

# Tutorial on Probabilistic Context-Free Grammars

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590AI, Winter 2009

# Outline

- PCFGs: Inference and Learning
- Parsing English
- Discriminative CFGs
- Grammar Induction

# Image Search for "pcfg"

Google™



Live Search

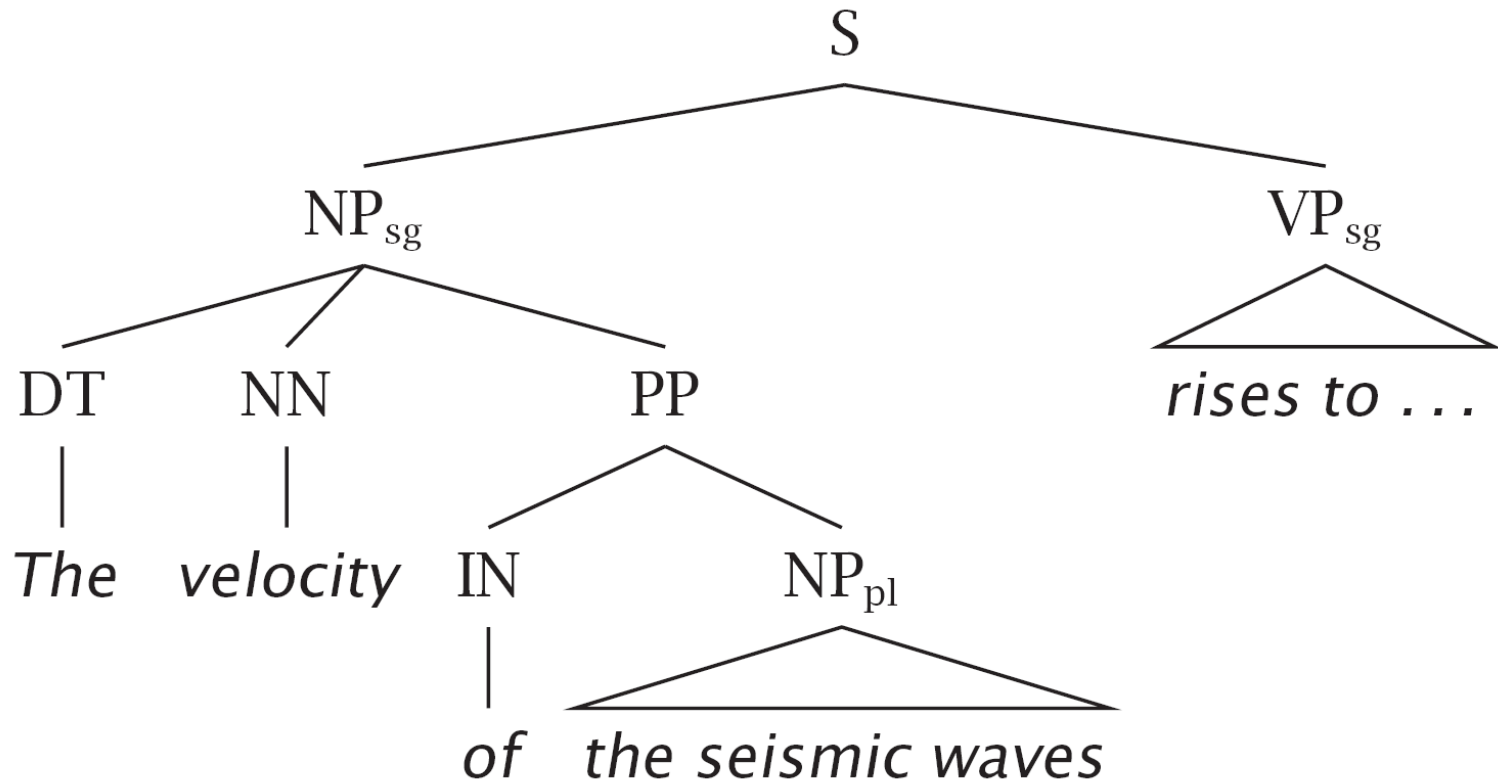


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The velocity of the seismic waves rises to ...



# A CFG consists of

- Terminals

$w^1, w^2, \dots, w^V$

- Nonterminals

$N^1, N^2, \dots, N^n$

- Start symbol

$N^1$

- Rules

$N^i \longrightarrow \zeta^j$

where  $\zeta^j$  is a sequence of terminals and nonterminals

# A (generative) PCFG consists of

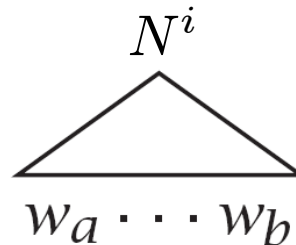
- Terminals  $w^1, w^2, \dots, w^V$
- Nonterminals  $N^1, N^2, \dots, N^n$
- Start symbol  $N^1$
- Rules  $N^i \longrightarrow \zeta^j$   
where  $\zeta^j$  is a sequence of terminals and nonterminals
- Rule probabilities such that  
$$\forall_i \sum_j P(N^i \longrightarrow \zeta^j) = 1$$

