Stereo II

CSE 576

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Several slides from Larry Zitnick and Steve Seitz

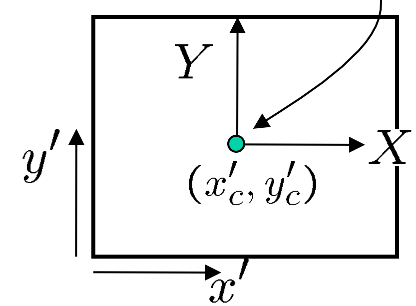
Camera parameters

A camera is described by several parameters

- Translation T of the optical center from the origin of world coords
- Rotation R of the image plane
- focal length f, principle point (x'_c, y'_c), pixel size (s_x, s_y)
- blue parameters are called "extrinsics," red are "intrinsics"

Projection equation

$$\mathbf{x} = \begin{bmatrix} wx \\ wy \\ w \end{bmatrix} = \begin{bmatrix} * & * & * & * \\ * & * & * & * \\ * & * & * & * \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} = \mathbf{\Pi X}$$



- The projection matrix models the cumulative effect of all parameters
- Useful to decompose into a series of operations

identity matrix

$$\mathbf{\Pi} = \underbrace{\begin{bmatrix} -fs_x & 0 & x'_c \\ 0 & -fs_y & y'_c \\ 0 & 0 & 1 \end{bmatrix}}_{\text{intrinsics}} \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}}_{\text{projection}} \underbrace{\begin{bmatrix} \mathbf{R}_{3x3} & \mathbf{0}_{3x1} \\ \mathbf{0}_{1x3} & 1 \end{bmatrix}}_{\text{rotation}} \underbrace{\begin{bmatrix} \mathbf{I}_{3x3} & \mathbf{T}_{3x1} \\ \mathbf{0}_{1x3} & 1 \end{bmatrix}}_{\text{translation}}$$

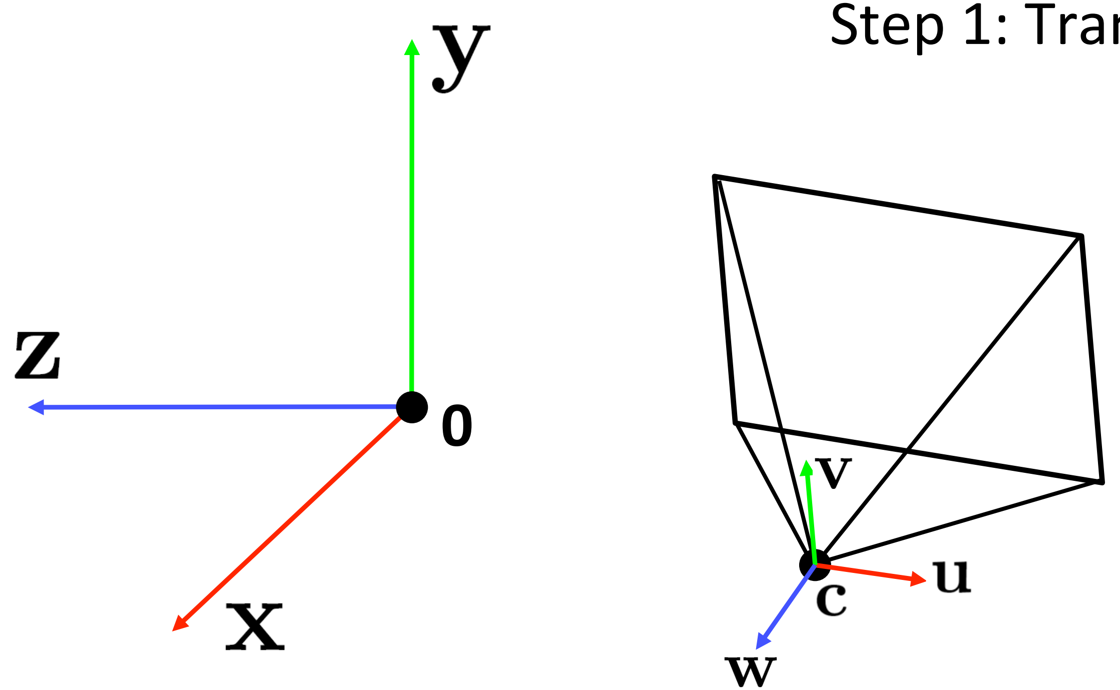
- The definitions of these parameters are not completely standardized
  - especially intrinsics—varies from one book to another

Extrinsics

How do we get the camera to “canonical form”?

- (Center of projection at the origin, x-axis points right, y-axis points up, z-axis points backwards)

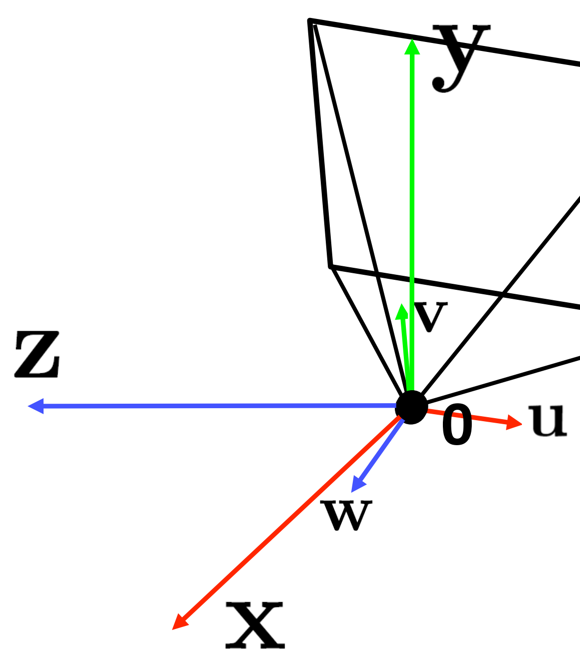
Step 1: Translate by -**c**



Extrinsics

How do we get the camera to "canonical form"?

- (Center of projection at the origin, x-axis points right, y-axis points up, z-axis points backwards)



Step 1: Translate by -**c**

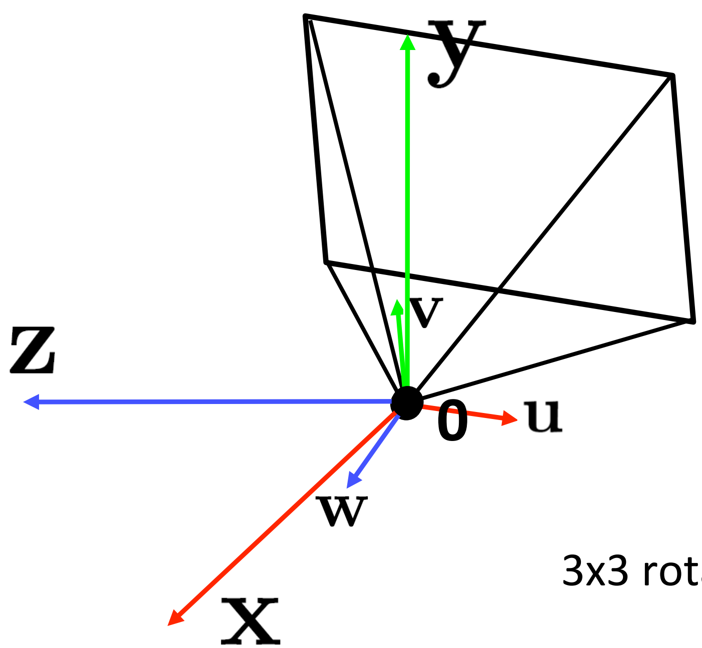
How do we represent translation as a matrix multiplication?

$$\mathbf{T} = \begin{bmatrix} \mathbf{I}_{3\times 3} & & & -\mathbf{c} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Extrinsics

How do we get the camera to "canonical form"?

- (Center of projection at the origin, x-axis points right, y-axis points up, z-axis points backwards)



Step 1: Translate by -**c**

Step 2: Rotate by **R**

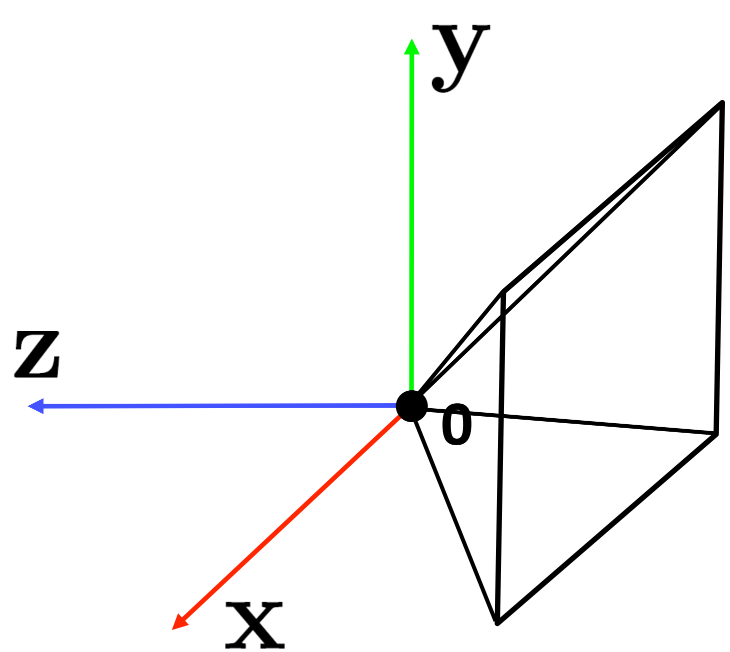
$$\mathbf{R} = \begin{bmatrix} \mathbf{u}^T \\ \mathbf{v}^T \\ \mathbf{w}^T \end{bmatrix}$$

3x3 rotation matrix

Extrinsics

How do we get the camera to "canonical form"?

