CSE 599F1, Winter 2017
Soft Constraints Assignment – Sample Solution

1. Consider the following constraints on real numbers x, y, and z.

required	$x + y + z = 10$
strong	$x = 6$
weak	$y = 0$
weak	$z = 0$

   (a) What are all the locally-predicate-better solutions?

   There are two: $\{x \mapsto 6, y \mapsto 0, z \mapsto 4\}$ and $\{x \mapsto 6, y \mapsto 4, z \mapsto 0\}$.

   We must satisfy the required constraint, and there are solutions that satisfy the strong $x = 6$ constraint, so any solution must also have $x = 6$. For locally-predicate-better, we only care whether the weak constraints are satisfied exactly or not. There are no solutions that satisfy both weak constraints. First consider $\{x \mapsto 6, y \mapsto 0, z \mapsto 4\}$. There are no other solutions that are better than it, since such a solution would need to also satisfy the $z = 0$ constraint. On the other hand, this solution is better than all other solutions (except for the second one) for example it is better than $\{x \mapsto 6, y \mapsto 2, z \mapsto 2\}$, since at least it satisfies one of the weak constraints. Similar reasoning holds for the second solution.

   (b) What are all the weighted-sum-metric-better solutions?

   There are infinitely many: all solutions such that $x \mapsto 6$, $y \in [0, 4]$, $z \in [0, 4]$, and $y + z = 4$.

   For example, $\{x \mapsto 6, y \mapsto 0, z \mapsto 4\}$, $\{x \mapsto 6, y \mapsto 4, z \mapsto 0\}$, $\{x \mapsto 6, y \mapsto 1.2, z \mapsto 2.8\}$, and $\{x \mapsto 6, y \mapsto 2, z \mapsto 2\}$ are all solutions — the weighted sum of the errors on the weak constraints is 4 for each of these. (Assume all weights are 1).

   On the other hand, if y or z is outside of the range $[0, 4]$ the error becomes larger than 4. For example, the error for $\{x \mapsto 6, y \mapsto -1, z \mapsto 5\}$ is 6, so this isn't a solution.

   (c) What are all the least-squares-metric-better solutions?

   There is just one: $\{x \mapsto 6, y \mapsto 2, z \mapsto 2\}$. The error for this solution is $2^2 + 2^2 = 4$. The error for any other solution is larger — for example the error for $\{x \mapsto 6, y \mapsto 1, z \mapsto 3\}$ is $1^2 + 3^2 = 10$.

2. Now consider the following constraints on real numbers a, b, and c. (Note that c has a read-only annotation in the first constraint.)

required	$a + b = c?$
required	$c = d$
strong	$a = 5$
medium	$b = 5$
weak	$d = 20$

   (a) What are all the locally-predicate-better solutions?

   (b) What are all the weighted-sum-metric-better solutions?

   (c) What are all the least-squares-metric-better solutions?

   The answer is the same for each of these, namely, there is one solution, $\{a \mapsto 5, b \mapsto 15, c \mapsto 20, d \mapsto 20\}$.

   There are two ways to find this solution.

   First, you can follow the definition in Section 3.1 of the paper. Form the set Q of all constraint hierarchies (i.e., sets of constraints with priorities) by substituting some arbitrary number for $c?$ in the first constraint. (Well, you can't actually write down all of them, since there are infinitely many, but eyeballing the problem indicates that 20 is likely to be an interesting number here.) So one element of Q is

required	$a+b=20$
required	$c=d$
strong	$a=5$
medium	$b=5$
weak	$d=20$

The solution to this is $\{a \mapsto 5, b \mapsto 15, c \mapsto 20, d \mapsto 20\}$ — we must satisfy the two required constraints, we can satisfy the strong constraint and the weak one, but the medium one can't be satisfied without violating the strong constraint. (This is the solution for all the above comparators.) Miraculously, c in this case is mapped to the number that we substituted for $c?$, namely 20, so this is a solution.

Another element of Q involves substituting 35 for $c?$. Here we get

required	$a+b=35$
required	$c=d$
strong	$a=5$
medium	$b=5$
weak	$d=20$

The solution to this is $\{a \mapsto 5, b \mapsto 30, c \mapsto 20, d \mapsto 20\}$ — alas in this case c is *not* mapped to the number that we substituted for $c?$, so this isn't a solution. And similarly for all the other numbers we might try for $c?$.

The other way to solve this problem is to partition the constraints into those upstream of the read-only variable and those downstream of it. This is much easier to think about — you can only do it if there are no cycles through a read-only variable, which is the case here.

So we have two separate sets of constraints. The upstream ones are

required	$c=d$
weak	$d=20$

Solve these to get $\{c \mapsto 20, d \mapsto 20\}$.

The downstream constraints are

required	$a+b=c?$
strong	$a=5$
medium	$b=5$

Plug in the solution we found for the upstream constraints (i.e., replace $c?$ with 20) to get

required	$a+b=20$
strong	$a=5$
medium	$b=5$

Solve these to get $\{a \mapsto 5, b \mapsto 15\}$. Put the two solutions together for the solution to the whole problem.

3. Suppose that we have a line that is constrained to be horizontal and to be contained within a box whose left is at 10, right at 100, top at 100, and bottom at 10. All of these constraints are required. The first endpoint p1 has x and y values p1.x and p1.y respectively, and the second endpoint p2 has x and y values p2.x and p2.y respectively.

   Initially p1.x=20, p1.y=35, p2.x=80, and p2.y=35. Suppose the user is trying to move p2 to the point x=200, y=50 using the mouse. This constraint is strong (not required). There are also weak stay constraints on the endpoints of the line. (Also see the attached sketch.)

   (a) Write down the set of constraints.

required	$p1x \geq 10$	(line in the box)
required	$p1x \leq 100$	
required	$p1y \geq 10$	
required	$p1y \leq 100$	
required	$p2x \geq 10$	
required	$p2x \leq 100$	
required	$p2y \geq 10$	
required	$p2y \leq 100$	
required	$p1y = p2y$	(this is the constraint keeping the line horizontal)
strong	$p2x = 200$	(follow the mouse)
strong	$p2y = 50$	
weak	$p1x = 20$	(stay constraints)
weak	$p1y = 35$	
weak	$p2x = 80$	
weak	$p2y = 35$	

Note that the inequality constraints to keep the line in the box need to be nonstrict rather than strict, in particular $p2x \leq 100$ rather than $p2x < 100$. If this were $p2x < 100$ there would be no solution using weighted-sum-metric-better!

(b) What are all the weighted-sum-metric-better solutions? (There might be just one, or might be several.)

There is just one weighted-sum-metric-better solution:
$\{p1x \mapsto 20, p1y \mapsto 50, p2x \mapsto 100, p2y \mapsto 50\}$.