# Notation

sentence: sequence of words  $w_1 w_2 \dots w_m$

$w_{ab}$  : the subsequence  $w_a \dots w_b$

$N_{ab}^i$  : nonterminal  $N^i$  dominates  $w_a \dots w_b$



$N^i \Longrightarrow^* \zeta$  : repeated derivation from  $N^i$  gives  $\zeta$



# Probability of a Sentence

$$P(w_{1n}) = \sum_t P(w_{1n}, t)$$

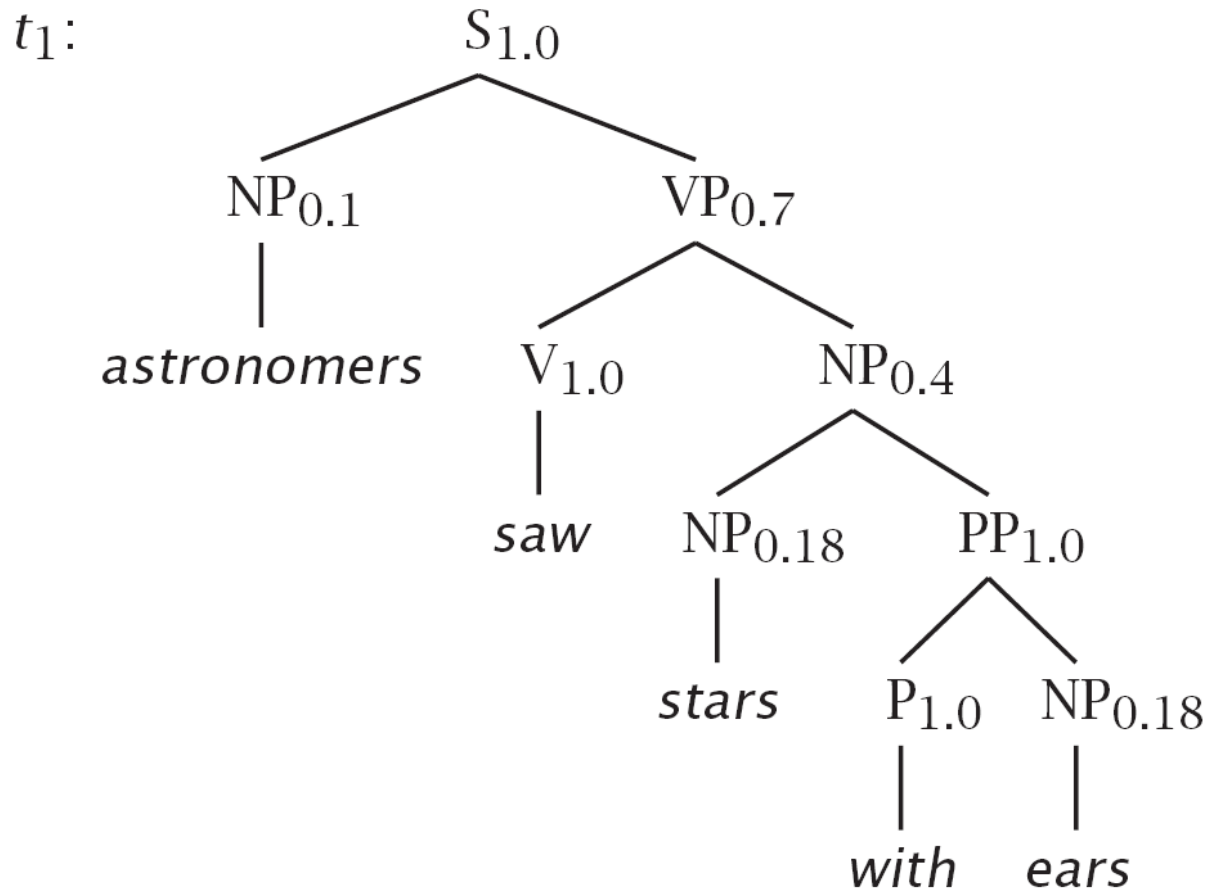
where  $t$  a parse tree of  $w_{1n}$

# Example

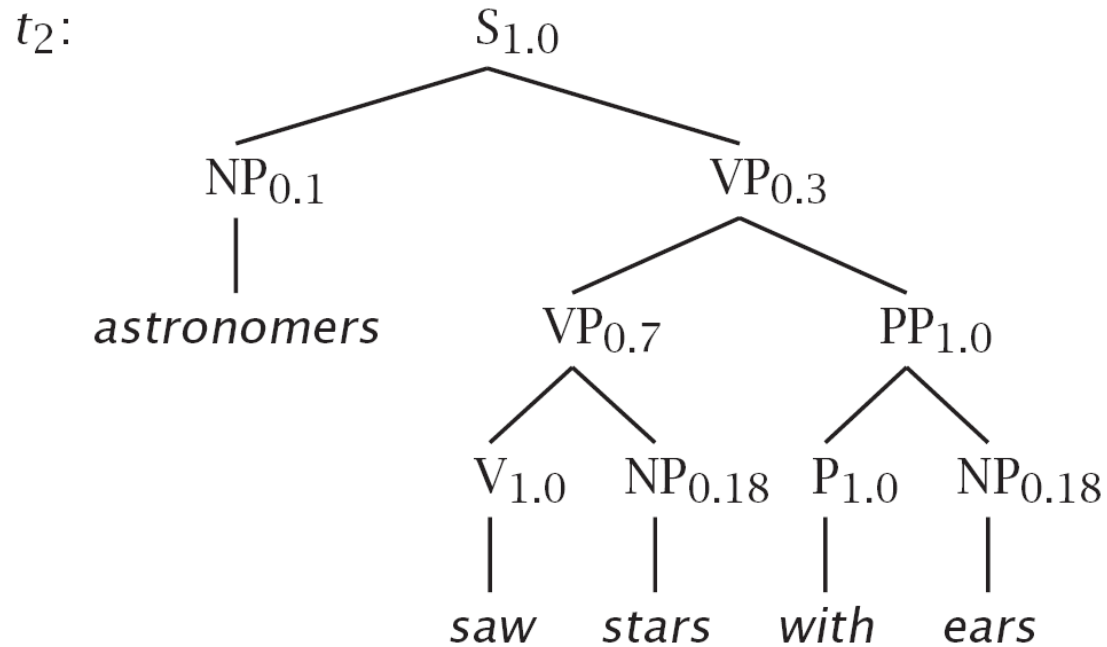
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- Terminals            *with, saw, astronomers, ears, stars, telescopes*
- Nonterminals        *S, PP, P, NP, VP, V*
- Start symbol         *S*

# astronomers saw stars with ears

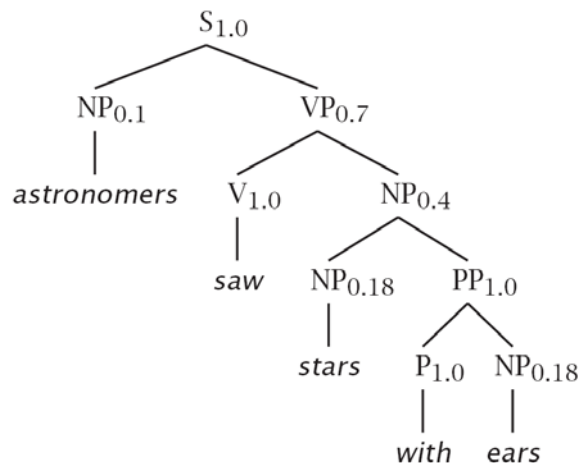


# astronomers saw stars with ears



# Probabilities

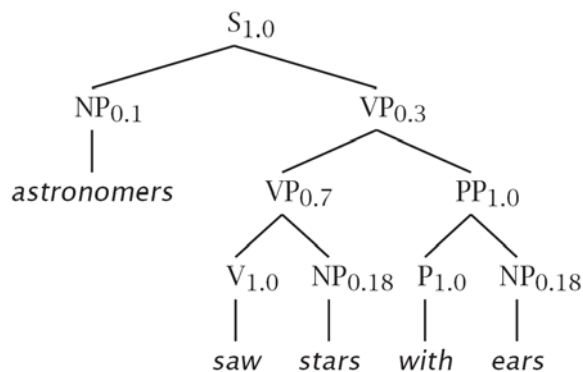
$t_1$ :



$$\begin{aligned}
 P(t_1) &= 1.0 \times 0.1 \times 0.7 \times 1.0 \times 0.4 \\
 &\quad \times 0.18 \times 1.0 \times 1.0 \times 0.18 \\
 &= 0.0009072
 \end{aligned}$$

$$\begin{aligned}
 P(t_2) &= 1.0 \times 0.1 \times 0.3 \times 0.7 \times 1.0 \\
 &\quad \times 0.18 \times 1.0 \times 1.0 \times 0.18 \\
 &= 0.0006804
 \end{aligned}$$

$t_2$ :



$$P(w_{15}) = P(t_1) + P(t_2) = 0.0015876$$

# Assumptions of PCFGs

1. Place invariance (like time invariance in HMMs)

$$\forall k \quad P(N_{k(k+c)}^j \longrightarrow \zeta) \text{ is the same}$$

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$$P(N_{kl}^j \longrightarrow \zeta \mid \text{words outside } w_k \dots w_l) = P(N_{kl}^j \longrightarrow \zeta)$$

3. Ancestor free

$$P(N_{kl}^j \longrightarrow \zeta \mid \text{ancestor nodes of } N_{kl}^j) = P(N_{kl}^j \longrightarrow \zeta)$$



# Some Features of PCFGs

- Partial solution for grammar ambiguity
- Can be learned from positive data alone  
(but grammar induction difficult)
- Robustness  
(admit everything with low probability)
- Gives a probabilistic language model
- Predictive power better than that for a HMM

# Some Features of PCFGs

- Not lexicalized (probabilities do not factor in lexical co-occurrence)
- PCFG is a worse language model for English than n-gram models
- Certain biases: smaller trees more probable (average WSJ sentence 23 words)

# Inconsistent Distributions

- $S \rightarrow \text{rhubarb} \quad P = \frac{1}{3}$

- $S \rightarrow S S \quad P = \frac{2}{3}$

- $\text{rhubarb} \quad \frac{1}{3}$

- $\text{rhubarb rhubarb} \quad \frac{2}{3} \times \frac{1}{3} \times \frac{1}{3} = \frac{2}{27}$

- $\text{rhubarb rhubarb rhubarb} \quad \left(\frac{2}{3}\right)^2 \times \left(\frac{1}{3}\right)^3 \times 2 = \frac{8}{243}$

...

- $P(\mathcal{L}) = \frac{1}{3} + \frac{2}{27} + \frac{8}{243} + \dots = \frac{1}{2}$

- Improper/inconsistent distribution

- Not a problem if you estimate from parsed treebank: Chi and Geman (1998).

# Questions

Let  $w_{1m}$  be a sentence,  $G$  a grammar,  $t$  a parse tree.

