Routing Algebras

What are routing algebras?

- Created to study properties of routing protocols
  - Does the routing protocol converge?
  - Does the routing protocol compute optimal paths?
- Separates the routing data & algorithm
  - Data → what routes are exchanged, how they are updated
  - Logic → how to compute paths (Dijkstra's, Bellman Ford, …)

What are routing algebras?

$$A = (R, F, \oplus, 0, \infty)$$

set of routes

route edge transformers

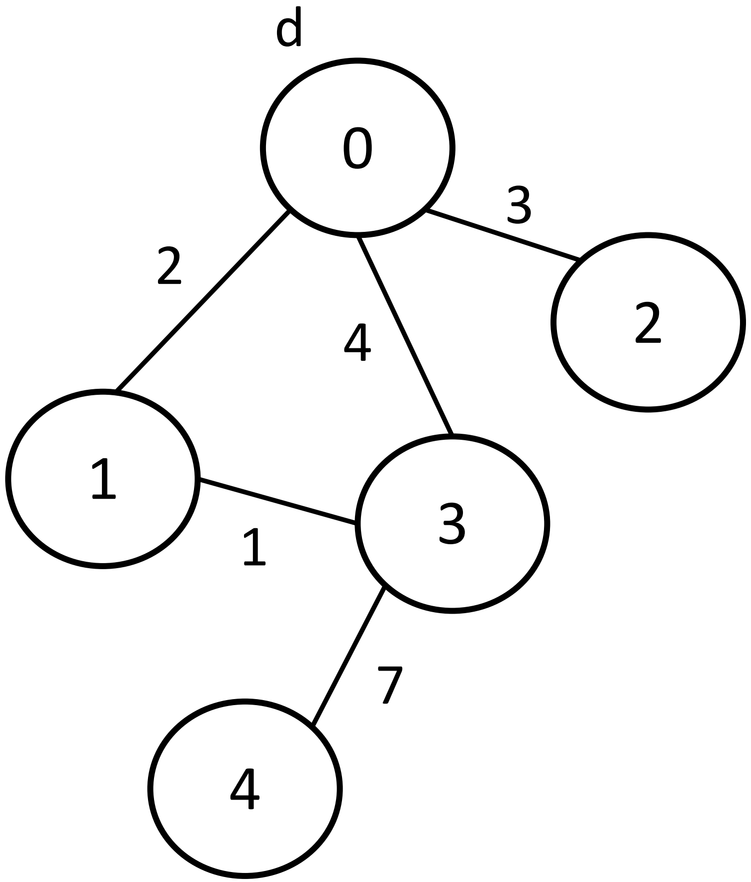
route merging

initial route

no route

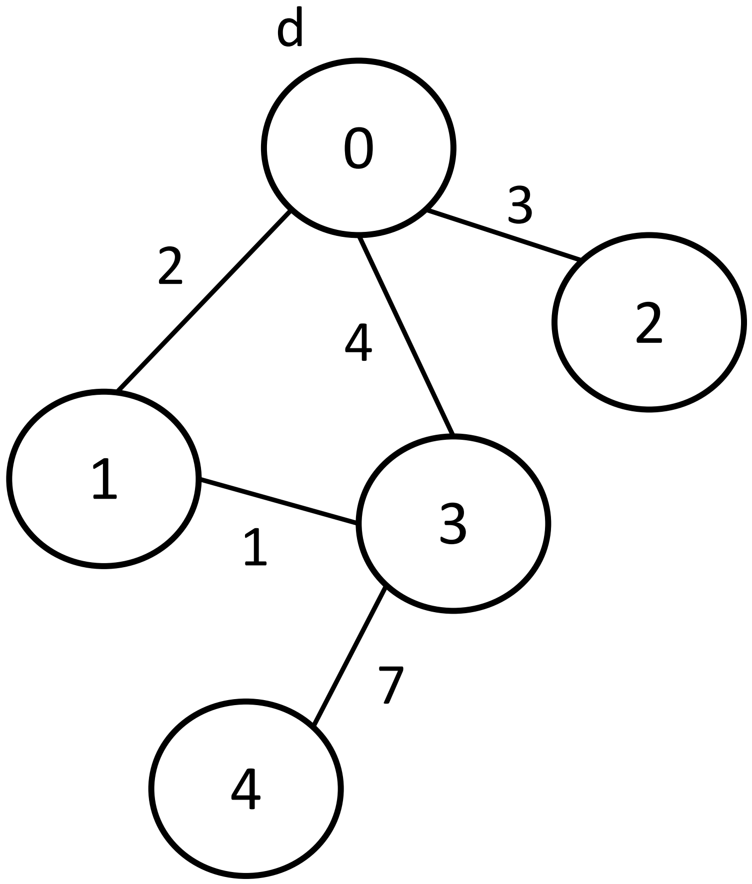
Computing shortest paths

```
def ROUTING(G, d):
  for each node v ∈ V do
    dist[v] ← INT.MAX
  end
  dist[d] ← INT.MIN
  Q ← {d}
  while Q ≠ ∅ do
    u ← EXTRACT(Q)
    for each node v ∈ N(u) do
      dist[v] = min(dist[v], dist[u] + weight(u, v))
      if dist[v] changed do
        Q ← Q ∪ {v}
      end
    end
  end
end
```



