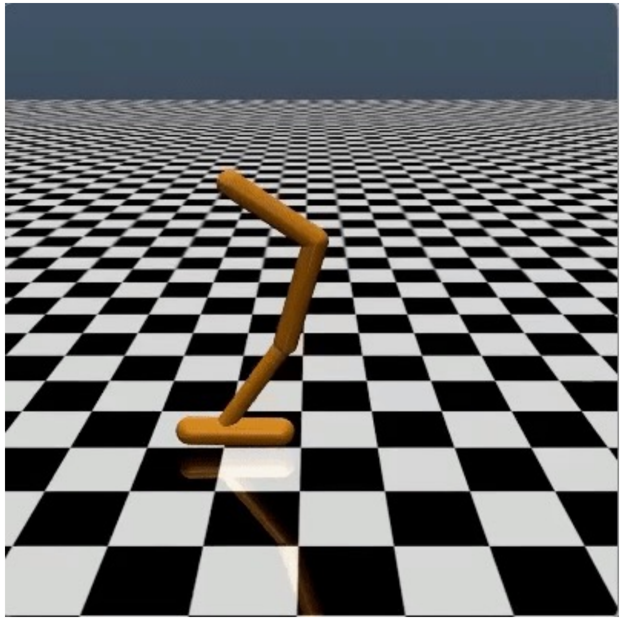
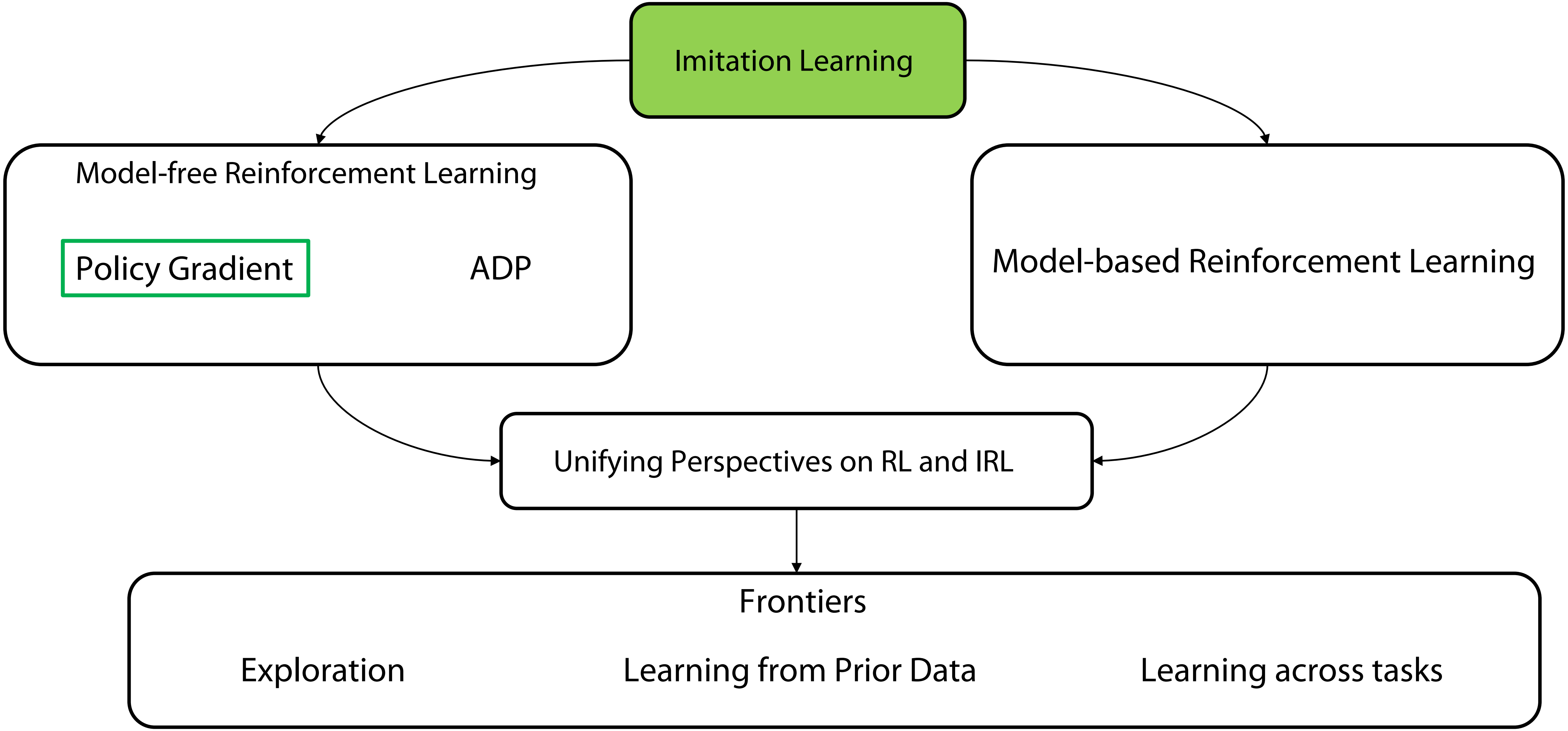
Reinforcement Learning
Autumn 2024

Abhishek Gupta

TA: Jacob Berg



Class Structure



Imitation Learning

Model-free Reinforcement Learning

Policy Gradient

ADP

Model-based Reinforcement Learning

Unifying Perspectives on RL and IRL

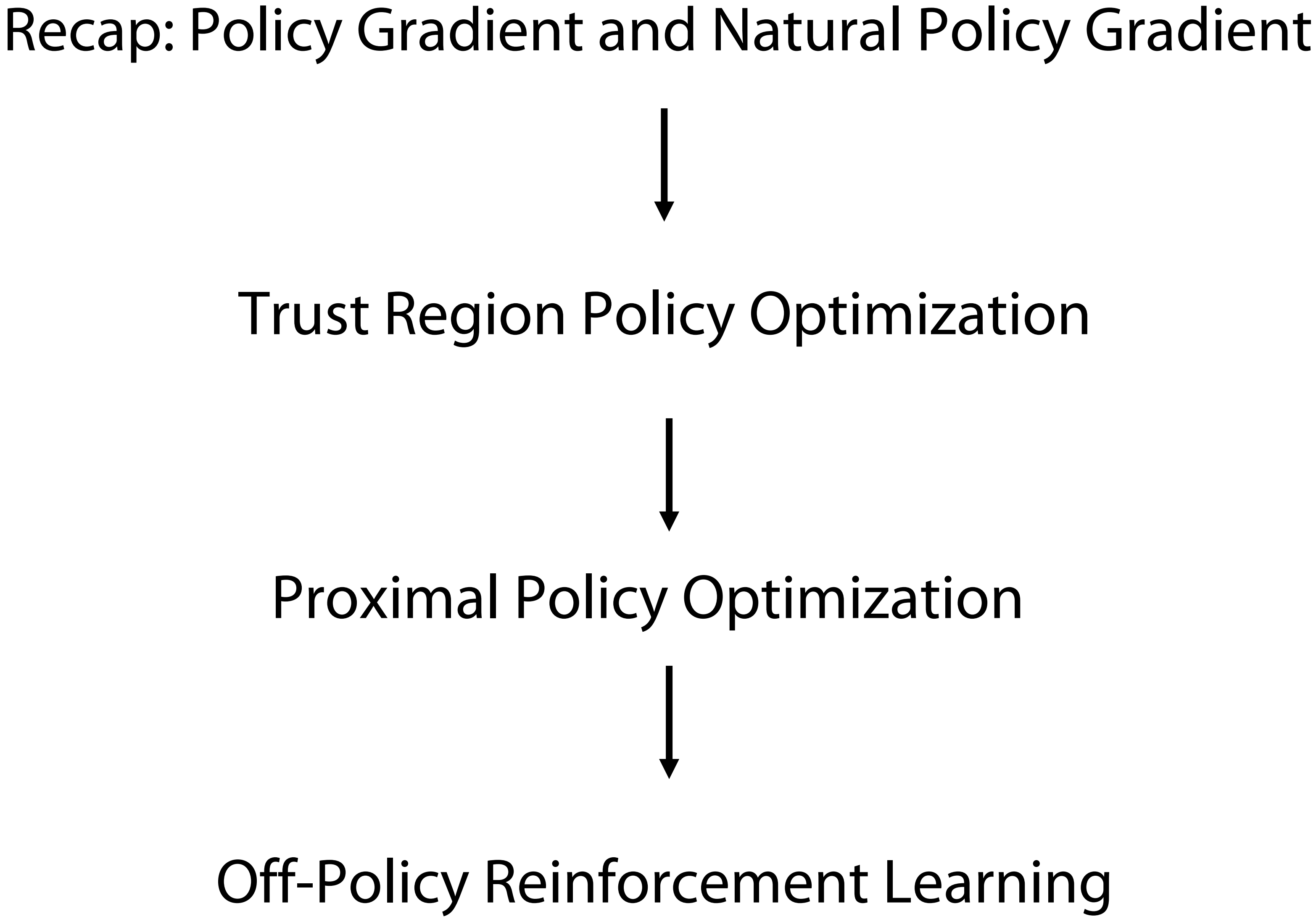
Frontiers

Exploration

Learning from Prior Data

Learning across tasks

Lecture outline



Recap: Policy Gradient and Natural Policy Gradient

Trust Region Policy Optimization

Proximal Policy Optimization

Off-Policy Reinforcement Learning

Taking the gradient of return

$$\nabla_\theta J(\theta) = \mathbb{E}_{\tau \sim p_\theta(\tau)} \left[\nabla_\theta \log p_\theta(\tau) \sum_{t=0}^{T} r(s_t, a_t) \right]$$

$$\nabla_\theta J(\theta) = \mathbb{E}_{\substack{s_0 \sim p(s_0) \\ s_{t+1} \sim p(s_{t+1}|s_t, a_t) \\ a_t \sim \pi(a_t|s_t)}} \left[\sum_{t=0}^{T} \nabla_\theta \log \pi_\theta(a_t|s_t) \sum_{t'=0}^{T} r(s_t, a_t) \right]$$

$$\approx \frac{1}{N} \sum_{i=0}^{N} \sum_{t=0}^{T} \nabla_\theta \log \pi_\theta(a_t^i|s_t^i) \sum_{t'=0}^{T} r(s_{t'}^i, a_{t'}^i)$$ (approximating using samples)

(Monte-Carlo approximation)

Take a deeper look at REINFORCE

$$\nabla_\theta J(\theta) = \int p_\theta(\tau) \nabla_\theta \log p_\theta(\tau) d\tau \approx \frac{1}{N} \sum_{i=0}^{N} \sum_{t=0}^{T} \nabla_\theta \log \pi_\theta(a_t^i | s_t^i) \sum_{t'=0}^{T} r(s_{t'}^i, a_{t'}^i)$$

Gradient ascent is steepest ascent on linear approximation under the Euclidean metric!

$$\max \;\; J(\theta_i) + \nabla_\theta J(\theta)|_{\theta=\theta_i}(\theta - \theta_i)$$

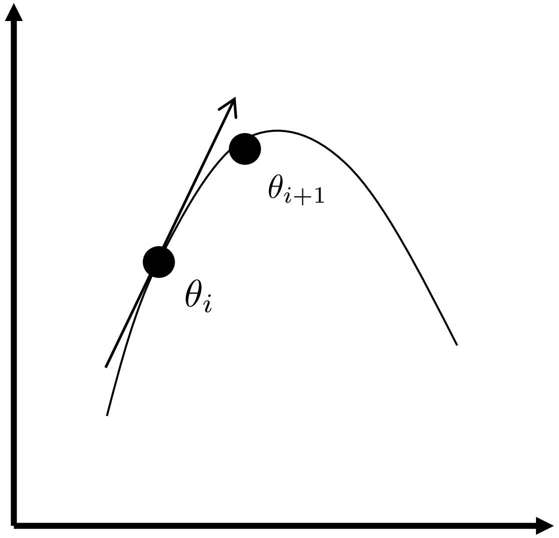
Linear approximation

$$(\theta - \theta_i)^T(\theta - \theta_i) \leq \epsilon$$

Quadratic Constraint

$$\big\downarrow$$

$$\theta = \theta_i + \alpha \nabla_\theta J(\theta)|_{\theta=\theta_i}$$



Covariant Policy Gradient Updates

$$\max \quad J(\theta_i) + \nabla_\theta J(\theta)|_{\theta=\theta_i}(\theta - \theta_i)$$
$$(\theta - \theta_i)^T G(\theta - \theta_i) \leq \epsilon$$

What should G be?

$$\max \quad J(\theta_i) + \nabla_\theta J(\theta)|_{\theta=\theta_i}(\theta - \theta_i)$$
$$D_{\mathrm{KL}}(\pi_\theta || \pi_{\theta_i}) \leq \epsilon$$

Let us use the constraint as KL divergence on the policy (2nd order Taylor expansion)

Measures functional distance, not parameter distance

Resulting "Natural" Policy Gradient

$$\max \quad J(\theta_i) + \nabla_\theta J(\theta)|_{\theta=\theta_i}(\theta - \theta_i)$$

$$D_{\mathrm{KL}}(\pi_\theta || \pi_{\theta_i}) \leq \epsilon$$

2nd order approximation of KL → Fisher Information Metric

$$F = \mathbb{E}_{\pi_\theta}\left[(\nabla_\theta \log \pi_\theta)(\nabla_\theta \log \pi_\theta)^T\right]$$

$$\max \quad J(\theta_i) + \nabla_\theta J(\theta)|_{\theta=\theta_i}(\theta - \theta_i)$$

$$(\theta - \theta_i)^T F(\theta - \theta_i) \leq \epsilon$$

Resulting update $\theta_{i+1} = \theta_i + \alpha F^{-1}\nabla_\theta J(\theta)|_{\theta=\theta_i}$ Covariant to parameterization

Lecture outline

Recap: Policy Gradient and Natural Policy Gradient

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Trust Region Policy Optimization

↓

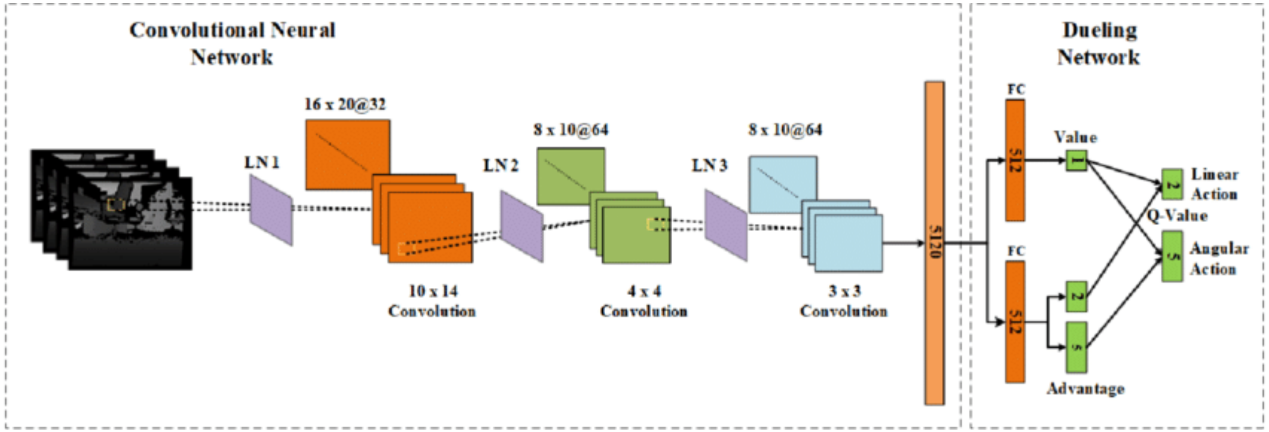
Proximal Policy Optimization

↓

Off-Policy Reinforcement Learning

Natural Policy Gradient - is it enough?

Huge matrix inversion

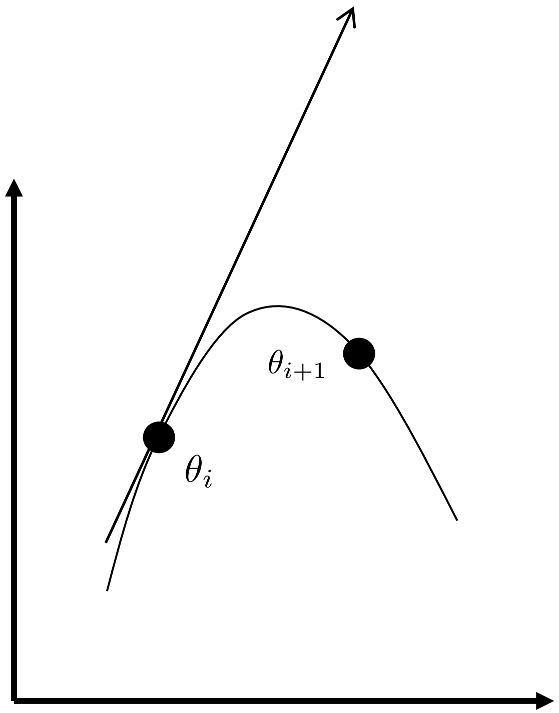


F – R^{dxd}

For a standard convnet – d is in the millions

Hessian is way out of memory / hard to invert!

Step-size?