- What is the probability of a sentence?

$$P(w_{1m}|G)$$

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$$\arg \max_t P(t|w_{1m}, G)$$

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$$P(w_{1m}|G)$$

- What is the most likely parse of sentence?

$$\arg \max_t P(t|w_{1m}, G)$$

- What rule probs. maximize probs. of sentences?

$$\text{Find } G \text{ that maximizes } P(w_{1m}|G)$$

# Chomsky Normal Form

- Any CFG grammar can be represented in CNF where all rules take the form

$$N^i \longrightarrow N^j N^k$$

$$N^i \longrightarrow w^j$$

# HMMs and PCFGs

- HMMs: distribution over strings of certain length

$$\forall n \sum_{w_{1n}} P(w_{1n}) = 1$$

- PCFGs: distribution over strings of language  $L$

$$\sum_{w \in L} P(w) = 1$$



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- Consider

$P(\text{John decided to bake a})$

high probability in HMM, low probability in PCFG

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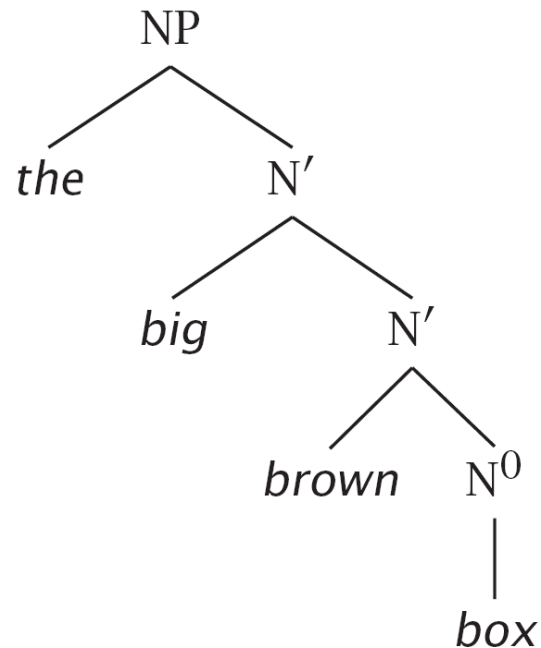
high probability in HMM, low probability in PCFG

- Probabilistic Regular Grammar

$$\begin{aligned} N^i &\longrightarrow w^j N^k \\ N^i &\longrightarrow w^j \end{aligned}$$

# HMMs and PCFGs

$X$ : NP  $\rightarrow$  N'  $\rightarrow$  N'  $\rightarrow$  N'  $\rightarrow$  sink  
| | | |  
 $O$ : *the* *big* *brown* *box*



# Inside and Outside Probabilities

- For HMMs we have

$$\text{Forwards} \quad \alpha_i(t) = P(w_{1(t-1)}, X_t = i)$$

$$\text{Backwards} \quad \beta_i(t) = P(w_{tT} | X_t = i)$$

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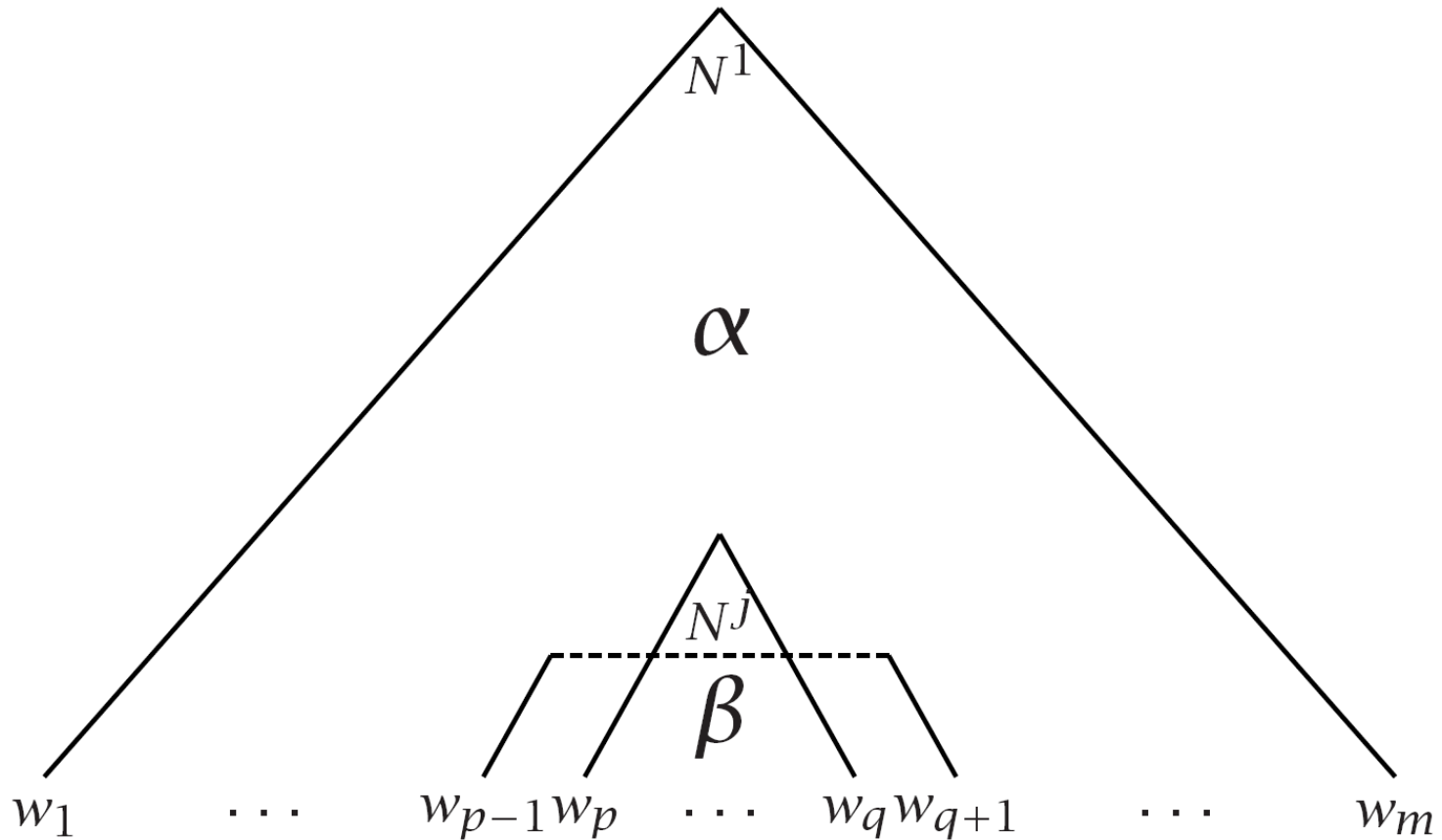
$$\text{Backwards} \quad \beta_i(t) = P(w_{tT} | X_t = i)$$

- For PCFGs we have

$$\text{Outside} \quad \alpha_j(p, q) = P(w_{1(p-1)}, N_{pq}^j, w_{(q+1)m} | G)$$

$$\text{Inside} \quad \beta_j(p, q) = P(w_{pq} | N_{pq}^j, G)$$

# Inside and Outside Probabilities



# Probability of a sentence

$$\text{Outside} \quad \alpha_j(p, q) = P(w_{1(p-1)}, N_{pq}^j, w_{(q+1)m} | G)$$

$$\text{Inside} \quad \beta_j(p, q) = P(w_{pq} | N_{pq}^j, G)$$

$$P(w_{1m} | G) = \beta_1(1, m)$$

$$P(w_{1m} | G) = \sum_j \alpha_j(k, k) P(N^j \longrightarrow w_k)$$

# Inside Probabilities

$$\beta_j(p, q) = P(w_{pq} | N_{pq}^j, G)$$

- Base case

$$\begin{aligned}\beta_j(k, k) &= P(w_{kk} | N_{kk}^j, G) \\ &= P(N^j \longrightarrow w_k | G)\end{aligned}$$



