Homework 1
CSE 599f

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\begin{array}{rcll}
x & \in & \mathit{Variable} & \\
P & \in & \mathit{Program} & = \ \mathit{assert}\ x \mid x{+}{+} \mid x{-}{-} \mid \\
 & & & \quad P_1; P_2 \mid \mathit{if}\ x\ \mathit{then}\ P_1\ \mathit{else}\ P_2 \mid \mathit{while}\ x\ P
\end{array}
$$

Figure 1: A simple programming language

Problem 1. Consider the programming language shown in Figure 1. The set *Variable* contains a finite set of integer variables initialized to 0. A program is constructed using the grammar shown in the figure. The various constructs in the language have intuitive semantics. *assert* x fails if $x = 0$ and otherwise has no effect. $x{+}{+}$ increments the value of x. $x{-}{-}$ decrements the value of x. $P_1; P_2$ computes as P_1 and then computes as P_2. *if* x *then* P_1 *else* P_2 computes as P_1 if $x \neq 0$ and as P_2 if $x = 0$. *while* x P iterates P as long as $x \neq 0$.

1. Show that it is undecidable to check whether a program in this language can fail. You can assume the undecidability of the halting problem for Turing machines.

2. What is the minimum cardinality of *Variable* for which this undecidability result holds?

3. Let us call the minimum cardinality from the last question c. If $c > 1$, provide an algorithm for the case when $|\mathit{Variable}| = c - 1$.

Problem 2. Let A be a set of atomic propositions. A state transition graph over A is a tuple $K = (Q, T, L)$ where Q is a finite set of states, $T \subseteq Q \times Q$ is a transition relation that is total over the first component, and $L : Q \rightarrow 2^A$ is the labeling function that provides for each $q \in Q$ the set of atomic propositions that hold in Q. A finite (or infinite) run of K is a finite (or infinite) sequence $q_0, q_1, q_2, \ldots$ such that $(q_i, q_{i+1}) \in T$ for all $i \geq 0$. A trace corresponding to a run $q_0, q_1, q_2, \ldots$ is the sequence $L(q_0), L(q_1), L(q_2), \ldots$ over 2^A. Let $\mathit{Finite}(q, K)$ be the set of finite traces that correspond to finite runs whose initial state is q. Let $\mathit{Infinite}(q, K)$ be the set of infinite traces that correspond to infinite runs whose initial state is q.

1. Suppose L is a language of finite strings over 2^A. Let the language $Limit(L)$ of infinite strings over 2^A be defined such that $\sigma \in Limit(L)$ iff every finite prefix of σ is in L. Show that $Limit(Finite(q, K)) = Infinite(q, K)$.

2. Let $K = (Q, T, L)$ and $K' = (Q', T', L')$ be two state transition graphs over A. Show that for any $q \in Q$ and $q' \in Q'$, we have $Finite(q, K) \subseteq Finite(q', K')$ if and only if $Infinite(q, K) \subseteq Infinite(q', K')$.