CSE 599c Homework Sheet

Modern Quantum Complexity Theory: Spring 2026

Problem 1. QMA(2), error reduction, and why the usual PP argument breaks.

First recall the definition of $\mathrm{QMA}(2)[c,s]$: a language L is in $\mathrm{QMA}(2)[c,s]$ if there is a polynomial-time uniform quantum verifier that, on input x, receives two polynomial-size quantum witnesses and satisfies completeness at least c and soundness at most s against all *product* witnesses.

Give a clean formal definition of this class. Then prove that for every polynomial $r(n)$,

$$\mathrm{QMA}(2)\left[\tfrac{2}{3}, \tfrac{1}{3}\right] = \mathrm{QMA}(2)\left[1 - 2^{-r(n)}, 2^{-r(n)}\right].$$

You may use any standard amplification theorem for QMA(2), provided you state clearly what theorem you are invoking and why it applies here.

After establishing this, explain why this does *not* immediately imply

$$\mathrm{QMA}(2) \subseteq \mathrm{PP}$$

by the same argument used in class to prove $\mathrm{QMA} \subseteq \mathrm{PP}$. Your answer should identify the exact step of the QMA proof that fails, and it should explain the conceptual difference between one quantum witness and two unentangled quantum witnesses that makes the QMA argument go through but blocks the analogous argument for QMA(2).

Problem 2. How well does an entropy bound alone control MPS approximation?

Let $|\psi\rangle \in (\mathbb{C}^d)^{\otimes n}$ be a pure state on a line of n qudits. Assume that for every cut between sites $1, \ldots, i$ and $i+1, \ldots, n$, the bipartite entanglement entropy satisfies

$$S(\rho_{1,\ldots,i}) \leq D,$$

where $S(\cdot)$ denotes the von Neumann entropy in bits.

Fix $\varepsilon \in (0,1)$. We will say that an MPS $|\phi\rangle$ is an ε-approximation to $|\psi\rangle$ if

$$\big\| \, |\psi\rangle - |\phi\rangle \, \big\|_2 \leq \varepsilon.$$

Determine the best upper bound you can give, as an explicit function of d, D, ε, and n, on the bond dimension χ required for such an ε-approximate MPS. Your answer should be correct up to universal constant factors in the exponent.

Then prove that this dependence is asymptotically tight, again up to universal constant factors in the exponent, by constructing an explicit family of states $|\psi\rangle$ that saturates your bound.