# Inside Probabilities

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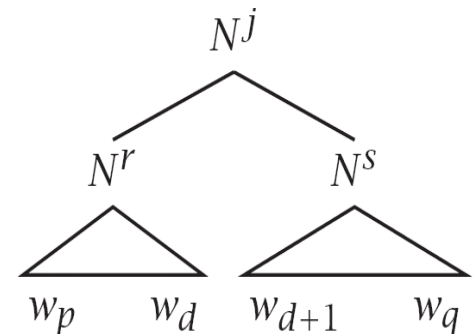
$$\begin{aligned}\beta_j(k, k) &= P(w_{kk} | N_{kk}^j, G) \\ &= P(N^j \longrightarrow w_k | G)\end{aligned}$$

- Induction

Want to find  $\beta_j(p, q)$  for  $p < q$

Since we assume Chomsky Normal Form,  
the first rule must be of the form  $N^j \longrightarrow N^r N^s$

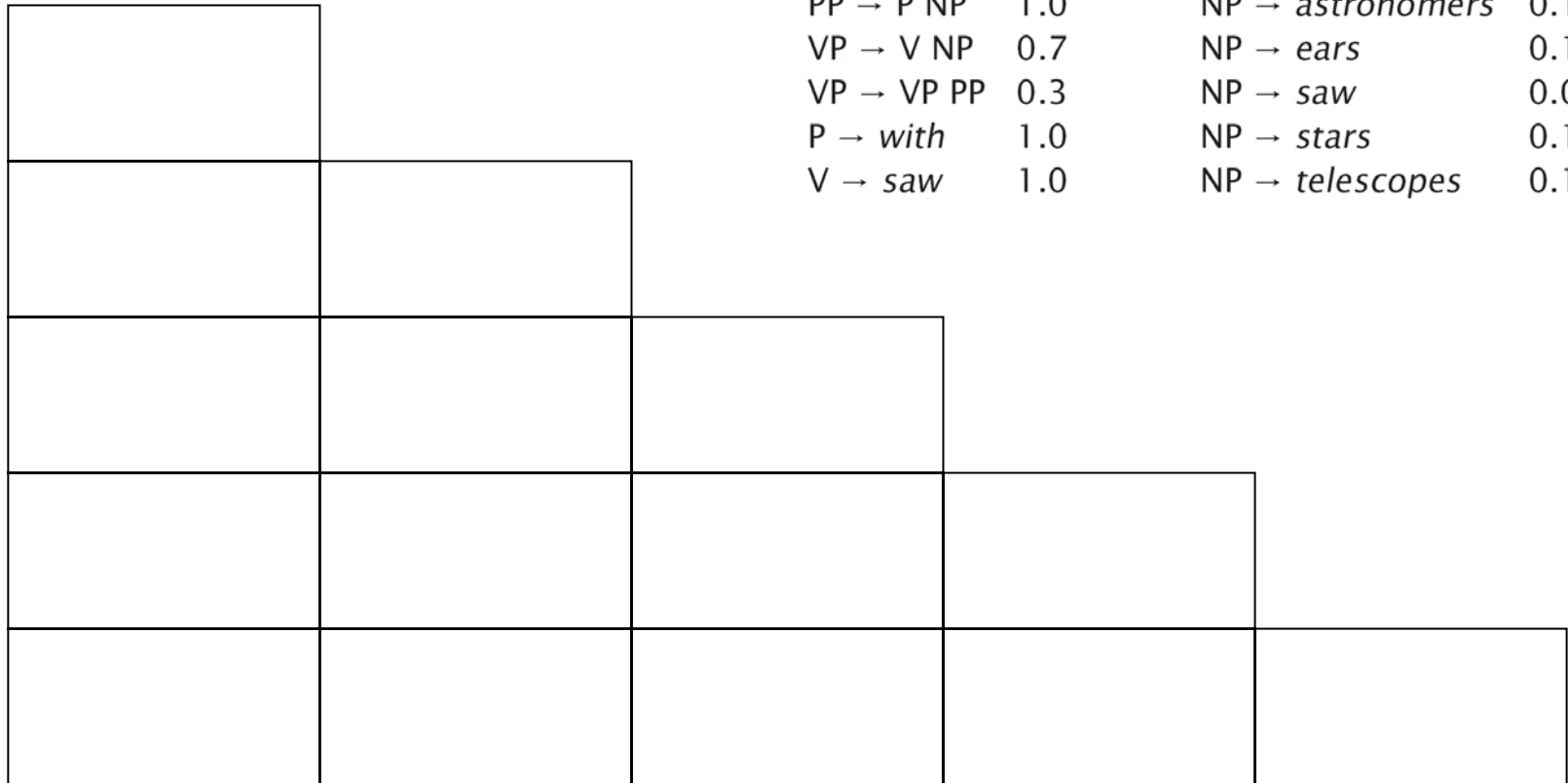
So we can divide the sentence in two in  
various places and sum the result



$$\beta_j(p, q) = \sum_{r,s} \sum_{d=p}^{q-1} P(N^j \longrightarrow N^r N^s) \beta_r(p, d) \beta_s(d+1, q)$$

# CYK Algorithm

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astronomers

saw

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$\beta_{NP} = 0.1$ $?$	$\beta_V = 1.0$ $?$ $\beta_{NP} = 0.04$	$\beta_{NP} = 0.18$ $?$	$\beta_P = 1.0$	$\beta_{NP} = 0.18$

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?	$\beta_{VP} = 0.126$ ?	?	$\beta_{PP} = 0.18$ ?	
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$\beta_S = 0.015876$				
	$\beta_{VP} = 0.015876$			
$\beta_S = 0.0126$ ?		$\beta_{NP} = 0.01296$		
	$\beta_{VP} = 0.126$		$\beta_{PP} = 0.18$	
$\beta_{NP} = 0.1$	$\beta_V = 1.0$ $\beta_{NP} = 0.04$		$\beta_P = 1.0$	$\beta_{NP} = 0.18$
astronomers	saw	stars	with	ears

# CYK Algorithm

Worst case:  $O(m^3r)$

$m$  = length of sentence

$r$  = number of rules in grammar

$n$  = number of non-terminals

If we consider all possible CNF rules:  $O(m^3n^3)$

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astronomers	saw	stars	with	ears

# Outside Probabilities

- Compute top-down (after inside probabilities)
- Base case

$$\alpha_1(1, m) = 1$$

$$\alpha_j(1, m) = 0, \text{ for } j \neq 1$$

- Induction

$$\alpha_j(p, q) = \left( \sum_{f, g} \sum_{e=q+1}^m \alpha_f(p, e) P(N^f \rightarrow N^j N^g) \beta_g(q+1, e) \right) + \left( \sum_{f, g} \sum_{e=1}^{p-1} \alpha_f(e, q) P(N^f \rightarrow N^g N^j) \beta_g(e, p-1) \right)$$

# Probability of a node existing

- As with a HMM, we can form a product of the inside and outside probabilities.

$$\alpha_j(p, q)\beta_j(p, q) = P(w_{1m}, N_{pq}^j | G)$$



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- Just in the cases of the root node and the preterminals, we know there will be some such constituent.

# Training

- If have data  $\rightarrow$  count

$$\hat{P}(N^j \rightarrow \zeta) = \frac{C(N^j \rightarrow \zeta)}{\sum_{\gamma} C(N^j \rightarrow \gamma)}$$

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- else use EM (Inside-Outside-Algorithm)

repeat


compute  $\alpha_j$ 's and  $\beta_j$ 's

compute  $\hat{P}$ 's

$$\hat{P}(N^j \rightarrow N^r N^s) = \dots$$

$$\hat{P}(N^j \rightarrow w^k) = \dots$$

end

  
two really long formulas with  $\alpha_j$ 's and  $\beta_j$ 's

# EM Problems

- Slow:  $O(m^3n^3)$  for each sentence and each iteration
- Local maxima (Charniak: 300 trials led to 300 different max.)
- In practice, need  $>3$  times more non-terminals than are theoretically needed
- No guarantee that learned non-terminals correspond to NP, VP, ...