- (Center of projection at the origin, x-axis points right, y-axis points up, z-axis points backwards)



Step 1: Translate by -**c**

Step 2: Rotate by **R**

$$\mathbf{R} = \begin{bmatrix} \mathbf{u}^T \\ \mathbf{v}^T \\ \mathbf{w}^T \end{bmatrix}$$

Perspective projection

$$\underbrace{\begin{bmatrix} -f & 0 & 0 \\ 0 & -f & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\mathbf{K}} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

(intrinsics)

(converts from 3D rays in camera coordinate system to pixel coordinates)

in general, $\mathbf{K} = \begin{bmatrix} -f & s & c_x \\ 0 & -\alpha f & c_y \\ 0 & 0 & 1 \end{bmatrix}$ (upper triangular matrix)

α : **aspect ratio** (1 unless pixels are not square)

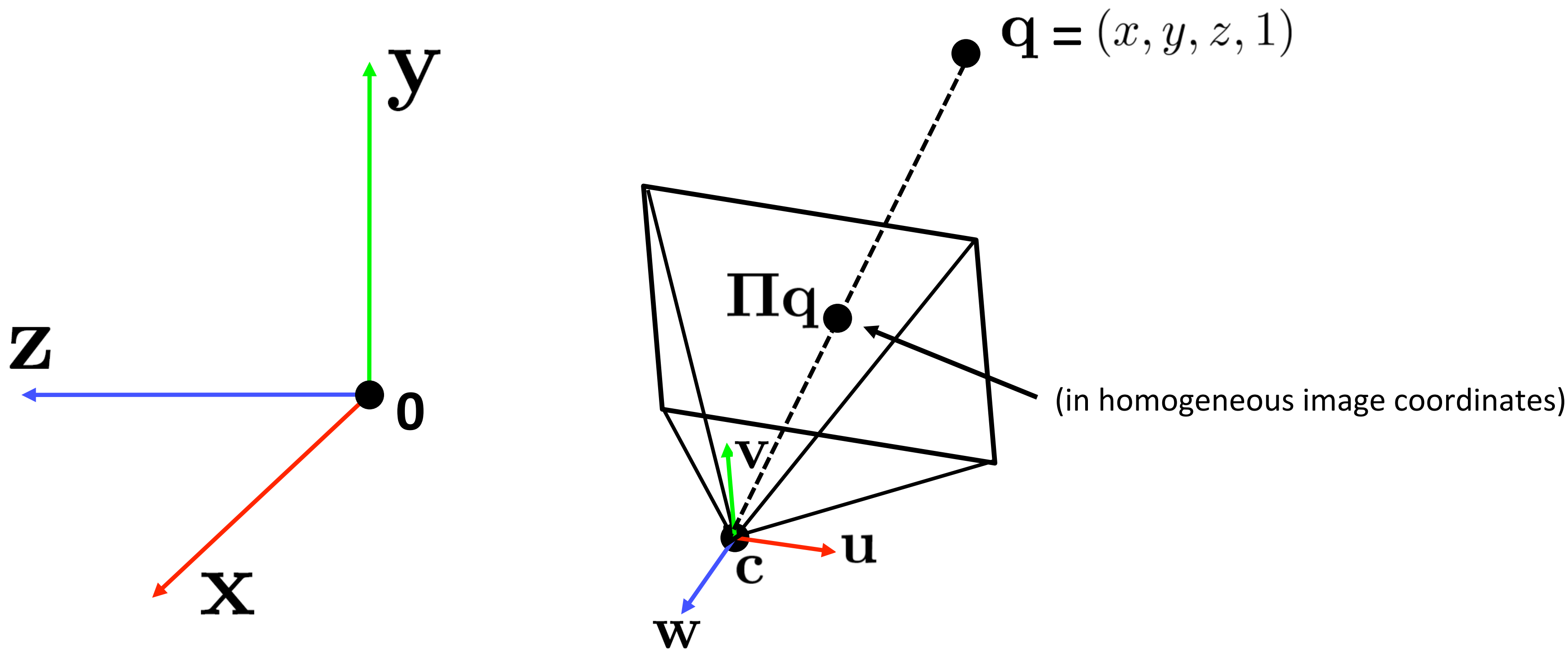
s : **skew** (0 unless pixels are shaped like rhombi/parallelograms)

(c_x, c_y) : **principal point** ((0,0) unless optical axis doesn't intersect projection plane at origin)

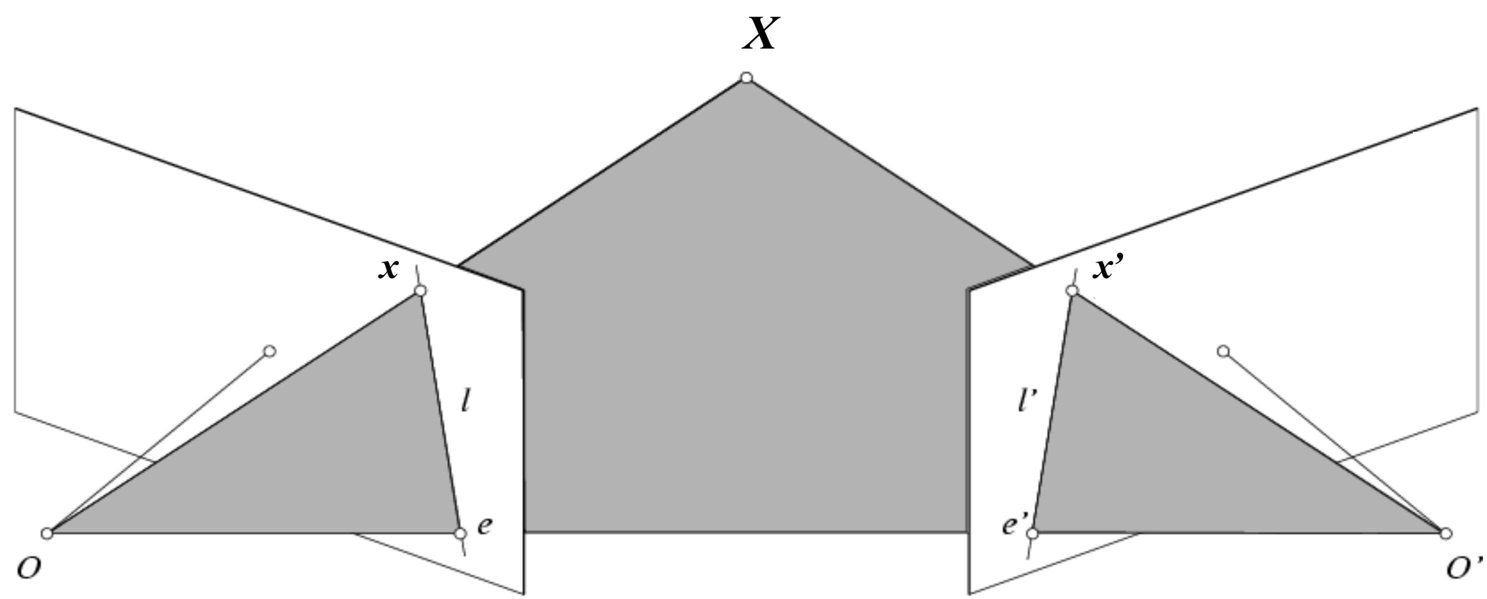
Projection matrix

$$\mathbf{\Pi} = \underbrace{\underset{\text{intrinsics}}{\mathbf{K}} \underset{\text{projection}}{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}} \underset{\text{rotation}}{\begin{bmatrix} & \mathbf{R} & & \begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \\ 0 & 0 & 0 & 1 \end{bmatrix}} \underset{\text{translation}}{\begin{bmatrix} & \mathbf{I}_{3\times 3} & & -\mathbf{c} \\ 0 & 0 & 0 & 1 \end{bmatrix}}}$$

