Homework 3
CSE 599f

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$dfsTable, ndfsTable : Set(S)$
$seed : S$

$dfsTable := \emptyset$
$ndfsTable := \emptyset$
for each s in S_0
    dfs(s)
return EMPTY

dfs(s) {
    add s to $dfsTable$
    for each successor t of s
        if ($t \notin dfsTable$) then dfs(t)
    if ($s \in BA$) then $seed := s$; ndfs(s)
}

ndfs(s) {
    add s to $ndfsTable$
    for each t such that $s \longrightarrow t$
        if ($t \notin ndfsTable$) then ndfs(t)
        else if ($t = seed$) then return NONEMPTY
}

Figure 1: Nested depth-first search

Problem 1. Let $A = (S, S_0, \longrightarrow, \phi, BA)$ be a Büchi automaton, that is provided as input to the nested depth-first search algorithm shown in Figure 1. The goal of the algorithm is to return NONEMPTY if A has an accepting run and to retun NONEMPTY otherwise. Prove that the algorithm is correct.

Problem 2. Let $B = (S, S_0, \longrightarrow, \phi, BA_1, \ldots, BA_n)$ be a multi-Büchi automaton. Generalize the algorithm in Figure 1 to check for the existence of an accepting run in B.

Problem 3. Construct the μ-calculus formulae equivalent to the following temporal logic formulae?

(a) $\exists\Diamond\Box p$

(b) $\forall\Diamond\Box p$

(c) $\forall\Diamond\exists\Box p$

(d) $\exists\Diamond\exists\Box p$

Problem 4. Given the BDD of a function f over variables $x_1, \ldots, x_n$, construct linear-time algorithms to compute the following:

(a) The number of disjoint paths to the *true* node in the BDD.

(b) The number of satisfying assignments for f.

Problem 5. A boolean function is *symmetric* if the value of the function depends only on the number of inputs that are assigned *true*; it does not matter which particular inputs they are. For example, the functions AND, OR, and XOR are symmetric. A symmetric boolean function f over n boolean variables is completely described by a set $S \subseteq \{0, 1, \ldots, n\}$, where for assignment $x \in \{0,1\}^n$, $f(x) = \mathit{true}$ iff the number of variables assigned to true in x is a member of S. Prove that, for any variable ordering, the BDD of a symmetric function f over n boolean variables has $O(n^2)$ nodes.