# Bracketing helps

Pereira/Schabes '92:

- Train on sentences:  
37% of predicted brackets correct
- Train on sentences + brackets:  
90% of predicted brackets correct

# Grammar Induction

- Rules typically selected by linguist
- Automatic induction is very difficult for context-free languages
- It is easy to find *some* form of structure, but little resemblance to that of linguistics/NLP

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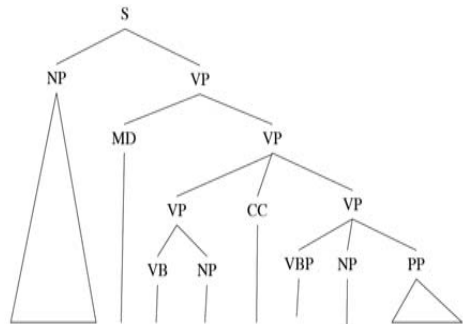
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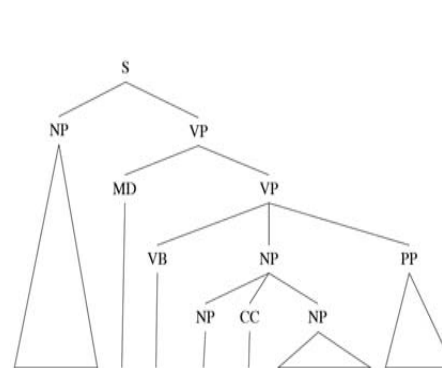


# Parsing for Disambiguation

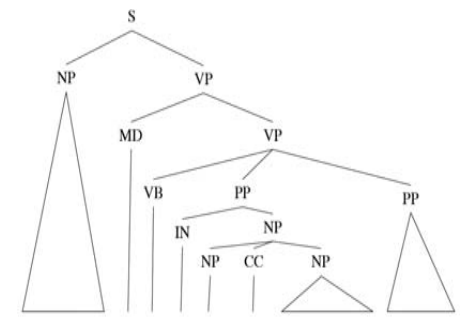
The post office will hold out discounts and service concessions as incentives.



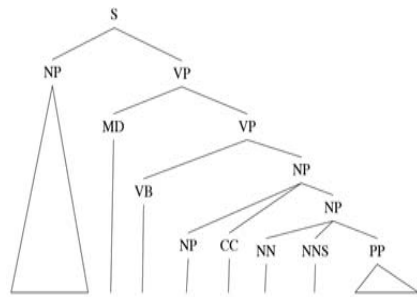
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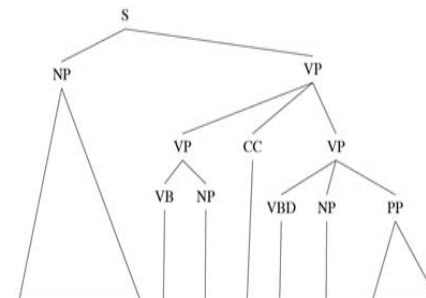
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# Parsing for Disambiguation

- There are typically many syntactically possible parses
- Want to find the most likely parses

# Treebanks

- If grammar induction does not work, why not count expansions in many parse trees?
- Penn Treebank

```
( (S
  (NP (NBAR (ADJP (ADJ "Battle-tested/JJ")
                 (ADJ "industrial/JJ")))
       (NPL "managers/NNS")))
  (? (ADV "here/RB"))
  (? (ADV "always/RB"))
  (AUX (TNS *))
  (VP (VPRES "buck/VBP")))
  (? (PP (PREP "up/RP")
         (NP (NBAR (ADJ "nervous/JJ")
                 (NPL "newcomers/NNS")))))
  (? (PP (PREP "with/IN")
         (NP (DART "the/DT")
            (NBAR (N "tale/NN")
                 (PP of/PREP
                    (NP (DART "the/DT")
                       (NBAR (ADJP
                              (ADJ "first/JJ"))))))))))))
```

# PCFG weaknesses

- No Context
  - (immediate prior context, speaker, ...)
- No Lexicalization
  - “VP NP NP” more likely if verb is “hand” or “tell”
  - fail to capture lexical dependencies (n-grams do)
- No Structural Context
  - How NP expands depends on position

# PCFG weaknesses

Expansion	% as Subj	% as Obj
NP → PRP	13.7%	2.1%
NP → NNP	3.5%	0.9%
NP → DT NN	5.6%	4.6%
NP → NN	1.4%	2.8%
NP → NP SBAR	0.5%	2.6%
NP → NP PP	5.6%	14.1%

Expansion	% as 1st Obj	% as 2nd Obj
NP → NNS	7.5%	0.2%
NP → PRP	13.4%	0.9%
NP → NP PP	12.2%	14.4%
NP → DT NN	10.4%	13.3%
NP → NNP	4.5%	5.9%
NP → NN	3.9%	9.2%
NP → JJ NN	1.1%	10.4%
NP → NP SBAR	0.3%	5.1%

# Other Approaches

- Challenge: use lexical and structural context, without too many parameters, sparse data
- Other Grammars
  - Probabilistic Left-Corner Grammars
  - Phrase Structure Grammars
  - Dependency Grammars
  - Probabilistic Tree Substitution Grammars
  - History-based Grammars

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# Generative vs Discriminative

- An HMM (or PCFG) is a *generative* model

$$P(y, w)$$

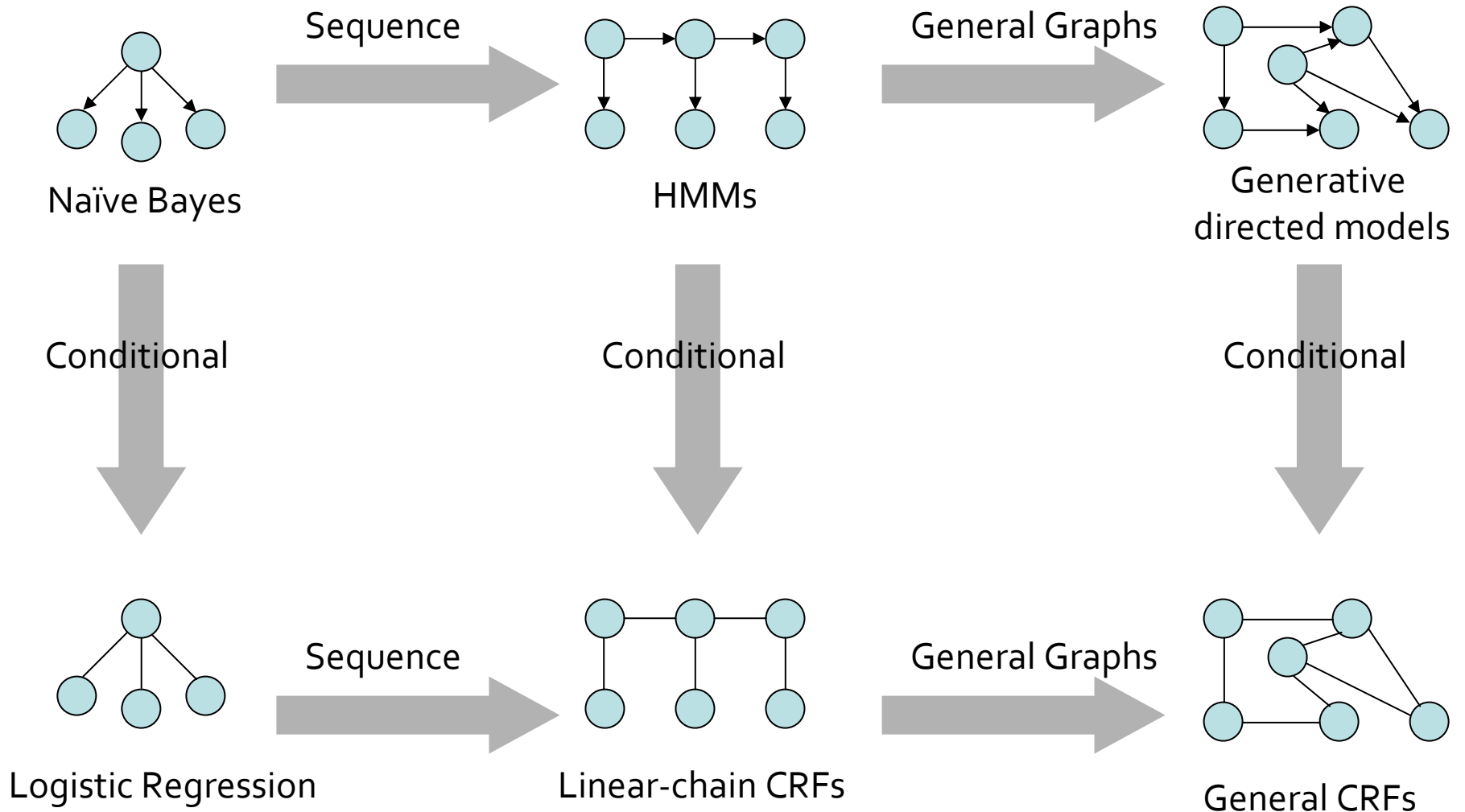
- Often sufficient is a *discriminative* model

