Query Optimization – Homework 4

January, 2026

Submit your answer in a pdf file on Canvas.

- Write your name in the file.
- Use this template `https://www.overleaf.com/project/67f01a081d8c577a12f22353`

Grading is done using credit/partial-credit/no-credit; ignore the points below.
An asterix * indicates that the question may be more challenging.

1 Tree Decomposition

1. (0 points)

   (a) For each query below indicate whether they are acyclic. (The head variables don't matter for this question and are not shown.)

$$
\begin{aligned}
Q_1 =& R(X, Y) \wedge S(Y, Z) \wedge T(Z, X) \\
Q_2 =& R(X, Y, Z) \wedge S(Y, Z, U) \wedge T(Z, U, V) \\
Q_3 =& A(X, Y, Z) \wedge R(X, Y) \wedge S(Y, Z) \wedge T(Z, X) \\
Q_4 =& A(X) \wedge B(Y) \wedge C(Z) \wedge R(X, Y) \wedge S(Y, Z) \wedge T(Z, X)
\end{aligned}
$$

   (b) Answer the following questions.

      i. Compute the treewidth of a cycle of length n.

      ii. Compute the treewidth of an $n \times n$ grid. The grid is defined by connecting each node (i, j) to the nodes $(i \pm 1, j)$ and $(i, j \pm 1)$.

      iii. Compute the treewidth of an $n \times n \times n$ cube. Here each node (i, j, k) is connected to $(i \pm 1, j, k)$, $(i, j \pm 1, k)$, $(i, j, k \pm 1)$.

2 The AGM Bound

2. (0 points)

   (a) Consider the following query:

   $$Q(x, y, z, u, v, w) = R(x, y) \wedge S(y, z) \wedge T(y, u) \wedge K(u, v) \wedge M(x, w)$$

   Assume that $|R| = |S| = |T| = |K| = |M| \leq N$.

   i. Find the maximum size of the output to the query Q

   ii. Find a worst-case database instance where the query Q has the bound you found above.

   (b) Consider the query:

   $$Q(x, y, z, u) = R(x, y) \wedge S(y, z) \wedge T(z, u) \wedge K(u, x)$$

   Suppose the four relations have cardinalities N_1, N_2, N_3, N_4.

   Give a formula that represents a tight upper bound on $|Q|$. Your formula should use the cardinalities N_1, N_2, N_3, N_4 and operations like $+, \times, /, \hat{}$, max, for example $\max(N_1/N_2, N_3^{3/2} + N_4)$ (not a real answer).

   (c) Consider the same query as above, and repeat your answer for the case when y is a key in S:

   $$Q(x, y, z, u) = R(x, y) \wedge S(\underline{y}, z) \wedge T(z, u) \wedge K(u, x)$$

3 Information Inequalities

3. (0 points)

   (a) Let Y, Z be two finites sets. Prove that

   $$|Y|^2 + |Z|^2 \leq |Y \cup Z|^2 + |Y \cap Z|^2$$

   Moreover, show that the inequality becomes an equality if and only if $Y \subseteq Z$ or $Z \subseteq Y$. (We used this property when we proved the generalized Shearer inequality.)

   (b) Consider the following query:

   $$Q(x, y, z, u) = R(x, y, z) \wedge S(y, z, u) \wedge T(z, u, x) \wedge K(u, x, y)$$

   Prove that the following inequalities hold:

   $$\begin{aligned} |Q| &\leq (|R| \cdot |S| \cdot |T| \cdot |K|)^{1/3} \\ |Q| &\leq |R| \cdot \max(\mathsf{deg}_S(u|yz)) \\ |Q| &\leq |T| \cdot \max(\mathsf{deg}_K(y|ux)) \end{aligned}$$

   (c) Consider the following query:

   $$\begin{aligned} Q(x, y, z, u, v, w) = &R(x, y, z) \wedge S(z, u, v) \wedge T(v, w, x) \\ &\wedge A(y, z, u) \wedge B(u, v, w) \wedge C(w, x, y) \end{aligned}$$

   Prove the following inequality:

   $$|Q| \leq \sqrt{|R| \cdot |S| \cdot |T| \cdot \max(\mathsf{deg}_A(y|zu)) \cdot \max(\mathsf{deg}_B(u|vw)) \cdot \max(\mathsf{deg}_C(w|xy))}$$

   (d) Prove the following inequality:

   $$\begin{aligned} &h(xyz) + h(zuv) + h(vwx) + h(yuw) + \\ &h(y|x) + h(z|y) + h(u|z) + h(v|u) + h(w|v) + h(x|w) \geq 3h(xyzuvw) \end{aligned}$$