Reinforcement Learning
Autumn 2024

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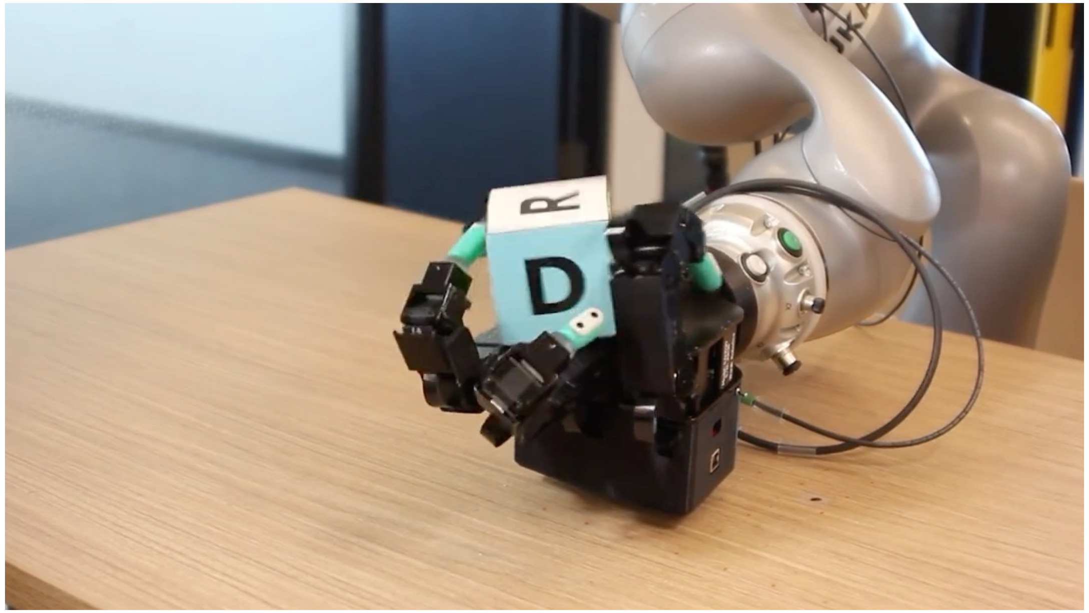

Lecture outline

Recap: Deriving the Policy Gradient

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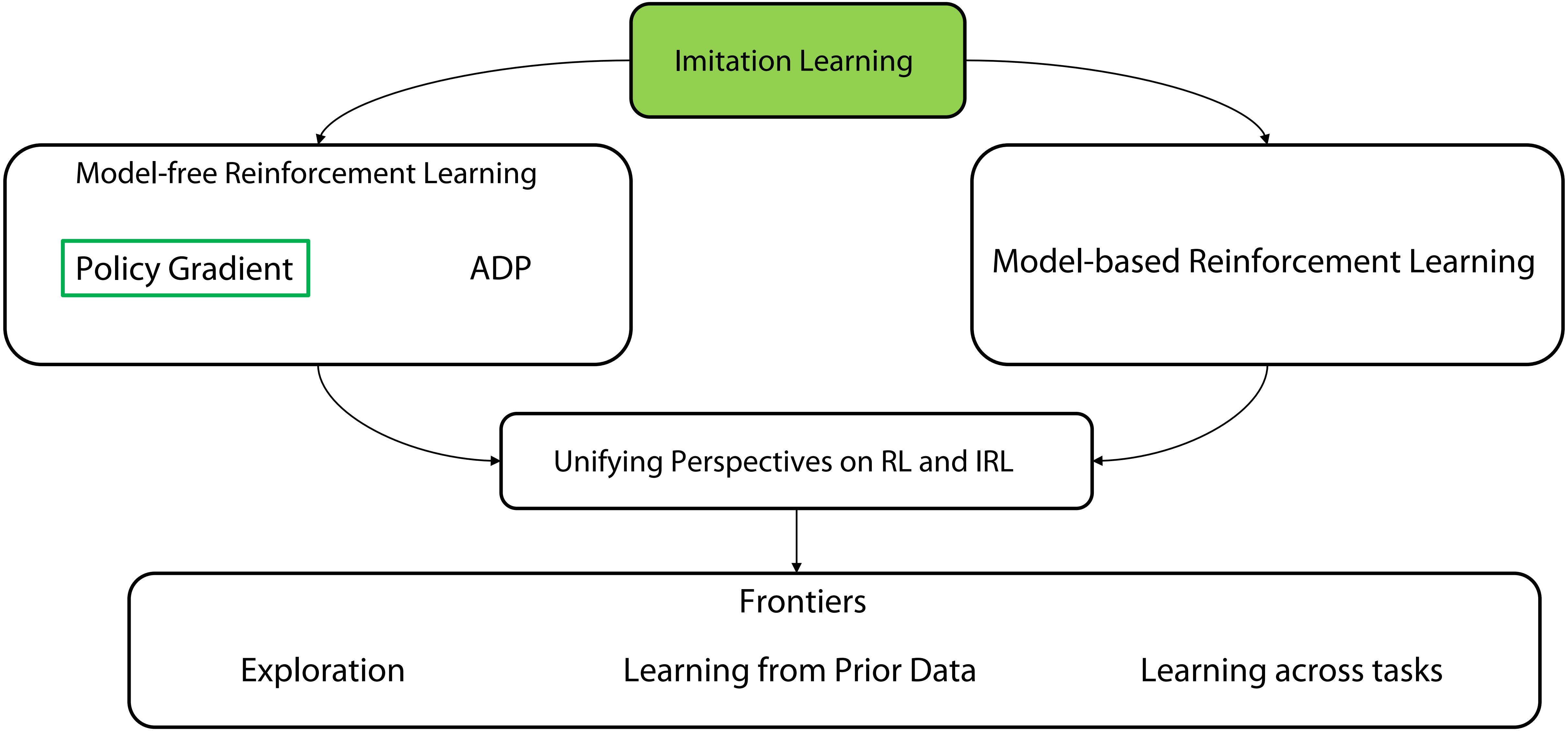
What makes the Policy Gradient Challenging? - Variance

↓

Natural Policy Gradients and Covariant Parameterization

Class Structure



Imitation Learning

Model-free Reinforcement Learning

Policy Gradient

ADP

Model-based Reinforcement Learning

Unifying Perspectives on RL and IRL

Frontiers

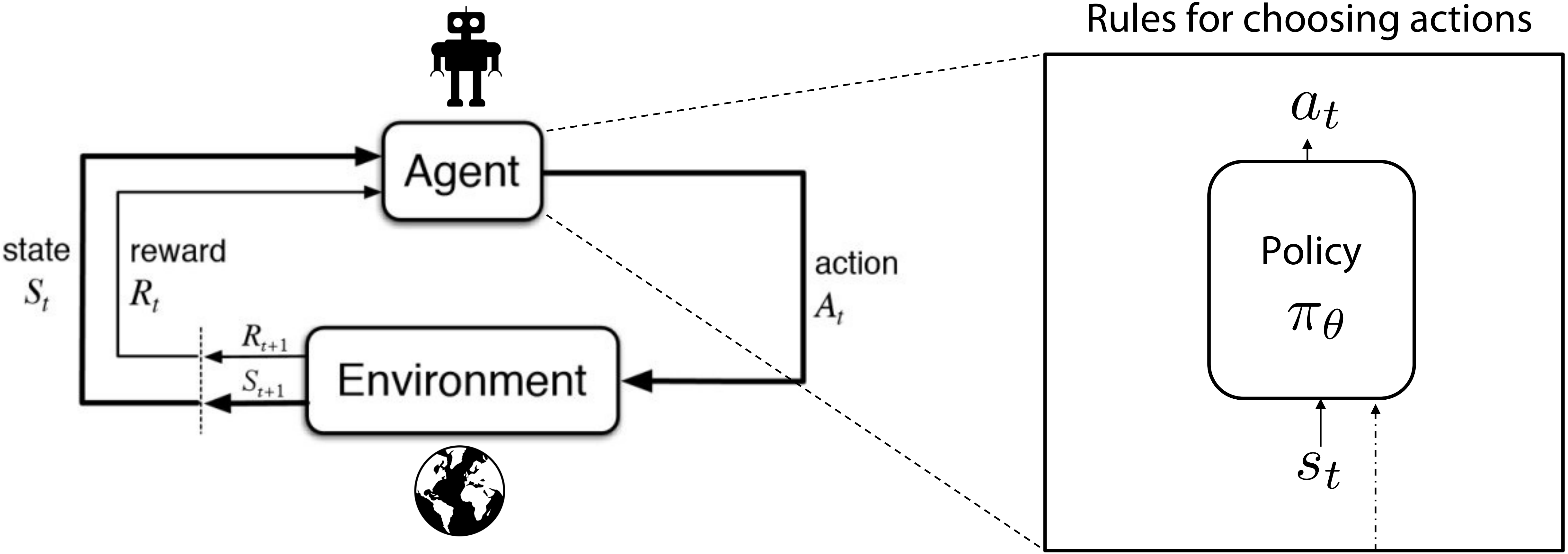
Exploration

Learning from Prior Data

Learning across tasks

Objective of Reinforcement Learning

Rules for choosing actions

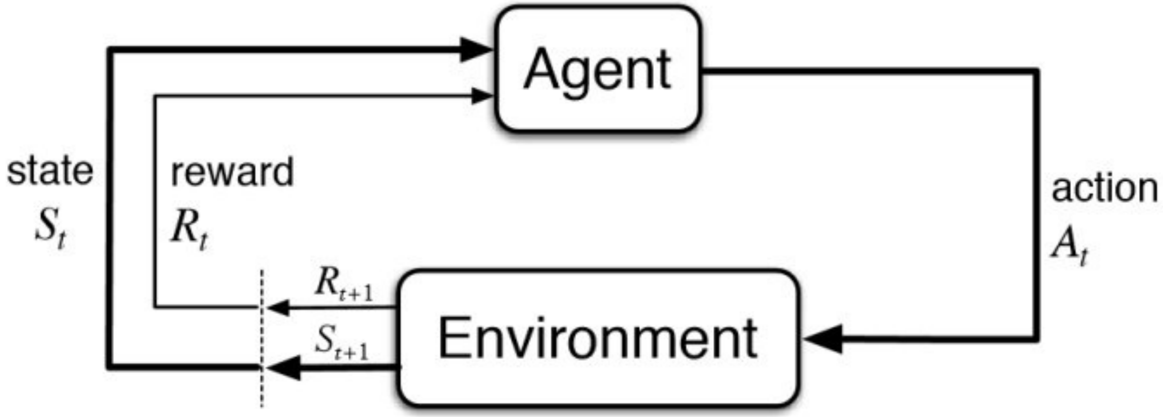


$$\max_{\theta} \mathbb{E}_{\tau \sim \pi_\theta} \left[\sum_{t=0}^{T} r(s_t, a_t) \right]$$

Trajectory sampled using policy

Needs to be learned

How should we optimize this objective?



$$\max_\theta \mathbb{E}_{\tau \sim \pi_\theta}\left[\sum_{t=0}^{T} r(s_t, a_t)\right]$$

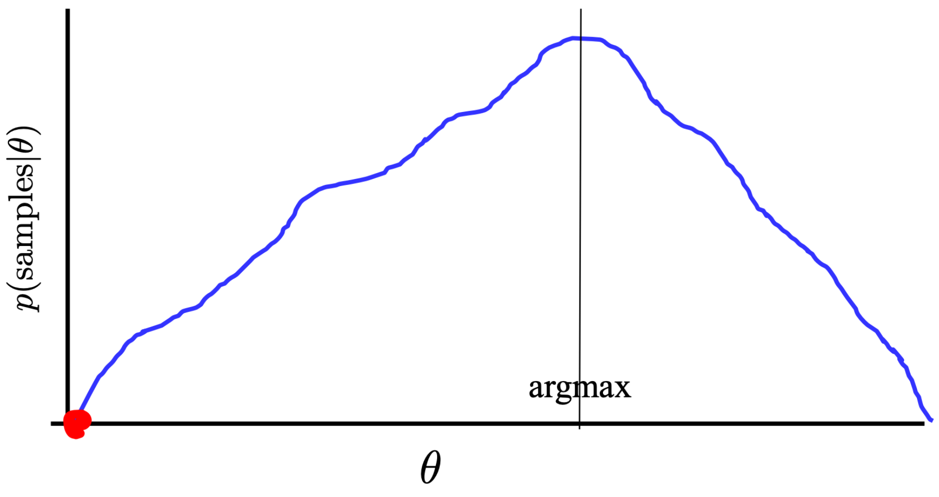
Gradient Ascent

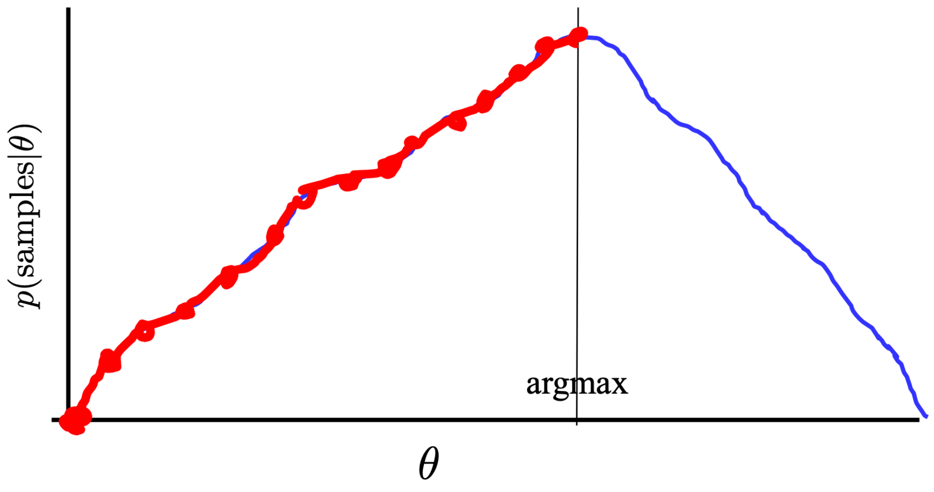
Dynamic Programming

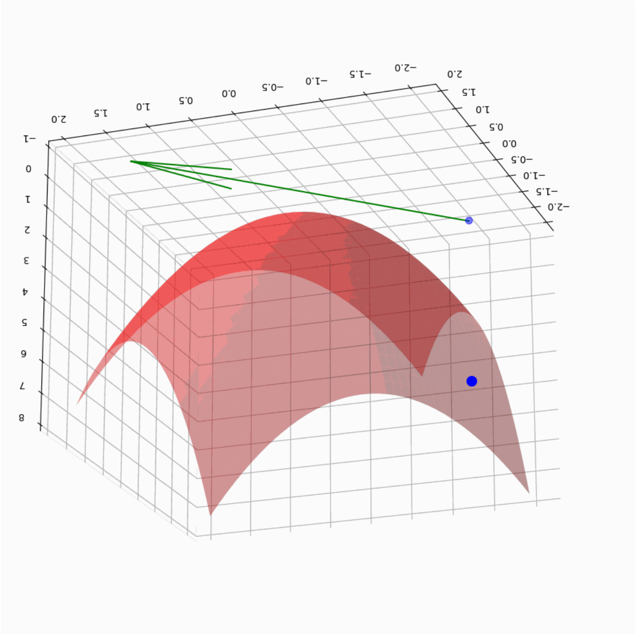
Model-Based Optimization

Each method has it’s own +/-

Gradient Ascent







Simple view – move the parameters in the direction of the gradient of the objective

$$\theta_{i+1} = \theta_i + \alpha \nabla_\theta J(\theta)|_{\theta=\theta_i}$$

More later: can be derived as steepest ascent in Euclidean norm

Ok let's do gradient ascent for the RL objective

$$\max_\theta \mathbb{E}_{\tau \sim \pi_\theta} \left[\sum_{t=0}^{T} r(s_t, a_t) \right]$$

$$= \int p_\theta(\tau) R(\tau) d\tau$$

<u>REINFORCE gradient descent (RL)</u>

$$\nabla_\theta \mathbb{E}_{x \sim p_\theta(x)} \left[f(x) \right]$$

(Cannot simply compute average of expectation)