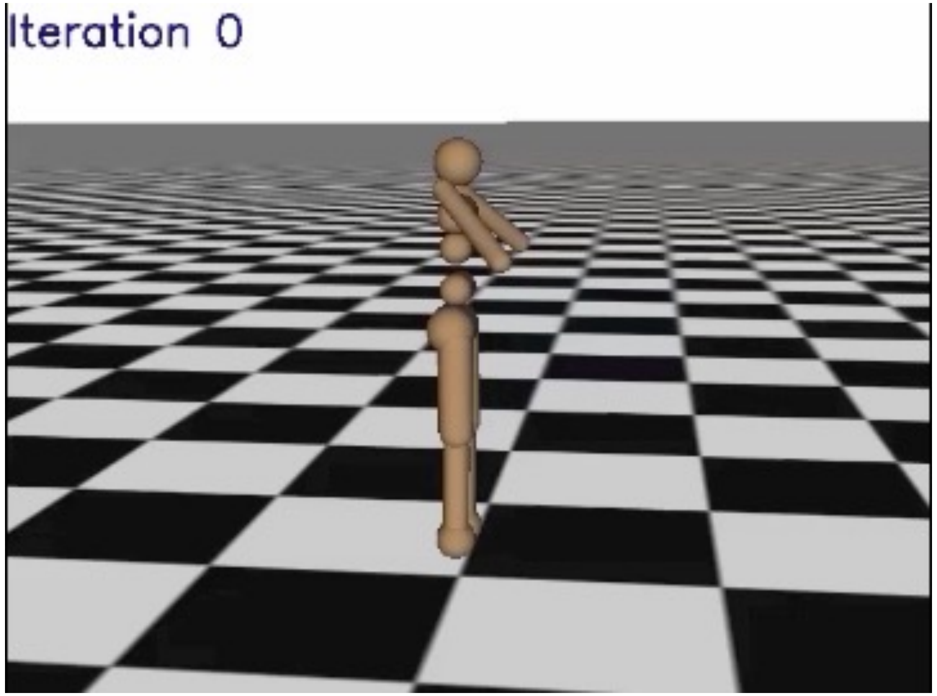
Can easily overstep and collapse performance

Also, only a single gradient step at a time before recollecting data!

Trust Region Policy Optimization

3 key ideas:

1. On-policy updates → importance sampled objective
2. Huge matrix inversion → conjugate gradient method
3. Step size may be too large → backtracking line search



TRPO, Schulman '15

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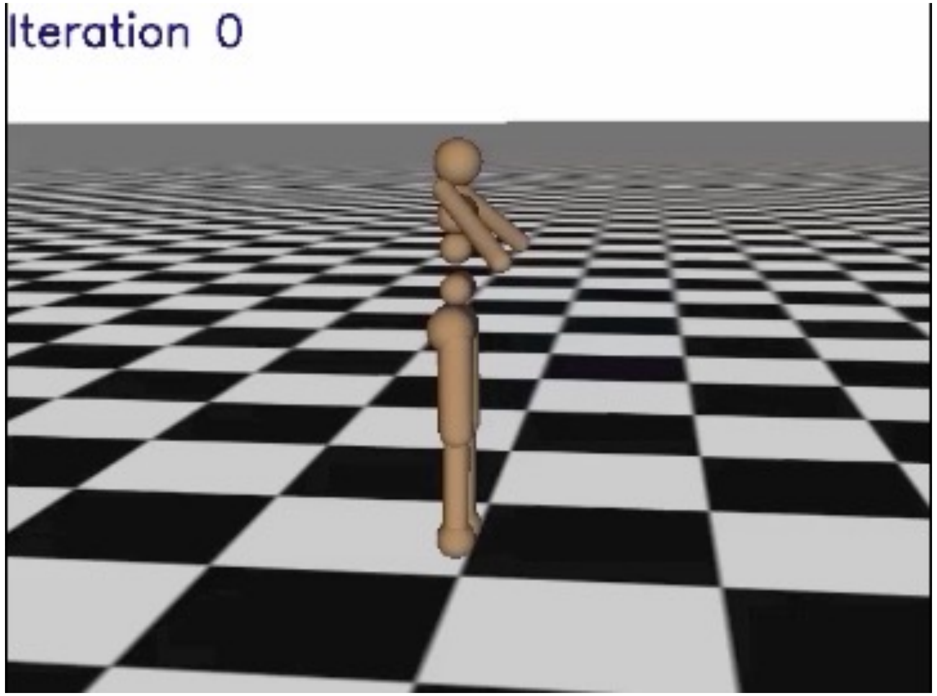

TRPO, Schulman '15

Off Policy Policy Gradient – Importance Sampling

Problem with original policy gradient objective – hard to evaluate without samples from θ

$$J(\theta) = \mathbb{E}_{\tau \sim p_\theta(\tau)}\left[R(\tau)\right] \quad \nabla_\theta J(\theta) = \mathbb{E}_{\tau \sim p_\theta(\tau)}\left[\nabla_\theta \log \pi_\theta(\tau) R(\tau)\right]$$

Must sample to evaluate

Can we be off-policy?

$$J(\theta) = \int p_\theta(\tau) R(\tau)$$

Can rederive policy gradient from this perspective

$$J(\theta) = \int \frac{q(\tau)}{q(\tau)} p_\theta(\tau) R(\tau)$$

Has numerical challenges

$$J(\theta) = \mathbb{E}_{\tau \sim q(\tau)}\left[\frac{p_\theta(\tau)}{q(\tau)} R(\tau)\right]$$

Expanding the Importance Sampling Objective

$$J(\theta) = \int p_\theta(\tau) R(\tau)$$

$$J(\theta) = \int \frac{q(\tau)}{q(\tau)} p_\theta(\tau) R(\tau)$$

$$J(\theta) = \mathbb{E}_{\tau \sim q(\tau)} \left[\frac{p_\theta(\tau)}{q(\tau)} R(\tau) \right]$$

$$J(\theta) = \mathbb{E}_{\tau \sim q(\tau)} \left[\left(\Pi_{t=1}^{T} \frac{\pi_\theta(a_t|s_t)}{q(a_t|s_t)} \right) \left(R(\tau) \right) \right]$$

Numerically unstable, high variance!

Trust Region Policy Optimization – Importance Sampling without the Pain

Cannot evaluate without resampling

Original Objective

$$J(\theta) = \mathbb{E}_{\tau \sim p_\theta(\tau)} \left[R(\tau) \right]$$

$$J(\theta) = \mathbb{E}_{s \sim p_\theta(s), a \sim \pi_\theta(a|s)} \left[Q(s, a) \right]$$

Importance Sampling (ish)

$$J(\theta) = \mathbb{E}_{s \sim p_{\theta_i}(s), a \sim \pi_{\theta_i}(a|s)} \left[\frac{p_\theta(s)}{p_{\theta_i}(s)} \frac{\pi_\theta(a|s)}{\pi_\theta(a|s)} Q(s, a) \right]$$

(Surrogate Objective)

$$\approx \mathbb{E}_{s \sim p_{\theta_i}(s), a \sim \pi_{\theta_i}(a|s)} \left[\frac{\pi_\theta(a|s)}{\pi_\theta(a|s)} Q(s, a) \right]$$

If policies are close, we can show that this is not so bad!

Often replaced by A = Q - V

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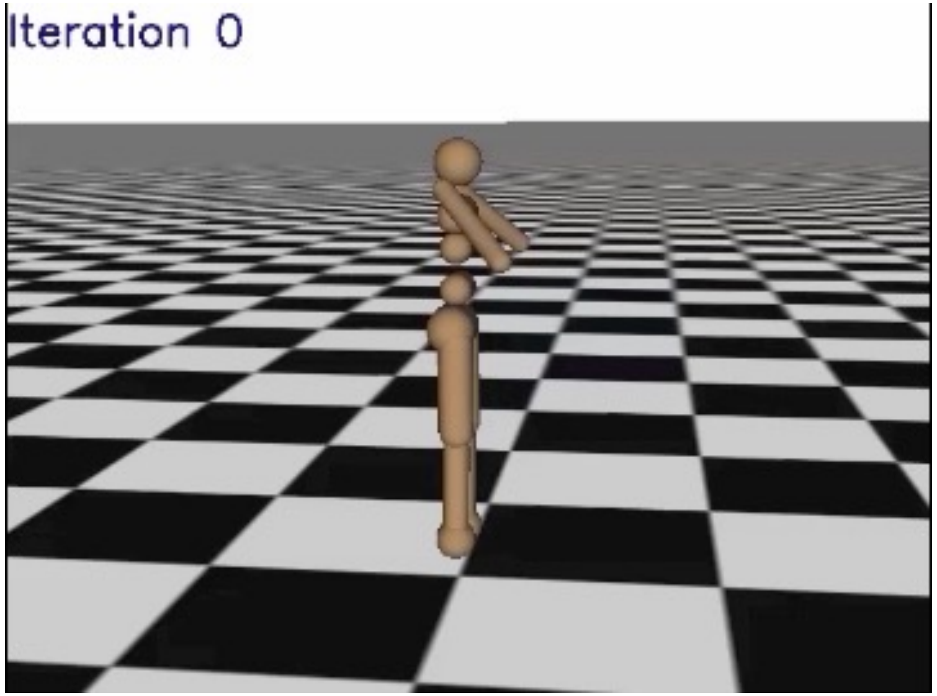


TRPO, Schulman '15

Trust Region Policy Optimization – Conjugate Gradient

Challenging to compute F^{-1} and then get $F^{-1}g$

⇩

Convert into an iterative minimization problem!

Solution to

$$Fx = g$$

same as

Solution to

$$\min_x \frac{1}{2} x^T F x - x^T g + c$$

Trust Region Policy Optimization – Conjugate Gradient

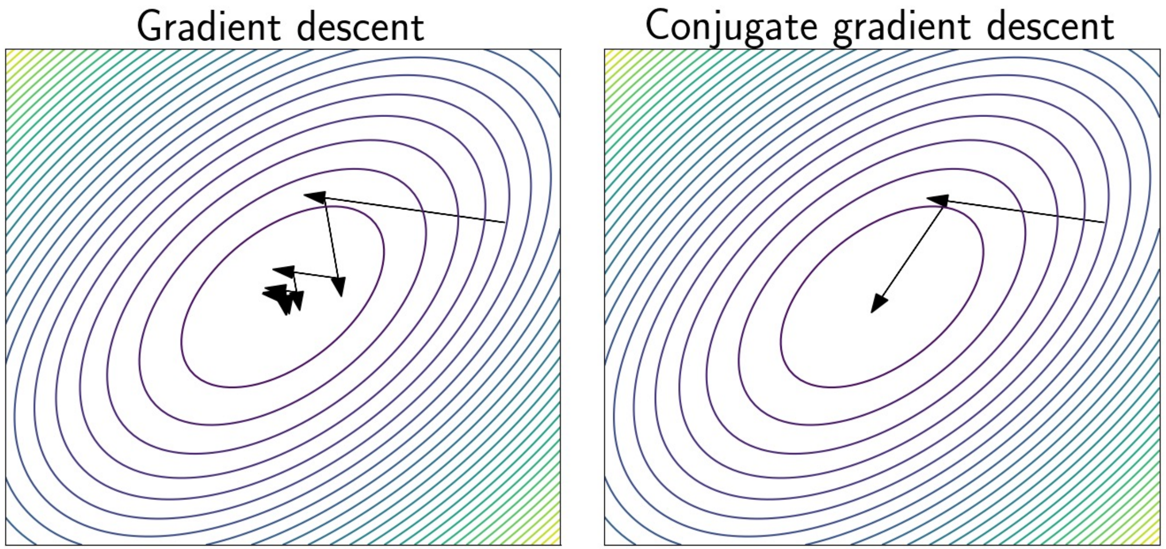
Challenging to compute F^{-1} and then get $F^{-1}g$

⇩

Convert into an iterative minimization problem!

⇩

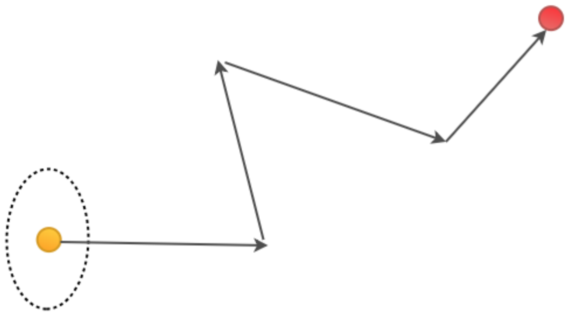
Solve with conjugate gradient



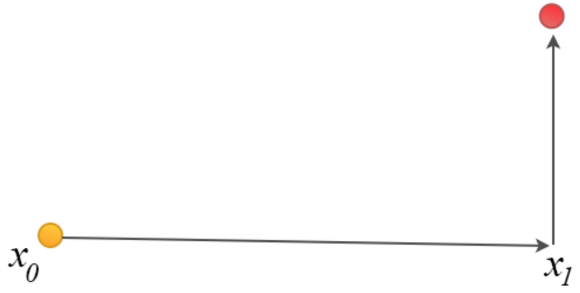
Do coordinate descent in geometry aligned orthogonal directions

https://www.cs.cmu.edu/~quake-papers/painless-conjugate-gradient.pdf

Trust Region Policy Optimization – Conjugate Gradient



Gradient ascent

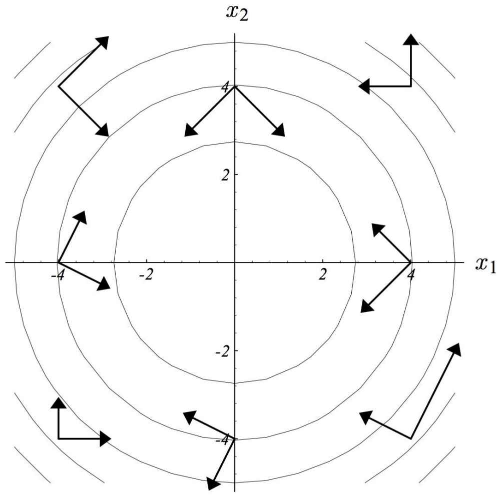


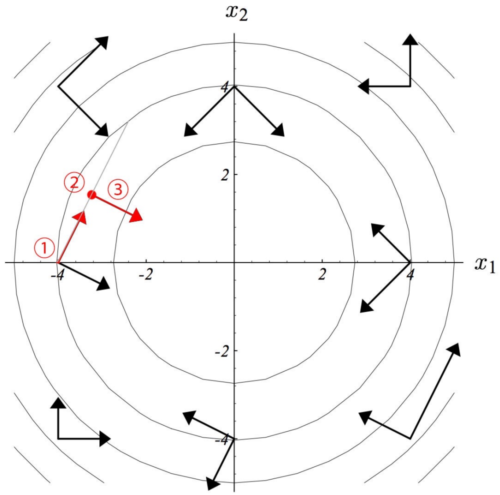
Conjugate gradient

$$\min_x \frac{1}{2} x^T F x - x^T g + c$$

Find search directions at ever step that are F-orthogonal with previous directions

$$d_{(i)}^T F d_{(i)} = 0$$





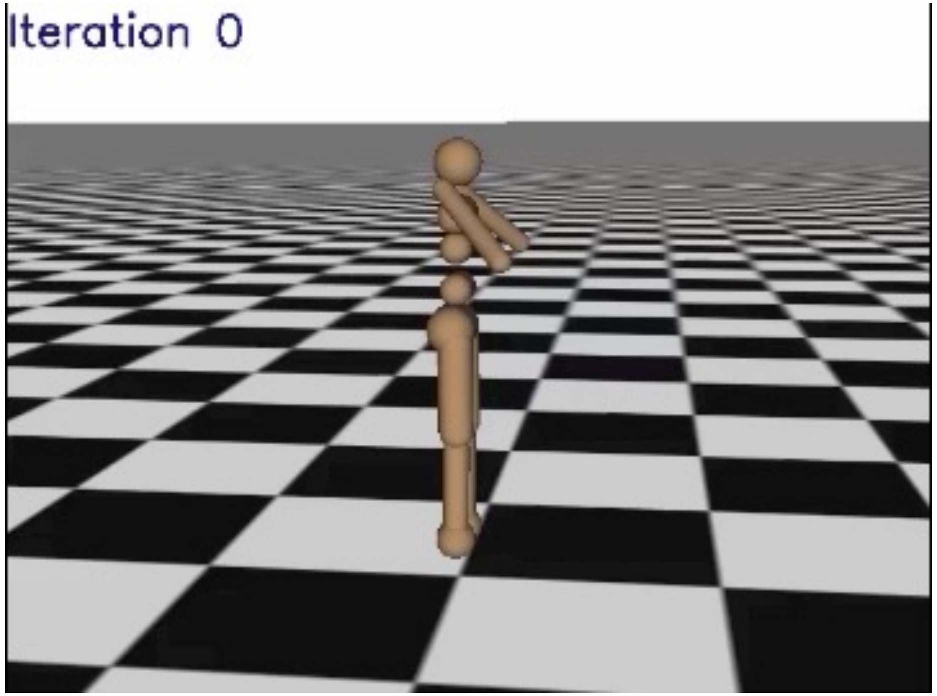
Converges in approx N steps!

