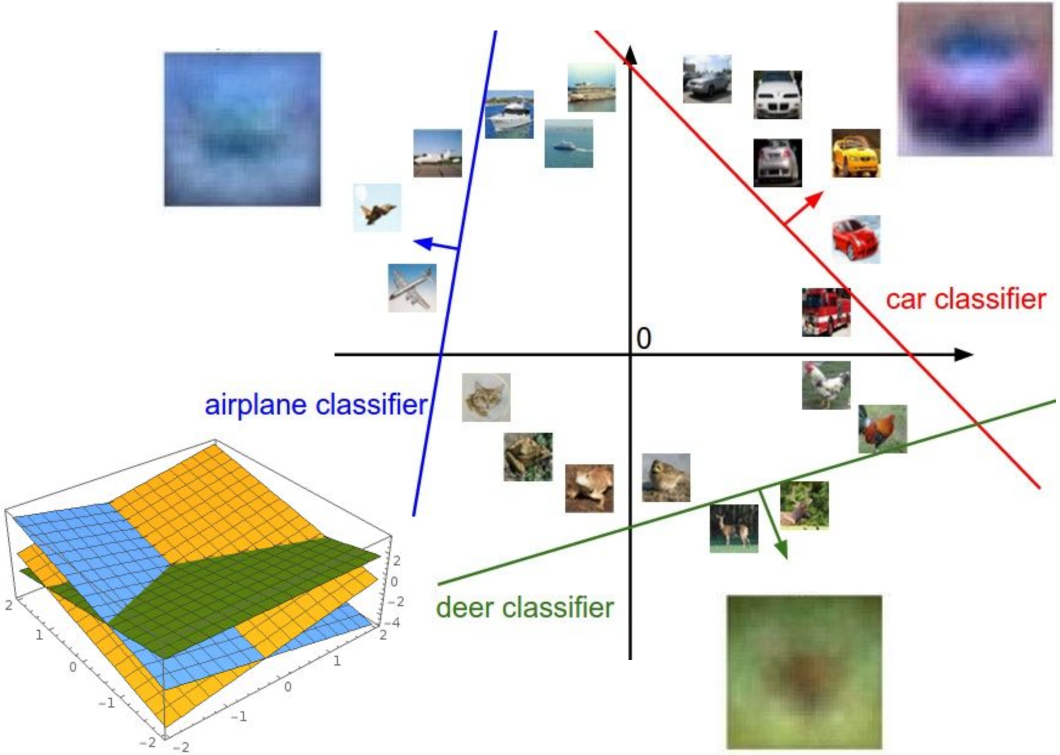
Geometric Viewpoint: linear decision boundaries



$$f(x,W) = Wx + b$$

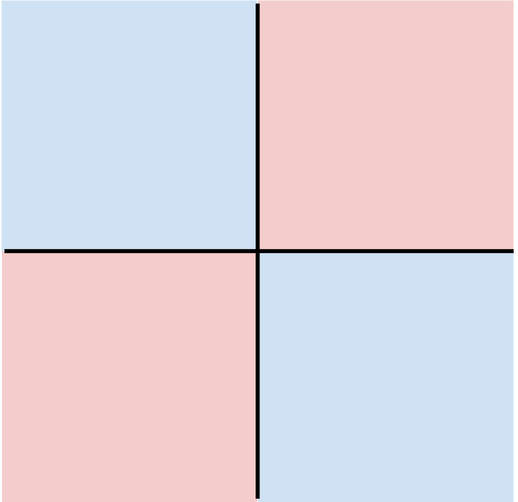
Array of **32x32x3** numbers
(3072 numbers total)