Projection matrix

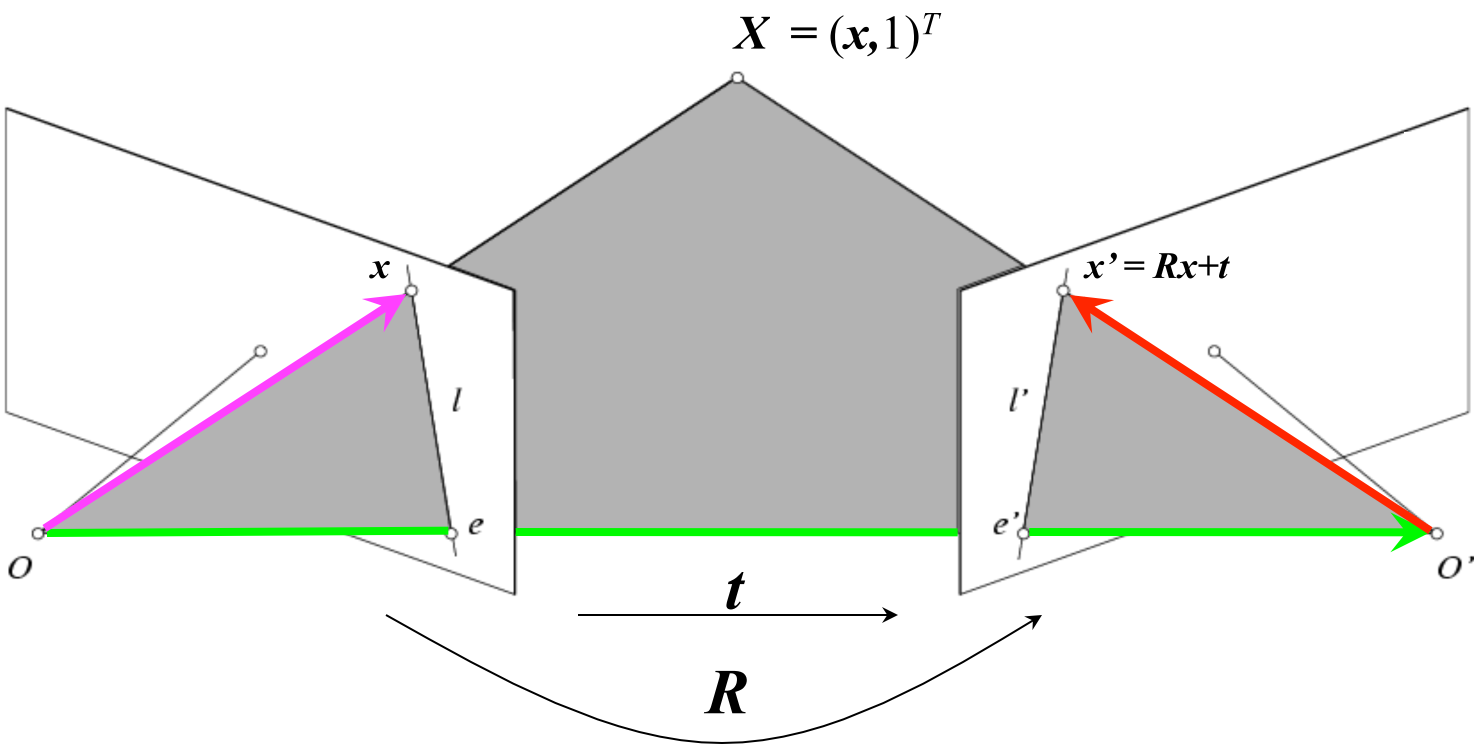


Epipolar constraint: Calibrated case



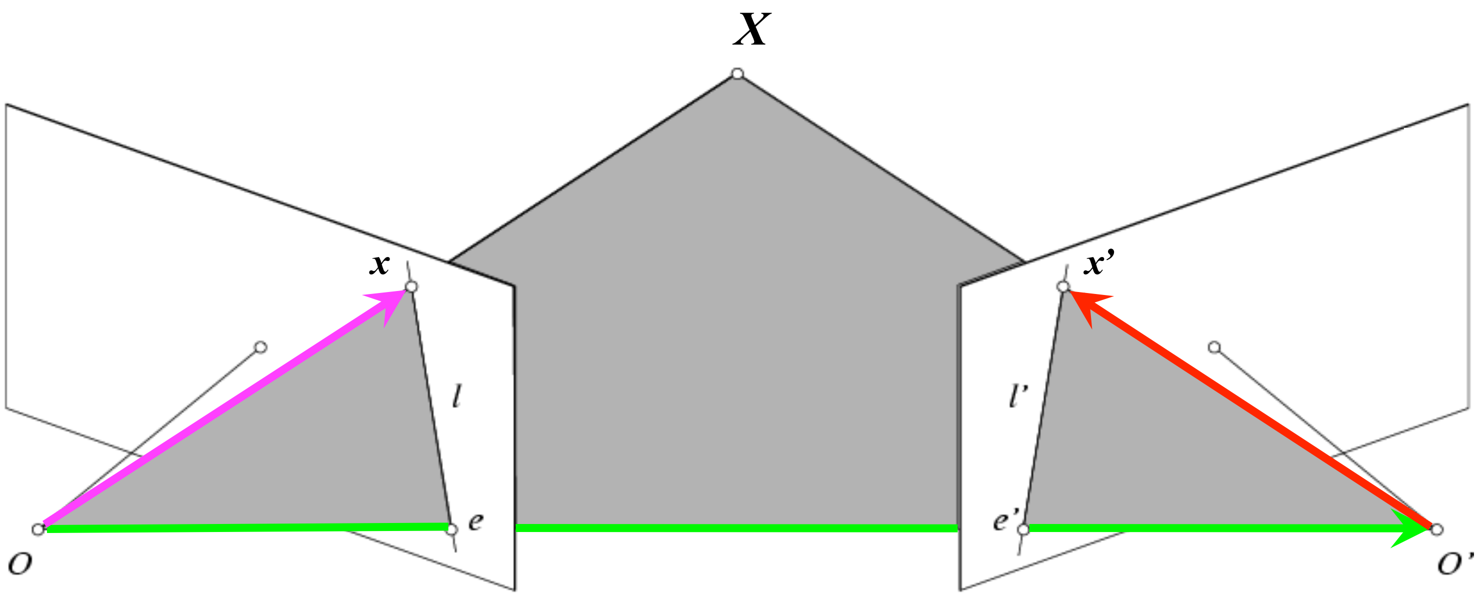
- Assume that the intrinsic and extrinsic parameters of the cameras are known
- We can multiply the projection matrix of each camera (and the image points) by the inverse of the calibration matrix to get *normalized* image coordinates
- We can also set the global coordinate system to the coordinate system of the first camera. Then the projection matrices of the two cameras can be written as $[\mathbf{I} \mid \mathbf{0}]$ and $[\mathbf{R} \mid \mathbf{t}]$

Epipolar constraint: Calibrated case



The vectors ***Rx***, ***t***, and ***x'*** are coplanar

Epipolar constraint: Calibrated case

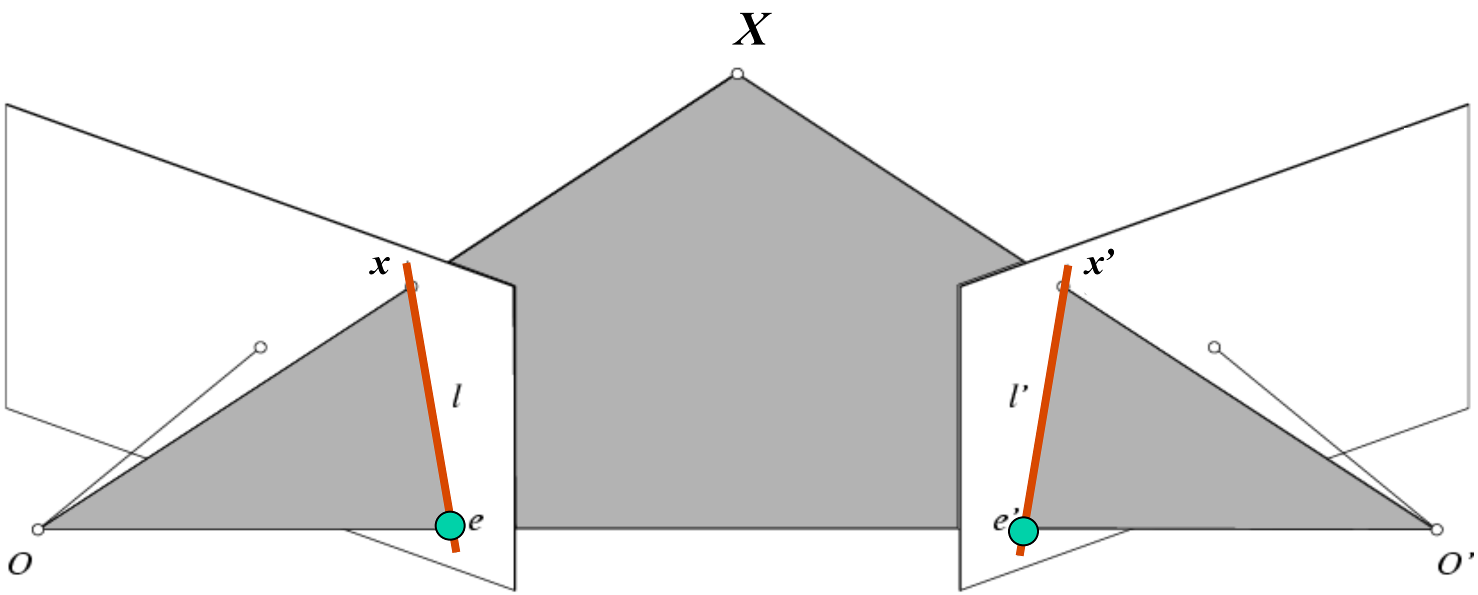


$$\boldsymbol{x}' \cdot [\boldsymbol{t} \times (\boldsymbol{R}\boldsymbol{x})] = 0 \quad \Rightarrow \quad \boldsymbol{x}'^T \boldsymbol{E} \boldsymbol{x} = 0 \quad \text{with} \quad \boldsymbol{E} = [\boldsymbol{t}_\times]\boldsymbol{R}$$

Essential Matrix
(Longuet-Higgins, 1981)

The vectors ***Rx***, ***t***, and ***x'*** are coplanar

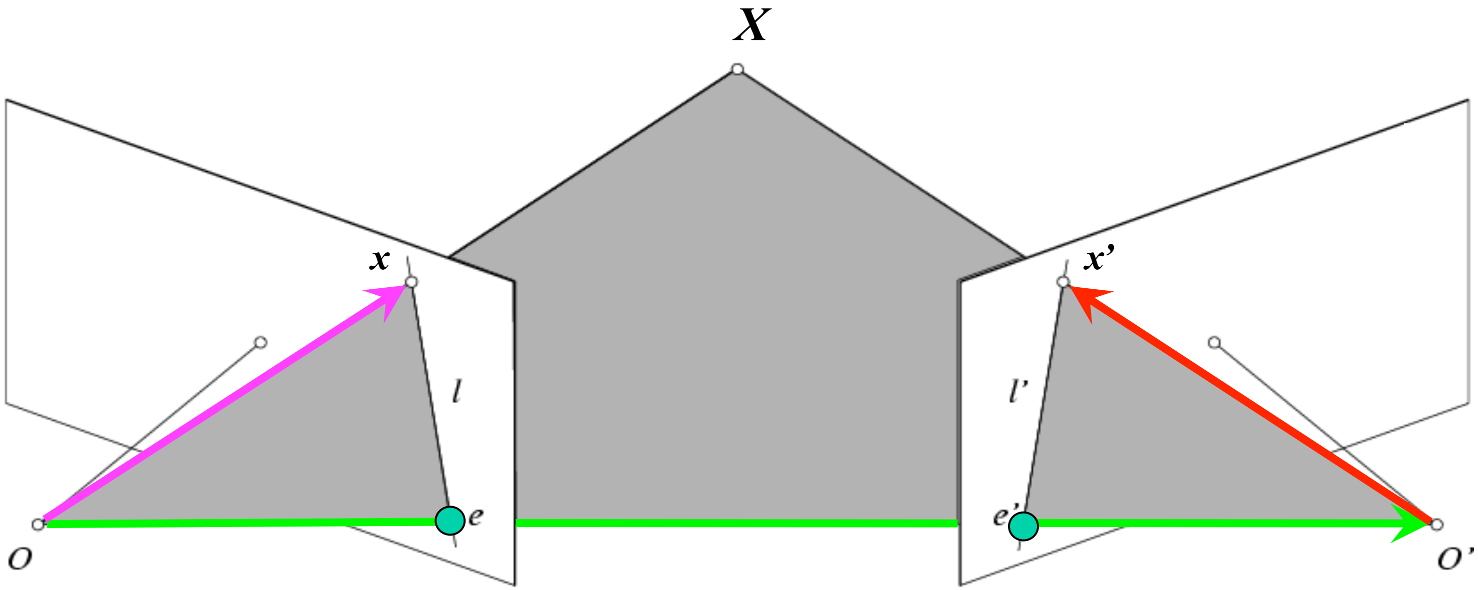
Epipolar constraint: Calibrated case



$$\boldsymbol{x}' \cdot [\boldsymbol{t} \times (\boldsymbol{R}\boldsymbol{x})] = 0 \quad \Rightarrow \quad \boldsymbol{x}'^T \boldsymbol{E}\, \boldsymbol{x} = 0 \quad \text{with} \quad \boldsymbol{E} = [\boldsymbol{t}_\times]\boldsymbol{R}$$

- ***E x*** is the epipolar line associated with ***x*** (***l'*** = ***E x***)
- $\boldsymbol{E}^T\boldsymbol{x}'$ is the epipolar line associated with ***x'*** ($\boldsymbol{l} = \boldsymbol{E}^T\boldsymbol{x}'$)
- $\boldsymbol{E}\,\mathbf{e} = 0$ and $\boldsymbol{E}^T\mathbf{e}' = 0$
- ***E*** is singular (rank two)
- ***E*** has five degrees of freedom