Only requires matrix-vector product

Trust Region Policy Optimization

3 key ideas:

1. On-policy updates → importance sampled objective
2. Huge matrix inversion → conjugate gradient method
3. Step size may be too large → backtracking line search

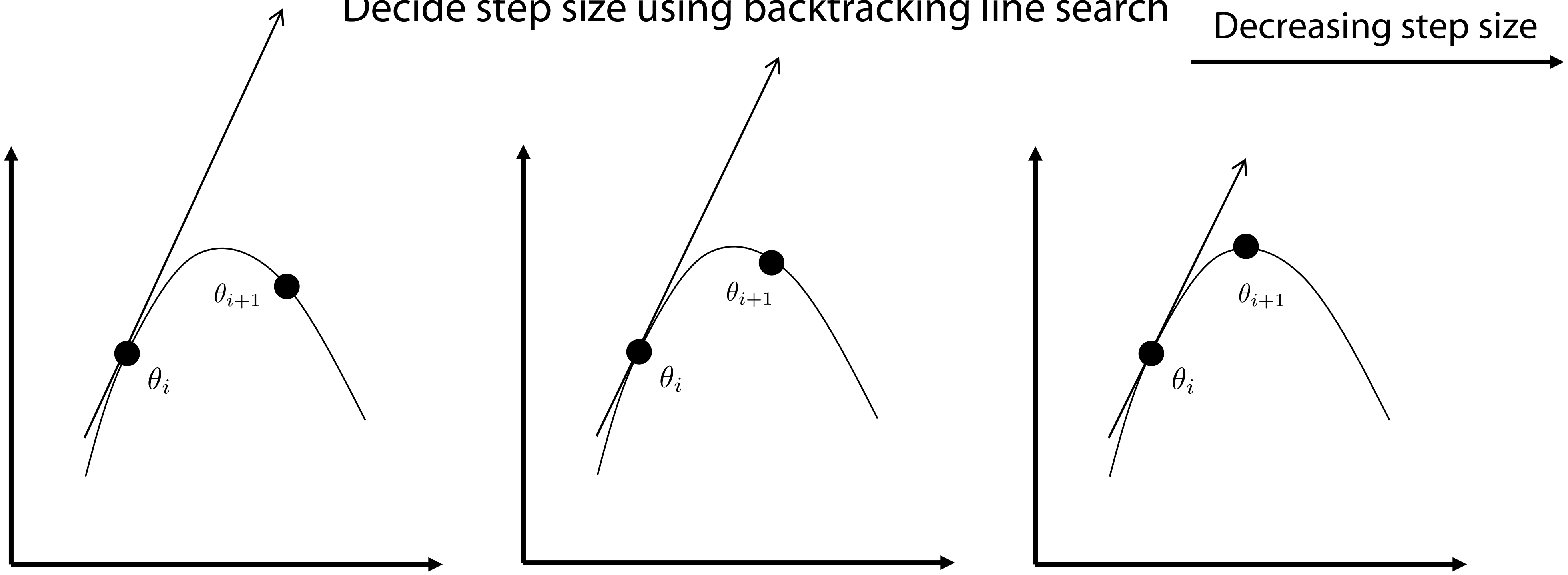


TRPO, Schulman '15

Trust Region Policy Optimization – Backtracking line search

Decide step size using backtracking line search

Decreasing step size



1. Choose parameter $\beta \in (0, 1)$, given search direction s = F^{-1}g
2. Compute maximal step size such that constraint is satisfied - $\frac{1}{2}(ts)^T F(ts) = \epsilon \rightarrow t = \sqrt{\frac{2\epsilon}{s^T F s}}$
3. While $J(\theta_i + ts) < J(\theta_i)$, set $t = \beta t$

Backtracking

Trust Region Policy Optimization

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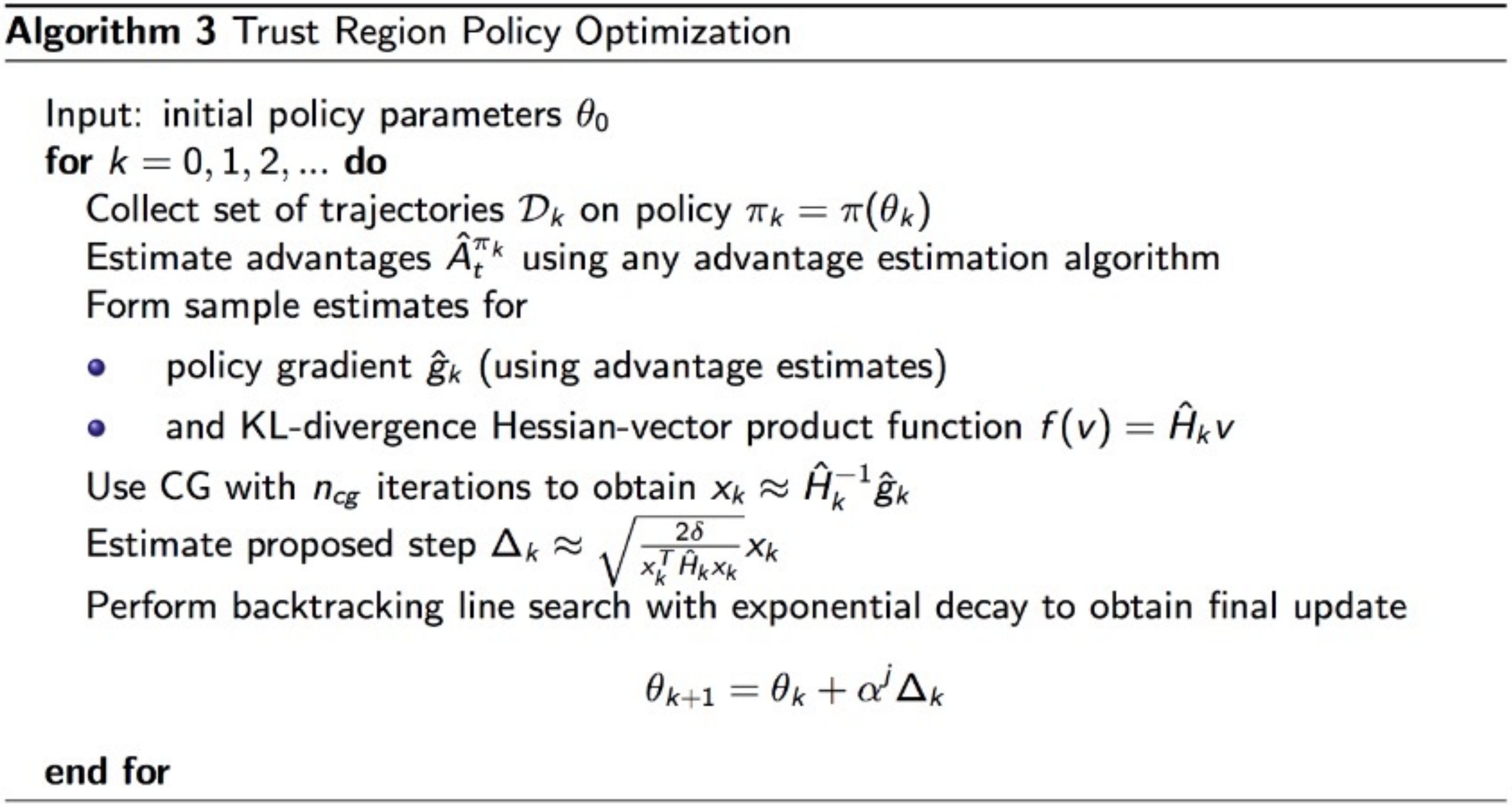

Algorithm 3 Trust Region Policy Optimization

```
Input: initial policy parameters $\theta_0$
for $k = 0, 1, 2, ...$ do
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    Use CG with $n_{cg}$ iterations to obtain $x_k \approx \hat{H}_k^{-1} \hat{g}_k$
    Estimate proposed step $\Delta_k \approx \sqrt{\frac{2\delta}{x_k^T \hat{H}_k x_k}} x_k$
    Perform backtracking line search with exponential decay to obtain final update

        $\theta_{k+1} = \theta_k + \alpha^j \Delta_k$

end for
```
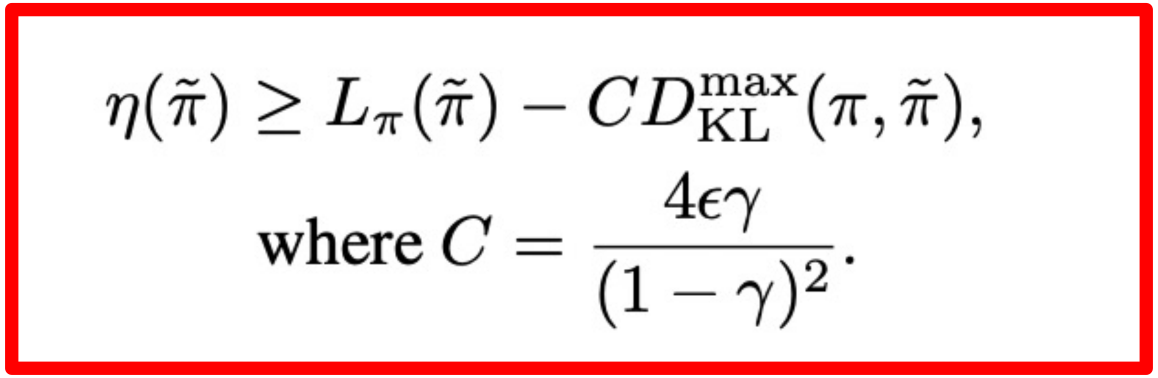

TRPO, Schulman '15

Can we say anything formal about updates?



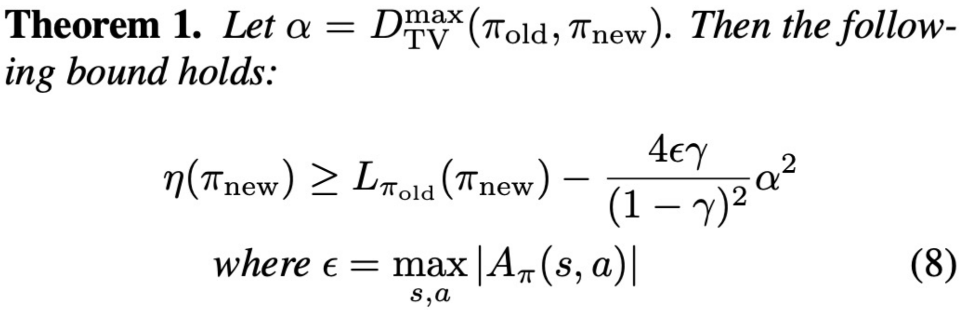
$$\eta(\tilde{\pi}) \geq L_{\pi}(\tilde{\pi}) - C D_{\mathrm{KL}}^{\max}(\pi, \tilde{\pi}),$$
$$\text{where } C = \frac{4\epsilon\gamma}{(1-\gamma)^2}.$$

Ensures that policies are non-decreasing in performance

Performance difference lemma

$$\eta(\tilde{\pi}) = \eta(\pi) + \mathbb{E}_{\tau \sim \tilde{\pi}}\left[\sum_{t=0}^{\infty} \gamma^t A_{\pi}(s_t, a_t)\right] \Rightarrow$$

Express advantage in terms of TVD



Theorem 1. *Let* $\alpha = D_{\mathrm{TV}}^{\max}(\pi_{\mathrm{old}}, \pi_{\mathrm{new}})$. *Then the following bound holds:*

$$\eta(\pi_{\mathrm{new}}) \geq L_{\pi_{\mathrm{old}}}(\pi_{\mathrm{new}}) - \frac{4\epsilon\gamma}{(1-\gamma)^2}\alpha^2$$
$$\textit{where } \epsilon = \max_{s,a} |A_{\pi}(s,a)| \qquad (8)$$

Key idea: by bounding how different the policies are, we can bound how different returns are

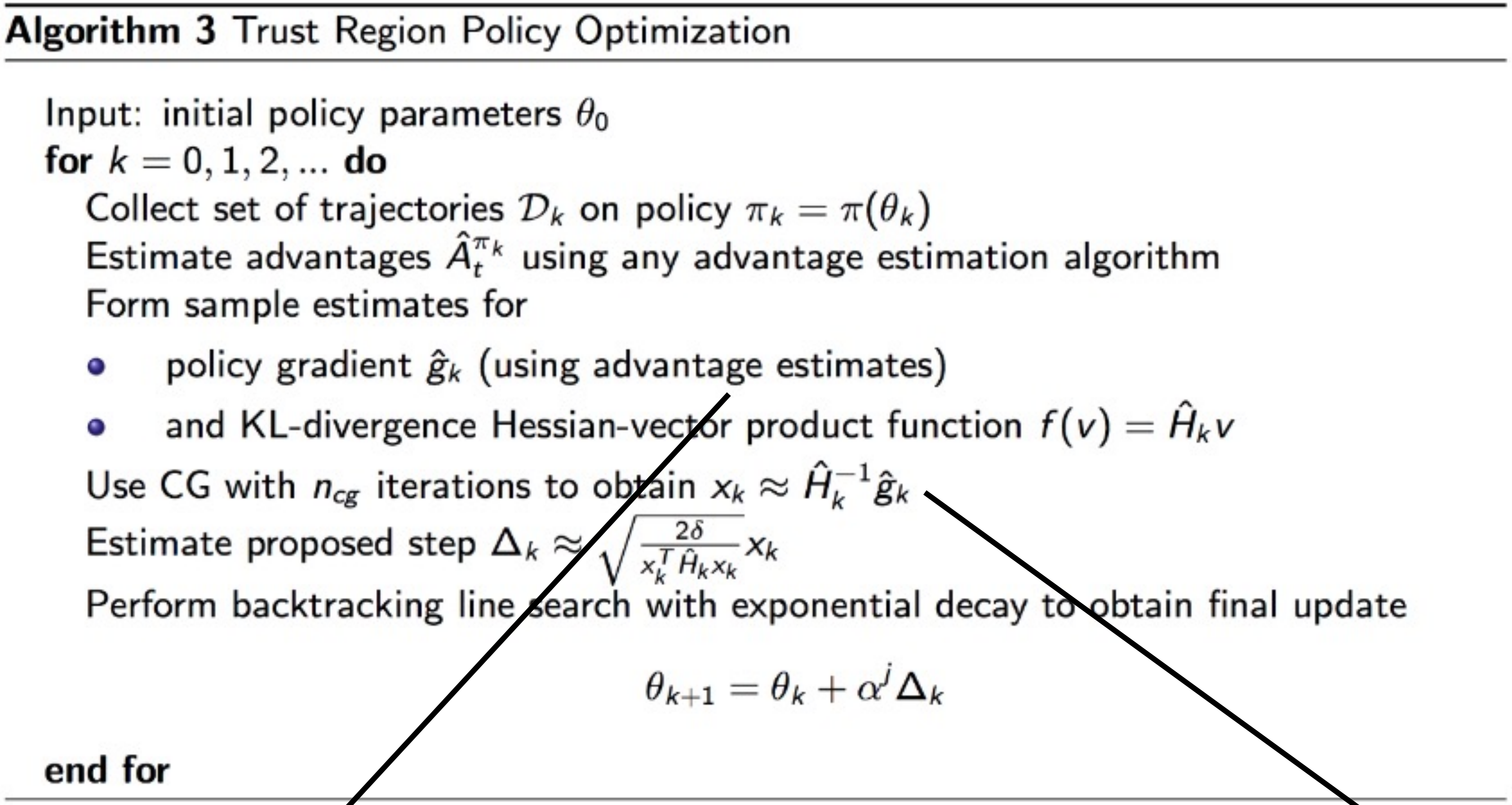
TRPO, Schulman '15

TRPO in action

Trust Region Policy Optimization

TRPO, Schulman ‘15

Why might TRPO not be enough?