Plot created using Wolfram Cloud

Hard cases for a linear classifier

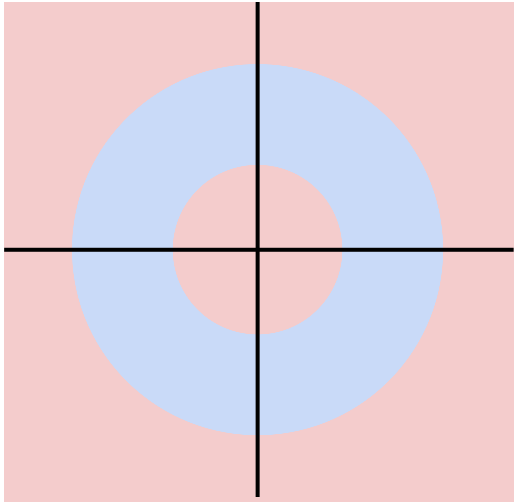
Class 1:
First and third quadrants

Class 2:
Second and fourth quadrants



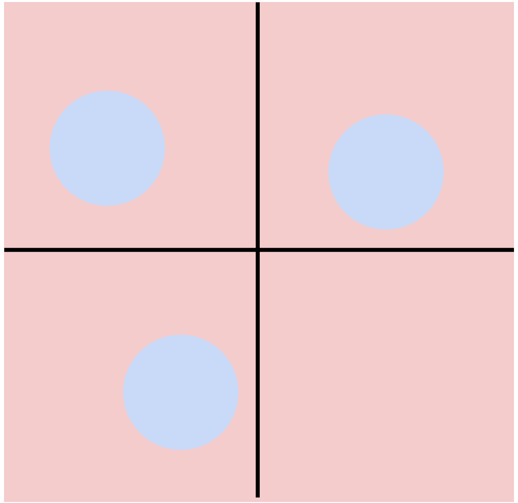
Class 1:
1 <= L2 norm <= 2

Class 2:
Everything else



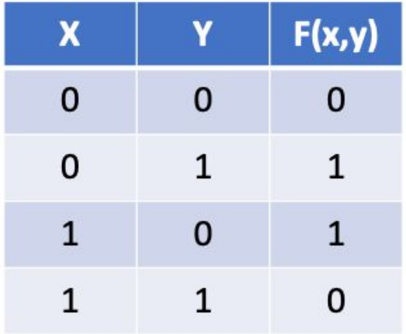
Class 1:
Three modes

Class 2:
Everything else

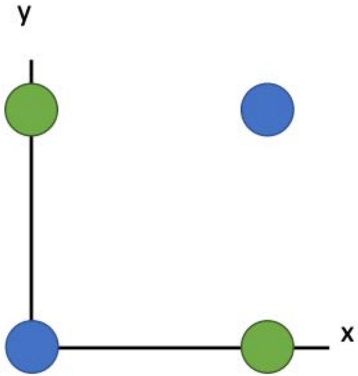


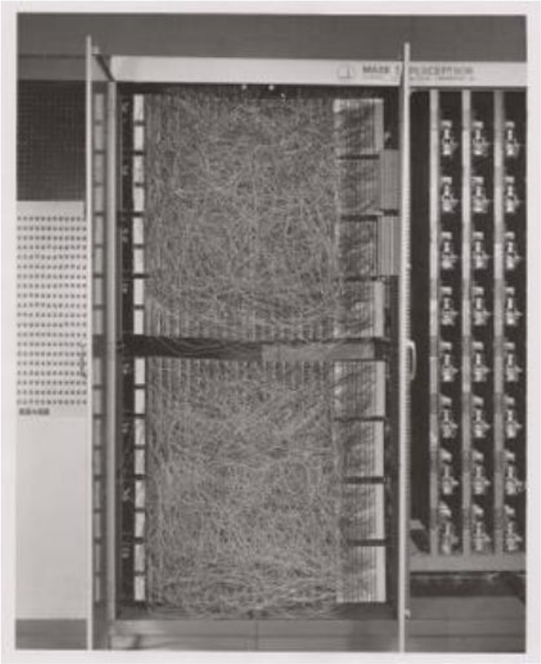
Recall the Minsky report 1969 from last lecture

Unable to learn the XNOR function



X	Y	F(x,y)
0	0	0
0	1	1
1	0	1
1	1	0

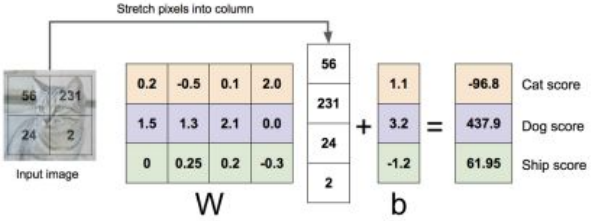




Three viewpoints for interpreting linear classifiers

Algebraic Viewpoint

f(x,W) = Wx

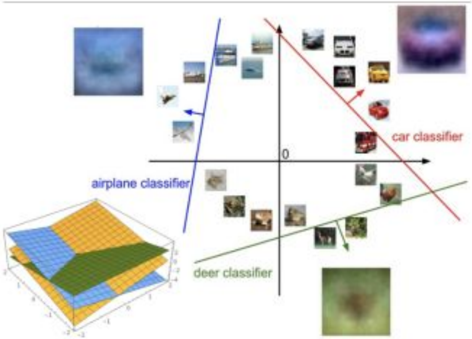


Visual Viewpoint

One template
per class

Geometric Viewpoint

Hyperplanes
cutting up space



$$f(x,W) = Wx + b$$

Coming up:

- Loss function
- Optimization
- Neural Networks

(quantifying what it means to have a "good" W)

(start with random W and find a W that minimizes the loss)

(tweak the functional form of f)