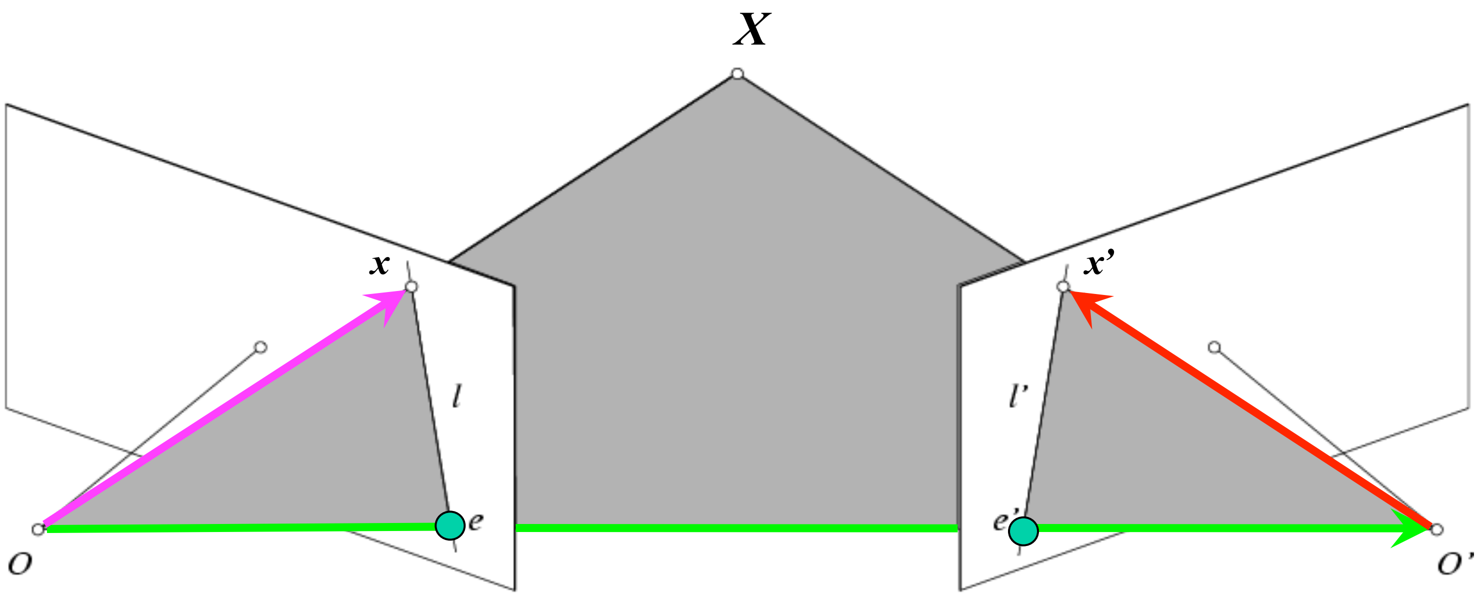
Epipolar constraint: Uncalibrated case



- The calibration matrices ***K*** and ***K'*** of the two cameras are unknown
- We can write the epipolar constraint in terms of *unknown* normalized coordinates:

$$\hat{\mathbf{x}}'^{T} \mathbf{E} \hat{\mathbf{x}} = 0 \qquad \hat{\mathbf{x}} = \mathbf{K}^{-1}\mathbf{x}, \quad \hat{\mathbf{x}}' = \mathbf{K}'^{-1}\hat{\mathbf{x}}'$$

Epipolar constraint: Uncalibrated case



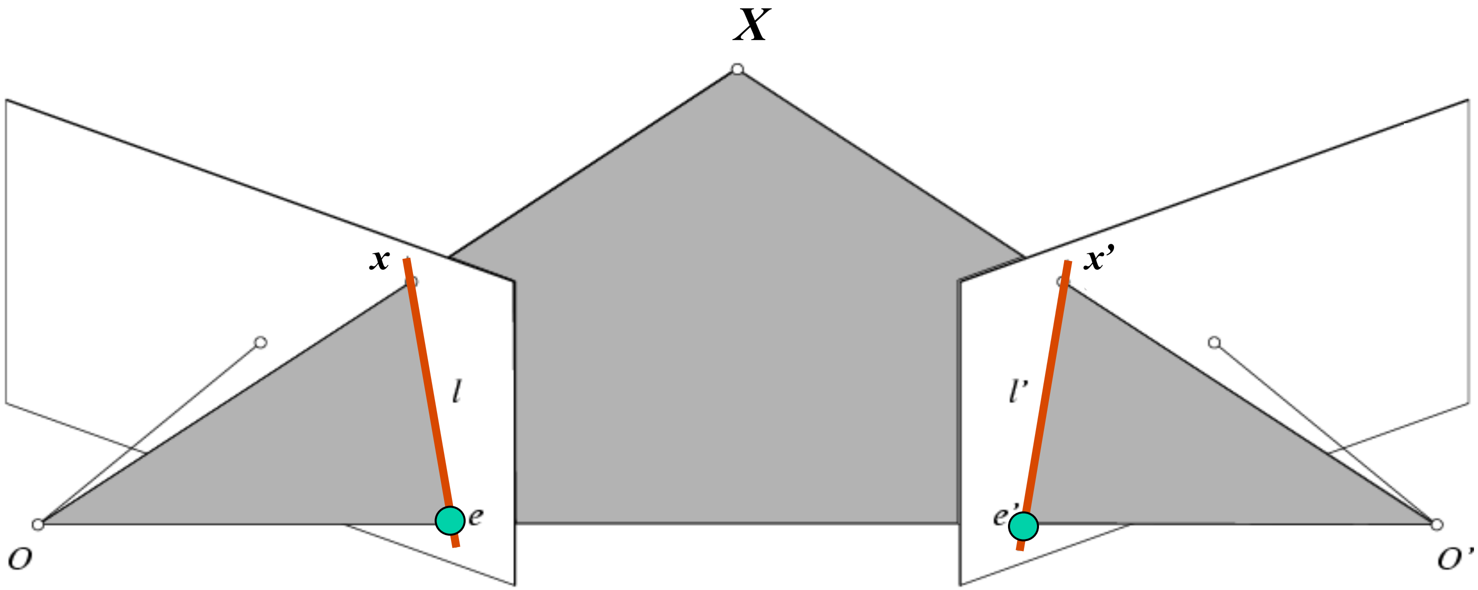
$$\hat{\boldsymbol{x}}'^{T} \boldsymbol{E} \hat{\boldsymbol{x}} = 0 \quad \Rightarrow \quad \boldsymbol{x}'^{T} \boldsymbol{F} \boldsymbol{x} = 0 \quad \text{with} \quad \boldsymbol{F} = \boldsymbol{K}'^{-T} \boldsymbol{E} \boldsymbol{K}^{-1}$$

$$\hat{\boldsymbol{x}} = \boldsymbol{K}^{-1} \boldsymbol{x}$$

$$\hat{\boldsymbol{x}}' = \boldsymbol{K}'^{-1} \boldsymbol{x}'$$

Fundamental Matrix
(Faugeras and Luong, 1992)

Epipolar constraint: Uncalibrated case



$$\hat{\boldsymbol{x}}'^{T} \boldsymbol{E}\, \hat{\boldsymbol{x}} = 0 \quad \Rightarrow \quad \boldsymbol{x}'^{T} \boldsymbol{F}\, \boldsymbol{x} = 0 \quad \text{with} \quad \boldsymbol{F} = \boldsymbol{K}'^{-T} \boldsymbol{E}\, \boldsymbol{K}^{-1}$$

- $\boldsymbol{F}\boldsymbol{x}$ is the epipolar line associated with $\boldsymbol{x}$ ($\boldsymbol{l}' = \boldsymbol{F}\boldsymbol{x}$)
- $\boldsymbol{F}^T\boldsymbol{x}'$ is the epipolar line associated with $\boldsymbol{x}'$ ($\boldsymbol{l}' = \boldsymbol{F}^T\boldsymbol{x}'$)
- $\boldsymbol{F}\boldsymbol{e} = 0$ and $\boldsymbol{F}^T\boldsymbol{e}' = 0$
- $\boldsymbol{F}$ is singular (rank two)
- $\boldsymbol{F}$ has *seven* degrees of freedom

The eight-point algorithm

$$\boldsymbol{x} = (u, v, 1)^T, \quad \boldsymbol{x}' = (u', v', 1)$$

$$\begin{bmatrix} u' & v' & 1 \end{bmatrix} \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix} \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = 0 \Rightarrow \underbrace{\begin{bmatrix} u'u & u'v & u' & v'u & v'v & v' & u & v & 1 \end{bmatrix}}_{\mathrm{A}} \begin{bmatrix} f_{11} \\ f_{12} \\ f_{13} \\ f_{21} \\ f_{22} \\ f_{23} \\ f_{31} \\ f_{32} \\ f_{33} \end{bmatrix} = 0$$

Minimize:

$$\sum_{i=1}^{N} (\boldsymbol{x}_i'^T \boldsymbol{F} \boldsymbol{x}_i)^2$$

under the constraint $||\boldsymbol{F}||^2 = 1$

Smallest eigenvalue of $\mathrm{A}^T\mathrm{A}$

The eight-point algorithm

- Meaning of error $$\sum_{i=1}^{N}(\boldsymbol{x}_i'^T \boldsymbol{F}\,\boldsymbol{x}_i)^2 :$$

  sum of squared *algebraic* distances between points $\boldsymbol{x}'_i$ and epipolar lines $\boldsymbol{F}\boldsymbol{x}_i$ (or points $\boldsymbol{x}_i$ and epipolar lines $\boldsymbol{F}^T\boldsymbol{x}'_i$)
- Nonlinear approach: minimize sum of squared *geometric* distances

$$\sum_{i=1}^{N}\left[\mathrm{d}^2(\boldsymbol{x}'_i, \boldsymbol{F}\,\boldsymbol{x}_i) + \mathrm{d}^2(\boldsymbol{x}_i, \boldsymbol{F}^T\boldsymbol{x}'_i)\right]$$

Problem with eight-point algorithm

$$\begin{bmatrix} u'u & u'v & u' & v'u & v'v & v' & u & v \end{bmatrix} \begin{bmatrix} f_{11} \\ f_{12} \\ f_{13} \\ f_{21} \\ f_{22} \\ f_{23} \\ f_{31} \\ f_{32} \end{bmatrix} = 0$$

Problem with eight-point algorithm

$$
\begin{bmatrix}
250906.36 & 183269.57 & 921.81 & 200931.10 & 146766.13 & 738.21 & 272.19 & 198.81 \\
2692.28 & 131633.03 & 176.27 & 6196.73 & 302975.59 & 405.71 & 15.27 & 746.79 \\
416374.23 & 871684.30 & 935.47 & 408110.89 & 854384.92 & 916.90 & 445.10 & 931.81 \\
191183.60 & 171759.40 & 410.27 & 416435.62 & 374125.90 & 893.65 & 465.99 & 418.65 \\
48988.86 & 30401.76 & 57.89 & 298604.57 & 185309.58 & 352.87 & 846.22 & 525.15 \\
164786.04 & 546559.67 & 813.17 & 1998.37 & 6628.15 & 9.86 & 202.65 & 672.14 \\
116407.01 & 2727.75 & 138.89 & 169941.27 & 3982.21 & 202.77 & 838.12 & 19.64 \\
135384.58 & 75411.13 & 198.72 & 411350.03 & 229127.78 & 603.79 & 681.28 & 379.48
\end{bmatrix}
\begin{bmatrix} f_{11} \\ f_{12} \\ f_{13} \\ f_{21} \\ f_{22} \\ f_{23} \\ f_{31} \\ f_{32} \end{bmatrix} = -1
$$

