Finite Model Theory – Homework 2

May 9, 2025

1 Query Containment

1. (0 points)
   (a) Indicate all containment or equivalence relationships between the following Boolean queries:

$$\begin{aligned} Q_1() =& R(x,y) \wedge R(z,y) \wedge R(x,u) \\ Q_2() =& R(x,y) \wedge R(y,z) \wedge R(z,u) \\ Q_3() =& R(x,y) \wedge R(y,z) \wedge R(z,x) \\ Q_4() =& R(x,y) \end{aligned}$$

   (b) Indicate all containment or equivalence relationships between the following queries:

$$\begin{aligned} Q_1 =& R(x,y) \wedge R(y,z) \wedge R(z,x) \\ Q_2 =& R(x,y) \wedge R(y,z) \wedge R(z,x) \wedge x \geq y \\ Q_3 =& R(x,y) \wedge R(y,z) \wedge R(z,x) \wedge x \leq y \leq z \end{aligned}$$

   (c) [1] Prove that $Q_1 \equiv Q_2$:

$$\begin{aligned} Q_1 =& R(x_1,x_2) \wedge R(x_2,x_3) \wedge R(x_3,x_4) \wedge R(x_4,x_5) \wedge R(x_5,x_1) \wedge x_1 \neq x_2 \\ Q_2 =& R(x_1,x_2) \wedge R(x_2,x_3) \wedge R(x_3,x_4) \wedge R(x_4,x_5) \wedge R(x_5,x_1) \wedge x_1 \neq x_3 \end{aligned}$$

   Hint: you may try the method in class, but I found it too tedious. Alternatively, you may try to find a more direct argument.

   (d) Consider the following two conjunctive queries, over the language of graphs:

$$\begin{aligned} Q_1(x) =& E(x,y) \wedge E(y,z) \wedge E(z,u) \\ Q_2(x) =& E(x,y) \wedge E(y,z) \end{aligned}$$

   i. Prove that $Q_1 \subseteq Q_2$ (Hint: it suffices to find a homomorphism $h : Q_2 \to Q_1$).
   ii. Find a query Q such that $Q_1 \subsetneq Q \subsetneq Q_2$. In other words, Q is between Q_1 and Q_2, but not equivalent to either of them.

2 (Hyper)-Treewidth

2. (0 points)

   (a) For each query below indicate whether they are acyclic. (The head variables don't matter for this question and are not shown.)

$$
\begin{aligned}
Q_1 =& R(X,Y) \wedge S(Y,Z) \wedge T(Z,X) \\
Q_2 =& R(X,Y,Z) \wedge S(Y,Z,U) \wedge T(Z,U,V) \\
Q_3 =& A(X,Y,Z) \wedge R(X,Y) \wedge S(Y,Z) \wedge T(Z,X) \\
Q_4 =& A(X) \wedge B(Y) \wedge C(Z) \wedge R(X,Y) \wedge S(Y,Z) \wedge T(Z,X)
\end{aligned}
$$

   (b) In the following questions we are considering tree decompositions of graphs: we are not considering hypertree decompositions.

      i. What is the treewidth of a cycle of length n?

      ii. What is the treewidth of an $n \times n$ grid? Node (i, j) is connected to $(i \pm 1, j)$ and to $(i, j \pm 1)$.

      iii. What is the treewidth of an $n \times n \times n$ cube? Node (i, j, k) is connected to $(i \pm 1, j, k)$, $(i, j \pm 1, k)$, $(i, j, k \pm 1)$.

References

[1] Y. Amsterdamer, D. Deutch, T. Milo, and V. Tannen. On provenance minimization. In *Proceedings of the 30th ACM SIGMOD-SIGACT-SIGART Symposium on Principles of Database Systems, PODS 2011, June 12-16, 2011, Athens, Greece*, pages 141–152, 2011.