Computing shortest paths

```
def ROUTING(G, d):
  for each node v ∈ V do
    dist[v] ← ∞
  end
  dist[d] ← 0
  Q ← {d}
  while Q ≠ ∅ do
    u ← EXTRACT(Q)
    for each node v ∈ N(u) do
      dist[v] = dist[v] ⊕ (f_(u,v)(dist[u]))
      if dist[v] changed do
        Q ← Q ∪ {v}
      end
    end
  end
end
```



Shortest path algebra

$R = \text{natural numbers}$

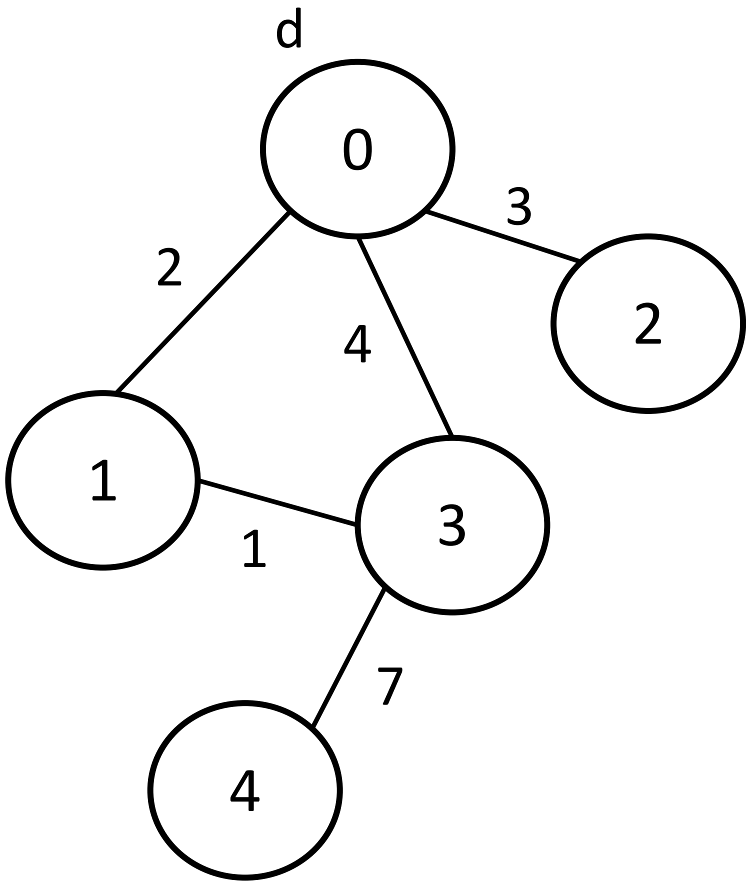
$f_e(x) = x + \text{weight}(e)$

$x \oplus y = \min(x, y)$

$\overline{0} = 0$

$\overline{\infty} = \infty$

$SP = (\text{Nat}, F_+, \min, 0, \infty)$



Other examples of algebras

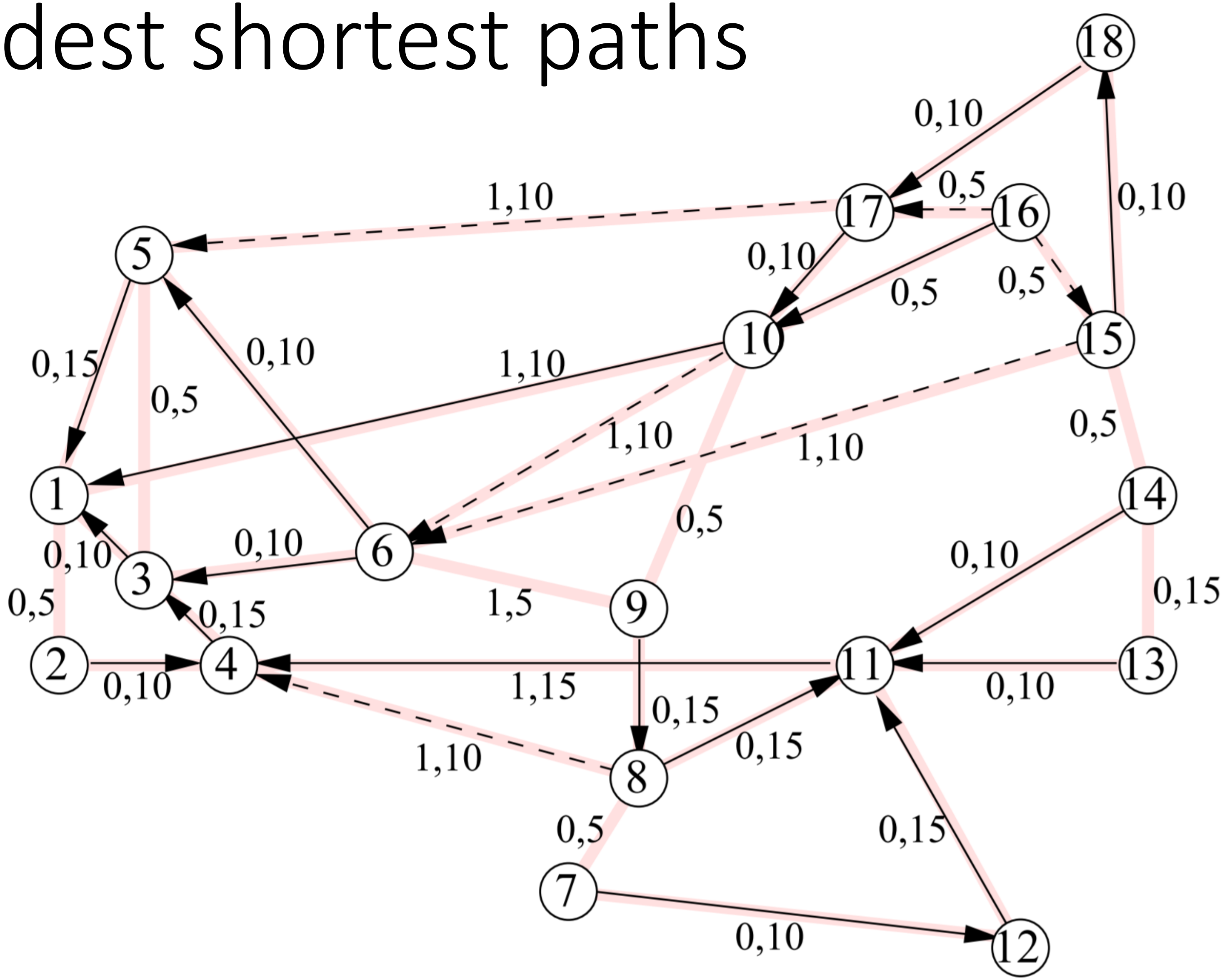
R	$\oplus$	F	$\overline{\infty}$	$\overline{0}$	Use
$\mathbb{N}_\infty$	min	F_+	∞	0	shortest paths
$\mathbb{N}_\infty$	max	F_+	0	∞	longest paths
$\mathbb{N}_\infty$	max	$F_{\min}$	0	∞	widest paths
$[0,\ 1]$	max	$F_\times$	0	1	most reliable paths

"Asynchronous Convergence of Policy Rich Distributed Bellman-Ford Routing Protocols"

Example: widest shortest paths

$$R = (\text{int}, \text{int})$$

length width



“Algebra and Algorithms for QoS Path Computation and Hop-by-Hop Routing in the Internet”

Properties of routing algebras

selectivity: $(a \oplus b) \in \{a, b\}$

we say $a \leq b \Leftrightarrow a \oplus b = a$

monotonicity: $a \leq f(a)$ or **strict monotonicity** $a < f(a)$

the cost of paths does not decrease or increases

isotonicity: $a \leq b \Rightarrow f(a) \leq f(b)$

relative cost of two paths will not flip when extended

Results from routing algebras

for path vector protocols:

1. always converge $\Leftrightarrow$ the algebra is strictly monotone
2. always optimal $\Leftrightarrow$ the algebra is strictly monotone & isotone

Convergence of protocols

Protocol type	SM	I	$\oplus$ assoc
path vector	✓		
link state with Dijkstra′s	✓	✓	✓
link state with LPVS	✓		

“Metarouting, Timothy Griffin 2005”