Poor numerical conditioning

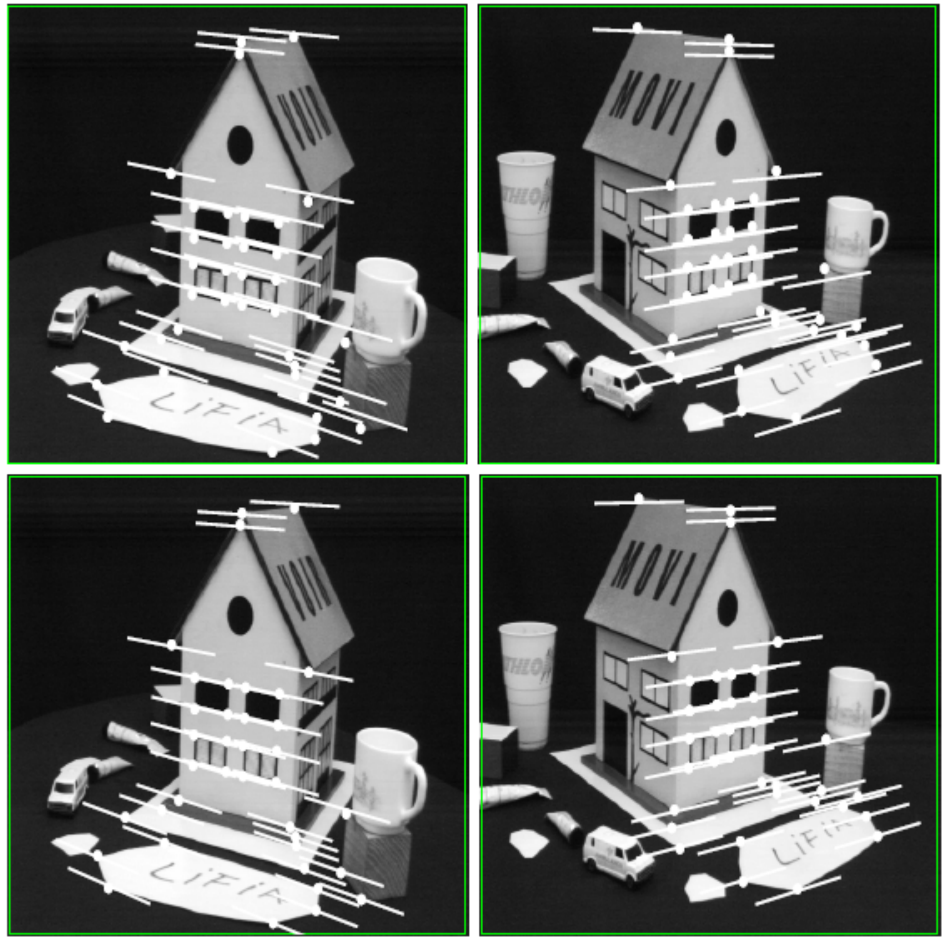
Can be fixed by rescaling the data

The normalized eight-point algorithm

(Hartley, 1995)

- Center the image data at the origin, and scale it so the mean squared distance between the origin and the data points is 2 pixels
- Use the eight-point algorithm to compute $\boldsymbol{F}$ from the normalized points
- Enforce the rank-2 constraint (for example, take SVD of $\boldsymbol{F}$ and throw out the smallest singular value)
- Transform fundamental matrix back to original units: if $\boldsymbol{T}$ and $\boldsymbol{T'}$ are the normalizing transformations in the two images, then the fundamental matrix in original coordinates is $\boldsymbol{T'}^{T} \boldsymbol{F} \boldsymbol{T}$

Comparison of estimation algorithms



	8-point	Normalized 8-point	Nonlinear least squares
Av. Dist. 1	2.33 pixels	0.92 pixel	0.86 pixel
Av. Dist. 2	2.18 pixels	0.85 pixel	0.80 pixel

Moving on to stereo…

Fuse a calibrated binocular stereo pair to produce a depth image

image 1

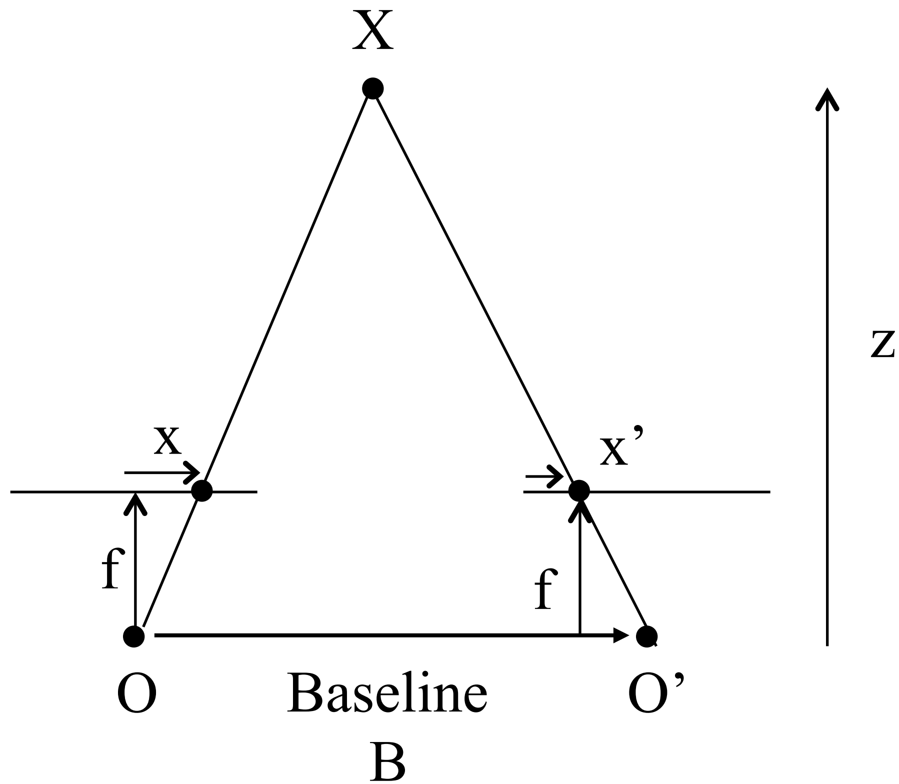
image 2

Dense depth map

Many of these slides adapted from Steve Seitz and Lana Lazebnik

Depth from disparity

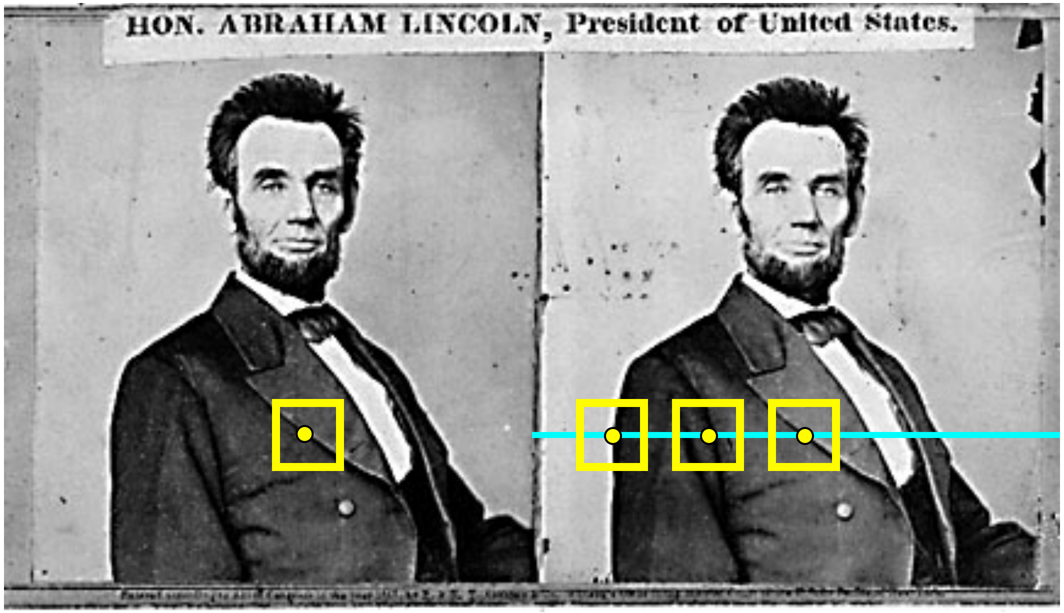
$$\frac{x - x'}{O - O'} = \frac{f}{z}$$



$$disparity = x - x' = \frac{B \cdot f}{z}$$

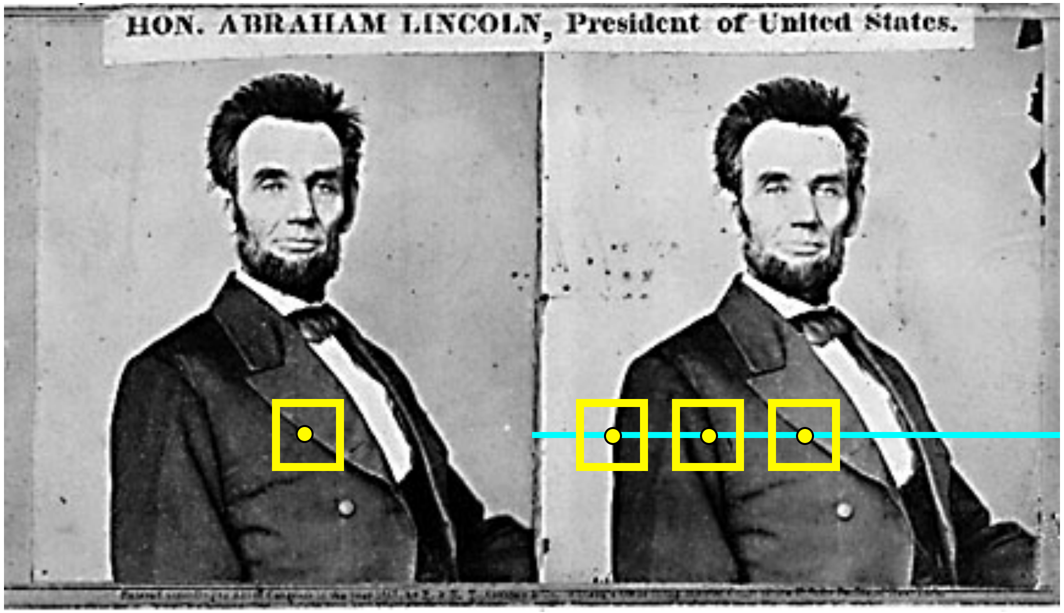
Disparity is inversely proportional to depth.