<u>Standard gradient descent (supervised learning)</u>

Gradient wrt expectation variable, not of integrand!

$$\nabla_\theta \mathbb{E}_{x \sim g(x)} \left[f_\theta(x) \right]$$

(Whiteboard)

(Gradient passes inside the expectation – compute gradient and average)

Taking the gradient of sum of rewards

$$\max_\theta \mathbb{E}_{\tau\sim\pi_\theta}\left[\sum_{t=0}^{T} r(s_t, a_t)\right]$$

Let's take the gradient of this objective

$$J(\theta) = \int p_\theta(\tau)R(\tau)d(\tau)$$

Let's think about this from the trajectory view

$$\nabla_\theta J(\theta) = \nabla_\theta \int p_\theta(\tau)R(\tau)d(\tau)$$

We need to express this in a way that we can evaluate with expectations

$$= \int \nabla_\theta p_\theta(\tau)R(\tau)d(\tau) \quad = \int \frac{p_\theta(\tau)}{p_\theta(\tau)}\nabla_\theta p_\theta(\tau)R(\tau)d(\tau)$$

$$= \int p_\theta(\tau)\nabla_\theta \log p_\theta(\tau)R(\tau)d(\tau) \;= \mathbb{E}_{p_\theta(\tau)}\left[\nabla_\theta \log p_\theta(\tau)R(\tau)\right]$$

REINFORCE trick

$$\boxed{\frac{d\log(x)}{d\theta} = \frac{d\log(x)}{dx}\frac{dx}{d\theta} = \frac{1}{x}\frac{dx}{d\theta}}$$

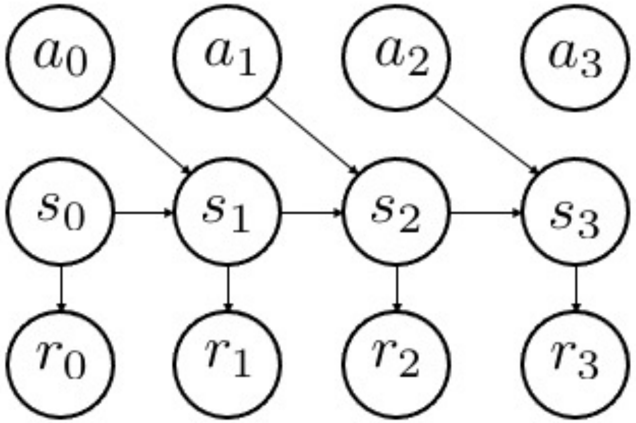
Use chain rule

Taking the gradient of return

Initial State Dynamics Policy

$$p_\theta(\tau) = p(s_0)\Pi_{t=0}^{T-1} p(s_{t+1}|s_t, a_t)\pi(a_t|s_t)$$

(Ancestral sampling)



$$\log p_\theta(\tau) = \log p(s_0) + \sum_{t=0}^{T-1} \log p(s_{t+1}|s_t, a_t) + \log \pi(a_t|s_t)$$

$$\nabla_\theta \log p_\theta(\tau) = \nabla_\theta \cancel{\log p(s_0)} + \sum_{t=0}^{T-1} \nabla_\theta \log \cancel{p(s_{t+1}|s_t, a_t)} + \nabla_\theta \log \pi(a_t|s_t)$$

$$\nabla_\theta \log p_\theta(\tau) = \sum_{t=0}^{T-1} \nabla_\theta \log \pi(a_t|s_t)$$

Model Free!!

Taking the gradient of return

$$\nabla_\theta J(\theta) = \mathbb{E}_{\tau \sim p_\theta(\tau)} \left[\nabla_\theta \log p_\theta(\tau) \sum_{t=0}^{T} r(s_t, a_t) \right]$$

$$\nabla_\theta J(\theta) = \mathbb{E}_{\substack{s_0 \sim p(s_0) \\ s_{t+1} \sim p(s_{t+1}|s_t, a_t) \\ a_t \sim \pi(a_t|s_t)}} \left[\sum_{t=0}^{T} \nabla_\theta \log \pi_\theta(a_t|s_t) \sum_{t'=0}^{T} r(s_t, a_t) \right]$$

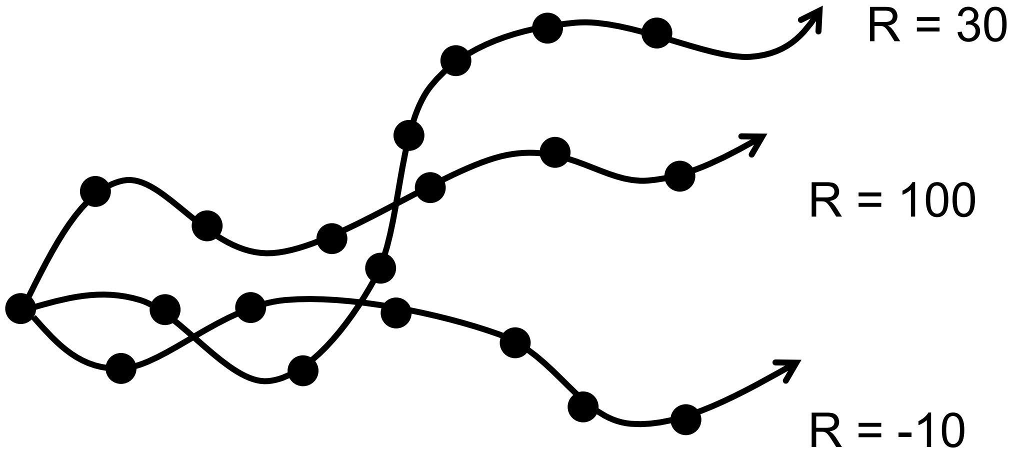
$$\approx \frac{1}{N} \sum_{i=0}^{N} \sum_{t=0}^{T} \nabla_\theta \log \pi_\theta(a_t^i|s_t^i) \sum_{t'=0}^{T} r(s_{t'}^i, a_{t'}^i)$$ (approximating using samples)

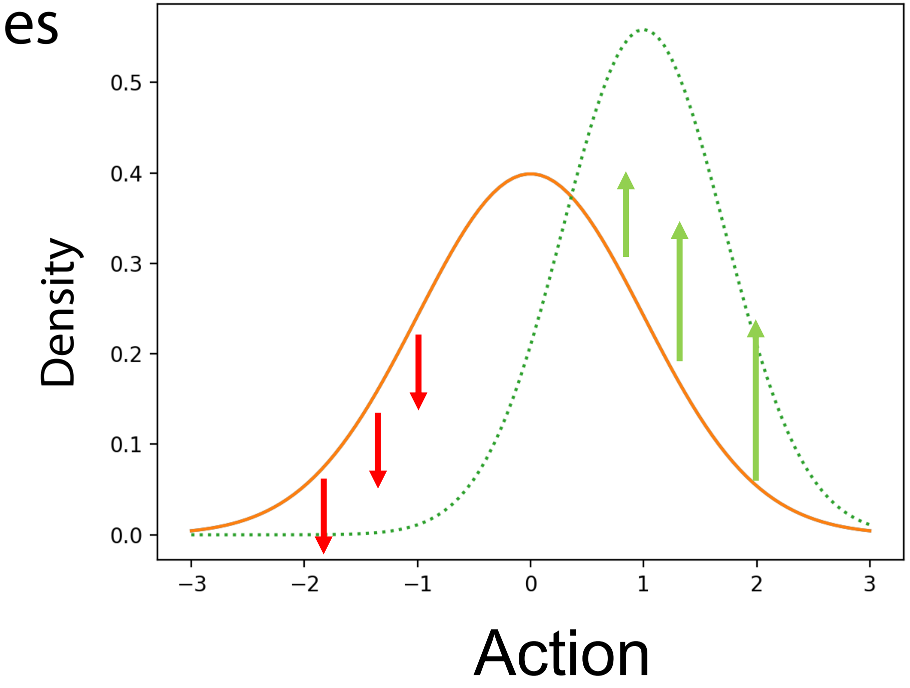
(Monte-Carlo approximation)

What does this mean?

$$\nabla_\theta J(\theta) = \int p_\theta(\tau) \nabla_\theta \log p_\theta(\tau) d\tau \approx \frac{1}{N} \sum_{i=0}^{N} \sum_{t=0}^{T} \nabla_\theta \log \pi_\theta(a_t^i | s_t^i) \sum_{t'=0}^{T} r(s_{t'}^i, a_{t'}^i)$$

Increase the likelihood of actions in high return trajectories

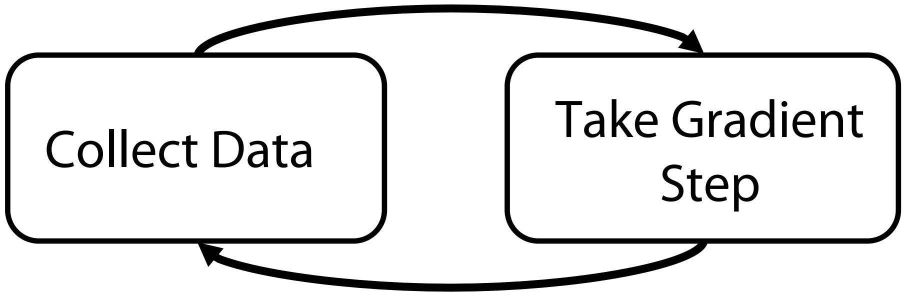




Resulting Algorithm (REINFORCE)

$$\nabla_\theta J(\theta) = \int p_\theta(\tau) \nabla_\theta \log p_\theta(\tau) d\tau$$

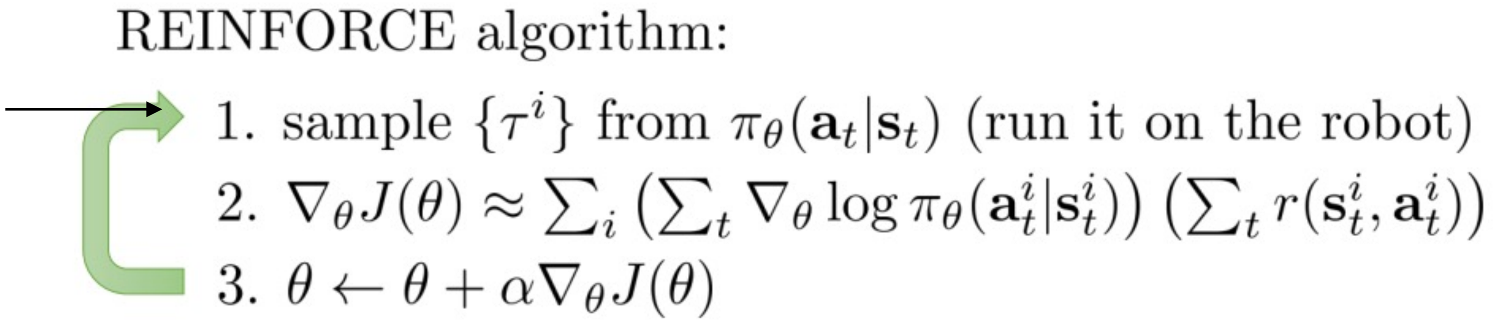
$$\theta_{i+1} = \theta_i + \alpha \nabla_\theta J(\theta)|_{\theta=\theta_i}$$



Collect Data

Take Gradient Step

On-policy



REINFORCE algorithm:

1. sample $\{\tau^i\}$ from $\pi_\theta(\mathbf{a}_t|\mathbf{s}_t)$ (run it on the robot)
2. $\nabla_\theta J(\theta) \approx \sum_i \left(\sum_t \nabla_\theta \log \pi_\theta(\mathbf{a}_t^i|\mathbf{s}_t^i)\right)\left(\sum_t r(\mathbf{s}_t^i, \mathbf{a}_t^i)\right)$
3. $\theta \leftarrow \theta + \alpha \nabla_\theta J(\theta)$

How is this related to supervised learning?

Reinforcement Learning

$$\nabla_\theta J(\theta) = \int p_\theta(\tau)\nabla_\theta \log p_\theta(\tau) d\tau$$

$$\approx \frac{1}{N}\sum_{i=0}^{N}\sum_{t=0}^{T}\nabla_\theta \log \pi_\theta(a_t^i|s_t^i)\sum_{t'=0}^{T} r(s_{t'}^i, a_{t'}^i)$$

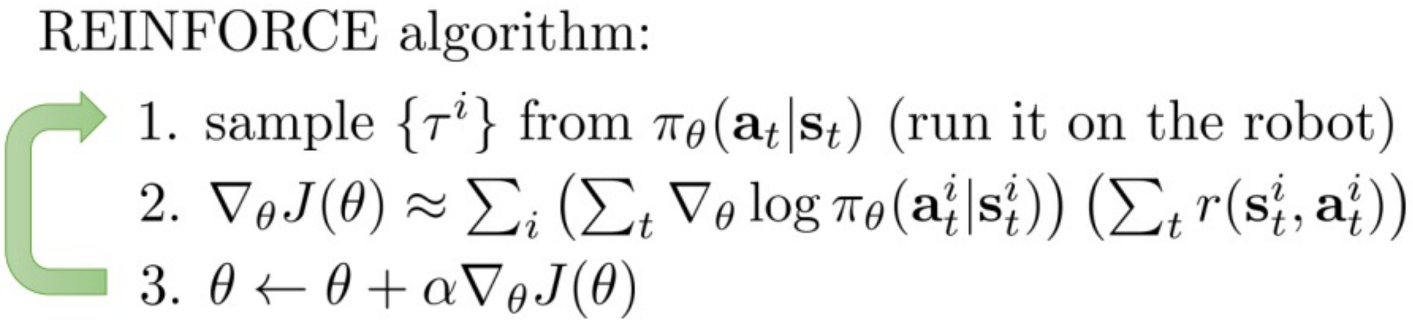
Supervised Learning

$$\max_\theta \mathbb{E}_{(x,y)\sim\mathcal{D}}\left[\log p_\theta(y|x)\right]$$

$$\approx \frac{1}{N}\sum_i \nabla_\theta \log p_\theta(y^i|x^i)$$

PG = select good data + increase likelihood of selected data

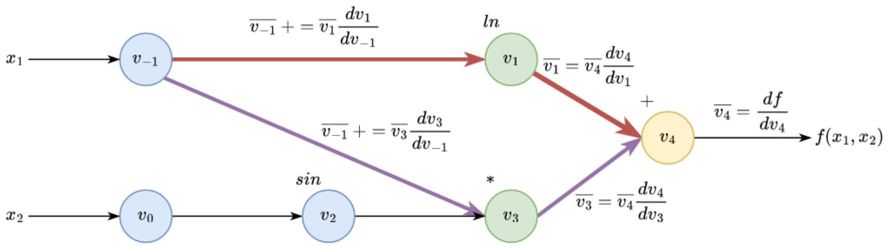
How do we implement this?