$$P(y|w)$$

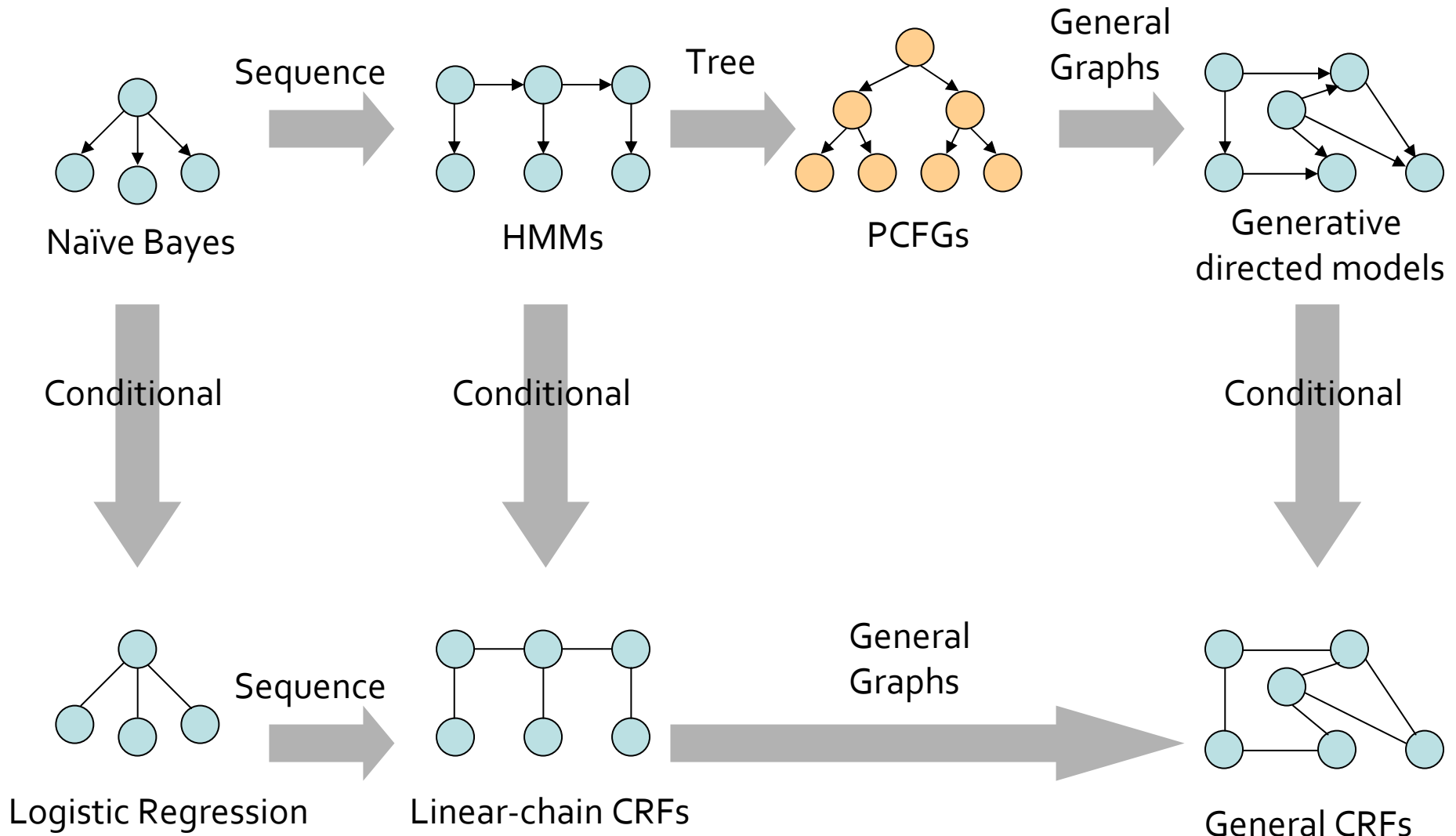
- Easier, because does not contain  $P(w)$
- Cannot model dependent features in HMM, so one only picks one feature: word's identity



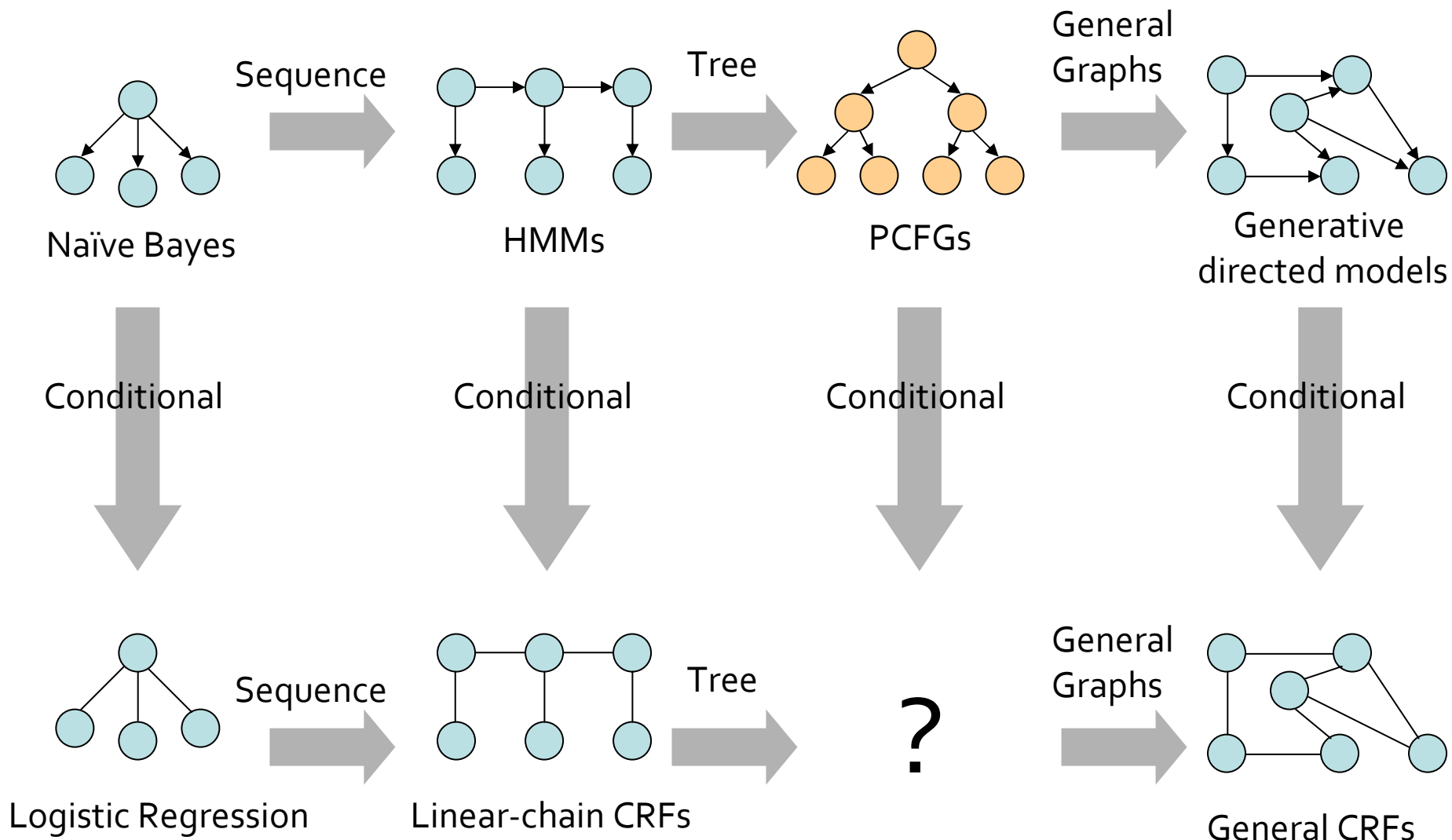
# Generative and Discriminative Models



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# Discriminative Context-Free Grammars