Basic stereo matching algorithm



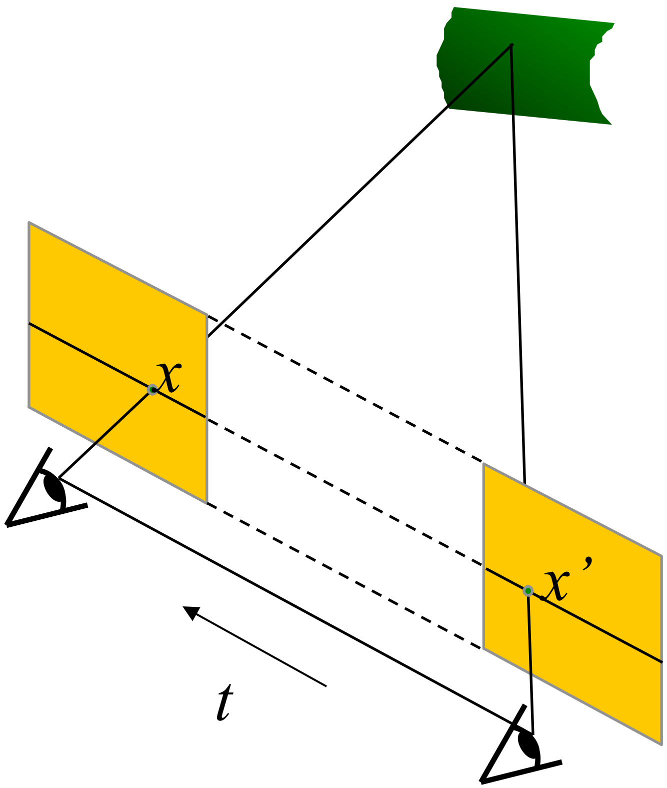
- If necessary, rectify the two stereo images to transform epipolar lines into scanlines
- For each pixel x in the first image
  - Find corresponding epipolar scanline in the right image
  - Search the scanline and pick the best match x’
  - Compute disparity x-x’ and set depth(x) = fB/(x-x’)

Basic stereo matching algorithm



- For each pixel in the first image
  - Find corresponding epipolar line in the right image
  - Search along epipolar line and pick the best match
  - Triangulate the matches to get depth information
- Simplest case: epipolar lines are scanlines
  - When does this happen?

Simplest Case: Parallel images



Epipolar constraint:

$$x^T E x' = 0, \quad E = t \times R$$

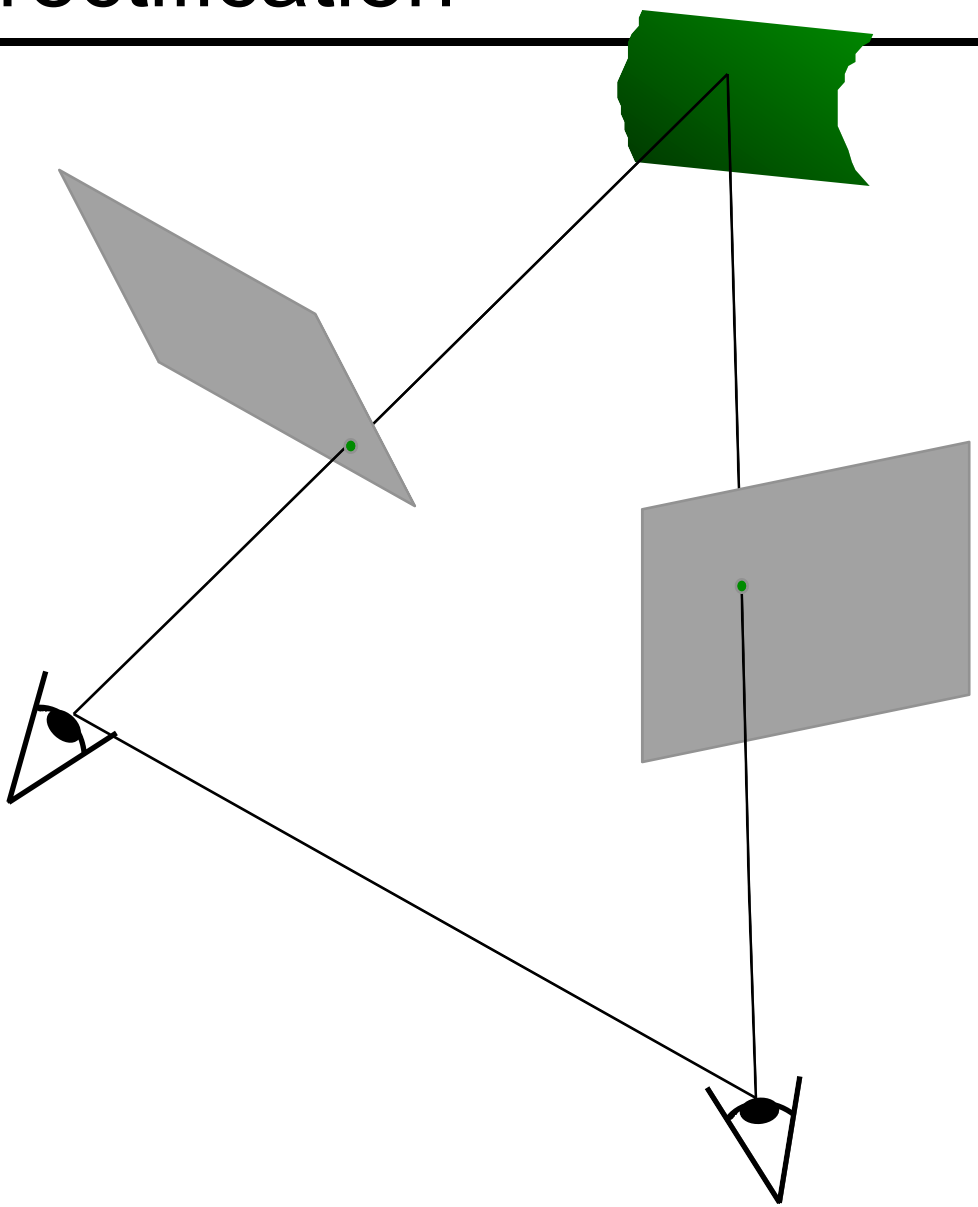
$$R = I \qquad t = (T, 0, 0)$$

$$E = t \times R = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -T \\ 0 & T & 0 \end{bmatrix}$$

$$\begin{pmatrix} u & v & 1 \end{pmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -T \\ 0 & T & 0 \end{bmatrix} \begin{pmatrix} u' \\ v' \\ 1 \end{pmatrix} = 0 \qquad \begin{pmatrix} u & v & 1 \end{pmatrix} \begin{pmatrix} 0 \\ -T \\ Tv' \end{pmatrix} = 0 \qquad Tv = Tv'$$