REINFORCE algorithm:

1. sample $\{\tau^i\}$ from $\pi_\theta(\mathbf{a}_t|\mathbf{s}_t)$ (run it on the robot)
2. $\nabla_\theta J(\theta) \approx \sum_i \left(\sum_t \nabla_\theta \log \pi_\theta(\mathbf{a}_t^i|\mathbf{s}_t^i)\right)\left(\sum_t r(\mathbf{s}_t^i, \mathbf{a}_t^i)\right)$
3. $\theta \leftarrow \theta + \alpha \nabla_\theta J(\theta)$

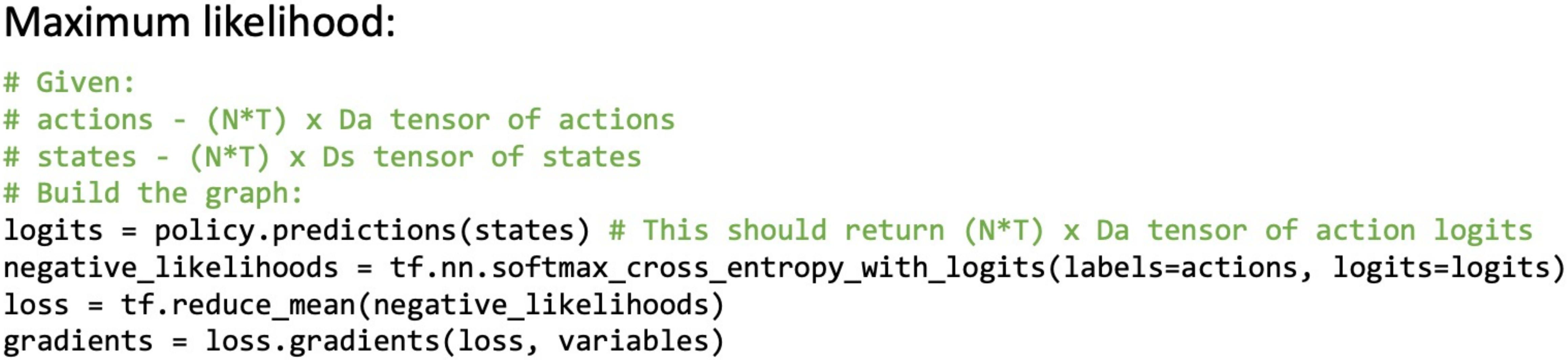
$$\nabla_\theta J(\theta) = \int p_\theta(\tau)\nabla_\theta \log p_\theta(\tau) d\tau \approx \frac{1}{N}\sum_{i=0}^{N}\sum_{t=0}^{T}\nabla_\theta \log \pi_\theta(a_t^i|s_t^i)\sum_{t'=0}^{T} r(s_{t'}^i, a_{t'}^i)$$


x_1
v_{-1}
$\overline{v_{-1}} += \overline{v_1}\frac{dv_1}{dv_{-1}}$
ln
v_1
$\overline{v_1} = \overline{v_4}\frac{dv_4}{dv_1}$
$+$
$\overline{v_4} = \frac{df}{dv_4}$
v_4
$f(x_1, x_2)$
$\overline{v_{-1}} += \overline{v_3}\frac{dv_3}{dv_{-1}}$
sin
$*$
x_2
v_0
v_2
v_3
$\overline{v_3} = \overline{v_4}\frac{dv_4}{dv_3}$

Compute gradients with autodiff

Sum up rewards in a trajectory

How do we implement this?

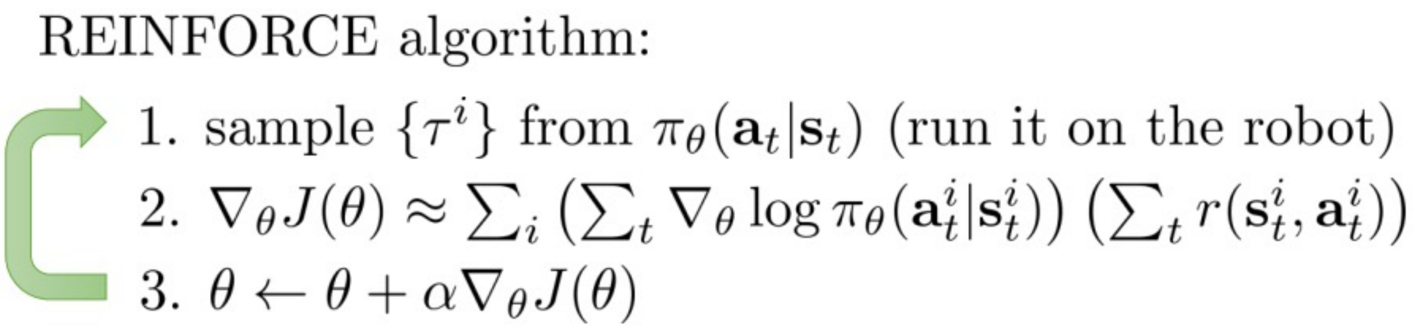


Maximum likelihood:

```
# Given:
# actions - (N*T) x Da tensor of actions
# states - (N*T) x Ds tensor of states
# Build the graph:
logits = policy.predictions(states) # This should return (N*T) x Da tensor of action logits
negative_likelihoods = tf.nn.softmax_cross_entropy_with_logits(labels=actions, logits=logits)
loss = tf.reduce_mean(negative_likelihoods)
gradients = loss.gradients(loss, variables)
```

^Standard maximum likelihood training

How do we implement this?



REINFORCE algorithm:

1. sample $\{\tau^i\}$ from $\pi_\theta(\mathbf{a}_t|\mathbf{s}_t)$ (run it on the robot)
2. $\nabla_\theta J(\theta) \approx \sum_i \left(\sum_t \nabla_\theta \log \pi_\theta(\mathbf{a}_t^i|\mathbf{s}_t^i)\right) \left(\sum_t r(\mathbf{s}_t^i, \mathbf{a}_t^i)\right)$
3. $\theta \leftarrow \theta + \alpha \nabla_\theta J(\theta)$

Policy gradient:

```
# Given:
# actions - (N*T) x Da tensor of actions
# states - (N*T) x Ds tensor of states
# q_values – (N*T) x 1 tensor of estimated state-action values
# Build the graph:
logits = policy.predictions(states) # This should return (N*T) x Da tensor of action logits
negative_likelihoods = tf.nn.softmax_cross_entropy_with_logits(labels=actions, logits=logits)
weighted_negative_likelihoods = tf.multiply(negative_likelihoods, q_values)
loss = tf.reduce_mean(weighted_negative_likelihoods)
gradients = loss.gradients(loss, variables)
```

→ Sum of rewards

Formalizes the notion of trial and error

How do we implement this?

$$\nabla_\theta \log \pi_\theta(a_t|s_t)$$

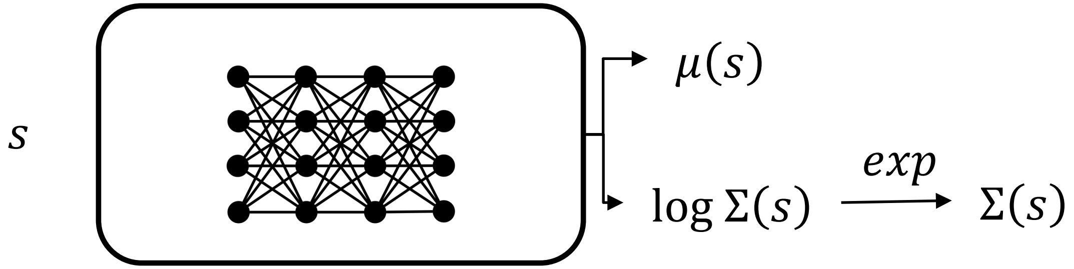
Let’s try it for a Gaussian

$$\pi(\mathbf{a} \mid \boldsymbol{s})$$

$$= \pi(\mathbf{a} \mid \boldsymbol{\mu_\theta}(\boldsymbol{s}), \boldsymbol{\Sigma_\theta}(\boldsymbol{s}))$$

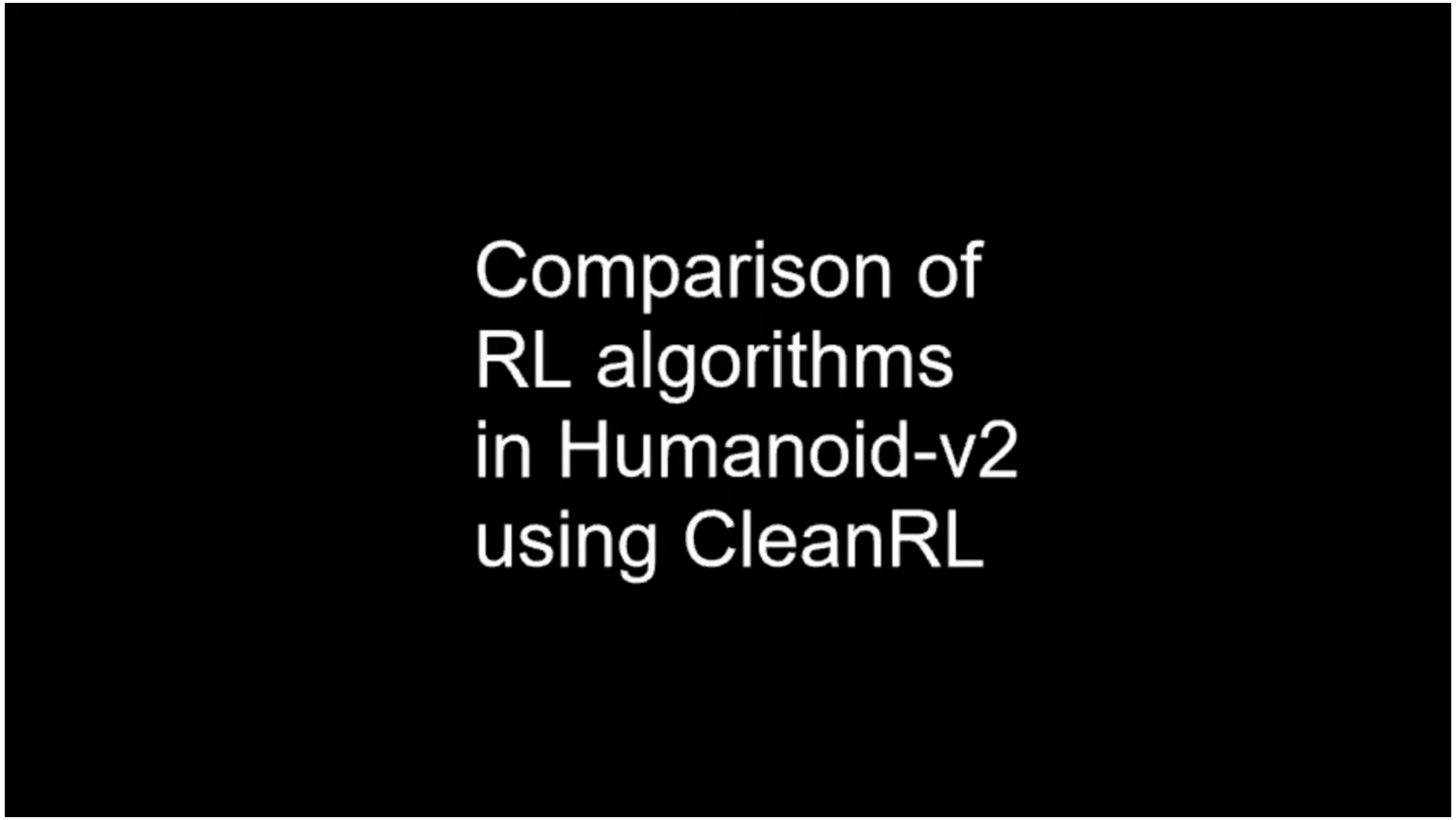
$$= \pi(\mathbf{a} \mid \boldsymbol{\mu_\theta}(\boldsymbol{s}), \boldsymbol{\Sigma_\theta}(\boldsymbol{s})) = \frac{1}{\sqrt{(2\pi)^k |\boldsymbol{\Sigma_\theta}(\boldsymbol{s})|}} \exp\left(-\frac{1}{2}(\mathbf{x} - \boldsymbol{\mu_\theta}(\boldsymbol{s}))^\top \boldsymbol{\Sigma_\theta}(\boldsymbol{s})^{-1}(\mathbf{x} - \boldsymbol{\mu_\theta}(\boldsymbol{s}))\right)$$

Similar for categorical or other distributions

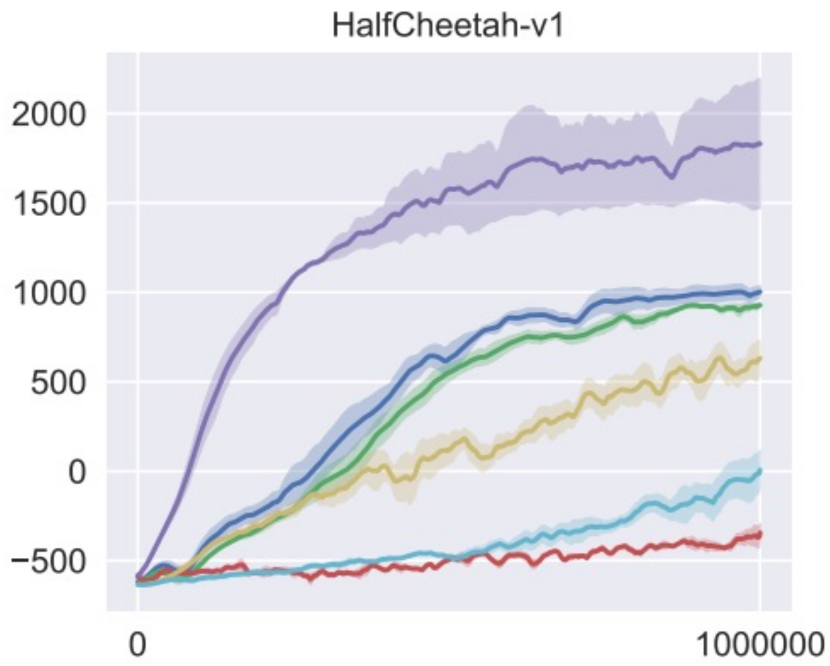


Easier for distributions where likelihood can be expressed symbolically

Does this work?



Comparison of RL algorithms in Humanoid-v2 using CleanRL



Kind of?

Lecture outline

Recap: Deriving the Policy Gradient

↓

What makes the Policy Gradient Challenging? - Variance

↓

Natural Policy Gradients and Covariant Parameterization

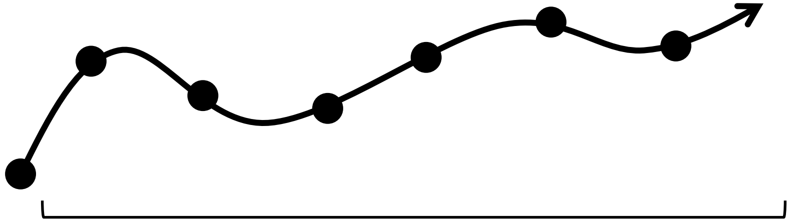
What makes policy gradient challenging?