- Terminals  $w^1, w^2, \dots, w^V$
- Nonterminals  $N^1, N^2, \dots, N^n$
- Start symbol  $N^1$
- Rules  $N^i \longrightarrow \zeta^j$   
where  $\zeta^j$  is a sequence of terminals and nonterminals

- Rule scores

$$S(N^i \longrightarrow \zeta^j, p, q) = \sum_{k=1}^F \lambda_k(N^i \longrightarrow \zeta^j) f_k(w_1 w_2 \dots w_m, p, q, N^i \longrightarrow \zeta^j)$$

# Features

$$S(N^i \longrightarrow \zeta^j, p, q) = \sum_{k=1}^F \lambda_k(N^i \longrightarrow \zeta^j) f_k(w_1 w_2 \dots w_m, p, q, N^i \longrightarrow \zeta^j)$$

- Features can depend on all tokens + span
- Consider feature “AllOnTheSameLine”

Mavis Wood  
Products

Mavis Wood Products

[compare to linear CRF  $f_k(s_t, t_{t-1}, w_1 w_2 \dots w_m, t)$  ]

# Features

$$S(N^i \longrightarrow \zeta^j, p, q) = \sum_{k=1}^F \lambda_k(N^i \longrightarrow \zeta^j) f_k(w_1 w_2 \dots w_m, p, q, N^i \longrightarrow \zeta^j)$$

- No independence between features necessary
- Can create features based on words, dictionaries, digits, capitalization, ...
- Can still do efficient Viterbi inference in  $O(m^3r)$

# Example

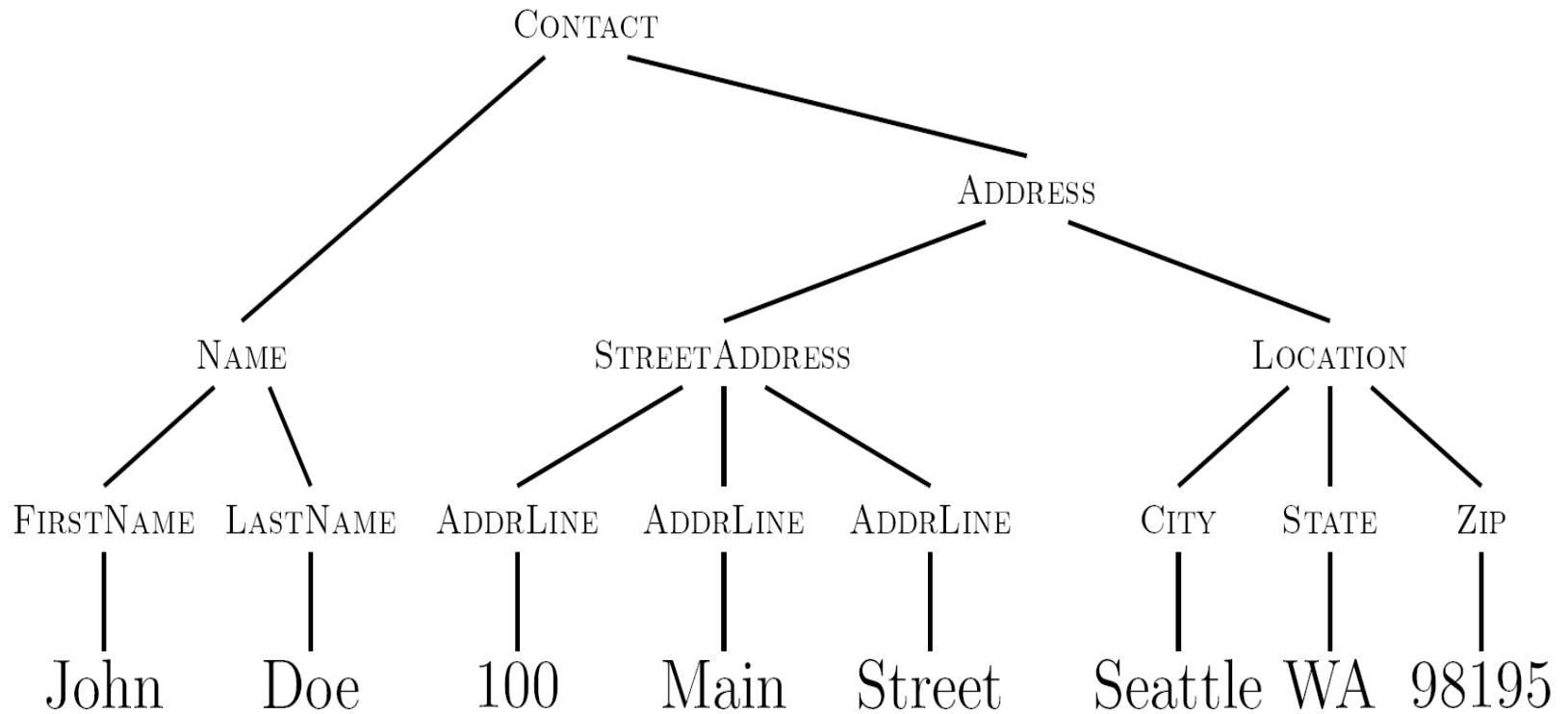
FIRSTNAME	LASTNAME	ADDRLINE	ADDRLINE	ADDRLINE	CITY	STATE	ZIP
John	Doe	100	Main	Street	Seattle	WA	98195

Fred Jones  
10 Main St.  
Cambridge, MA 02146  
(425) 994-8021

Boston College  
10 Main St.  
Cambridge MA 02146  
(425) 994-8021

BizContact	→	BizName	Address	BizPhone
PersonalContact	→	BizName	Address	HomePhone

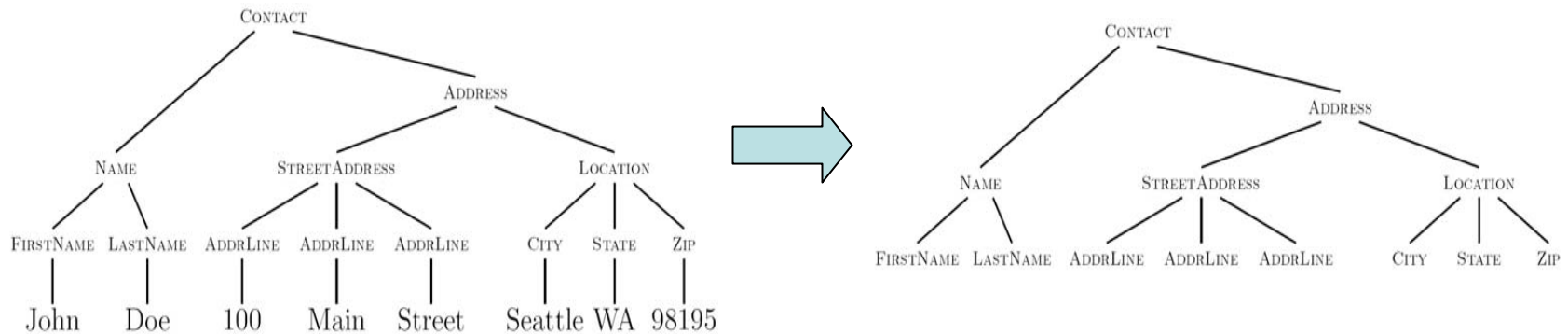
# Example





# Training

- Train feature weight vector for each rule  $\lambda_j(R)$
- Have labels, but not parse trees; efficiently create trees by ignoring leaves



# Collins' Averaged Perceptron

```
for  $r \leftarrow 1 \dots numRounds$  do
  for  $i \leftarrow 1 \dots m$  do
     $T \leftarrow$  optimal parse of  $w^i$  with current parameters
    if  $T \neq T^i$  then
      for each rule  $R$  used in  $T$  but not in  $T^i$  do
        if feature  $f_j$  is active in  $w^i$  then
           $\lambda_j(R) \leftarrow \lambda_j(R) - 1;$ 
        endif
      endfor
      for each rule  $R$  used in  $T^i$  but not in  $T$  do
        if feature  $f_j$  is active in  $w^i$  then
           $\lambda_j(R) \leftarrow \lambda_j(R) + 1;$ 
        endif
      endfor
    endif
  endfor
endfor
```

# Results

	Linear CRF	Discriminative CFG	Improvement
Word Error Rate	11.57%	6.29%	45.63%
Record Error Rate	54.71%	27.13%	50.41%

# Outline

- PCFGs: Inference and Learning
- Parsing English
- Discriminative CFGs
- Grammar Induction



# Gold's Theorem '67

“Any formal language which has hierarchical structure capable of infinite recursion is unlearnable from positive evidence alone.”