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    Perform backtracking line search with exponential decay to obtain final update
        $$\theta_{k+1} = \theta_k + \alpha^j \Delta_k$$
end for
```

Advantage estimation is too high variance

Optimization expensive/unstable

TRPO, Schulman '15

Better Advantage Estimation - Generalized Advantage Estimation

Advantage estimator

$$A_N^\theta(s_1, a_1) = r_1 + \gamma r_2 + \cdots + \gamma^{N-1} r_N - V(s_1)$$

High variance!

N step advantage estimator

$$A_N^\theta(s_1, a_1) = r_1 + \gamma r_2 + \cdots + \gamma^{N-1} r_N - V(s_1)$$

N-1 step advantage estimator

$$A_{N-1}^\theta(s_1, a_1) = r_1 + \gamma r_2 + \cdots + \gamma^{N-2} V(s_{N-1}) - V(s_1)$$

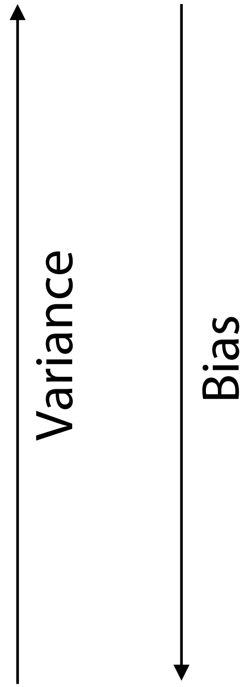
⋮

2 step advantage estimator

$$A_2^\theta(s_1, a_1) = r_1 + \gamma r_2 + \cdots + \gamma^2 V(s_3) - V(s_1)$$

1 step advantage estimator

$$A_1^\theta(s_1, a_1) = r_1 + \gamma V(s_2) - V(s_1)$$



Variance

Bias

GAE, Schulman ‘15

Generalized Advantage Estimation

Sum up all the estimators in a geometric sum

$$\left.\begin{aligned}
A^\theta_N(s_1,a_1) &= r_1 + \gamma r_2 + \cdots + \gamma^{N-1} r_N - V(s_1) \\
A^\theta_{N-1}(s_1,a_1) &= r_1 + \gamma r_2 + \cdots + \gamma^{N-2} V(s_{N-1}) - V(s_1) \\
\\
\\
A^\theta_2(s_1,a_1) &= r_1 + \gamma r_2 + \cdots + \gamma^{2} V(s_3) - V(s_1) \\
A^\theta_1(s_1,a_1) &= r_1 + \gamma V(s_2) - V(s_1)
\end{aligned}\right\}$$

Geometric sum

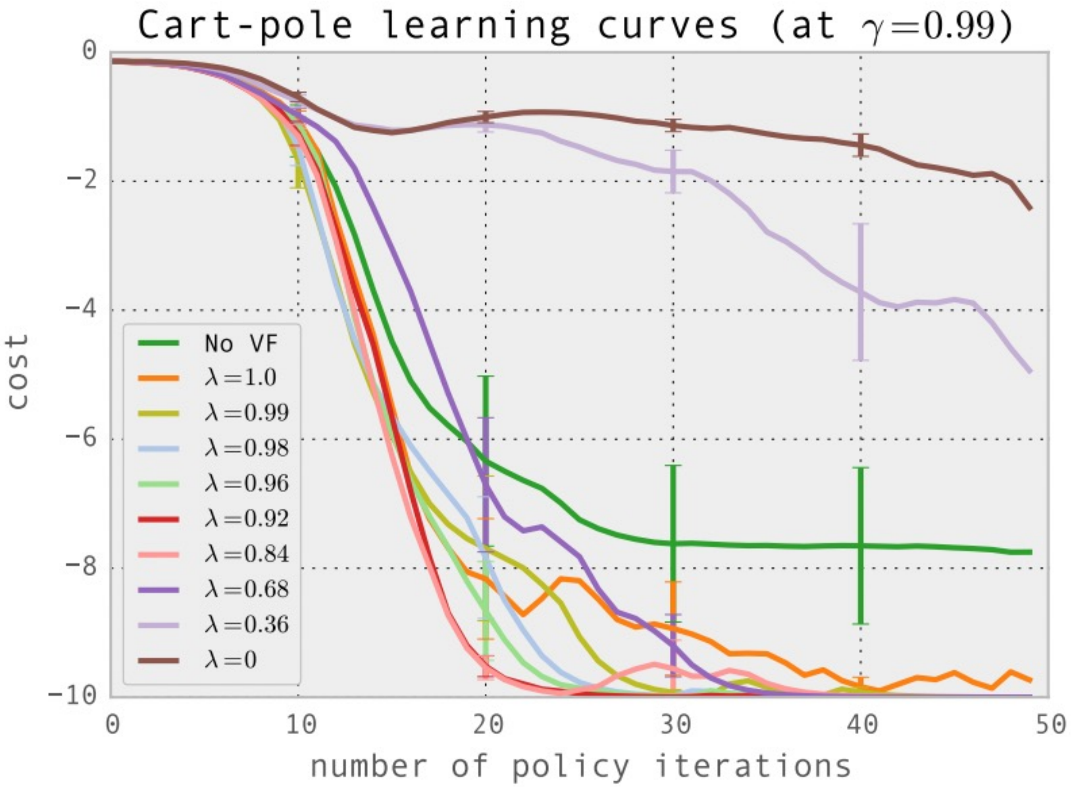
$$A^\theta_\lambda(s_1,a_1) = \sum_{j=1}^{N} \lambda^j A^\theta_j(s,a)$$

λ controls bias-variance tradeoff

Best of both worlds – very similar idea to eligibility traces

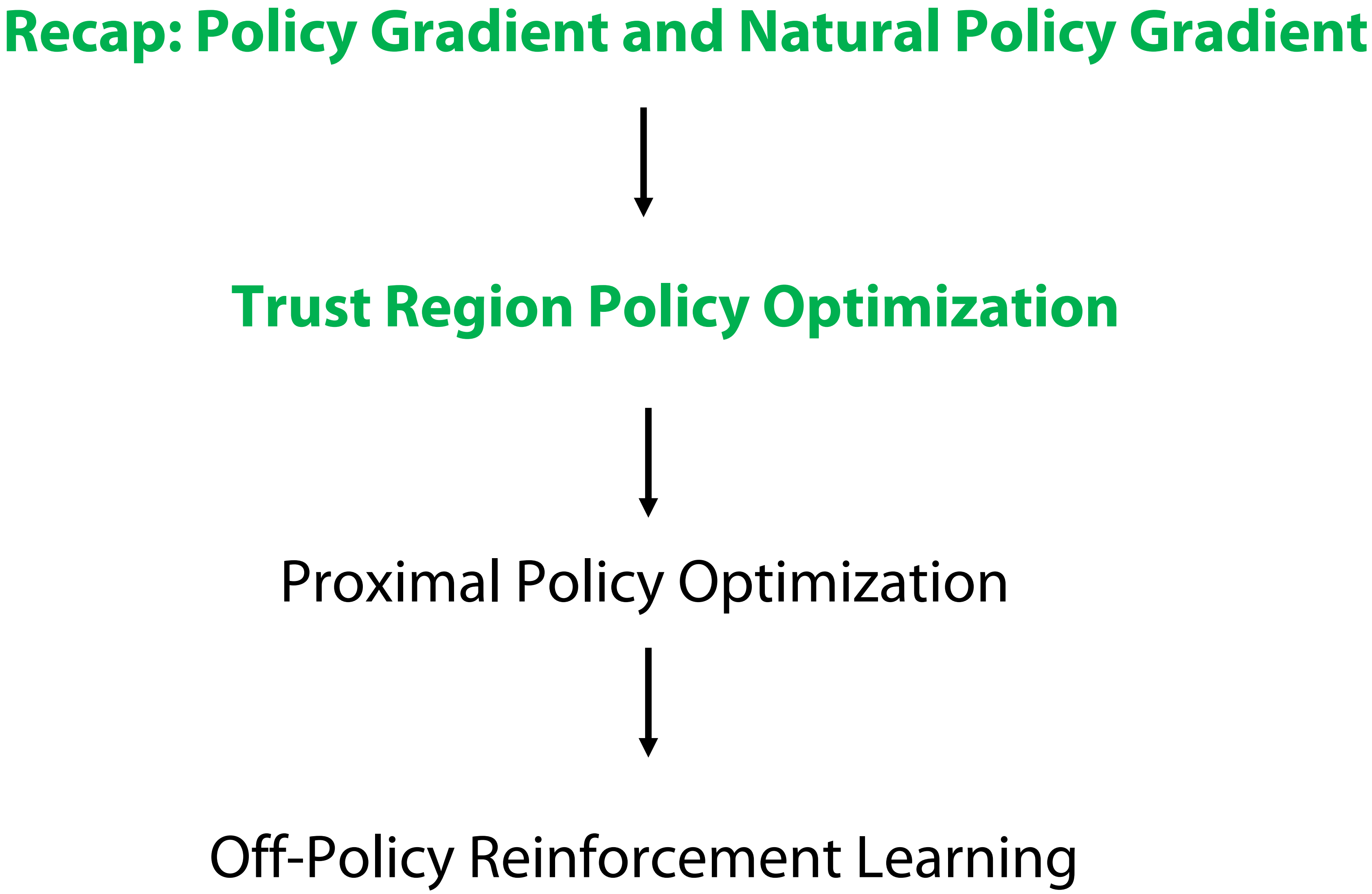
GAE, Schulman '15

Generalized Advantage Estimation in Action



GAE, Schulman '15

Lecture outline



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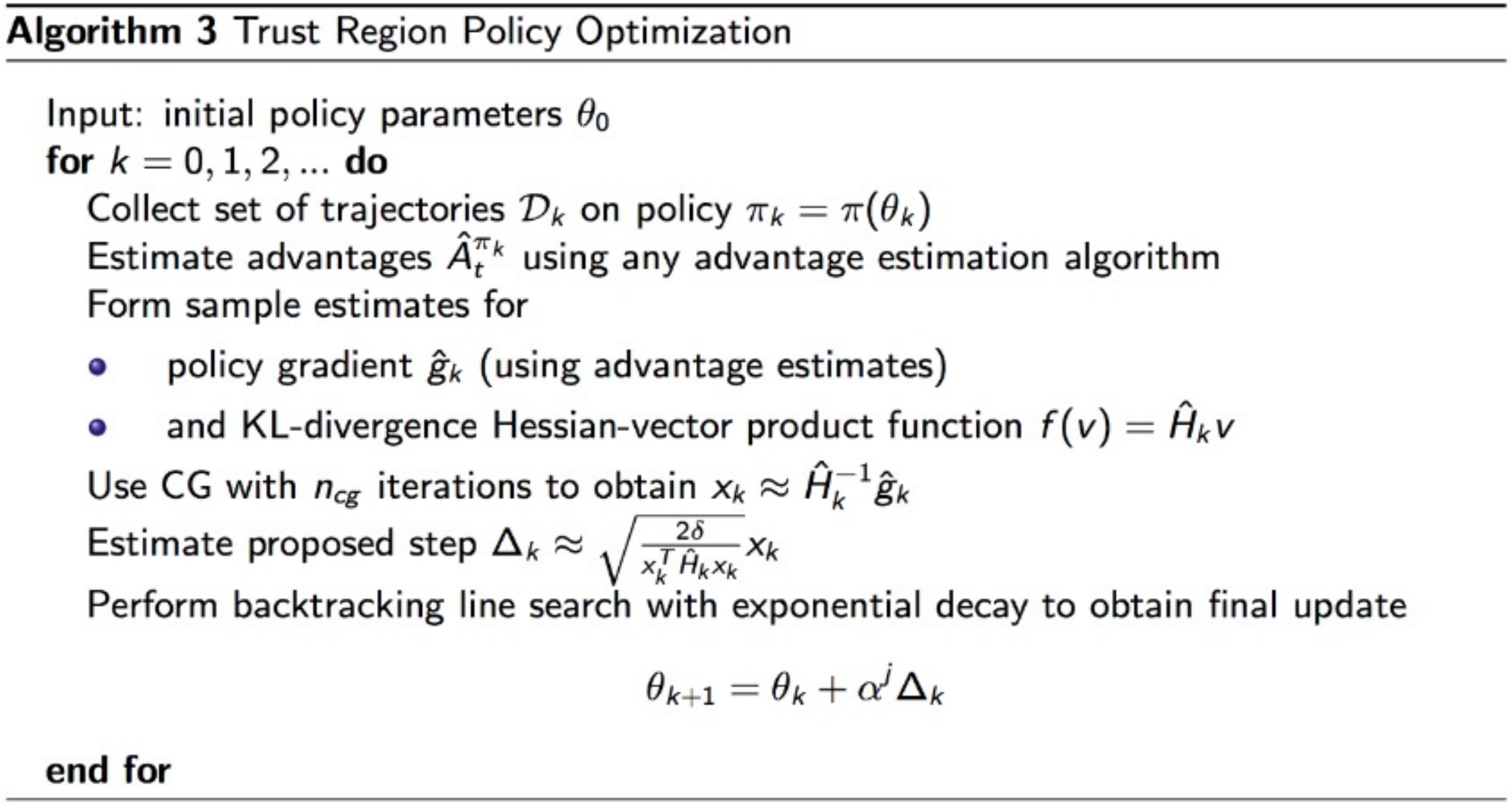
Proximal Policy Optimization

Off-Policy Reinforcement Learning

Avoiding Second Order Optimization



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        $\theta_{k+1} = \theta_k + \alpha^j \Delta_k$
end for
```

Expensive second order optimization, can we avoid?

⇩

What if we just restricted how much the policy changes directly!

Proximal Policy Optimization Update

Trust Region Policy Optimization

$$\max \; J(\theta_i) + \nabla_\theta J(\theta)|_{\theta=\theta_i}(\theta - \theta_i)$$

$$D_{\mathrm{KL}}(\pi_\theta || \pi_{\theta_i}) \leq \epsilon$$

Restrict the amount the policy moves

Proximal Policy Optimization

$$\mathcal{L}(s, a, \theta_i, \theta) = \min\left(\frac{\pi_\theta(a|s)}{\pi_{\theta_i}(a|s)} A(s,a), \operatorname{clip}\left(\frac{\pi_\theta(a|s)}{\pi_{\theta_i}(a|s)}, 1-\epsilon, 1+\epsilon\right) A(s,a)\right)$$

PPO, Schulman '17

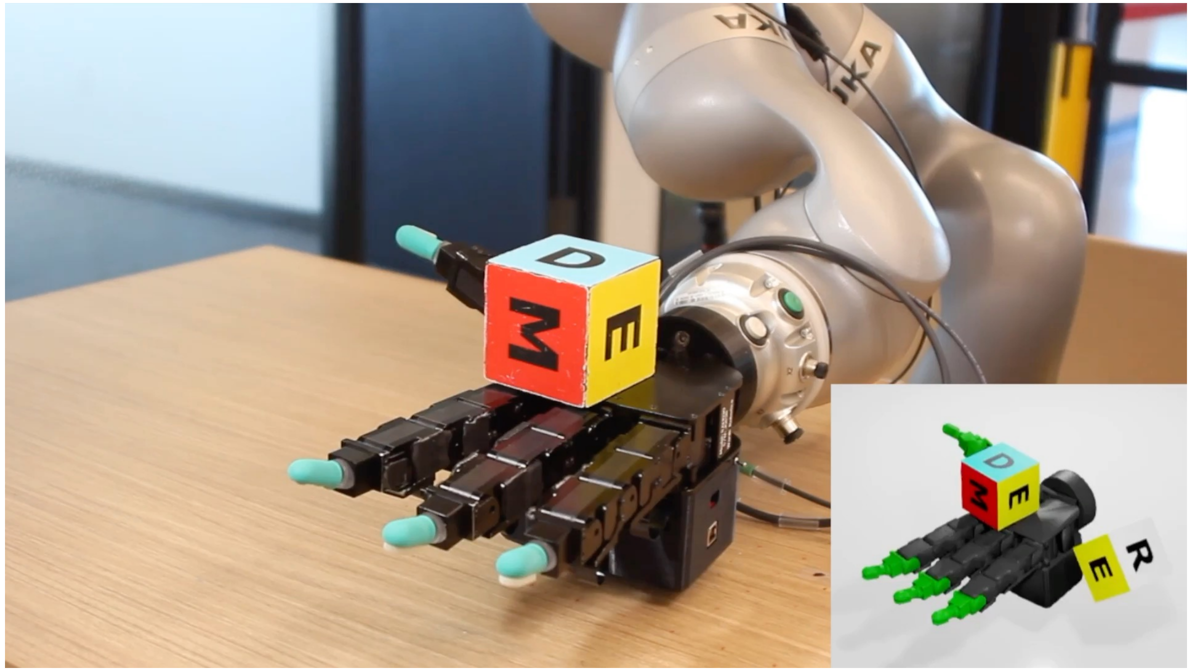
Proximal Policy Optimization Algorithm

$$\mathcal{L}(s, a, \theta_i, \theta) = \min\left(\frac{\pi_\theta(a|s)}{\pi_{\theta_i}(a|s)} A(s,a), \mathrm{clip}\left(\frac{\pi_\theta(a|s)}{\pi_{\theta_i}(a|s)}, 1-\epsilon, 1+\epsilon\right) A(s,a)\right)$$

✓ Multiple minibatch gradient steps
✓ No second order optimization
✓ Simple and stable, without huge updates

PPO, Schulman ‘17

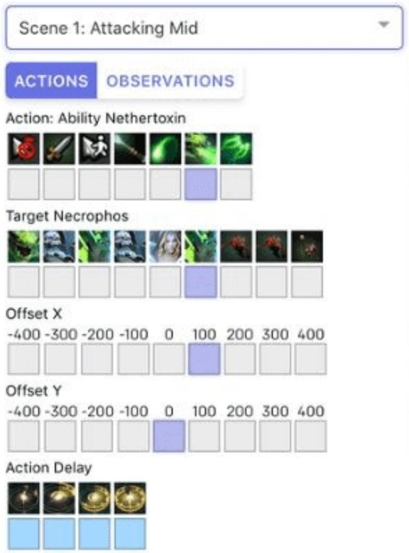
PPO in Action



The goal here is to reorient the object to a target orientation

Dextreme, Handa ’22
A system for general in-hand reorientation, Chen ’21

PPO in Action



Scene 1: Attacking Mid

ACTIONS OBSERVATIONS

Action: Ability Nethertoxin

Target Necrophos

Offset X

-400 -300 -200 -100 0 [✓ 100] 200 300 400

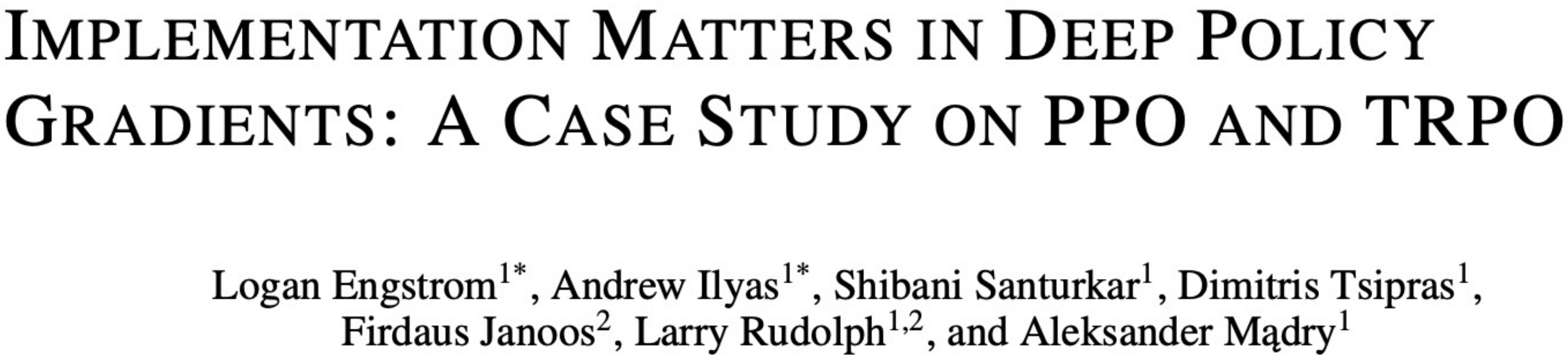
Offset Y

-400 -300 -200 -100 [✓ 0] 100 200 300 400

Action Delay

PPO, Schulman ‘17

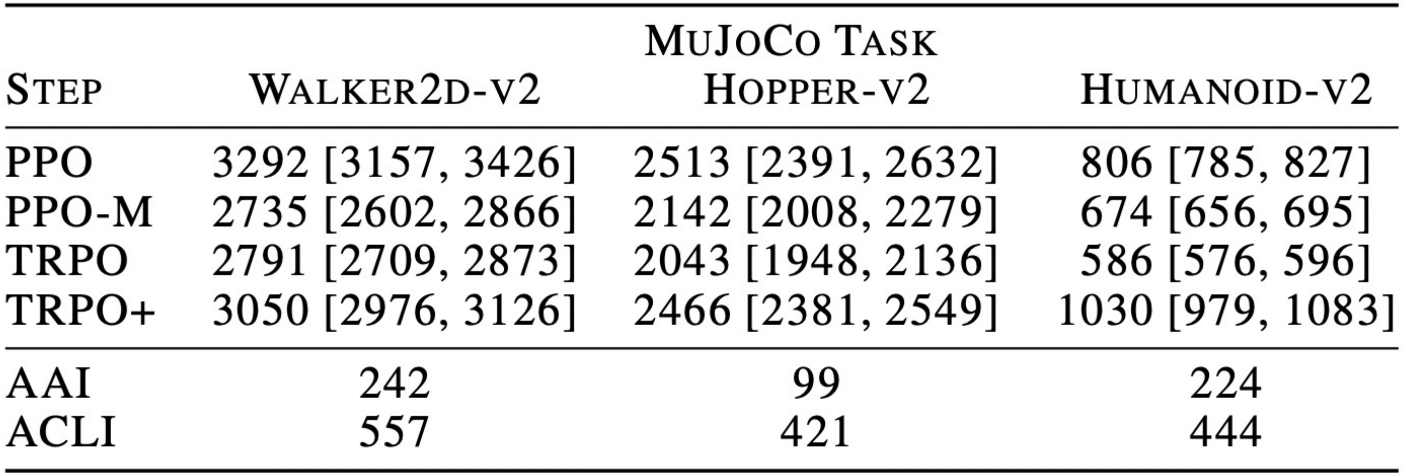
So should we just use PPO for everything?