Hard to tell what matters without many samples

$$\nabla_\theta J(\theta) = \int p_\theta(\tau)\nabla_\theta \log p_\theta(\tau)d\tau$$

$$\approx \frac{1}{N}\sum_{i=0}^{N}\sum_{t=0}^{T}\nabla_\theta \log \pi_\theta(a_t^i|s_t^i)\sum_{t'=0}^{T} r(s_{t'}^i, a_{t'}^i)$$

What we do



Single sample estimate

For every (s, a) pair, weight by only the sum of rewards in the current trajectory

Couples together all actions

Susceptible to scale variations

Susceptible to lucky samples

Makes policy gradient unstable, requires huge numbers of samples and huge batch size

What makes policy gradient challenging?

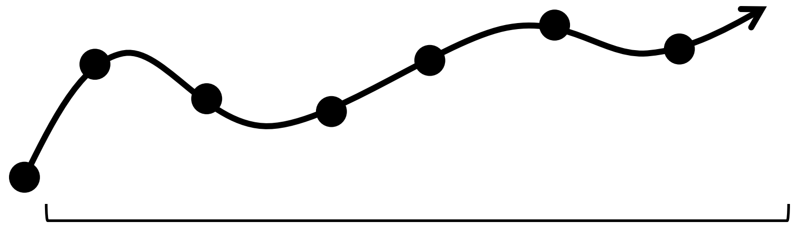
$$\nabla_\theta J(\theta) = \int p_\theta(\tau) \nabla_\theta \log p_\theta(\tau) d\tau$$

$$\approx \frac{1}{N} \sum_{i=0}^{N} \sum_{t=0}^{T} \nabla_\theta \log \pi_\theta(a_t^i | s_t^i) \sum_{t'=0}^{T} r(s_{t'}^i, a_{t'}^i)$$

High variance estimator!!

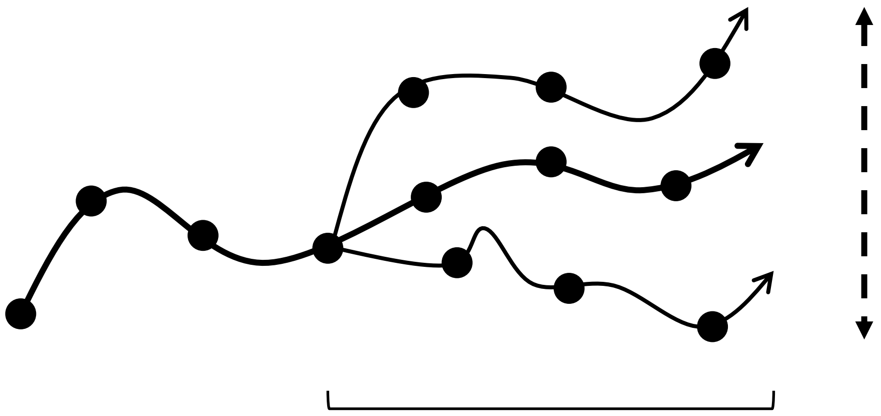
Hard to tell what matters without many samples

What we do



Single sample estimate

What we actually want



Averaged return estimate

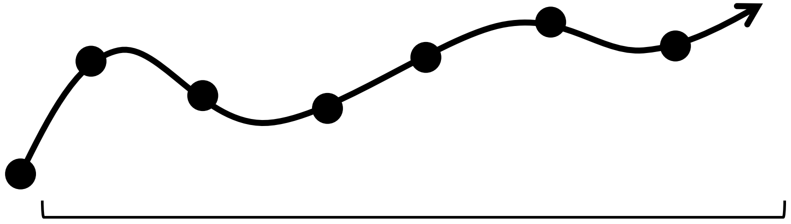
What makes policy gradient challenging?

Hard to tell what matters without many samples

$$\nabla_\theta J(\theta) = \int p_\theta(\tau) \nabla_\theta \log p_\theta(\tau) d\tau$$

$$\approx \frac{1}{N} \sum_{i=0}^{N} \sum_{t=0}^{T} \nabla_\theta \log \pi_\theta(a_t^i | s_t^i) \sum_{t'=0}^{T} r(s_{t'}^i, a_{t'}^i)$$

What we do



Single sample estimate

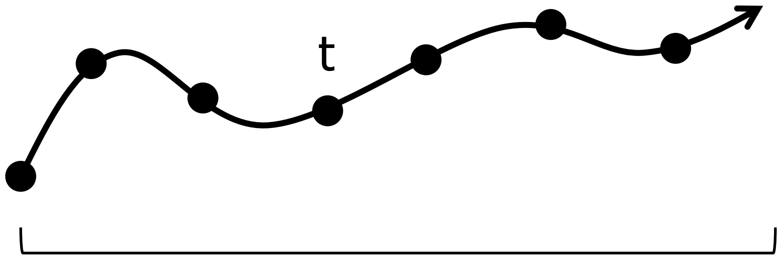
For every (s, a) pair, weight by only the sum of rewards in the current trajectory

Couples together all actions

Variance Reduction with Causality

Idea: Trajectory returns depend on past and future, but we only care about the future, since actions cannot affect the past. Instead, consider **“return-to-go”**

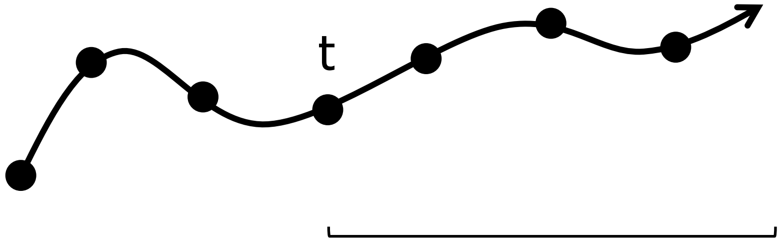
$$\approx \frac{1}{N}\sum_{i=0}^{N}\sum_{t=0}^{T}\nabla_\theta \log \pi_\theta(a_t^i|s_t^i)\underbrace{\sum_{t'=0}^{T} r(s_{t'}^i, a_{t'}^i)}_{\text{Includes t' < t}}$$



Full trajectory return

Ignore past terms ⇩

$$\frac{1}{N}\sum_{i=0}^{N}\sum_{t=0}^{T}\nabla_\theta \log \pi_\theta(a_t^i|s_t^i)\sum_{t'=t}^{T} r(s_t^i, a_t^i)$$



Return to go

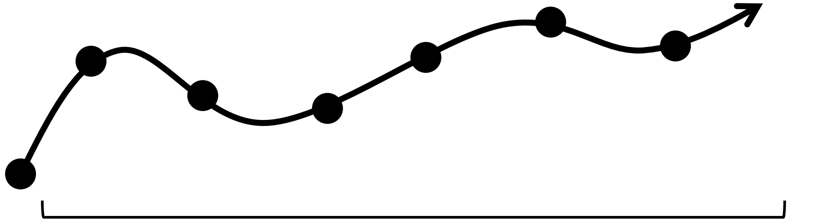
What makes policy gradient challenging?

Hard to tell what matters without many samples

$$\nabla_\theta J(\theta) = \int p_\theta(\tau) \nabla_\theta \log p_\theta(\tau) d\tau$$

$$\approx \frac{1}{N} \sum_{i=0}^{N} \sum_{t=0}^{T} \nabla_\theta \log \pi_\theta(a_t^i | s_t^i) \sum_{t'=0}^{T} r(s_{t'}^i, a_{t'}^i)$$

What we do

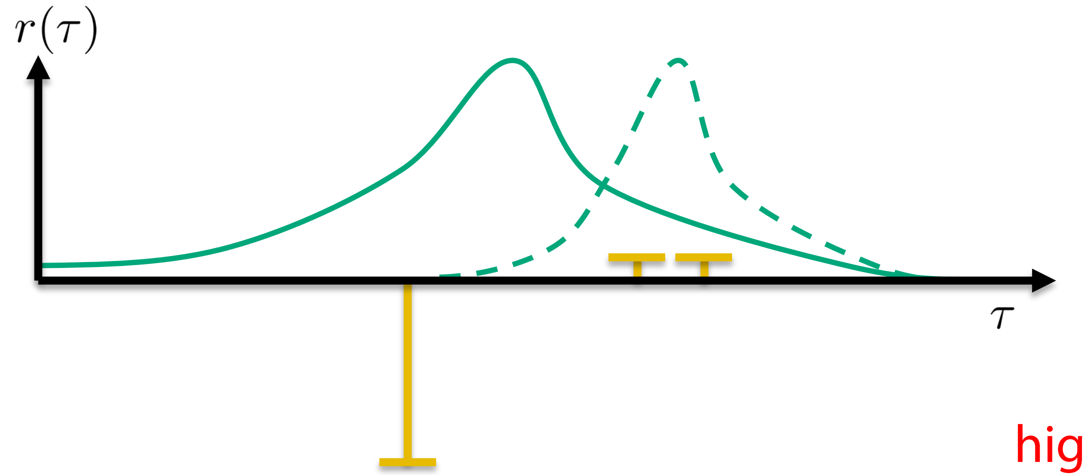


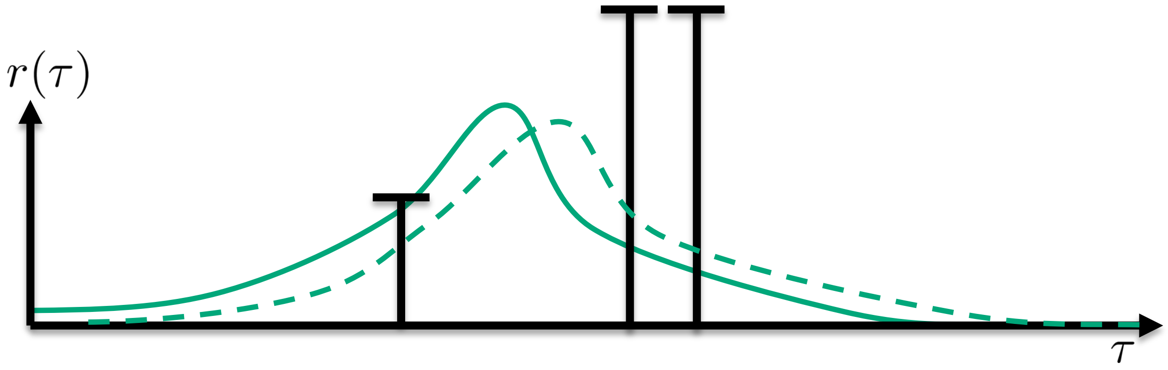
Single sample estimate

For every (s, a) pair, weight by only the sum of rewards in the current trajectory

Susceptible to scale variations

Policy gradient is susceptible to scale variations





high variance

Arbitrarily uncentered, scaled returns can lead to huge variance:

→ Imagine all rewards were positive, every action would be pushed up, some more than others

→ What if instead, we pushed down some actions and pushed up some others (even if rewards are positive)

Credit: Sergey Levine

Variance Reduction with a Baseline

Idea: We can reduce variance by subtracting a current state dependent function from the policy gradient return

$$\frac{1}{N}\sum_{i=1}^{N}\sum_{t=0}^{T}\nabla_\theta \log \pi_\theta(a_t^i|s_t^i)\left[\sum_{t'=t}^{T} r(s_{t'}^i, a_{t'}^i) - b(s_t)\right]$$

Baseline: Centers the returns, reduces variance

But does this increase bias??

Variance Reduction with a Baseline

$$\int_{\mathcal{S}} \int_{\mathcal{A}} p(s_t, a_t) \nabla_\theta \log \pi_\theta(a_t | s_t) \left[\sum_{t'=t}^{T} r(s_{t'}, a_{t'}) - b(s_t) \right] \, ds_t \, da_t$$

$$\int_{\mathcal{S}} \int_{\mathcal{A}} p(s_t, a_t) \nabla_\theta \log \pi_\theta(a_t | s_t) \left[\sum_{t'=t}^{T} r(s_{t'}, a_{t'}) \right] \, ds_t \, da_t - \boxed{\int_{\mathcal{S}} \int_{\mathcal{A}} p(s_t, a_t) \nabla_\theta \log \pi_\theta(a_t | s_t) b(s_t) \, ds_t \, da_t}$$

Let us show this is 0!

Variance Reduction with a Baseline

$$\int\int p(s_t, a_t)\nabla_\theta \log \pi_\theta(a_t|s_t)\left[b(s_t)\right] ds_t da_t = \int\int p(s_t)\pi_\theta(a_t|s_t)\nabla_\theta \log \pi_\theta(a_t|s_t)\left[b(s_t)\right] ds_t da_t$$

$$= \int p(s_t)b(s_t)\int \pi_\theta(a_t|s_t)\nabla_\theta \log \pi_\theta(a_t|s_t) da_t ds_t$$

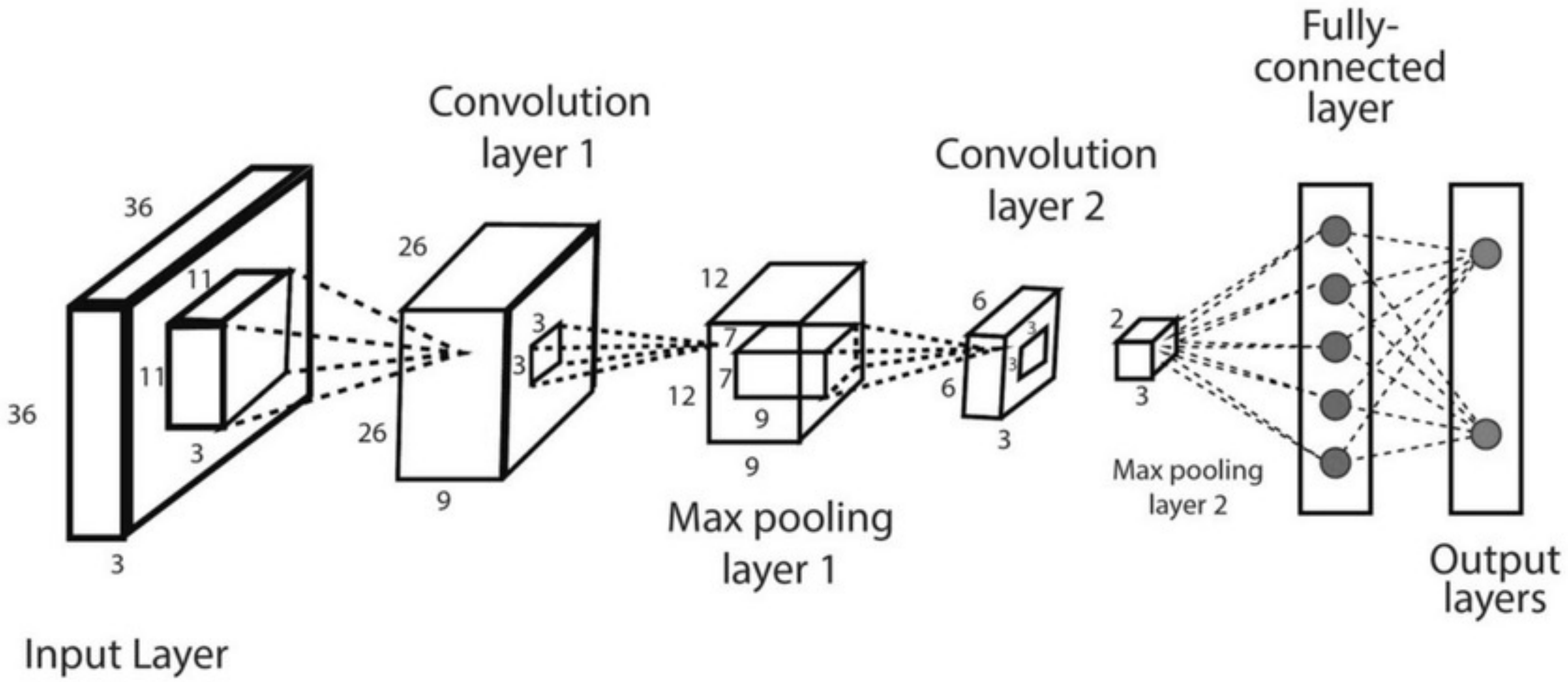
$$= \int p(s_t)b(s_t)\int \nabla_\theta \pi_\theta(a_t|s_t) da_t ds_t$$

$$= \int p(s_t)b(s_t)\nabla_\theta \int \pi_\theta(a_t|s_t) da_t ds_t = \int p(s_t)b(s_t)\nabla_\theta(1) ds_t = 0$$

Unbiased!

