CSE 599S Proof Complexity Fall 2020

Exercises: Due Friday, October 16

1. The GT_n formula formalizes properties of "greater than" on n elements (intuitively x_{ij} states that element i is greater than element j) and makes the claim that there is no maximum element among them. It contains the following clauses.

   - Totality: $x_{ij} \vee x_{ji}$ for all $i \neq j \in [n]$.
   - Anti-symmetry: $\neg x_{ij} \vee \neg x_{ji}$ for all $i \neq j \in [n]$.
   - Transitivity: $\neg x_{ij} \vee \neg x_{jk} \vee x_{ik}$ for all distinct $i, j, k \in [n]$.
   - Non-maximality: $\bigvee_{i \neq j} x_{ij}$ for all $j \in [n]$.

   Describe polynomial size resolution refutations for the GT_n formulas.
   Hint: derive the non-maximality axioms of GT_{n-1} from those of GT_n by resolving out all variables that touch vertex n, one after another.

2. Show that for any CNF formula F you can convert any tree resolution refutation R of F into a Nullstellensatz refutation of F (over any field) whose degree is at most the height of R. (Recall that you can assume without loss of generality that R is regular also, so it looks like a DPLL tree.)