# Empirical Problems

- Even finite search spaces can be too big
- Noise
- Insufficient data
- Many local optima

# Common Approach

- Minimize total description length
- Simulated Annealing

Initial state

$D := \text{pabikugolatuda}\dots$

$T := T_0$

$G := \begin{cases} \gamma \rightarrow p \gamma \\ \gamma \rightarrow a \gamma \\ \gamma \rightarrow b \gamma \\ \vdots \end{cases}$

Repeat:

$G' := \text{random\_neighbor}(G)$

$\Delta := \text{Energy}(G', D) - \text{Energy}(G, D)$

$p := \begin{cases} 1 & \Delta \leq 0 \\ e^{-\frac{\Delta}{T}} & \Delta > 0 \end{cases}$

$G := G'$  with probability  $p$

$T := \alpha T$

# random\_neighbor(G)

- Insert:

$$G := \begin{cases} A \rightarrow B C \\ B \rightarrow D E \end{cases} \Rightarrow G := \begin{cases} A \rightarrow B X C \\ B \rightarrow D E \end{cases}$$

- Delete

$$G := \begin{cases} A \rightarrow B C \\ B \rightarrow D E \end{cases} \Rightarrow G := \begin{cases} A \rightarrow B \\ B \rightarrow D E \end{cases}$$

- New Rule

$$G := \begin{cases} A \rightarrow B C \\ B \rightarrow D E \end{cases} \Rightarrow G := \begin{cases} A \rightarrow B C \\ B \rightarrow D E \\ Y \rightarrow \end{cases}$$

- Split

$$G := \begin{cases} A \rightarrow B C \\ B \rightarrow D E \end{cases} \Rightarrow G := \begin{cases} A \rightarrow D E C \\ B \rightarrow D E \end{cases}$$

- Substitute

$$G := \begin{cases} A \rightarrow B C \\ B \rightarrow D E \end{cases} \Rightarrow G := \begin{cases} A \rightarrow Z C \\ B \rightarrow D E \\ Z \rightarrow B \end{cases}$$



# Energy

$$\text{Energy}(G, D) := |G| + |\text{code}(D|G)|$$

Define binary representation for  $G$ ,  $\text{code}(D|G)$

$$G := \left\{ \begin{array}{l} A \rightarrow B A \\ A \rightarrow B \\ B \rightarrow C D \\ \vdots \\ E \rightarrow F G \end{array} \right.$$

$$G := \text{ABA\#AB\#BCD\#\dots\#EFG\#\#}$$

# Experiment 1

- Word segmentation by 8-month old infants
- Vocabulary: pabiku, golatu, daropi, tibudo
- Saffran '96: use speech synthesizer, no word breaks, 2 minutes = 180 words
- Infants can distinguish words from non-words
- Now try grammar induction (60 words)

# Experiment 1

Step 37: current temp. = 14.994450998883487 Grammar:

$\gamma \rightarrow g \gamma; \gamma \rightarrow l \gamma; \gamma \rightarrow a \gamma; \gamma \rightarrow p \gamma; \gamma \rightarrow k \gamma; \gamma \rightarrow . \gamma; \gamma \rightarrow t \gamma; \gamma \rightarrow u \gamma; \gamma \rightarrow o \gamma; \gamma \rightarrow d \gamma; \gamma \rightarrow b \gamma; \gamma \rightarrow i \gamma; \gamma \rightarrow r \gamma; l \rightarrow$

Grammar length: 146 Encoding length: 1442

Step 618: current temp. = 14.907585393190937

Grammar:

$k \rightarrow k g 23; i \rightarrow p; 23 \rightarrow o; \gamma \rightarrow; \gamma \rightarrow g \gamma; \gamma \rightarrow p a \gamma; \gamma \rightarrow a \gamma; \gamma \rightarrow t \gamma; \gamma \rightarrow u \gamma; \gamma \rightarrow o \gamma; \gamma \rightarrow k u \gamma; \gamma \rightarrow o l \gamma; \gamma \rightarrow o p \gamma; \gamma \rightarrow d \gamma; \gamma \rightarrow b \gamma; \gamma \rightarrow i \gamma; \gamma \rightarrow r \gamma; o \rightarrow; t \rightarrow g$

Grammar length: 200 Encoding length: 1199

Step 5837: current temp. = 14.149508793558308

Grammar:  $k \rightarrow p; \gamma \rightarrow t i b u d o \gamma; \gamma \rightarrow \gamma; \gamma \rightarrow$

$p a b i k u \gamma; \gamma \rightarrow i a \gamma \gamma g; \gamma \rightarrow d a r o p i \gamma; \gamma \rightarrow$

$g o l a t u \gamma; b \rightarrow u d o o \gamma; b \rightarrow k u l; d \rightarrow; l \rightarrow; t \rightarrow \gamma$

Grammar length: 179 Encoding length: 183

# Experiment 2

$S \rightarrow NP VP$

$NP \rightarrow Nm \mid D N$

$Nm \rightarrow max \mid sam \mid kim \mid bill \mid mary$

$D \rightarrow the \mid a$

$N \rightarrow man \mid dog \mid cat \mid turtle$

$VP \rightarrow Vin \mid Vtr NP \mid Vtl NP CP \mid Vsy CP$

$Vin \rightarrow walks \mid runs$

$Vtr \rightarrow kills \mid hits$

...

► Sample sentences:

aturtleknowsthatsamtellskimthattheturtlewalksands  
amkillsmax.kimknowsthatsamtellsmaxthatkimtellsk  
imthatkimhitstheman.kimtellsaturtlethatmaxruns

# Experiment 2

Step : 423000 Temperature : 26.2

1 → *aman*1; 1 → *hits*1o; m → h; 1 → *bill*1m; m → au; m → ε; 1 → or1; 1 → *knowsthat*1u; 1 → *urtle*1; a → n; 1 → *eman*; m → a; 1 → *heui*; 1 → *edog*; 1 → *saysthat*1; t → e; a → x; 1 → *wak*1e; a → o; 1 → *and*1; 1 → ε; 1 → *tells*1; 1 → *walks*1c; 1 → *raac*; 1 → *runs*1; a → uu; 1 → *acat*1cx; 1 → x; 1 → *kim*1ky; m → o; a → ε; 1 → *th*11; 1 → *at*1w; m → r; 1 → *ma*11; 1 → *et*; 1 → r; 1 → *ecat*; 1 → *adog*1ty; 1 → *inksthat*; 1 → ry; 1 → *sam*1os; 1 → *kills*1; 1 → a

Grammar length: 778 Encoding length: 10620 Energy: 11398

- Accurate segmentation
- Inaccurate structural learning

# Prototype-Driven Grammar Induction

- Semi-supervised approach
- Give only a few dozen prototypical examples (for NP e.g. determiner noun, pronouns, ...)
- On English Penn Treebank:  $F_1 = 65.1$   
(52% reduction over naïve PCFG induction)

Aria Haghighi and Dan Klein.  
*Prototype-Driven Grammar Induction.*  
ACL 2006

Dan Klein and Chris Manning.  
*A Generative Constituent-Context Model for Improved Grammar Induction.*  
ACL 2001

That's it!