Learning Baselines

Baselines are typically learned as deep neural nets from R^s → R^1



$$\arg\min_{\hat{V}} \frac{1}{N}\sum_{j=1}^{N} \|\hat{V}(s_t^j) - \sum_{t=1}^{H} r(s_t^j, a_t^j)\|$$

$$\nabla_\theta J(\theta) = \mathbb{E}_\pi\left[\sum_{t=0}^{T} \nabla_\theta \log \pi_\theta(a_t|s_t)\left(\sum_{t'=t}^{T} r(s_{t'}, a_{t'}) - \hat{V}(s_t)\right)\right]$$

Minimize with Monte-Carlo regression at every iteration, club with policy gradient

Why do baselines really reduce variance?

Let’s define variance: $\text{Var}[x] = E[x^2] - E[x]^2$

$$\nabla_\theta J(\theta) = E_{\tau \sim p_\theta(\tau)}[\nabla_\theta \log p_\theta(\tau)(r(\tau) - b)]$$

Whiteboard

Lecture outline

Recap: Deriving the Policy Gradient

↓

What makes the Policy Gradient Challenging? - Variance

↓

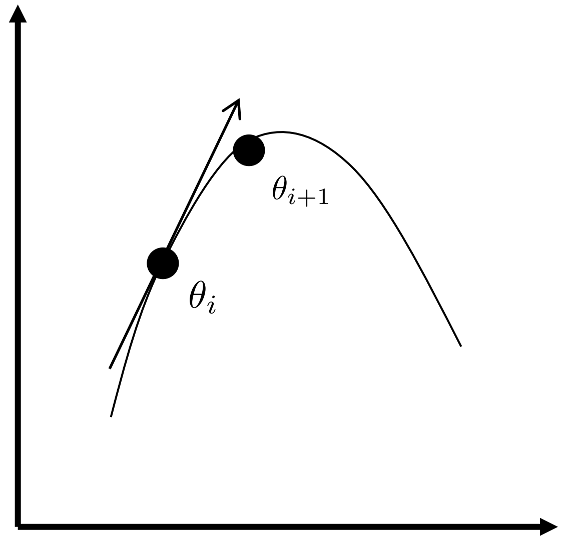
Natural Policy Gradients and Covariant Parameterization

Take a deeper look at REINFORCE

$$\nabla_\theta J(\theta) = \int p_\theta(\tau)\nabla_\theta \log p_\theta(\tau) d\tau \approx \frac{1}{N}\sum_{i=0}^{N}\sum_{t=0}^{T}\nabla_\theta \log \pi_\theta(a_t^i|s_t^i)\sum_{t'=0}^{T} r(s_{t'}^i, a_{t'}^i)$$

Gradient ascent is steepest ascent on linear approximation under the Euclidean metric!

$$\max_\theta \mathbb{E}_{\tau\sim\pi_\theta}\left[\sum_{t=0}^{T} r(s_t, a_t)\right]$$
$$= J(\theta)$$



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Gradient ascent is steepest ascent on linear approximation under the Euclidean metric!

$$\max \quad J(\theta_i) + \nabla_\theta J(\theta)|_{\theta=\theta_i} (\theta - \theta_i)$$

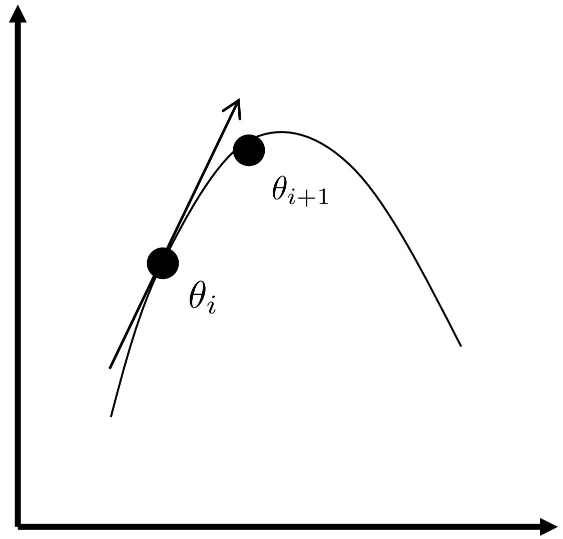
Linear approximation

$$(\theta - \theta_i)^T (\theta - \theta_i) \leq \epsilon$$

Quadratic Constraint

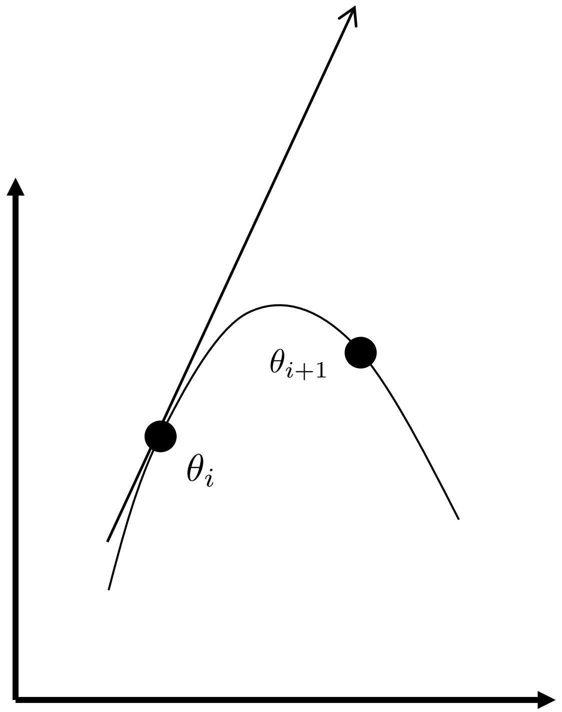
$$\downarrow$$

$$\theta = \theta_i + \alpha \nabla_\theta J(\theta)|_{\theta=\theta_i}$$



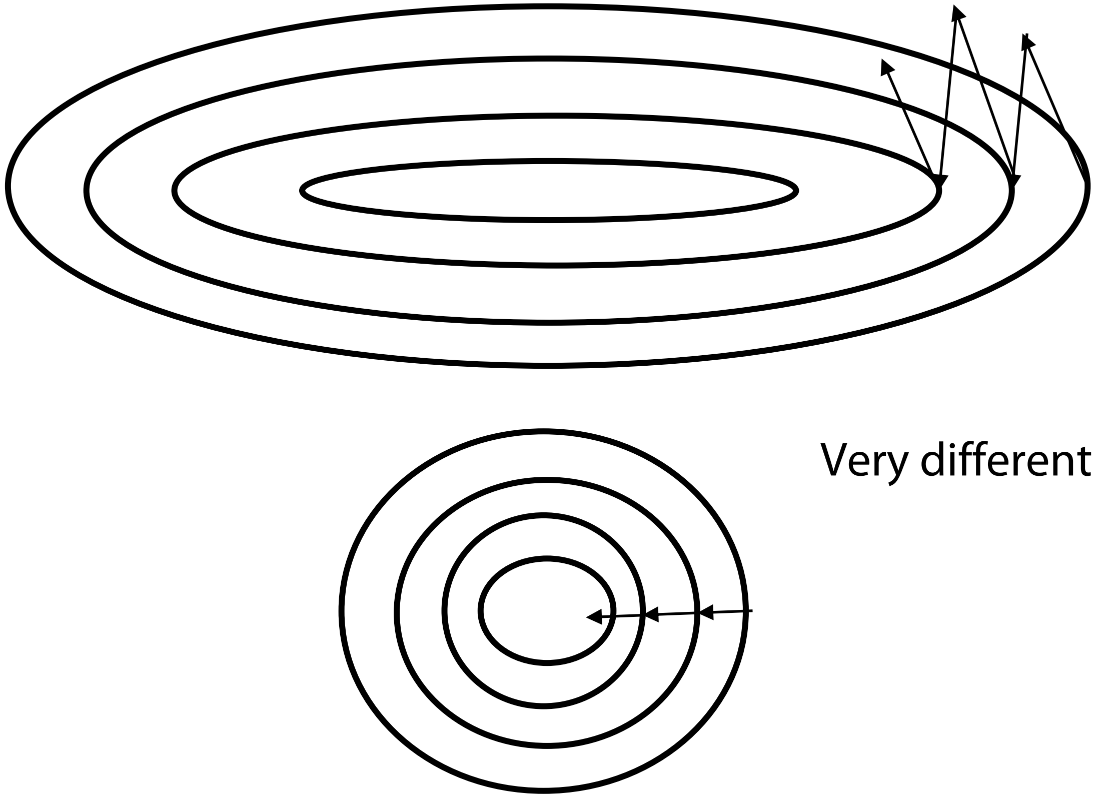
When might this fail?

Large step sizes may cause collapse



Must use very small step sizes, slow!

Sensitive to Policy Parameterization



Can struggle for a deep neural network!

Parameterization dependence of PG

Sensitive to Policy Parameterization

$$L(\theta) = \theta_1 + \theta_2$$

$$L(\phi) = \phi_1^{0.5} + \phi_2^{-1}$$

$$\phi_1 = \theta_1^2$$

$$\phi_2 = \theta_2^{-1}$$

$$\nabla_{\theta_1} L = 1$$

$$\nabla_{\theta_2} L = 1$$

$$\longleftrightarrow$$

Not covariant!

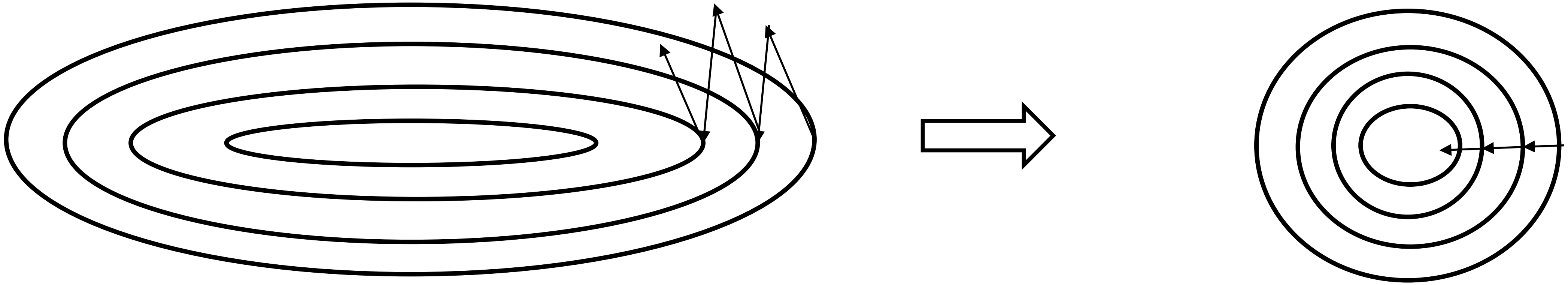
$$\nabla_{\phi_1} L = 0.5\phi_1^{-0.5} = 0.5\theta_1^{-1}$$

$$\nabla_{\phi_2} L = -\phi_2^{-2} = -\theta_2^2$$

Modified Constraint on Policy Gradient

$$\begin{aligned} \max \quad & J(\theta_i) + \nabla_\theta J(\theta)|_{\theta=\theta_i}(\theta - \theta_i) \\ & (\theta - \theta_i)^T(\theta - \theta_i) \leq \epsilon \end{aligned}$$

$$\begin{aligned} \max \quad & J(\theta_i) + \nabla_\theta J(\theta)|_{\theta=\theta_i}(\theta - \theta_i) \\ & (\theta - \theta_i)^T G(\theta - \theta_i) \leq \epsilon \end{aligned}$$



$$\theta_{i+1} = \theta_i + \alpha G^{-1} \nabla_\theta J(\theta)|_{\theta=\theta_i}$$

Rescales according to G^{-1}

Adaptive choice of G can avoid sensitivity to policy parameterization!

Covariant Policy Gradient Updates

$$\begin{aligned}\max \quad & J(\theta_i) + \nabla_\theta J(\theta)|_{\theta=\theta_i}(\theta - \theta_i) \\ & (\theta - \theta_i)^T G(\theta - \theta_i) \leq \epsilon\end{aligned}$$

What should G be?

$$\begin{aligned}\max \quad & J(\theta_i) + \nabla_\theta J(\theta)|_{\theta=\theta_i}(\theta - \theta_i) \\ & D_{\mathrm{KL}}(\pi_\theta || \pi_{\theta_i}) \leq \epsilon\end{aligned}$$

Let us use the constraint as KL divergence on the policy (2nd order Taylor expansion)

Measures functional distance, not parameter distance

Resulting "Natural" Policy Gradient

$$\max \; J(\theta_i) + \nabla_\theta J(\theta)|_{\theta=\theta_i}(\theta - \theta_i)$$

$$D_{\mathrm{KL}}(\pi_\theta || \pi_{\theta_i}) \leq \epsilon$$

2nd order approximation of KL → Fisher Information Metric

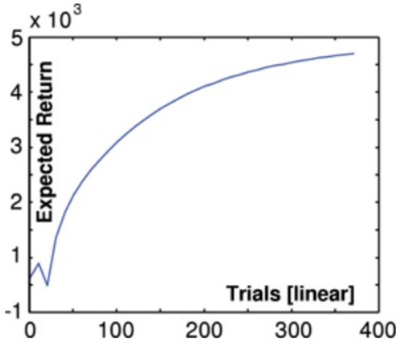
$$F = \mathbb{E}_{\pi_\theta}\left[(\nabla_\theta \log \pi_\theta)(\nabla_\theta \log \pi_\theta)^T\right]$$

$$\max \; J(\theta_i) + \nabla_\theta J(\theta)|_{\theta=\theta_i}(\theta - \theta_i)$$

$$(\theta - \theta_i)^T F(\theta - \theta_i) \leq \epsilon$$

Resulting update $\theta_{i+1} = \theta_i + \alpha F^{-1} \nabla_\theta J(\theta)|_{\theta=\theta_i}$ Covariant to parameterization

Natural Policy Gradient in Action

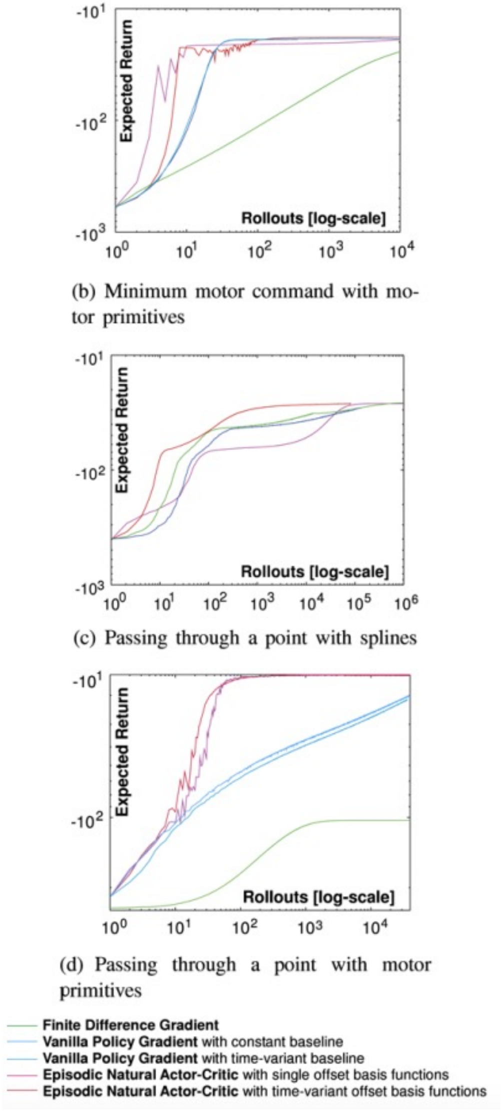


(a) Performance.