The y-coordinates of corresponding points are the same

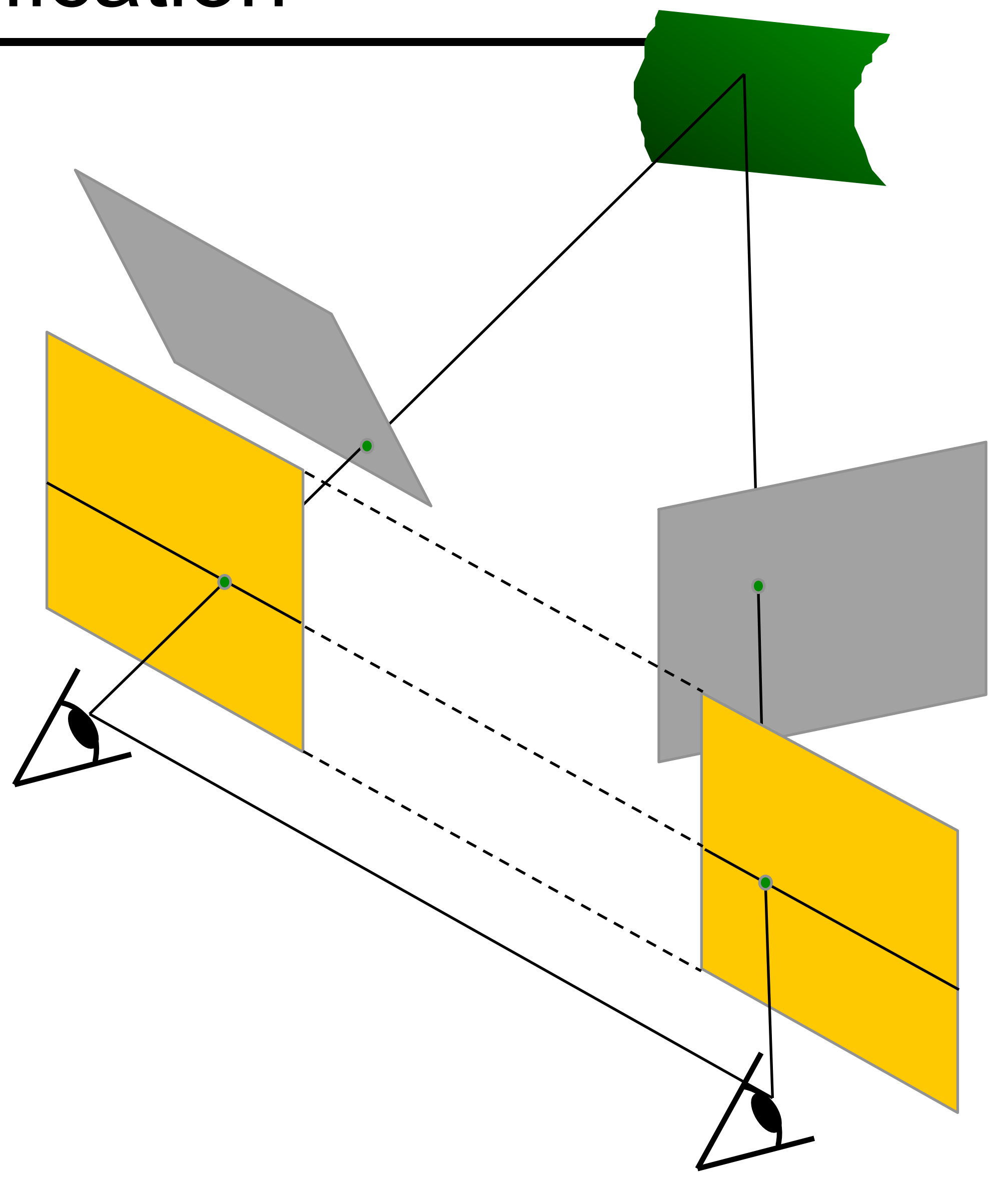
Stereo image rectification



Stereo image rectification

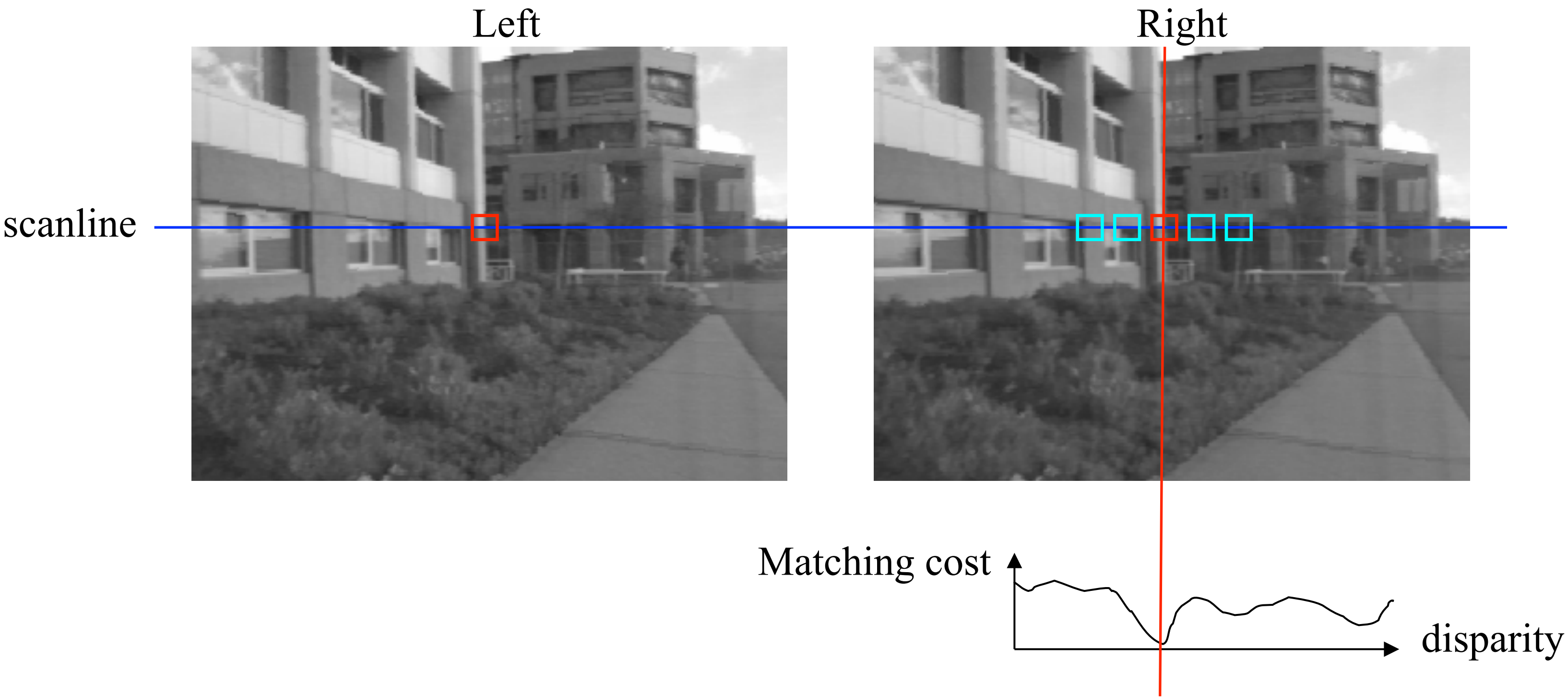
- Reproject image planes onto a common plane parallel to the line between camera centers
- Pixel motion is horizontal after this transformation
- Two homographies (3x3 transform), one for each input image reprojection

➢ C. Loop and Z. Zhang. [Computing Rectifying Homographies for Stereo Vision](). IEEE Conf. Computer Vision and Pattern Recognition, 1999.



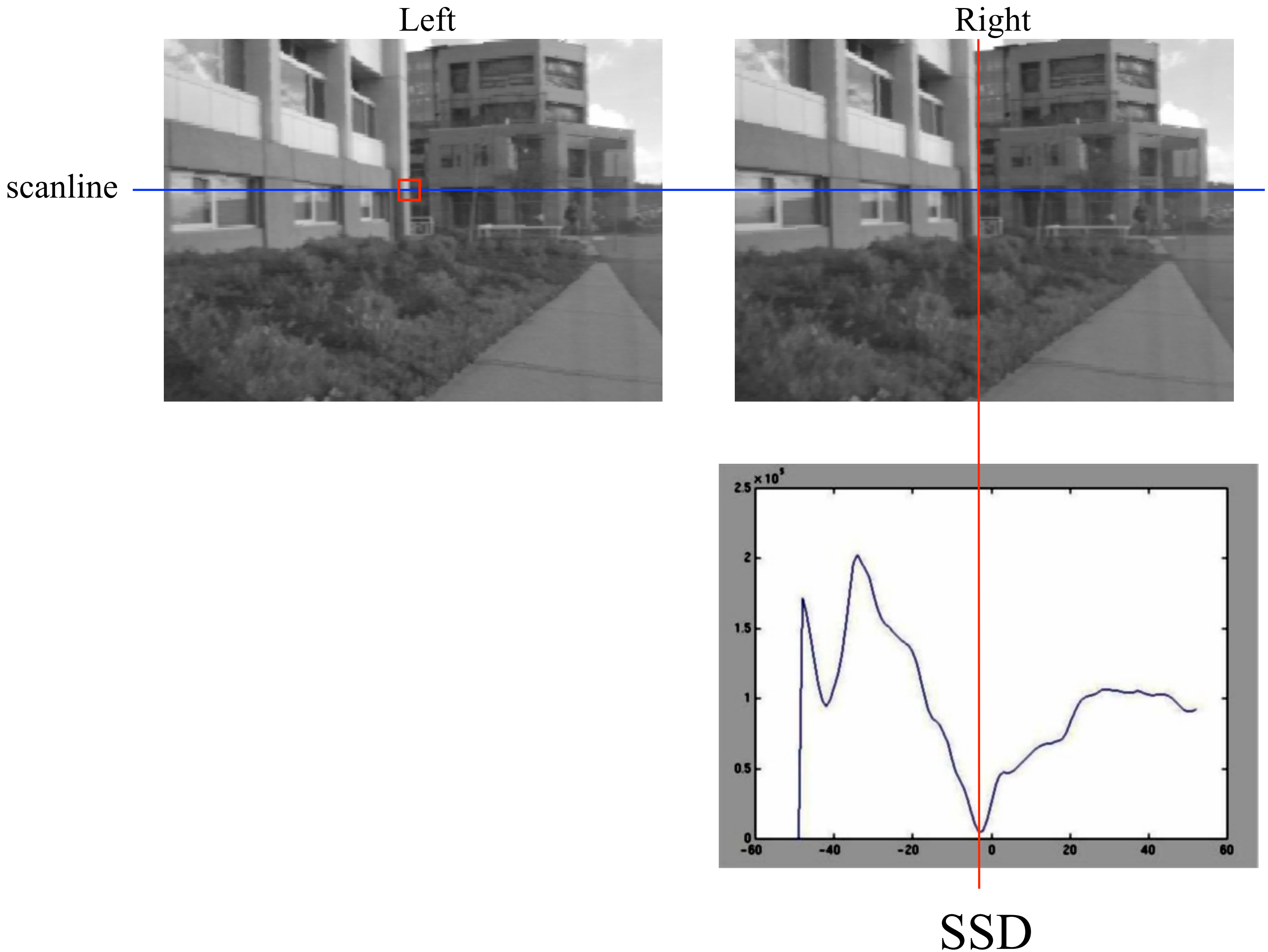
Example

Correspondence search

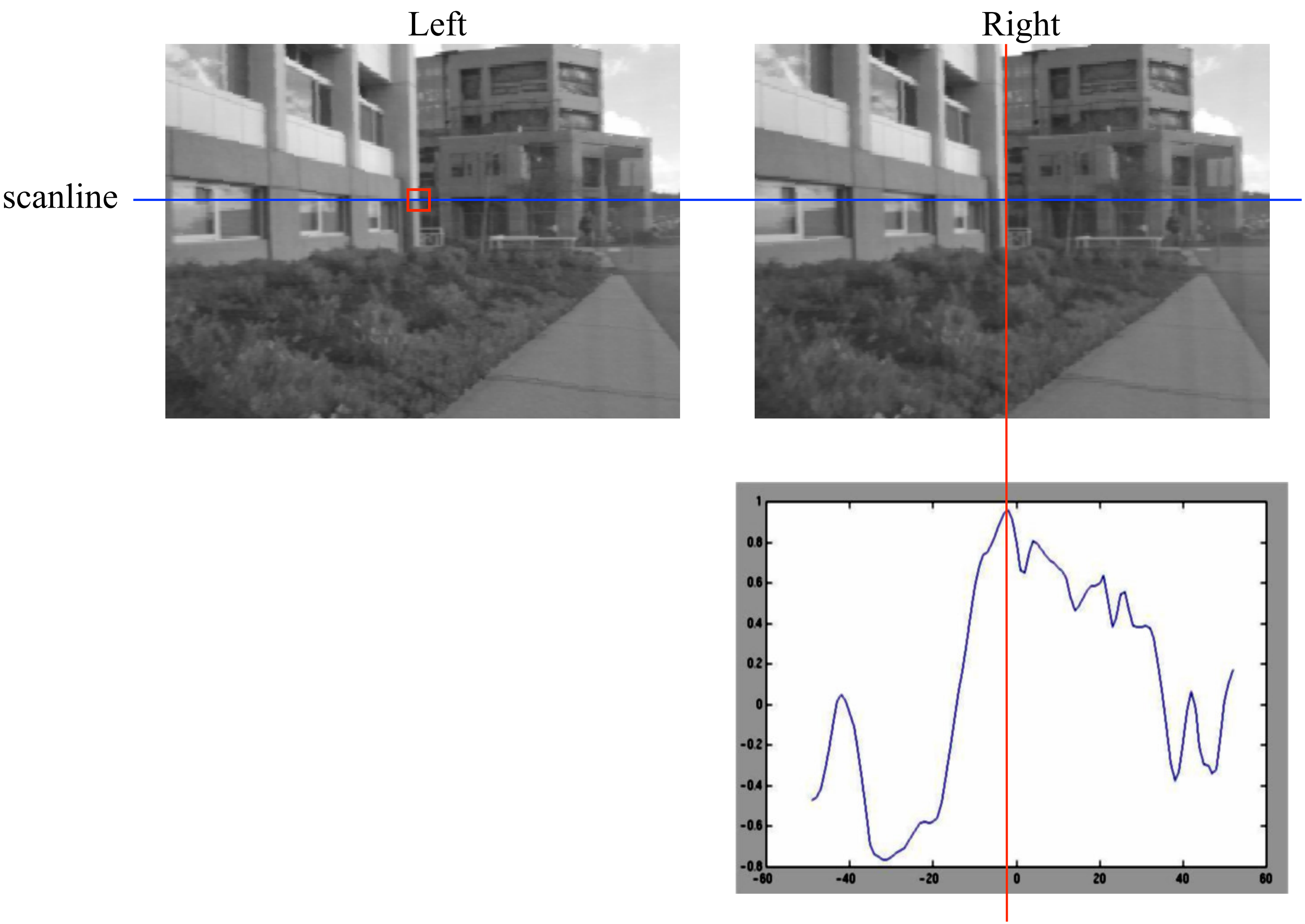


- Slide a window along the right scanline and compare contents of that window with the reference window in the left image
- Matching cost: SSD or normalized correlation