Implementation Matters in Deep Policy Gradients: A Case Study on PPO and TRPO

Logan Engstrom[1*], Andrew Ilyas[1*], Shibani Santurkar[1], Dimitris Tsipras[1],
Firdaus Janoos[2], Larry Rudolph[1,2], and Aleksander Mądry[1]

Open question!



		MuJoCo Task	
Step	Walker2d-v2	Hopper-v2	Humanoid-v2
PPO	3292 [3157, 3426]	2513 [2391, 2632]	806 [785, 827]
PPO-M	2735 [2602, 2866]	2142 [2008, 2279]	674 [656, 695]
TRPO	2791 [2709, 2873]	2043 [1948, 2136]	586 [576, 596]
TRPO+	3050 [2976, 3126]	2466 [2381, 2549]	1030 [979, 1083]
AAI	242	99	224
ACLI	557	421	444

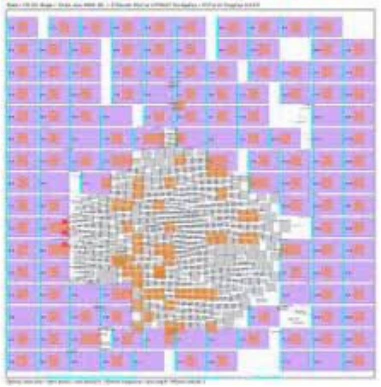
Code optimizations may make a bigger impact than algorithmic ones
→ needs more investigation!

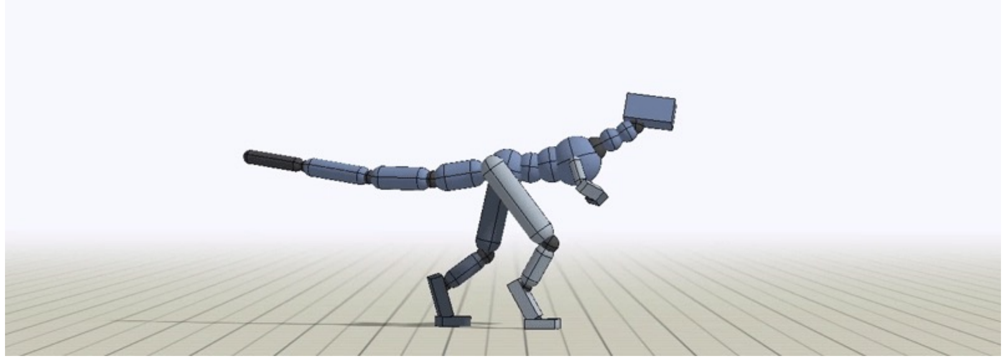
Engstrom et al, '20

Pros/Cons of Policy Gradient Methods

Pros

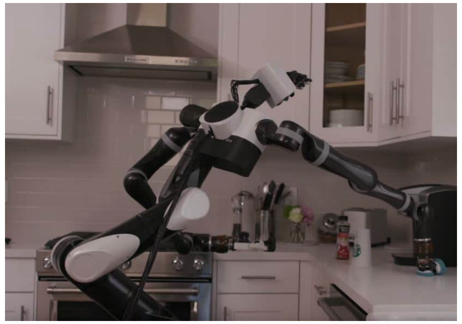
- Conceptually simple, easy to implement
- Stable, good asymptotic performance
- Compatible with deep models
- Require minimal modeling

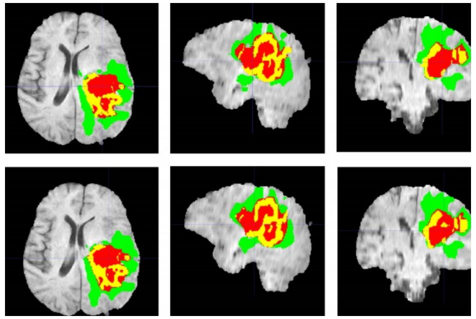




Cons

- Sample inefficient
- Unable to reuse prior data effectively → on-policy
- Blackbox, can be hard to debug





Frontiers of Policy Gradient Research

Major open challenges in policy gradient research:

Convergence guarantees

Asynchronous/Parallel Methods

Better Variance Reduction

Learning from high-dimensional inputs

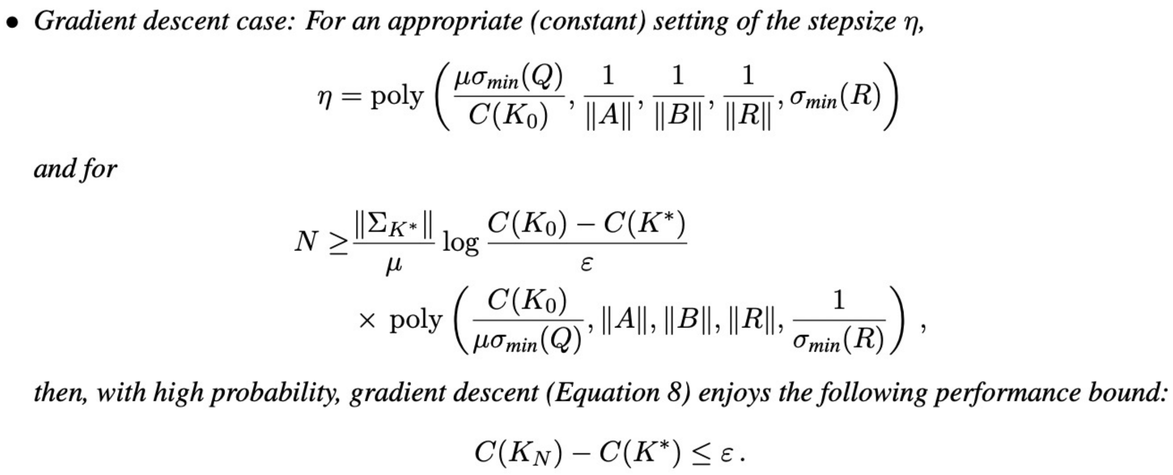
Bootstrapping from prior data

Multi-agent Policy Gradient

Frontiers of Policy Gradient Research

Convergence guarantees and empirical investigations

Globally Convergent in LQR/LQG Case



- *Gradient descent case: For an appropriate (constant) setting of the stepsize η,*

$$\eta = \text{poly}\left(\frac{\mu\sigma_{min}(Q)}{C(K_0)}, \frac{1}{\|A\|}, \frac{1}{\|B\|}, \frac{1}{\|R\|}, \sigma_{min}(R)\right)$$

and for

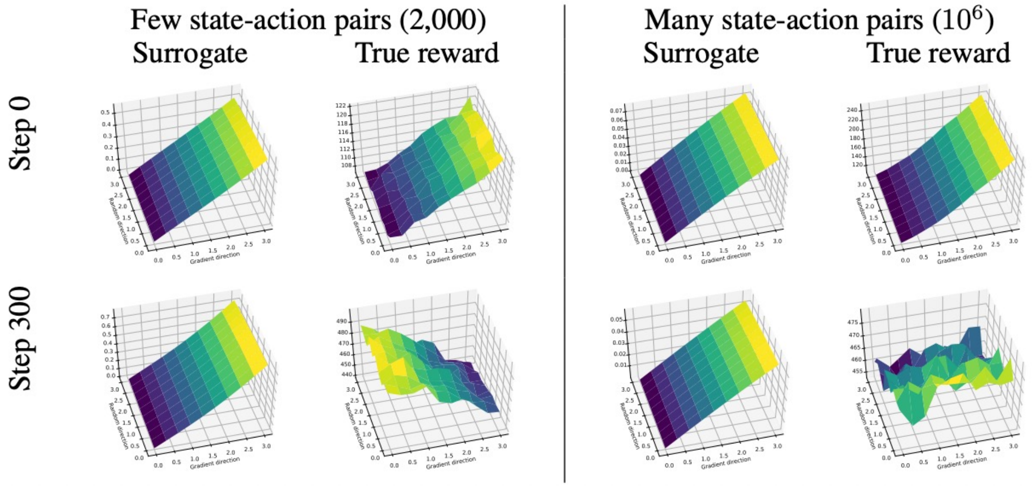
$$N \geq \frac{\|\Sigma_{K^*}\|}{\mu} \log \frac{C(K_0) - C(K^*)}{\varepsilon} \times \text{poly}\left(\frac{C(K_0)}{\mu\sigma_{min}(Q)}, \|A\|, \|B\|, \|R\|, \frac{1}{\sigma_{min}(R)}\right),$$

then, with high probability, gradient descent (Equation 8) enjoys the following performance bound:

$$C(K_N) - C(K^*) \leq \varepsilon\,.$$

Global Convergence of Policy Gradient Methods for the Linear Quadratic Regulator, Fazel et al ’19
Global Convergence of Policy Gradient Methods to (Almost) Locally Optimal Policies, Zhang et al, ‘19
Globally convergent policy search over dynamic filters for output estimation, Umenberger ‘21

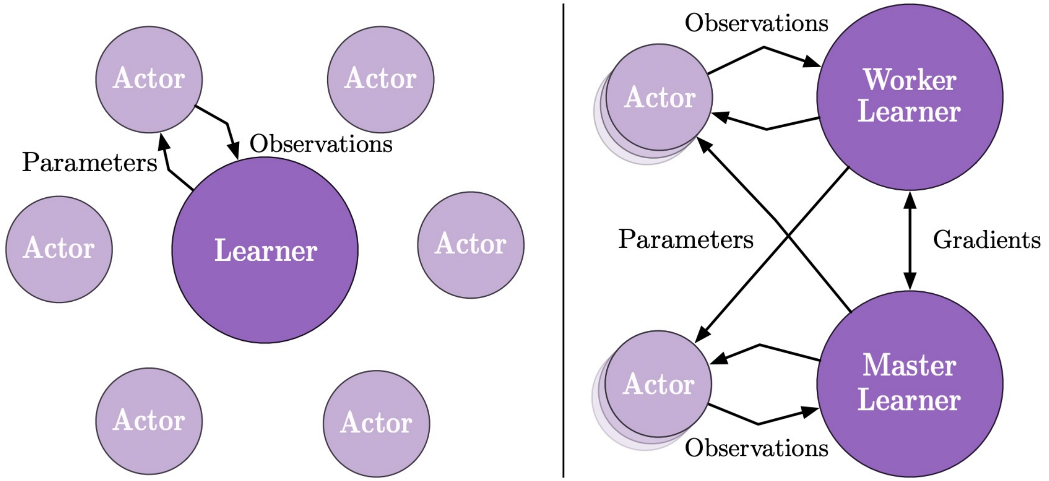
Practical Algorithms Deviate from Theory



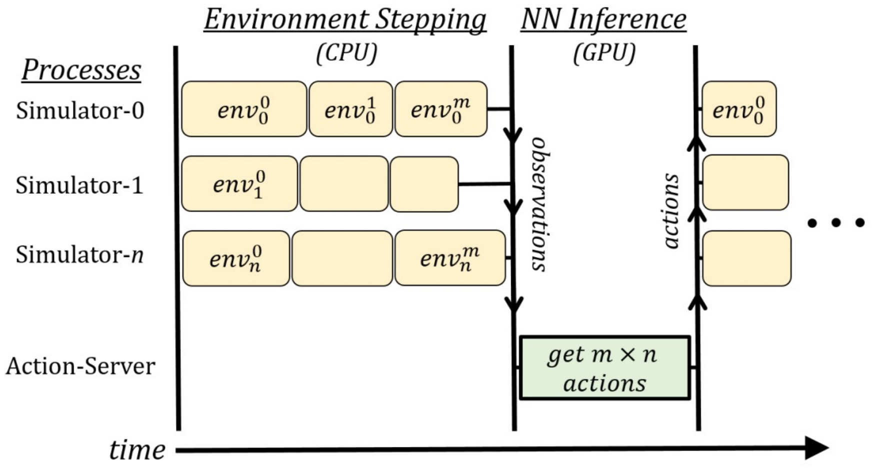
Is the Policy Gradient a Gradient?, Nota et al, ’19
A Closer Look at Deep Policy Gradients, Ilyas et al ’19
An Empirical Analysis of Proximal Policy Optimization with Kronecker-factored Natural Gradients, Song et al ‘18
What Matters In On-Policy Reinforcement Learning? A Large-Scale Empirical Study, Andrychowicz et al ‘20

Frontiers of Policy Gradient Research

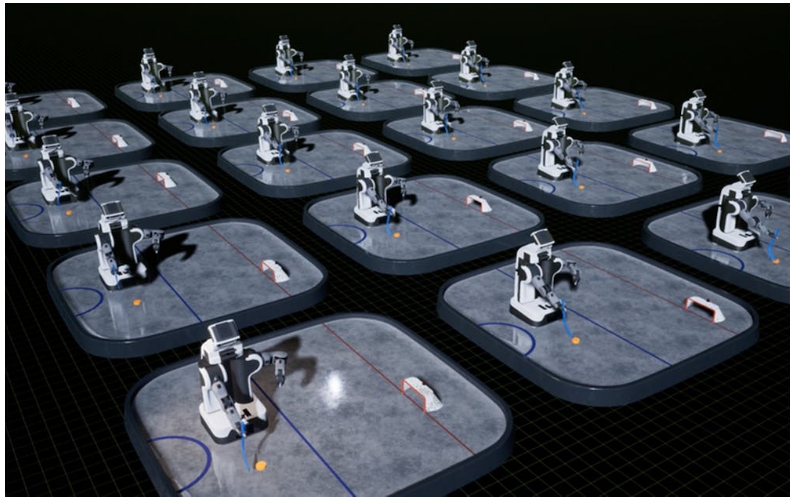
Asynchronous methods for large scale speedup



IMPALA: Scalable Distributed Deep-RL with Importance Weighted Actor-Learner Architectures, Espeholt '18



Accelerated Methods for Deep Reinforcement Learning, Stooke et al '19



Frontiers of Policy Gradient Research

Better Variance Reduction Methods

Action dependent baselines

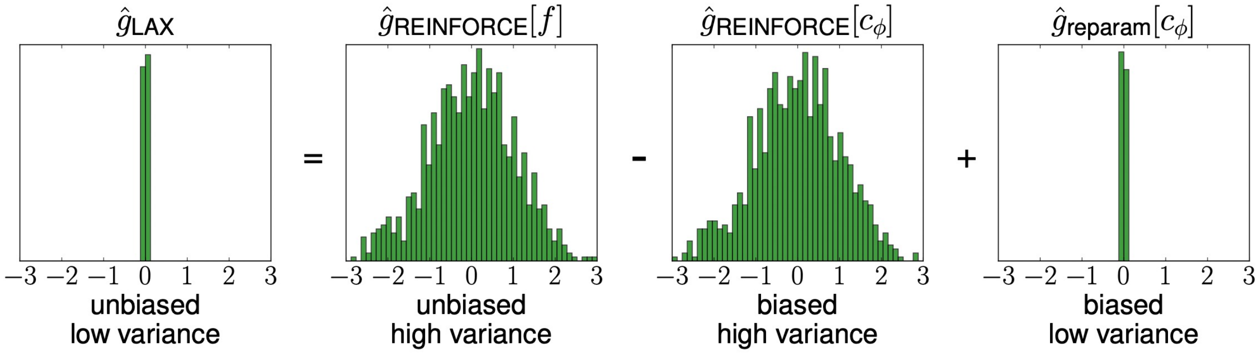
$$\pi_\theta(a_t|s_t) = \prod_{i=1}^m \pi_\theta(a_t^i|s_t)$$

$$\nabla_\theta \eta(\pi_\theta) = \mathbb{E}_{\rho_\pi,\pi}\left[\sum_{i=1}^m \nabla_\theta \log \pi_\theta(a_t^i|s_t)\left(\hat{Q}(s_t,a_t) - b_i(s_t,a_t^{-i})\right)\right]$$

For factorized spaces, baselines can depend on independent action factors

The Mirage of Action-Dependent Baselines in Reinforcement Learning, Tucker et al '18
Variance Reduction for Policy Gradient with Action-Dependent Factorized Baselines, Wu et al '18

Alternative Estimators



Q-Prop: Sample-Efficient Policy Gradient with An Off-Policy Critic, Gu et al '16
Backpropagation through the Void: Optimizing control variates for black-box gradient estimation, Grathwohl et al '17
Categorical Reparameterization with Gumbel-Softmax, Jang et al '16

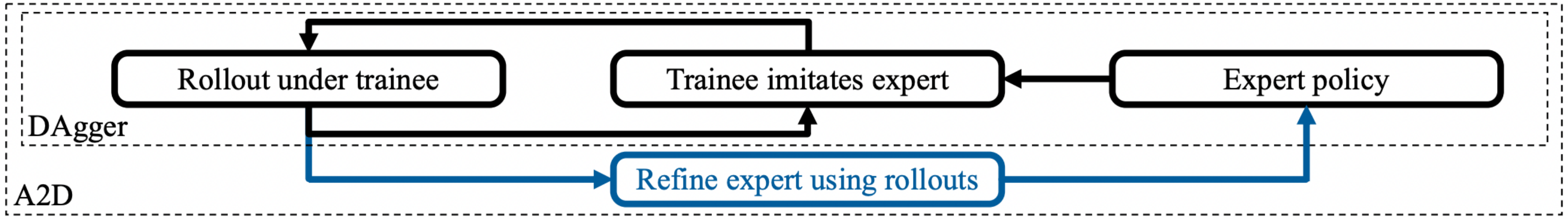
Frontiers of Policy Gradient Research

Learning from High Dimensional Observations

Learning Quadrupedal Locomotion over Challenging Terrain, Lee et al ‘20

A System for General In-Hand Reorientation, Chen et al ‘21

Challenging to provide guarantees in partially observed settings!