(b) Imitation learning.

(c) Initial reproduction.

(d) After reinforcement learning.



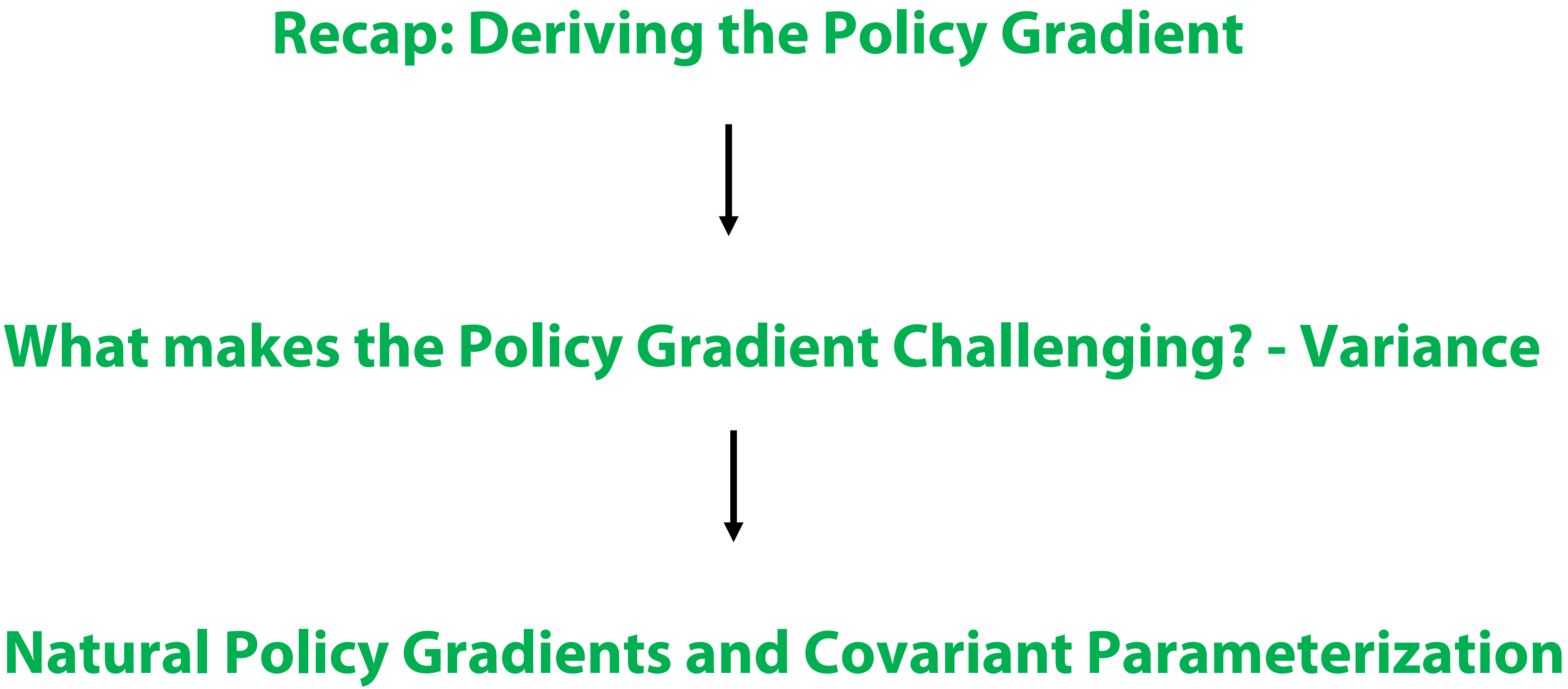
(b) Minimum motor command with motor primitives

(c) Passing through a point with splines

(d) Passing through a point with motor primitives

Peters, Schaal '08

Lecture outline



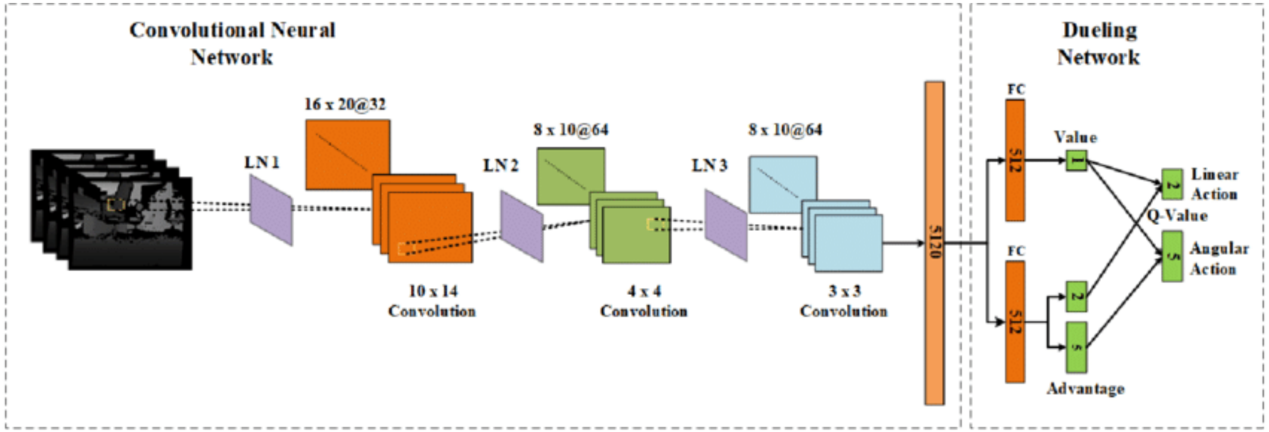
Recap: Deriving the Policy Gradient

What makes the Policy Gradient Challenging? - Variance

Natural Policy Gradients and Covariant Parameterization

Natural Policy Gradient - is it enough?

Huge matrix inversion

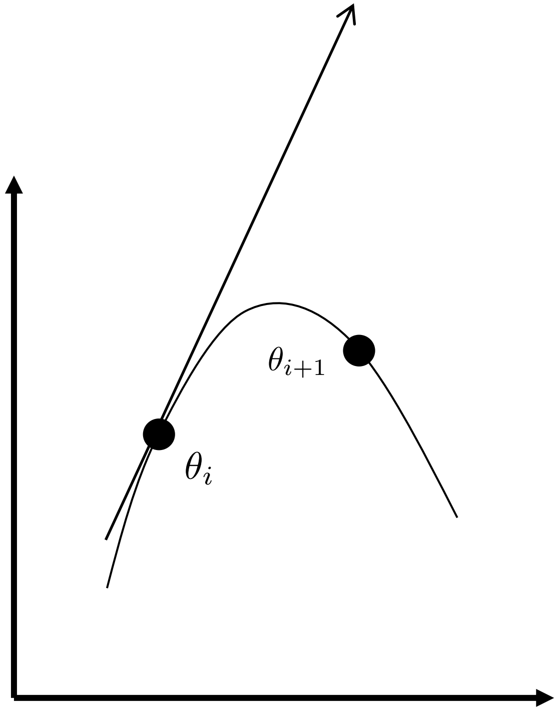


F – R^{dxd}

For a standard convnet – d is in the millions

Hessian is way out of memory / hard to invert!

Step-size?



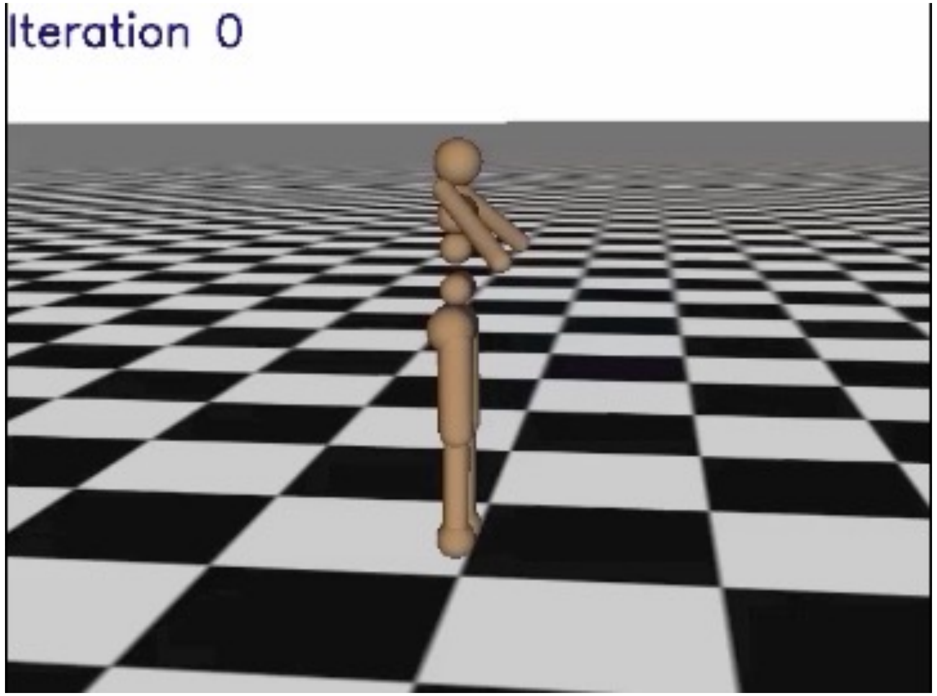
Can easily overstep and collapse performance

Also, only a single gradient step at a time before recollecting data!

Trust Region Policy Optimization

3 key ideas:

1. On-policy updates → importance sampled objective
2. Huge matrix inversion → conjugate gradient method
3. Step size may be too large → backtracking line search



TRPO, Schulman ‘15

Trust Region Policy Optimization

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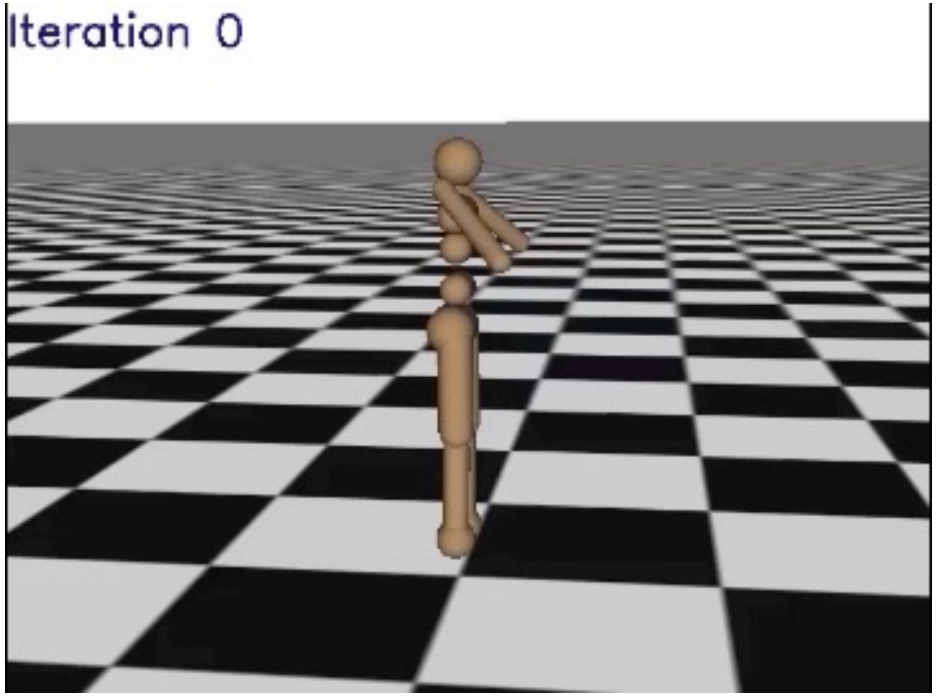

TRPO, Schulman '15

Trust Region Policy Optimization – Importance Sampling

Cannot evaluate without resampling

Original Objective

$$J(\theta) = \mathbb{E}_{s \sim d_\pi(\theta), a \sim \theta} \left[A(s,a) \right]$$

$$= \mathbb{E}_{s \sim d_\pi(\theta), a \sim \theta_i} \left[\frac{\pi_\theta}{\pi_{\theta_i}} A(s,a) \right]$$

Importance Sampling (ish)

$$\approx \mathbb{E}_{s \sim d_\pi(\theta_i), a \sim \theta_i} \left[\frac{\pi_\theta}{\pi_{\theta_i}} A(s,a) \right]$$

$\uparrow$

If policies are close, we can show that this is not so bad!

Trust Region Policy Optimization

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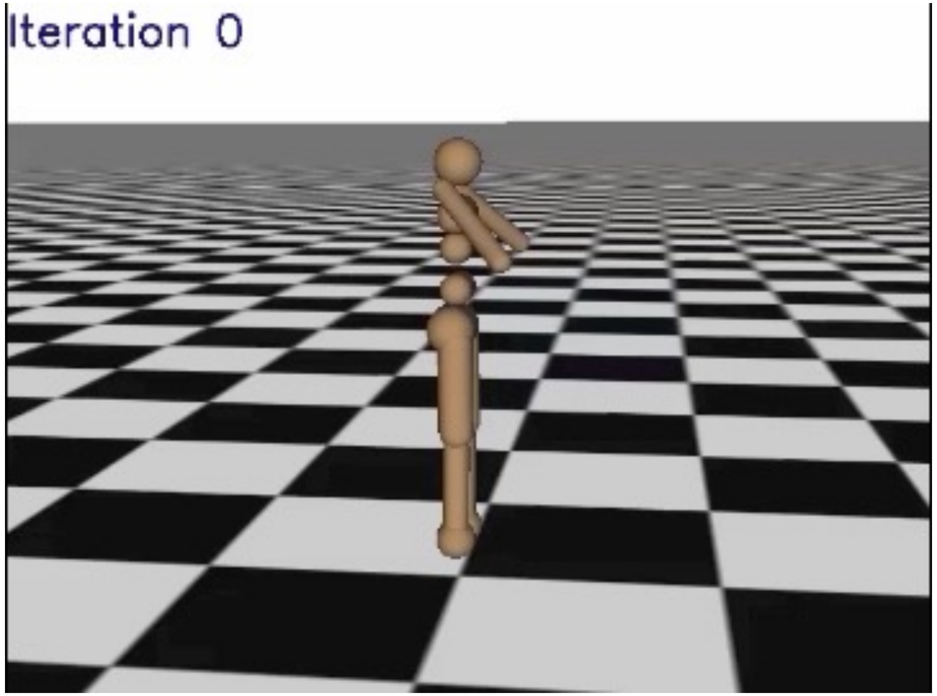

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Trust Region Policy Optimization – Conjugate Gradient

Challenging to compute F^{-1} and then get $F^{-1}g$

⇩

Convert into an iterative minimization problem!

