Exercises: Due Tuesday November 10

4. Define a modified version of polynomial calculus mPC where we have the following more general version of the multiplication rule:

$$\frac{p}{qp} \qquad \text{where } \deg(q) \leq 1,$$

   Prove that if an unsatisiable system of polynomials $f_1, \ldots, f_m$ has a *tree-like* mPC refutation of height h over some field $\mathbb{F}$, then it has a Nullstellensatz refutation over $\mathbb{F}$ of degree at most $h + \max_{i \in [m]} \deg(f_i)$.

5. Let $\text{char}(\mathbb{F}) \neq 2$. Prove that any unsatisfiable set of binomials $f_1, \ldots, f_m$ in variables $z_1, \ldots, z_n$ has a polynomial-size $\mathbf{PC}^{\pm}_{\mathbb{F}}$ refutation. (Recall that such refutations have equations $z_i^2 - 1 = 0$ for all $i \in [k]$.)
   Hint: Use the relationships proved in class and the notes.