Rollout under trainee

Trainee imitates expert

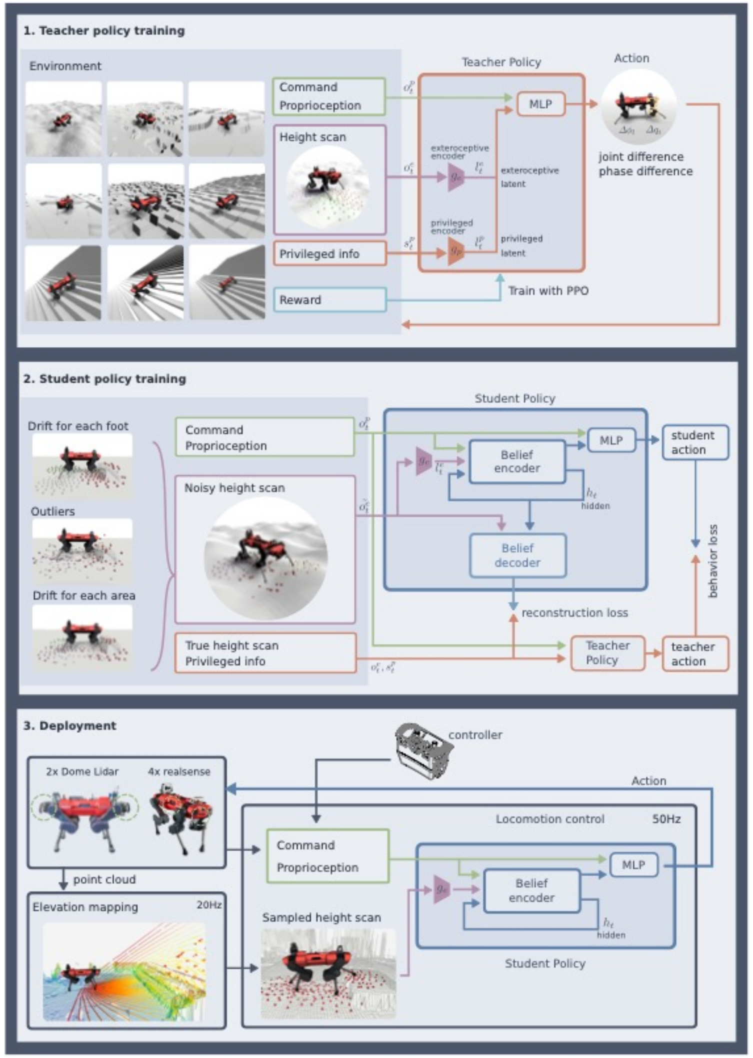
Expert policy

DAgger

Refine expert using rollouts

A2D

Robust Asymmetric Learning in POMDPs, Warrington et al ‘20



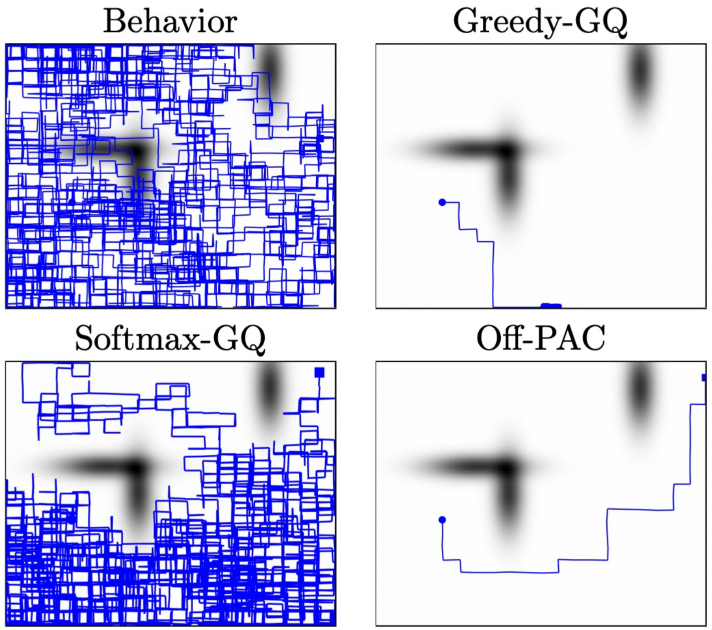
Learning robust perceptive locomotion for quadrupedal robots in the wild, Miki et al ‘22

Frontiers of Policy Gradient Research

Bootstrapping from Prior/Off-Policy Data

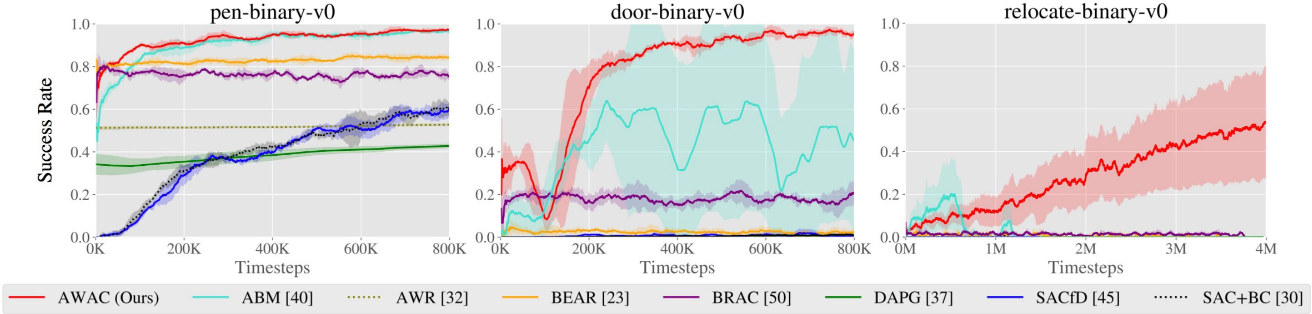
Off-policy policy gradient

$$\mathbb{E}_\beta\left[\frac{\pi_\theta(a|s)}{\beta(a|s)}Q^\pi(s,a)\nabla_\theta \ln \pi_\theta(a|s)\right]$$



Off-Policy Actor-Critic, Degris et al ‘13

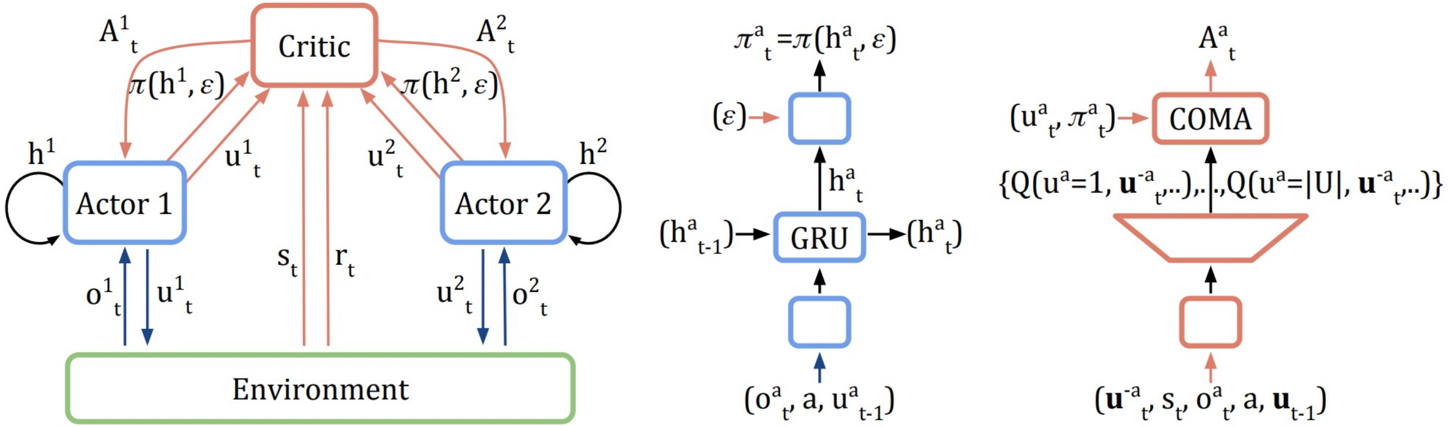
Learning from Prior Data



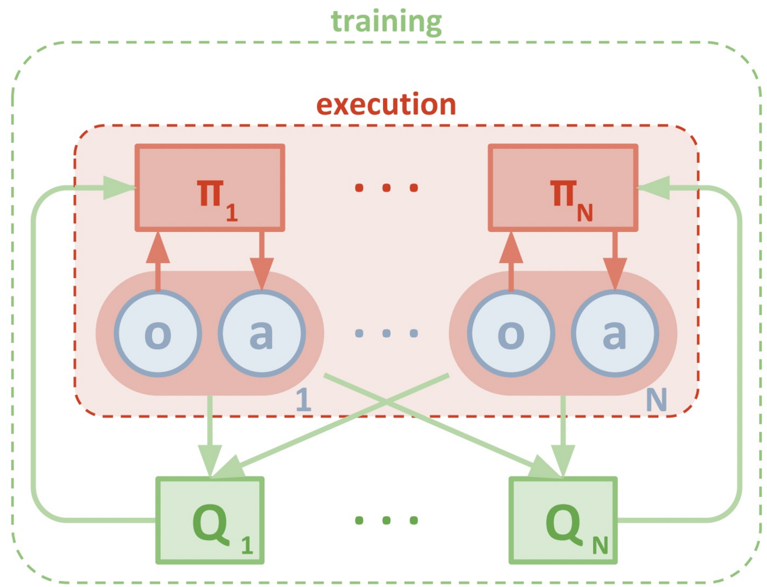
Advantage Weighted Actor Critic, Nair et al ’20
DDPGfD, Vecerik ‘17
DAPG, Rajeswaran ‘17

Frontiers of Policy Gradient Research

Multi-agent policy gradient



Counterfactual Multi-Agent Policy Gradients, Foerster et al '17



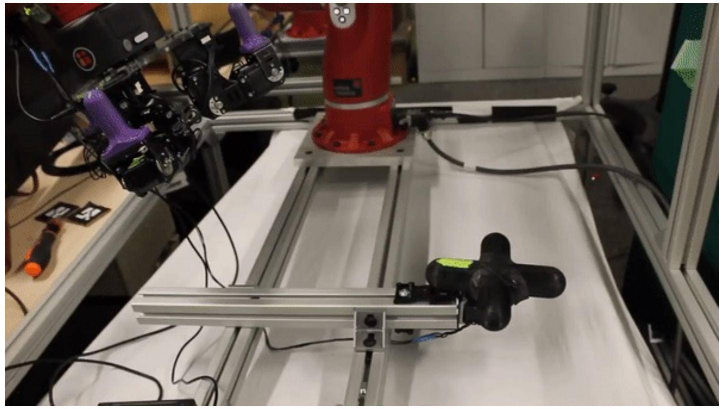
Multi-Agent Actor-Critic for Mixed Cooperative-Competitive Environments, Lowe et al '17

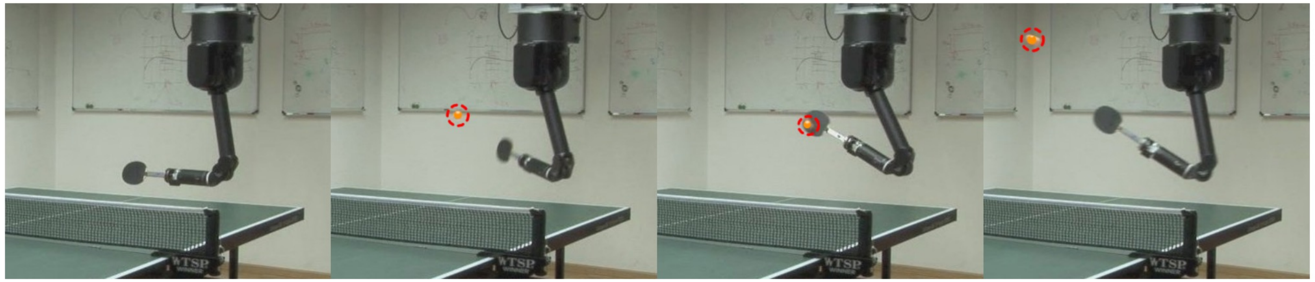
Primary challenges:

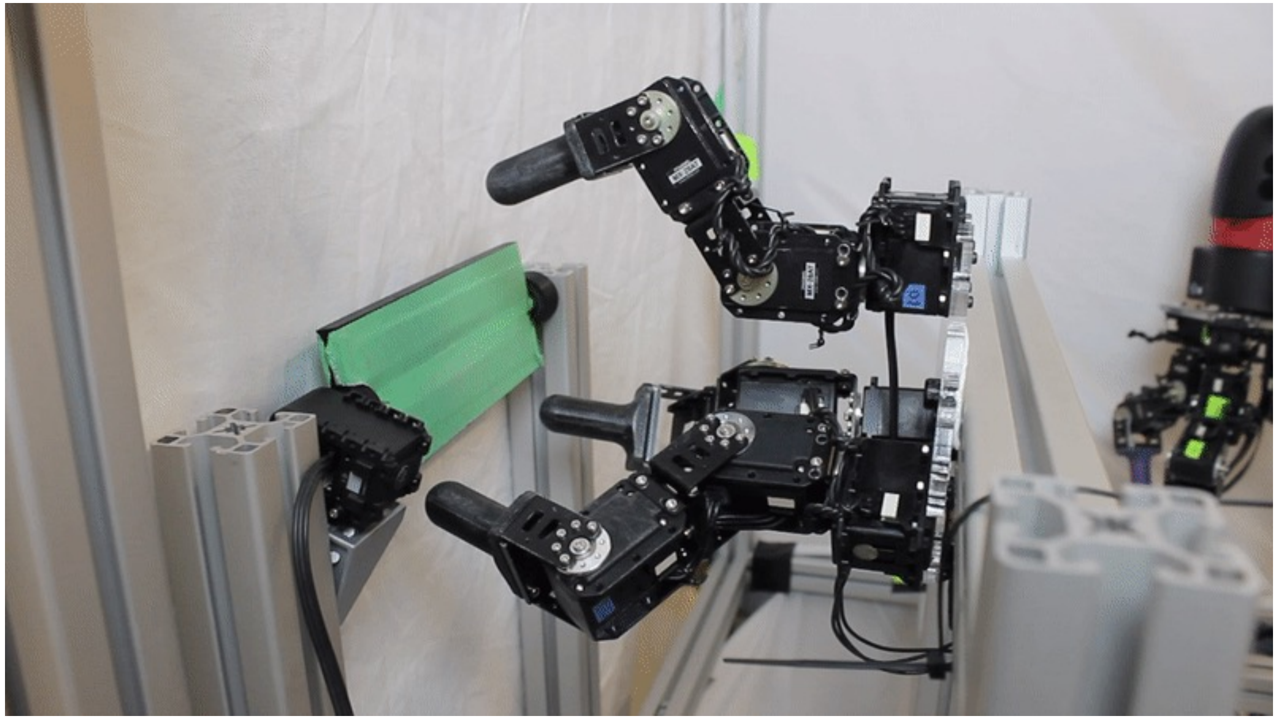
1. Non-stationarity
2. Data-efficiency
3. Communication

How is this useful for robotics?

Can be used to train robots in the real world but only in limited settings

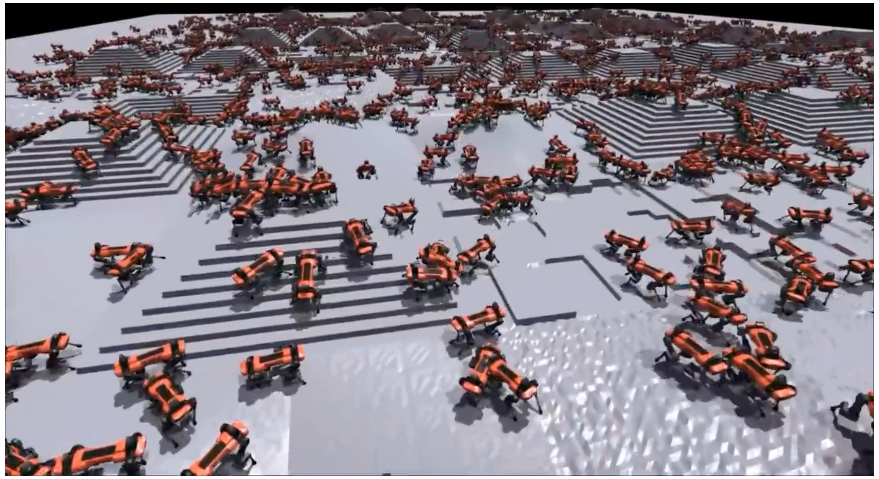


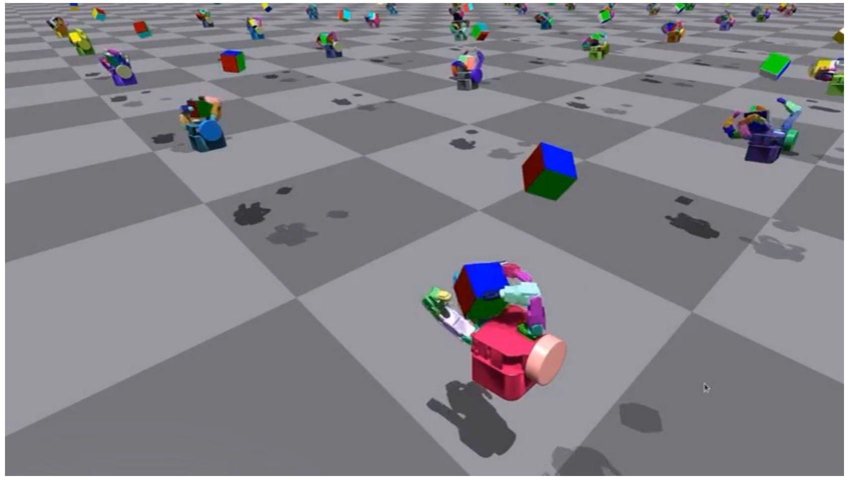


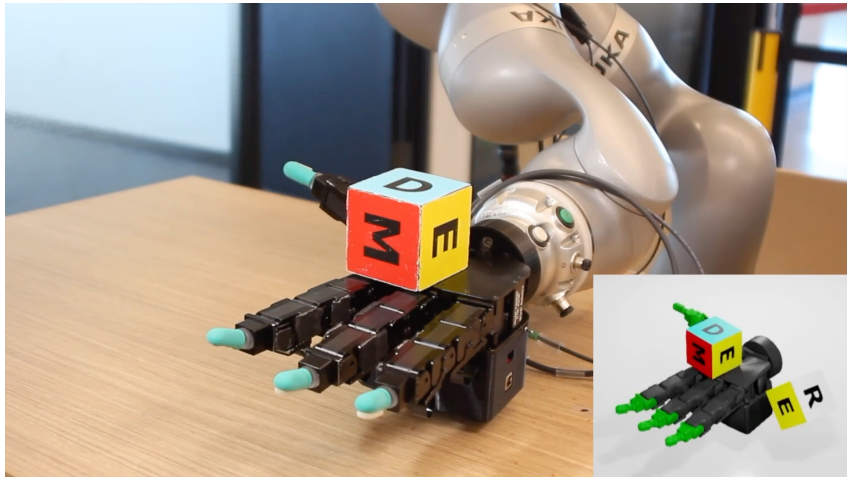


How is this useful for robotics?