Solution to

$$Fx = g$$

same as

Solution to

$$\min_x \frac{1}{2} x^T F x - x^T g + c$$

Trust Region Policy Optimization – Conjugate Gradient

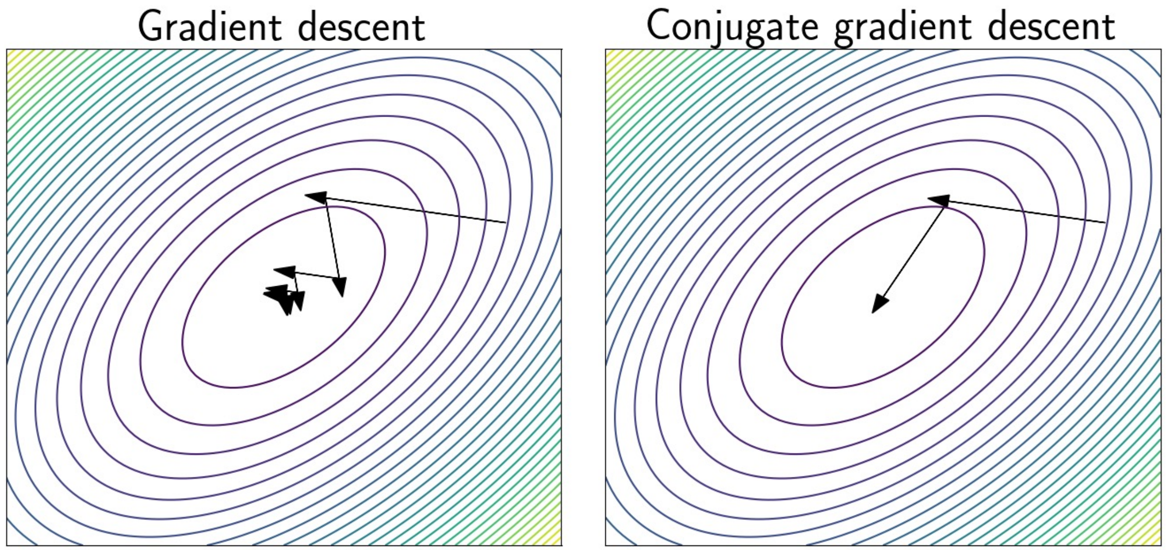
Challenging to compute F^{-1} and then get $F^{-1}g$

⇩

Convert into an iterative minimization problem!

⇩

Solve with conjugate gradient



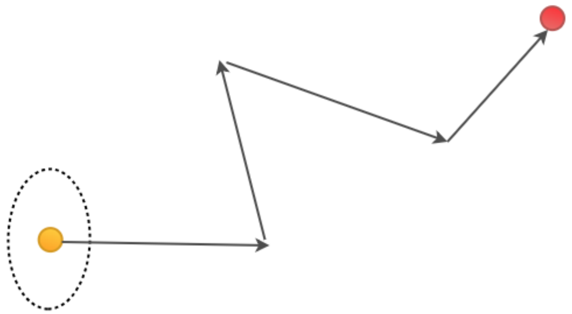
Gradient descent

Conjugate gradient descent

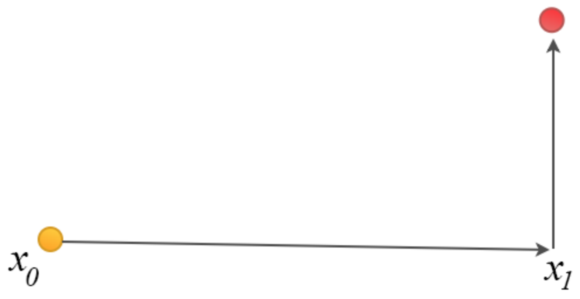
Do coordinate descent in geometry aligned orthogonal directions

https://gregorygundersen.com/blog/2022/03/20/conjugate-gradient-descent/

Trust Region Policy Optimization – Conjugate Gradient



Gradient ascent

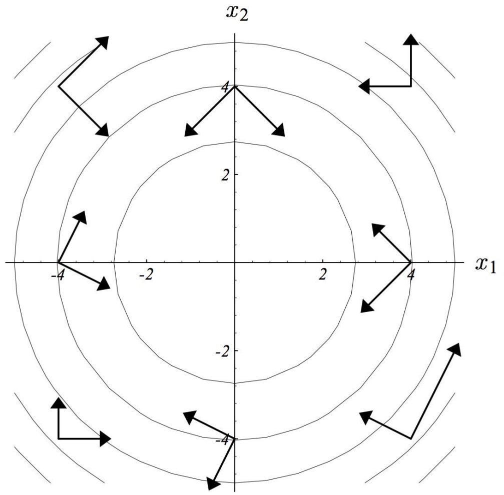


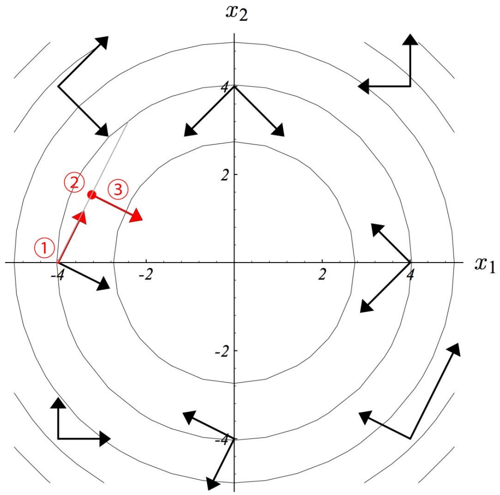
Conjugate gradient

$$\min_x \frac{1}{2} x^T F x - x^T g + c$$

Find search directions at ever step that are F-orthogonal with previous directions

$$d_{(i)}^T F d_{(i)} = 0$$





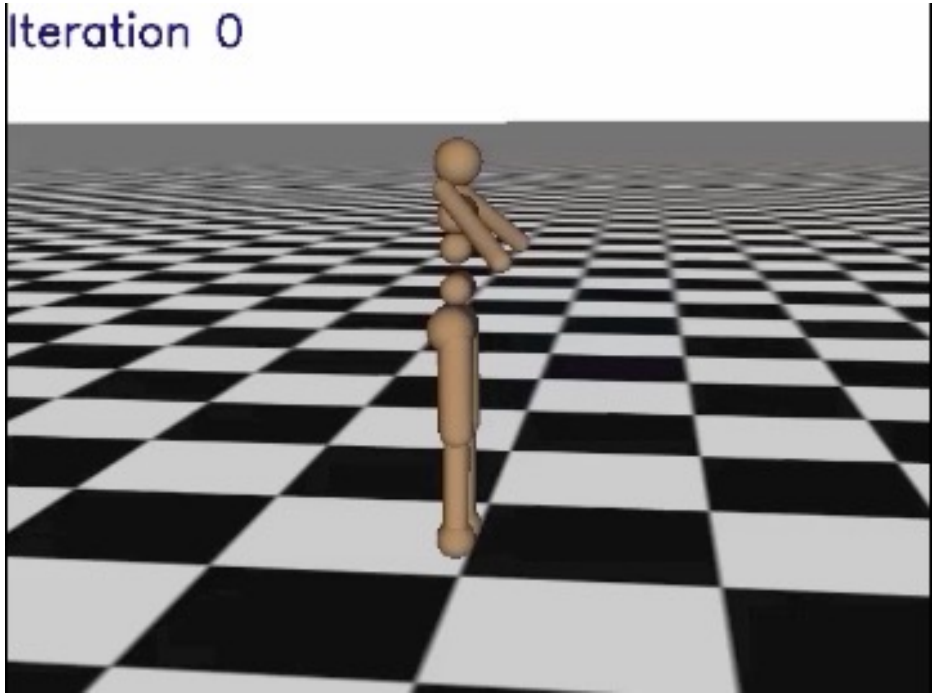
Converges in approx N steps!

Only requires matrix-vector product

Trust Region Policy Optimization

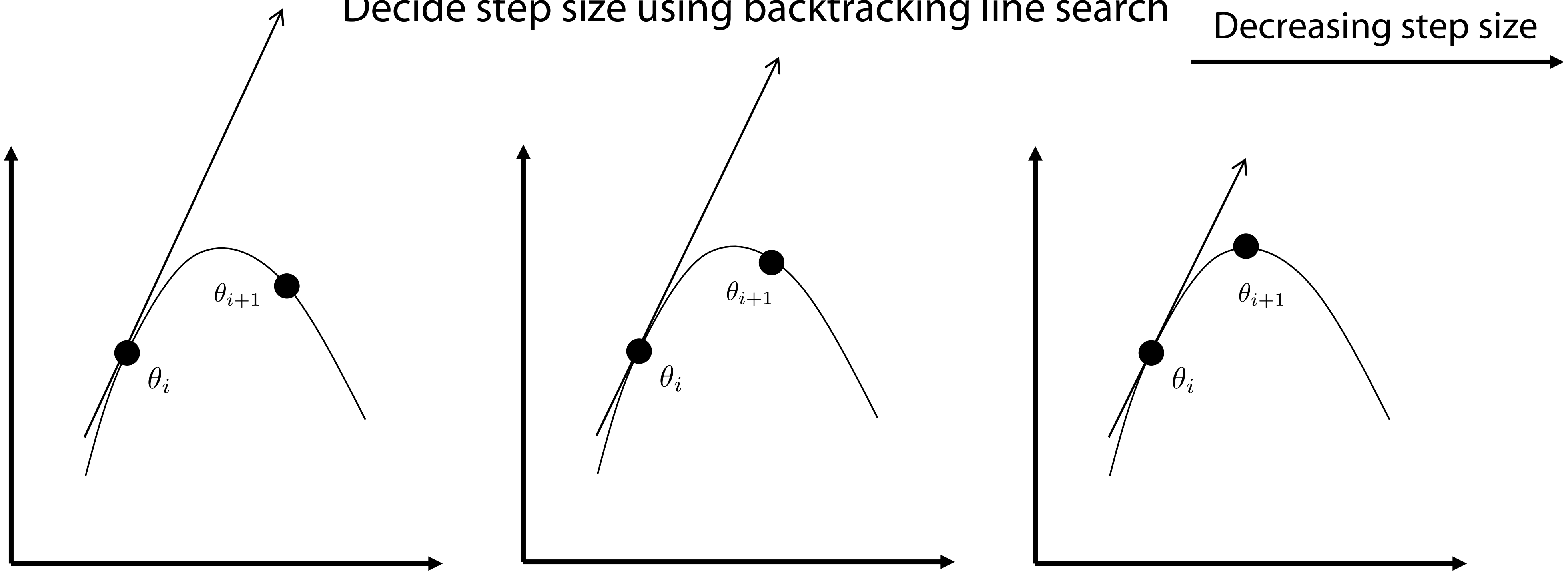
3 key ideas:

1. On-policy updates → importance sampled objective
2. Huge matrix inversion → conjugate gradient method
3. Step size may be too large → backtracking line search



TRPO, Schulman '15

Trust Region Policy Optimization – Backtracking line search



Decide step size using backtracking line search

Decreasing step size

1. Choose parameter $\beta \in (0, 1)$, given search direction s = F^{-1}g
2. Compute maximal step size such that constraint is satisfied - $\frac{1}{2}(ts)^T F(ts) = \epsilon \rightarrow t = \sqrt{\frac{2\epsilon}{s^T F s}}$
3. While $J(\theta_i + ts) < J(\theta_i)$, set $t = \beta t$

Backtracking

Trust Region Policy Optimization

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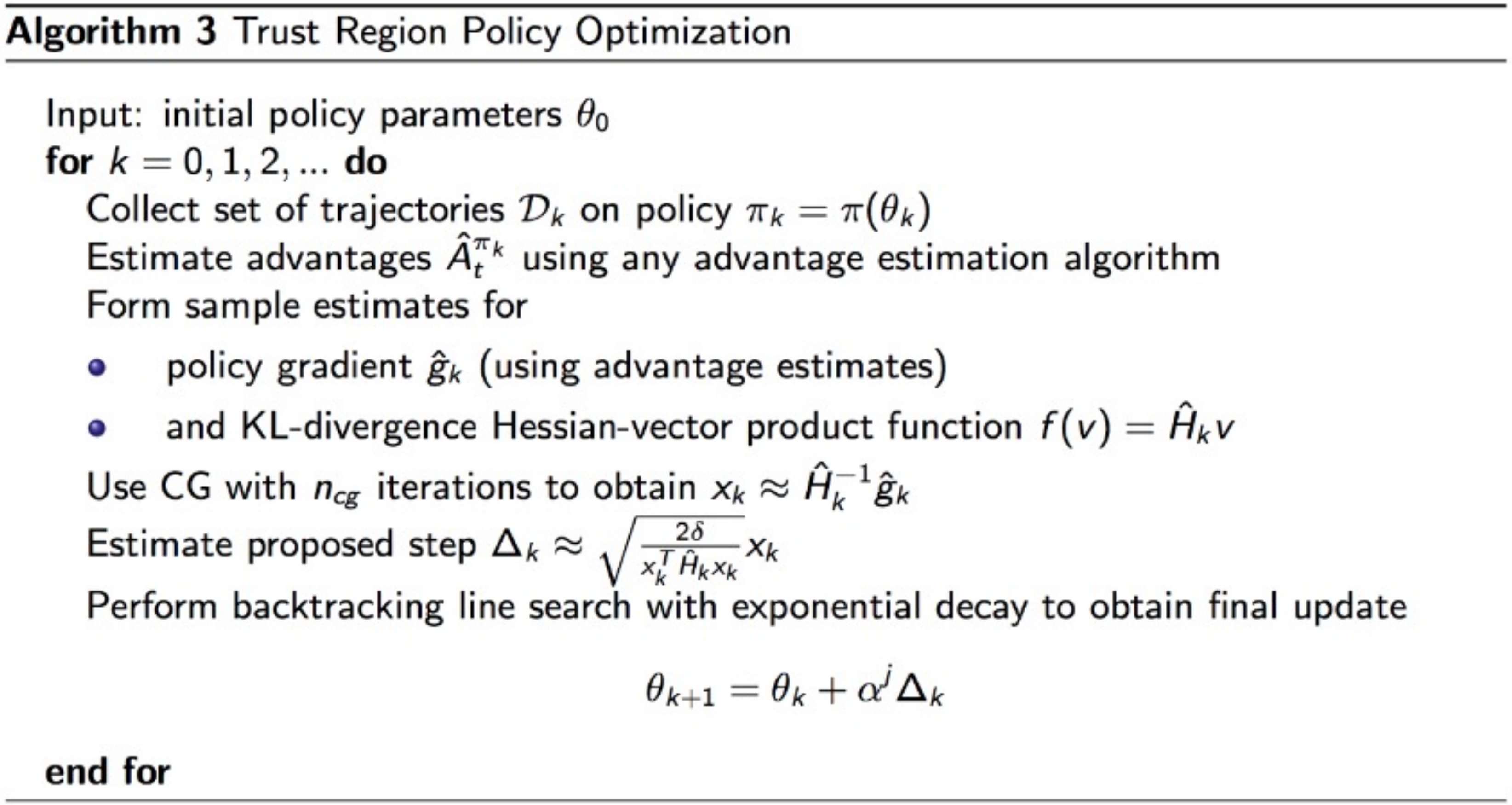


