Homework Assignment #1

Due Tuesday 10/15

(All assignments will be turned in at start of lecture.)

1. Write and test a recursive Scheme function `range` that takes two arguments: `start` and `stop`. It should return the list of integers between `start` and `stop` inclusive. Examples:

```
(range 1 5)    → (1 2 3 4 5)
(range 0 0)    → (0)
(range 5 4)    → ()
```

You don't need to check for non-numeric arguments.

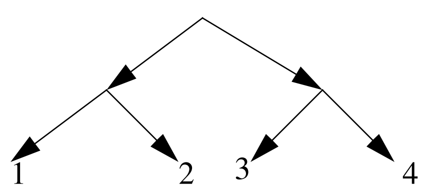
2. Is your `range` procedure tail recursive? If not, can you write a version that is tail recursive? If you can, show it working; if not, explain why one can't write a tail recursive version.

3. In Scheme, binary trees can be represented using cons cells for internal nodes to hold pointers to the left subtree (in `car`) and right subtree (in `cdr`); empty subtrees can be represented with nil, leaf nodes with non-list values (like an integer). The following expression, for example, constructs a simple tree with four leaf nodes:

```
(cons (cons 1 2) (cons 3 4))    → ((1 . 2) 3 . 4))
```

(Scheme tries to print it as a list which it isn't.)

The tree looks like this:



The following predicates are used to work with trees represented this way:

```
(pair? s)    true if s is an internal node
(null? s)    true if s is an empty subtree
(leaf? s)    true if s is a leaf; it should be defined as:
                  (define (leaf? s) (not (or (pair? s) (null? s))))
```

Consider the following Scheme procedure:

```
;;; sum the numbers at the leaves of a tree
(define (sum-tree s)
   (cond ((null? s) 0)
         ((leaf? s) s)
         ((pair? s) (+ sum-tree (car s)) (sum-tree (cdr s))))
         (else (error "non-numeric element in tree"))))
```

For `sum-tree`, can you write a tail recursive version? If so, write and test it; if not, explain why not.

4. Write a `tree-reduce` function that works on trees rather than lists. It should have four arguments: the reduction function; its identity element, returned in case of an empty tree; the leaf-value function which, given a leaf of the tree returns the value to be reduced; and the tree itself. Given this function you should be able to define the `tree-sum` function from the previous question as:

```
(define (sum-tree s) (tree-reduce + 0 (lambda (leaf) leaf) s))
```

5. In class, we saw a definition of `mapc`, the curried form of `map`. Instead of defining `mapc` in terms of `map`, define it in terms of `reducec`, the curried form of `reduce` (also discussed in class).

6. Write a function `num-occurrences` that takes a list and returns a list of two-element lists, each inner list being an element and the number of occurrences of that element. Examples:

```
(num-occurrences '(7 8 9 8 7 9 8 7))   → ((7 3) (8 3) (9 2))
(num-occurrences '(3.5 3.5 9.5))        → ((3.5 2) (9.5 1))
```

`(The result need not be sorted by element as these are.)`

7. Consider the following code:

```
(define (add-n n)
   (lambda (x) (+ x n)))

(define f1 (add-n 3))
```

When computing `(f1 5)` , what is the value of `n` in the expression `(+ x n)` , and where is it set? What would evaluating `(f1 5) do` if Scheme were dynamically scoped, and why?

8. In class we saw a version of the ASSOC function which used EQUAL? to test for equality of keys. Write a version of ASSOC which takes an equality function as an extra parameter and then uses this function to compare keys. Now use this ASSOC function to define ASSOC-EQUAL? and ASSOC-EQV?, which use EQUAL? and EQV? as the equality function, respectively.