Largely useful for pretraining in simulation





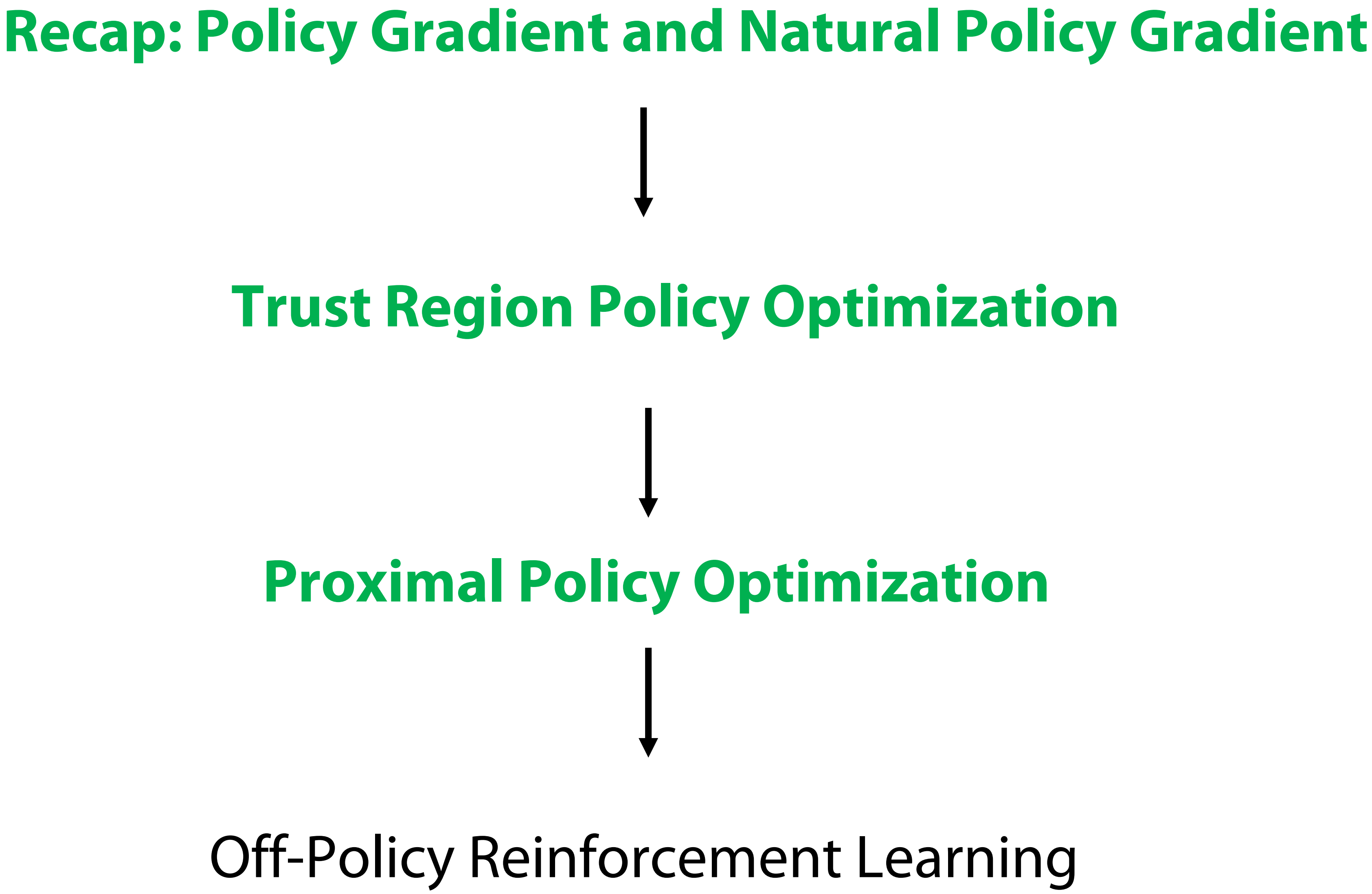


More in the sim2real lecture!

Perspective on Policy Gradients

- Policy gradient methods form an effective solution technique to the RL problem
- Techniques range from vanilla policy gradient to NPG to PPO, each with it's own pros and cons
- PG can be very adept at solving black-box optimization asymptotically, but can be very slow
- Several open frontiers still exist for research into PG methods
- Most promising use of PG methods in robotics is through simulation to reality transfer

Lecture outline



Recap: Policy Gradient and Natural Policy Gradient

Trust Region Policy Optimization

Proximal Policy Optimization

Off-Policy Reinforcement Learning

What can we do to make PG suitable for robots?

Why is Policy Gradient sample inefficient?

$$\nabla_\theta J(\theta) = \int p_\theta(\tau) \nabla_\theta \log p_\theta(\tau) d\tau$$

$$\approx \frac{1}{N} \sum_{i=1}^{N} \sum_{t} \nabla_\theta \log \pi_\theta(a_t^i | s_t^i) \sum_{t'=t}^{T} r(s_t^i, a_t^i)$$

On-policy, unable to effectively use past data

High Variance Estimator

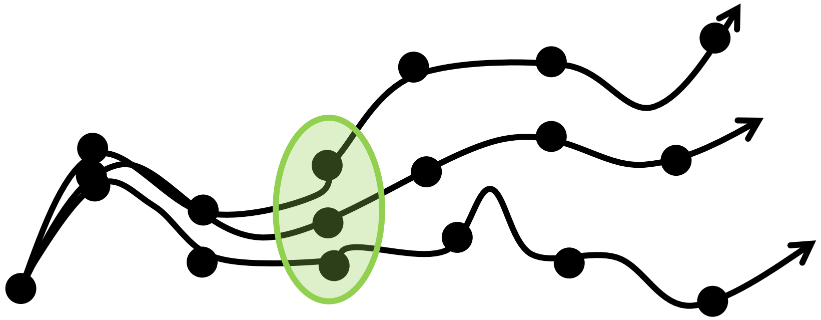
Can we develop a **low variance** **off-policy** RL algorithm that can bootstrap from prior data?

What can we do to lower variance?

$$\nabla_\theta J(\theta) = \int p_\theta(\tau) \nabla_\theta \log p_\theta(\tau) d\tau$$

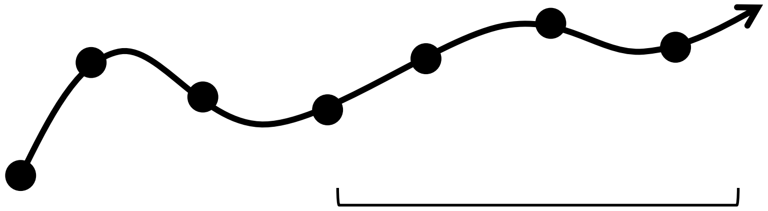
$$\approx \frac{1}{N} \sum_{i=1}^{N} \sum_{t} \nabla_\theta \log \pi_\theta(a_t^i | s_t^i) \underbrace{\sum_{t'=t}^{T} r(s_t^i, a_t^i)}_{}$$

Idea: bundle this across many (s, a) with a function approximator



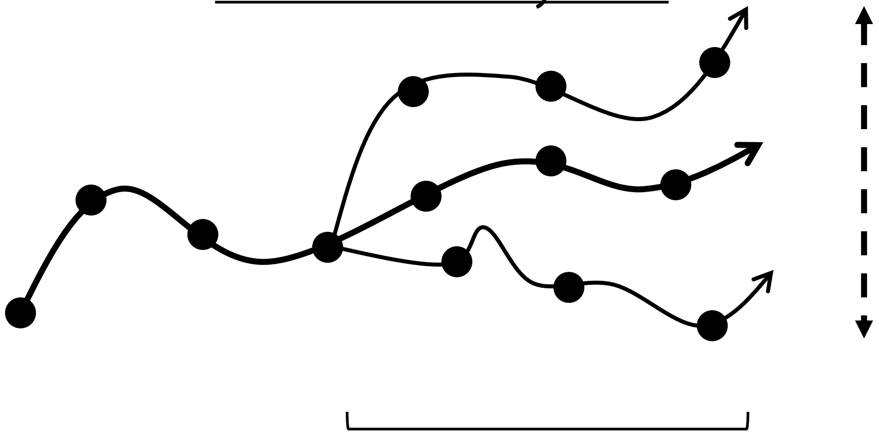
Function approximator bundles return estimates across states

What we do



Single sample estimate

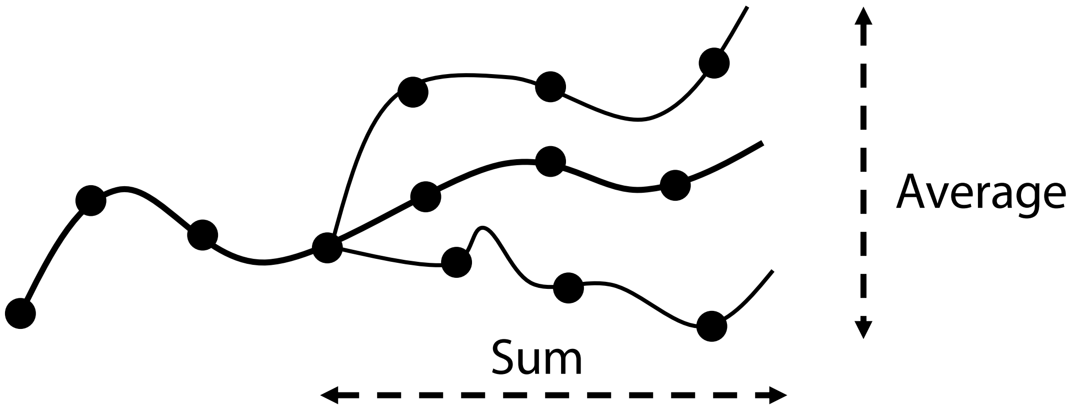
What we actually want



Averaged return estimate

Notation: Q functions

$$\frac{1}{N}\sum_{i=1}^{N}\sum_{t}\nabla_\theta \log \pi_\theta(a_t^i|s_t^i)\sum_{t'=t}^{T} r(s_t^i, a_t^i)$$



Expected sum of rewards in the future, starting from (s, a) on first step, then π

$$Q^{\pi}(s_t, a_t) = \mathbb{E}_{\pi_\theta}\left[\sum_{t'=t}^{T} r(s'_t, a'_t)|s_t, a_t\right]$$

Bundles estimates across (s, a)

Use the magic of (deep) function approximation

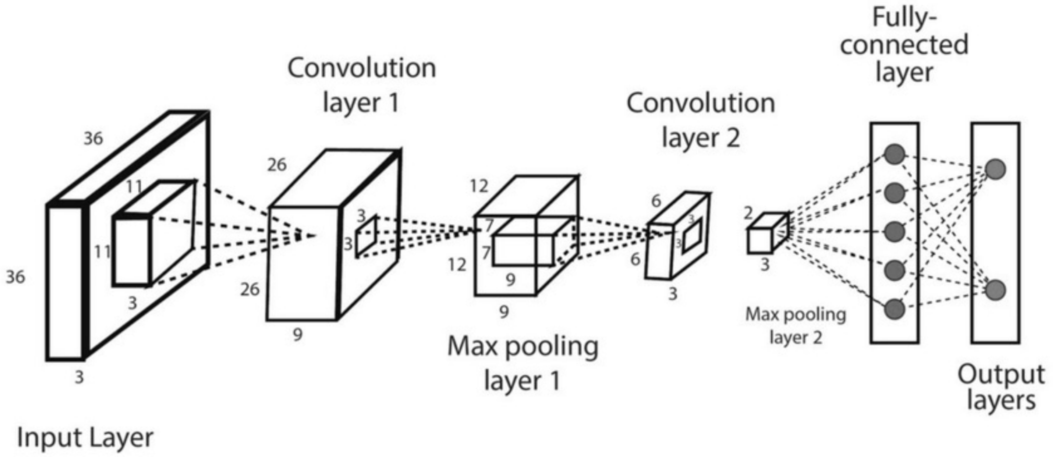
Attempt 0: Monte-Carlo Estimation of Q-Functions

$$\frac{1}{N}\sum_{i=0}^{N}\sum_{t=0}^{T}\nabla_\theta \log \pi_\theta(a_t^i|s_t^i)Q^\pi(s_{t'}^i, a_{t'}^i)$$

$$Q^\pi(s_t, a_t) = \mathbb{E}_{\pi_\theta}\left[\sum_{t'=t}^{T} r(s_t', a_t')|s_t, a_t\right]$$ ← Monte-carlo approximation

Idea: Regression from (s, a) to Monte-Carlo estimate

State
Action



Return to Go

Unbiased, but high variance!

On-Policy

Can we do better?

$$\frac{1}{N}\sum_{i=1}^{N}\sum_{t}\nabla_\theta \log \pi_\theta(a_t^i|s_t^i)\sum_{t'=t}^{T} r(s_t^i, a_t^i)$$

$$\Downarrow$$

Much lower variance if estimated well

Can be learned off-policy!

$$\frac{1}{N}\sum_{i=0}^{N}\sum_{t=0}^{T}\nabla_\theta \log \pi_\theta(a_t^i|s_t^i)Q^\pi(s_t^i, a_t^i)$$

Has special structure we can exploit!!

Attempt 1: Using Recursive Structure

$$\frac{1}{N}\sum_{i=0}^{N}\sum_{t=0}^{T}\nabla_\theta \log \pi_\theta(a_t^i|s_t^i)Q^\pi(s_t^i,a_t^i)$$

$$Q^\pi(s_t,a_t) = \mathbb{E}_{\pi_\theta}\left[\sum_{t'=t}^{T} r(s_t',a_t')|s_t,a_t\right]$$

Note the definition of a value function $$V^\pi(s_t) = \mathbb{E}_{\pi_\theta}\left[\sum_{t'=t}^{T} r(s_{t'},a_{t'})|s_t\right] = \mathbb{E}_{a_t\sim\pi_\theta(\cdot|s_t)}\left[Q(s_t,a_t)\right]$$

Average Q-function over actions sampled from policy

Value functions are recursive

$$V^\pi(s_t) = \mathbb{E}_{\pi_\theta}\left[r(s_t,a_t) + \sum_{t'=t+1}^{T} r(s_{t'},a_{t'})|s_t\right]$$

$$V^\pi(s_t) = \mathbb{E}_{\pi_\theta}\left[r(s_t,a_t) + \mathbb{E}_{\pi_\theta}\left[\sum_{t'=t+1}^{T} r(s_{t'},a_{t'})|s_{t+1}\right]\right] \longleftarrow \text{VF!}$$

$$V^\pi(s_t) = \mathbb{E}_{\substack{a_t\sim\pi_\theta(a_t|s_t)\\ s_{t+1}\sim p(\cdot|s_t,a_t)}}\left[r(s_t,a_t) + V^\pi(s_{t+1})\right]$$

Q-values via 1-step lookahead $$Q^\pi(s_t,a_t) = \mathbb{E}_{s_{t+1}\sim p(\cdot|s_t,a_t)}\left[r(s_t,a_t) + V^\pi(s_{t+1})|s_t=s_t,a_t=a_t\right]$$

Attempt 1: Using Recursive Structure

$$\frac{1}{N}\sum_{i=0}^{N}\sum_{t=0}^{T}\nabla_\theta \log \pi_\theta(a_t^i|s_t^i)Q^\pi(s_t^i,a_t^i)$$

$$Q^\pi(s_t,a_t)=\mathbb{E}_{\pi_\theta}\left[\sum_{t'=t}^{T} r(s_t',a_t')|s_t,a_t\right]$$