Algorithm 3 Trust Region Policy Optimization

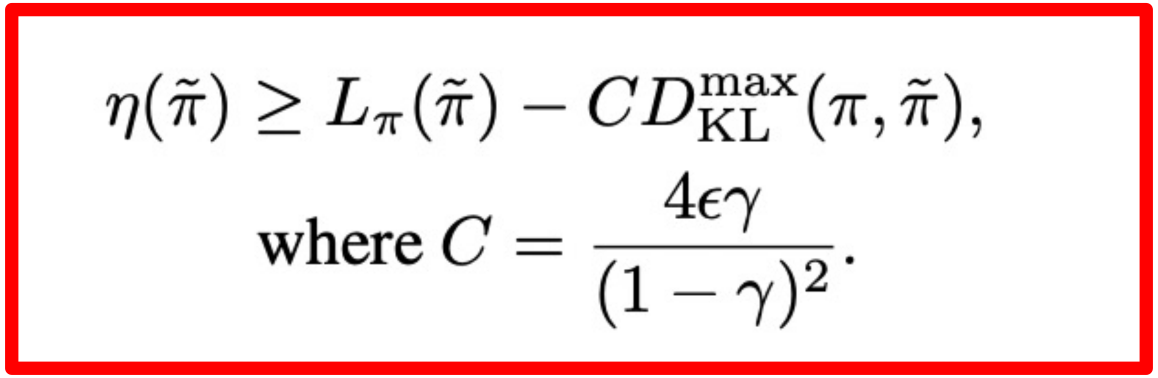
```
Input: initial policy parameters $\theta_0$
for $k = 0, 1, 2, ...$ do
    Collect set of trajectories $\mathcal{D}_k$ on policy $\pi_k = \pi(\theta_k)$
    Estimate advantages $\hat{A}_t^{\pi_k}$ using any advantage estimation algorithm
    Form sample estimates for
    •   policy gradient $\hat{g}_k$ (using advantage estimates)
    •   and KL-divergence Hessian-vector product function $f(v) = \hat{H}_k v$
    Use CG with $n_{cg}$ iterations to obtain $x_k \approx \hat{H}_k^{-1}\hat{g}_k$
    Estimate proposed step $\Delta_k \approx \sqrt{\frac{2\delta}{x_k^T \hat{H}_k x_k}} x_k$
    Perform backtracking line search with exponential decay to obtain final update

        $\theta_{k+1} = \theta_k + \alpha^j \Delta_k$

end for
```

TRPO, Schulman '15

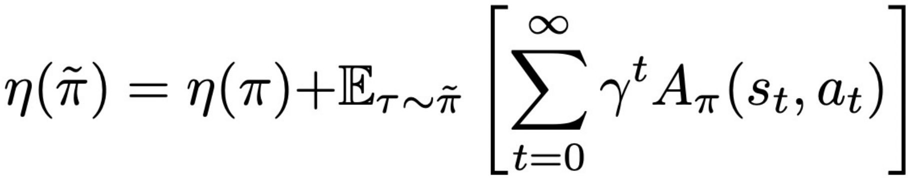
Can we say anything formal about updates?



$$\boxed{\begin{gathered}\eta(\tilde{\pi}) \geq L_\pi(\tilde{\pi}) - C D_{\mathrm{KL}}^{\max}(\pi, \tilde{\pi}),\\ \text{where } C = \frac{4\epsilon\gamma}{(1-\gamma)^2}.\end{gathered}}$$

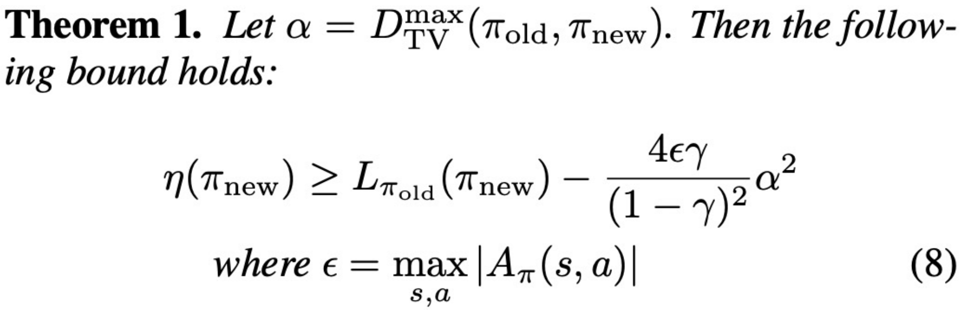
Ensures that policies are non-decreasing in performance

Performance difference
lemma



$$\eta(\tilde{\pi}) = \eta(\pi) + \mathbb{E}_{\tau\sim\tilde{\pi}}\left[\sum_{t=0}^{\infty}\gamma^t A_\pi(s_t, a_t)\right] \Rightarrow$$

Express advantage
in terms of TVD



Theorem 1. *Let* $\alpha = D_{\mathrm{TV}}^{\max}(\pi_{\mathrm{old}}, \pi_{\mathrm{new}})$. *Then the following bound holds:*

$$\begin{aligned}\eta(\pi_{\mathrm{new}}) &\geq L_{\pi_{\mathrm{old}}}(\pi_{\mathrm{new}}) - \frac{4\epsilon\gamma}{(1-\gamma)^2}\alpha^2\\ \textit{where } \epsilon &= \max_{s,a}|A_\pi(s,a)|\end{aligned} \qquad (8)$$

Key idea: by bounding how different the policies are, we can bound how different returns are

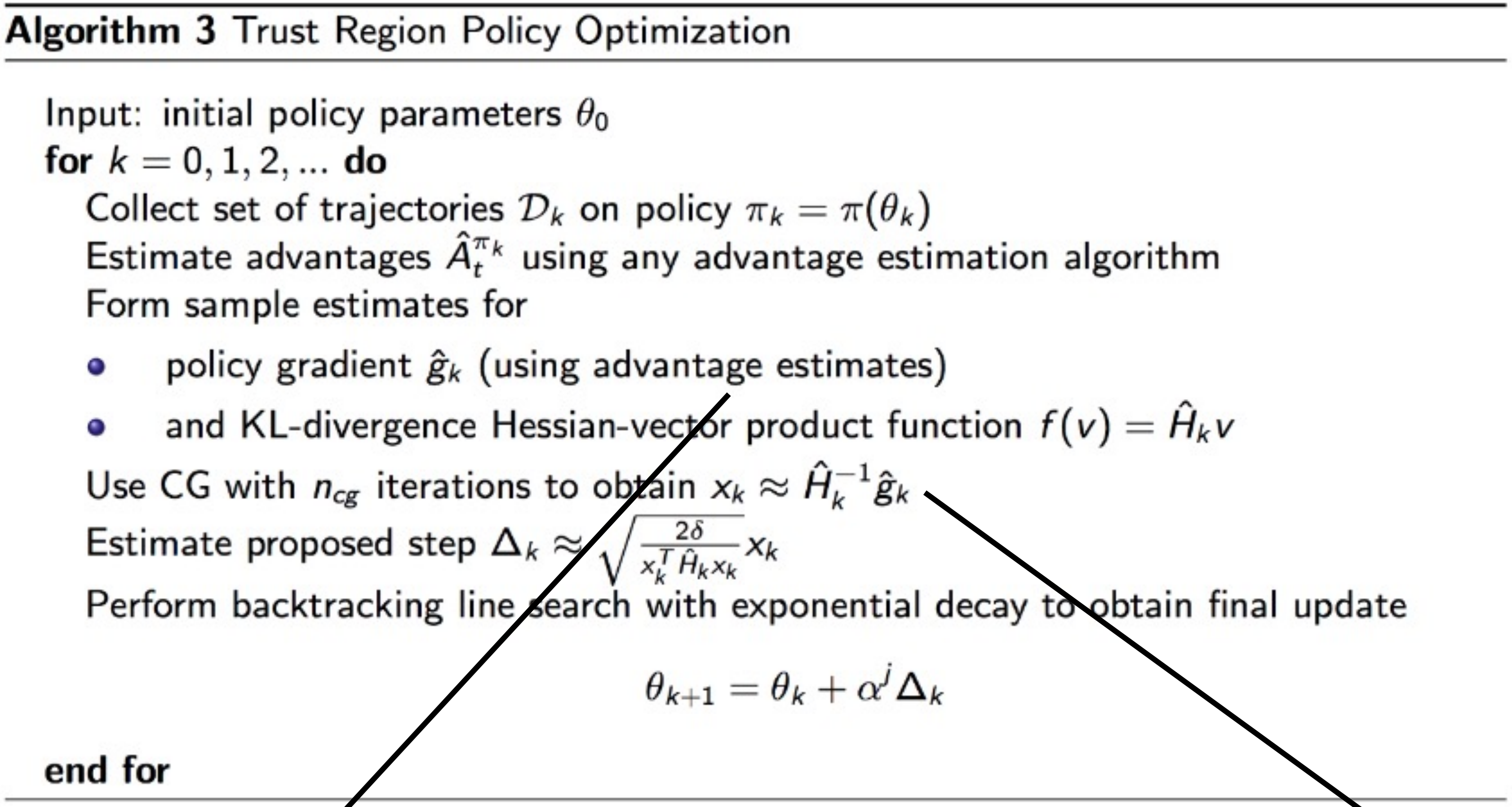
TRPO, Schulman ‘15

TRPO in action

Trust Region Policy Optimization

TRPO, Schulman '15

Why might TRPO not be enough?



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    Perform backtracking line search with exponential decay to obtain final update
        $$\theta_{k+1} = \theta_k + \alpha^j \Delta_k$$
end for
```

Advantage estimation is too high variance

Optimization expensive/unstable

TRPO, Schulman '15

Better Advantage Estimation - Generalized Advantage Estimation

Advantage estimator $A^{\theta}_{N}(s_1, a_1) = r_1 + \gamma r_2 + \cdots + \gamma^{N-1} r_N - V(s_1)$

High variance!

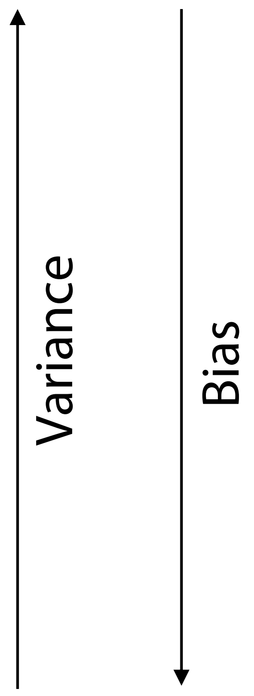
N step advantage estimator $A^{\theta}_{N}(s_1, a_1) = r_1 + \gamma r_2 + \cdots + \gamma^{N-1} r_N - V(s_1)$

N-1 step advantage estimator $A^{\theta}_{N-1}(s_1, a_1) = r_1 + \gamma r_2 + \cdots + \gamma^{N-2} V(s_{N-1}) - V(s_1)$

⋮

2 step advantage estimator $A^{\theta}_{2}(s_1, a_1) = r_1 + \gamma r_2 + \cdots + \gamma^{2} V(s_3) - V(s_1)$

1 step advantage estimator $A^{\theta}_{1}(s_1, a_1) = r_1 + \gamma V(s_2) - V(s_1)$



Variance

Bias

GAE, Schulman ‘15

Generalized Advantage Estimation

Sum up all the estimators in a geometric sum

$$\left.\begin{aligned}
A^{\theta}_{N}(s_1, a_1) &= r_1 + \gamma r_2 + \cdots + \gamma^{N-1} r_N - V(s_1) \\
A^{\theta}_{N-1}(s_1, a_1) &= r_1 + \gamma r_2 + \cdots + \gamma^{N-2} V(s_{N-1}) - V(s_1) \\
\\
\\
A^{\theta}_{2}(s_1, a_1) &= r_1 + \gamma r_2 + \cdots + \gamma^{2} V(s_3) - V(s_1) \\
A^{\theta}_{1}(s_1, a_1) &= r_1 + \gamma V(s_2) - V(s_1)
\end{aligned}\right\}$$

Geometric sum

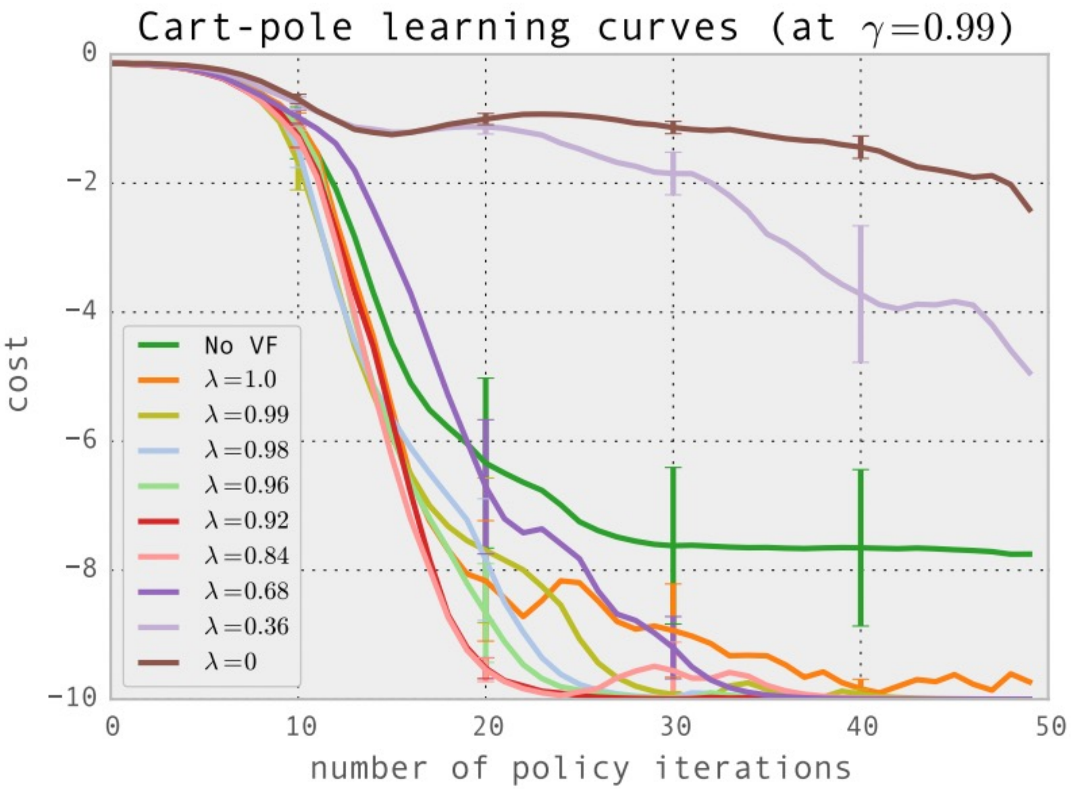
$$A^{\theta}_{\lambda}(s_1, a_1) = \sum_{j=1}^{N} \lambda^j A^{\theta}_{j}(s, a)$$

λ controls bias-variance tradeoff

Best of both worlds – very similar idea to eligibility traces

GAE, Schulman '15

Generalized Advantage Estimation in Action



GAE, Schulman ‘15

Fin.