Value functions are recursive

$$V^\pi(s_t)=\mathbb{E}_{\pi_\theta}\left[r(s_t,a_t)+V^\pi(s_{t+1})\right]$$

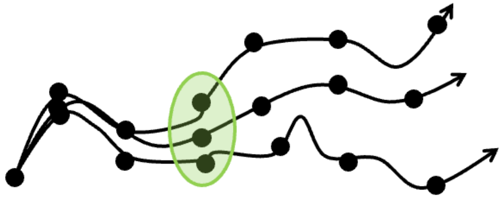
Recipe for policy gradient

$$\min_\phi \mathbb{E}_{(s_i,a_i,{s_i}')\sim\pi}\left[(V_\phi^\pi(s_i)-y_i)^2\right]$$

$$y_i = r(s_i,a_i)+V({s_i}^{'})$$

Value Bellman equation

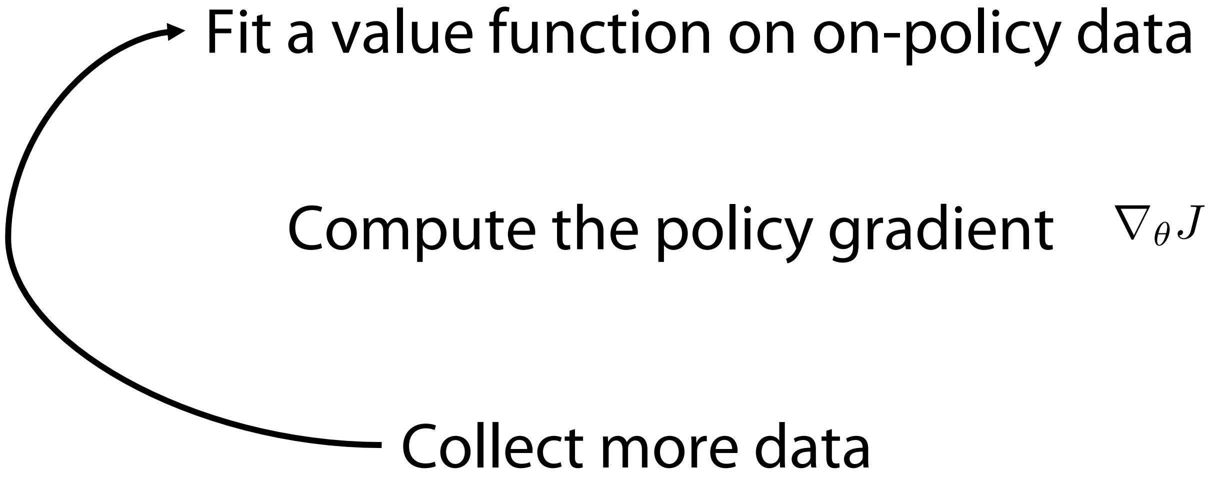
$$\nabla_\theta J(\theta)=\frac{1}{N}\sum_{i=0}^{N}\sum_{t=0}^{T}\nabla_\theta \log \pi_\theta(a_t^i|s_t^i)(\underbrace{r(s_t,a_t)+V(s_{t+1})}_{\text{1-step lookahead - better estimate of future return}}-V(s_t))$$



Attempt 1: Using Recursive Structure

TODO replace this $\frac{1}{N}\sum_{i=0}^{N}\sum_{t=0}^{T}\nabla_\theta \log \pi_\theta(a_t^i|s_t^i)Q^\pi(s_{t'}^i, a_{t'}^i)$

$$Q^\pi(s_t, a_t) = \mathbb{E}_{\pi_\theta}\left[\sum_{t'=t}^{T} r(s_t', a_t')|s_t, a_t\right]$$



Fit a value function on on-policy data

$$\min_\phi \mathbb{E}_{(s_i, a_i, s_i{}')\sim\pi}\left[(V_\phi^\pi(s_i) - y_i)^2\right]$$

$$y_i = r(s_i, a_i) + V(s_i{}')$$

Compute the policy gradient $\nabla_\theta J(\theta) = \frac{1}{N}\sum_{i=0}^{N}\sum_{t=0}^{T}\nabla_\theta \log \pi_\theta(a_t^i|s_t^i)(r(s_t, a_t) + V(s_{t+1}) - V(s_t))$

Collect more data

+ lowers variance

- Still on-policy

Revisit: Generalized Advantage Estimation

Sum up all the estimators in a geometric sum

$$A^{\theta}_{N}(s_1, a_1) = r_1 + \gamma r_2 + \cdots + \gamma^{N-1} r_N - V(s_1)$$

$$A^{\theta}_{N-1}(s_1, a_1) = r_1 + \gamma r_2 + \cdots + \gamma^{N-2} V(s_{N-1}) - V(s_1)$$

$$A^{\theta}_{2}(s_1, a_1) = r_1 + \gamma r_2 + \cdots + \gamma^{2} V(s_3) - V(s_1)$$

$$A^{\theta}_{1}(s_1, a_1) = r_1 + \gamma V(s_2) - V(s_1)$$

Geometric sum

$$A^{\theta}_{\lambda}(s_1, a_1) = \sum_{j=1}^{N} \lambda^{j} A^{\theta}_{j}(s, a)$$

λ controls bias-variance tradeoff

Best of both worlds – very similar idea to eligibility traces

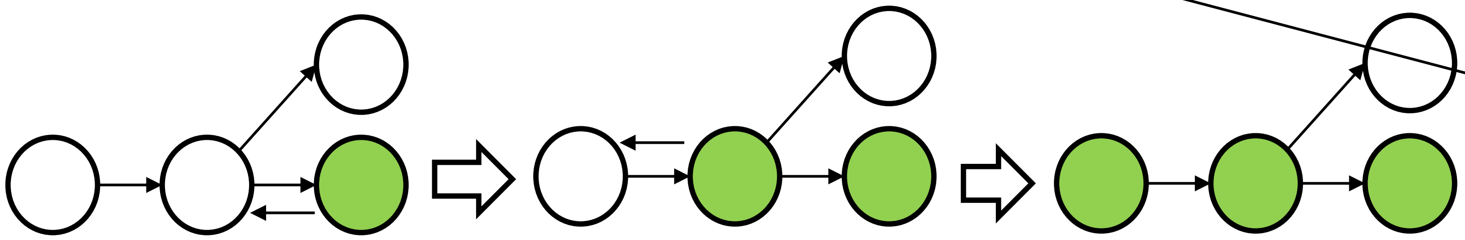
Attempt 2: Recursive structure in Q functions directly

Q functions have special recursive structure themselves!

$$Q^\pi(s_t, a_t) = \mathbb{E}_{\pi_\theta}\left[\sum_{t'=t}^{T} r(s'_t, a'_t) | s_t, a_t\right]$$

$$= r(s_t, a_t) + \mathbb{E}_\pi\left[\sum_{t'=t+1} r(s_{t'}, a_{t'}) | s_{t+1}, a_{t+1} \sim \pi(.|s_{t+1})\right]$$

Bellman equation $$Q^\pi(s_t, a_t) = r(s_t, a_t) + \mathbb{E}_{\substack{s_{t+1} \sim p(.|s_t, a_t) \\ a_{t+1} \sim \pi_\theta(.|s_{t+1})}} \left[Q^\pi(s_{t+1}, a_{t+1})\right]$$



Can be from different policies

Decompose temporally via dynamic programming

Off-policy!

Learning Q-functions via Dynamic Programming

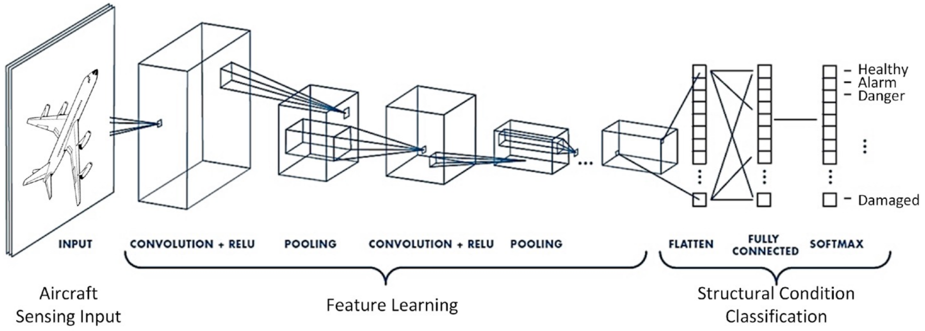
Policy Evaluation: Try to minimize Bellman Error (almost)

Bellman equation

$$Q^\pi(s_t, a_t) = r(s_t, a_t) + \mathbb{E}_{\substack{s_{t+1}\sim p(.|s_t, a_t) \\ a_{t+1}\sim \pi_\theta(.|s_{t+1})}} [Q^\pi(s_{t+1}, a_{t+1}]$$

Same function approximator

How can we convert this recursion into an off-policy learning objective?

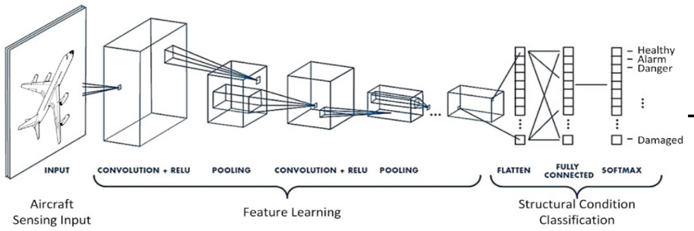


Why is this not just the gradient of the Bellman Error?

$$\min_{\phi} \mathbb{E}_{(s_t, a_t, s_{t+1}) \sim \mathcal{D}} \left(Q^{\pi}_{\phi}(s_t, a_t) - \left(r(s_t, a_t) + \mathbb{E}_{a_{t+1} \sim \pi_\theta(a_{t+1}|s_{t+1})} \left[Q^{\pi}_{\hat{\phi}}(s_{t+1}, a_{t+1}) \right] \right) \right)^2$$

Approximate using stochastic optimization

$$\min_{\phi} \mathbb{E}_{(s_t, a_t, s_{t+1}) \sim \mathcal{D}} \left(Q^{\pi}_{\phi}(s_t, a_t) - \left(r(s_t, a_t) + Q^{\pi}_{\hat{\phi}}(s_{t+1}, a_{t+1}) \right) \right)^2 \quad a_{t+1} \sim \pi(\cdot|s_{t+1})$$



"Target" moves too much

Often tough empirically with function approximators

Expectation inside the square, hard to be unbiased

Note: this may look like gradient descent on Bellman error, it is not!

Improving Policies with Learned Q-functions

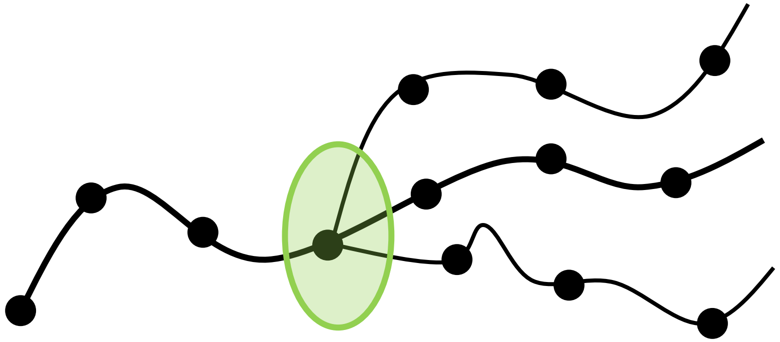
Policy Improvement: Improve policy with **policy gradient**

$$\max_\theta \mathbb{E}_{s\sim\mathcal{D},a\sim\pi_\theta(a|s)}\left[Q^{\pi_\theta}(s,a)\right]$$

Replace Monte-Carlo sum of rewards with learned Q function

Lowers variance compared to policy gradient!

+ off-policy



Policy Updates – REINFORCE or Reparameterization

Let's look a little deeper into the policy update

$$\max_\theta J(\theta) = \max_\theta \mathbb{E}_{s\sim\mathcal{D}} \mathbb{E}_{a\sim\pi_\theta(.|s)} \left[Q^\pi(s,a)\right]$$

<u>Likelihood Ratio/Score Function</u>

$$\nabla_\theta J(\theta) = \mathbb{E}_{s\sim\mathcal{D}} \mathbb{E}_{a\sim\pi_\theta(.|s)} \left[\nabla_\theta \log \pi_\theta(a|s) Q^\pi(s,a)\right]$$

Easier to Apply to Broad Policy Class

<u>Pathwise derivative/Reparameterization</u>

$$\nabla_\theta J(\theta) = \mathbb{E}_{s\sim\mathcal{D}} \mathbb{E}_{z\sim p(z)} \left[\nabla_a Q^\pi(s,a)|_{a=\mu_\theta+z\sigma_\theta} \nabla_\theta(\mu_\theta + z\sigma_\theta)\right]$$

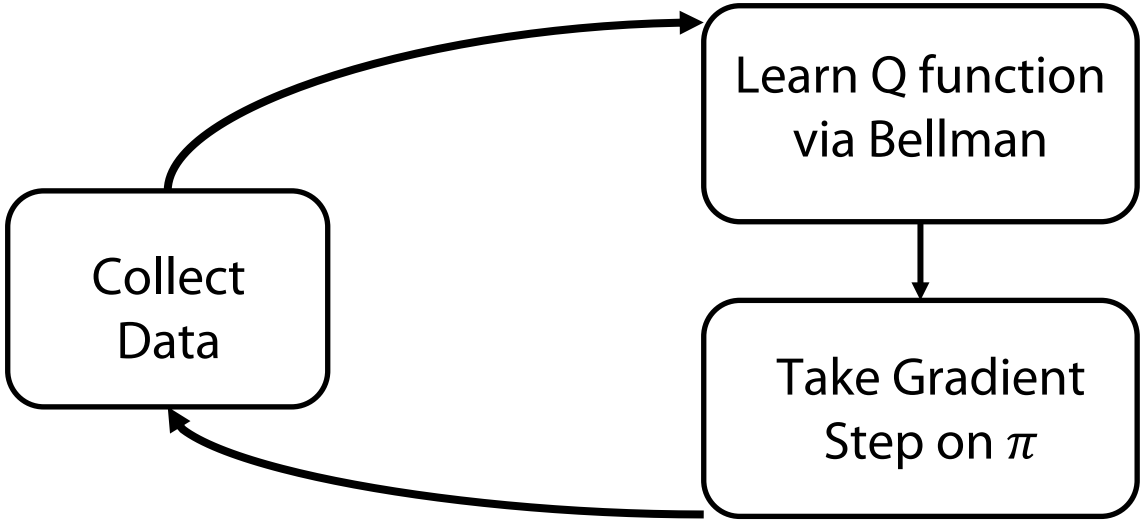
Lower variance (empirically)

Remember Lecture 2 and discussion of when gradients can be moved inside

Actor-Critic: Policy Gradient in terms of Q functions

Critic: learned via the Bellman update (Policy Evaluation)

$$\min_{\phi} \mathbb{E}_{(s_t, a_t, s_{t+1}) \sim \mathcal{D}} \left(Q^{\pi}_{\phi}(s_t, a_t) - (r(s_t, a_t) + Q^{\pi}_{\hat{\phi}}(s_{t+1}, a_{t+1})) \right)^2 \quad a_{t+1} \sim \pi(\cdot | s_{t+1})$$

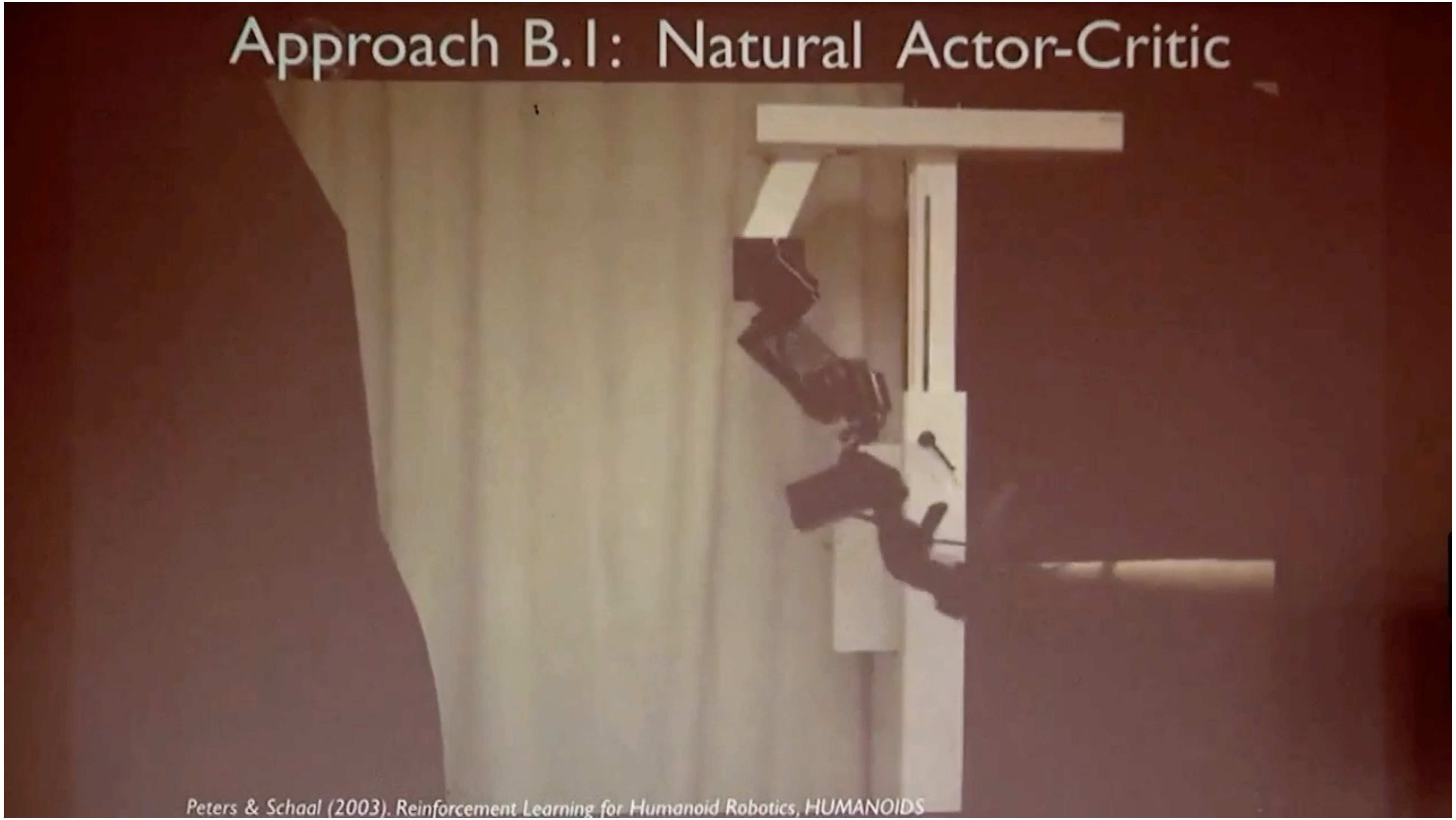


Lowers variance and is off-policy!

Actor: updated using learned critic (Policy Improvement)

$$\max_{\pi} \mathbb{E}_{s \sim \mathcal{D}} \mathbb{E}_{a \sim \pi(.|s)} \left[Q^{\pi}(s, a) \right]$$

Actor-Critic in Action



Approach B.1: Natural Actor-Critic

Peters & Schaal (2003). Reinforcement Learning for Humanoid Robotics, HUMANOIDS

Fin.