Correspondence search



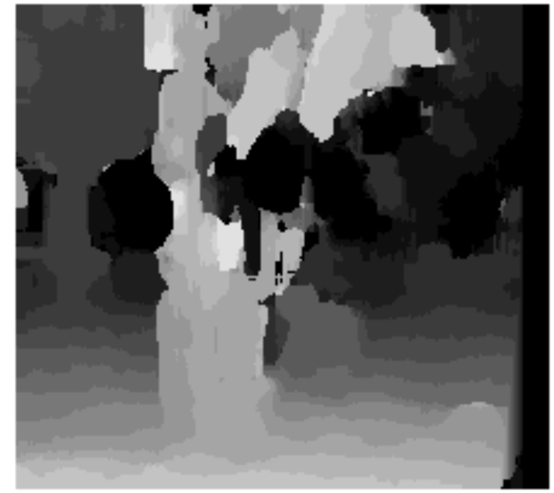
Correspondence search



Norm. corr

Effect of window size

W = 3

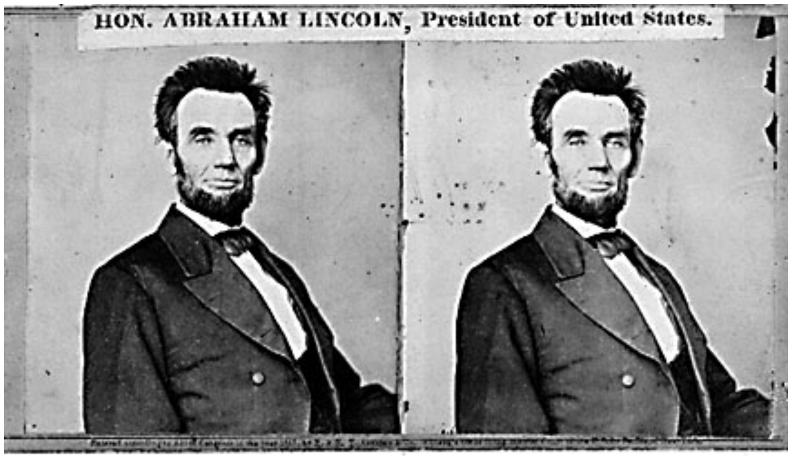


W = 20

- Smaller window
  + More detail
  – More noise

- Larger window
  + Smoother disparity maps
  – Less detail
  – Fails near boundaries

Failures of correspondence search



Textureless surfaces

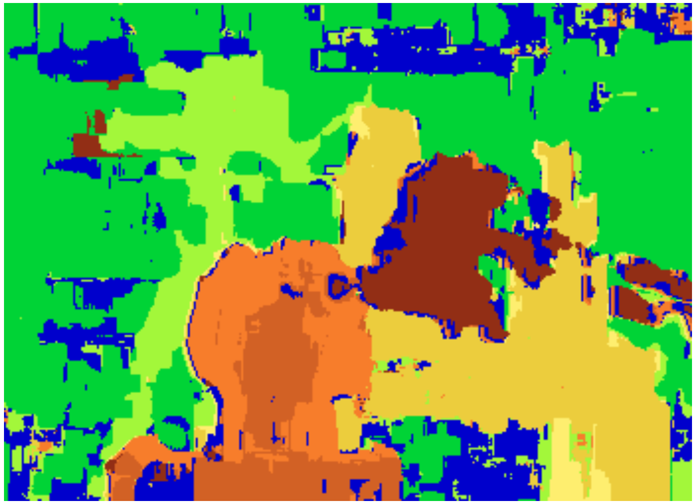
Occlusions, repetition

Non-Lambertian surfaces, specularities

Results with window search

Data

Window-based matching



Ground truth

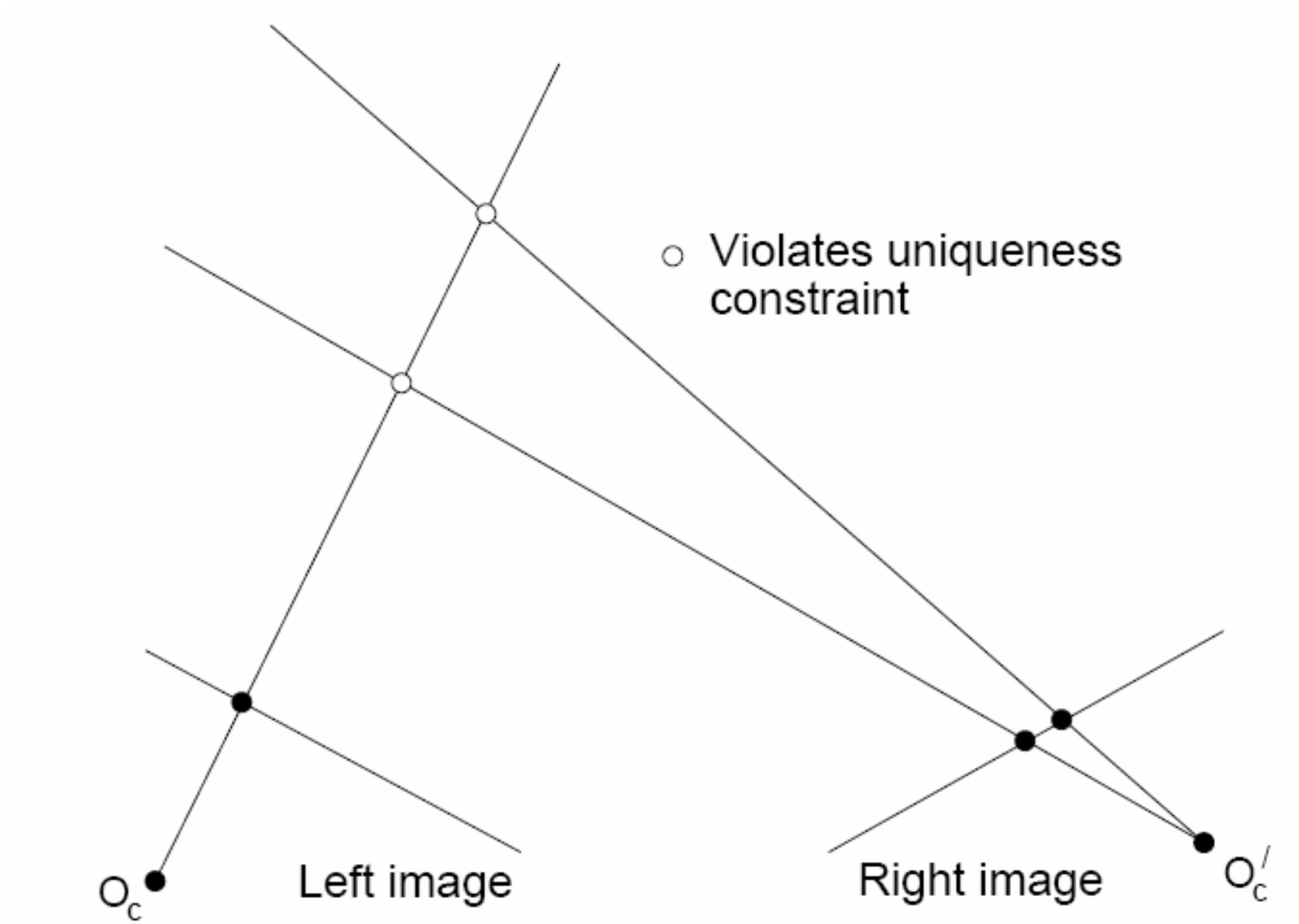
How can we improve window-based matching?

So far, matches are independent for each point

What constraints or priors can we add?

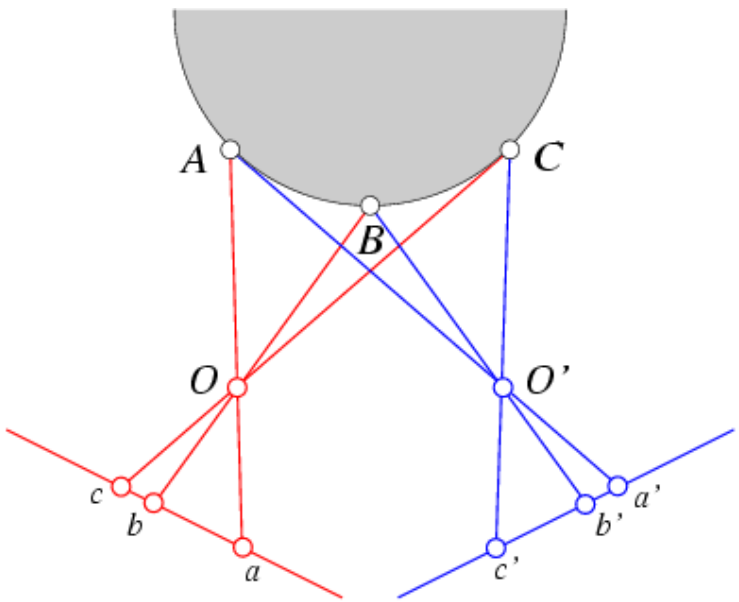
Stereo constraints/priors

- Uniqueness
  - For any point in one image, there should be at most one matching point in the other image



Stereo constraints/priors

- Uniqueness
  - For any point in one image, there should be at most one matching point in the other image
- Ordering
  - Corresponding points should be in the same order in both views



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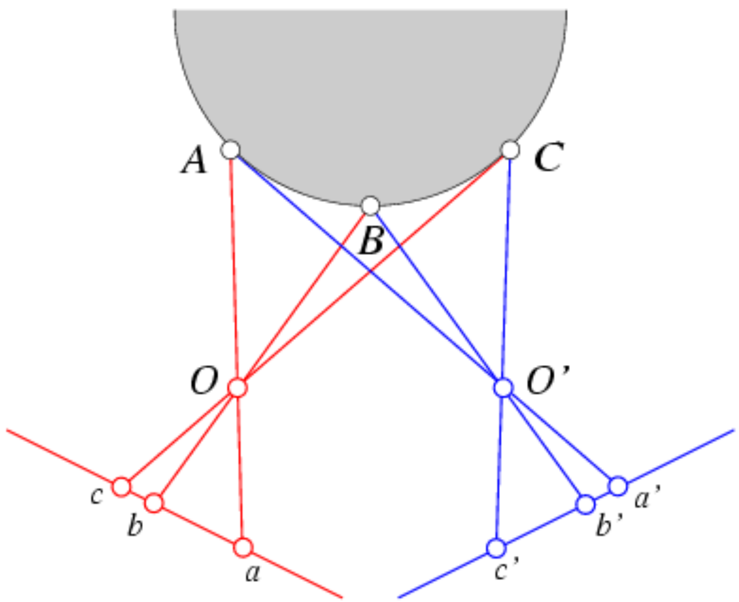
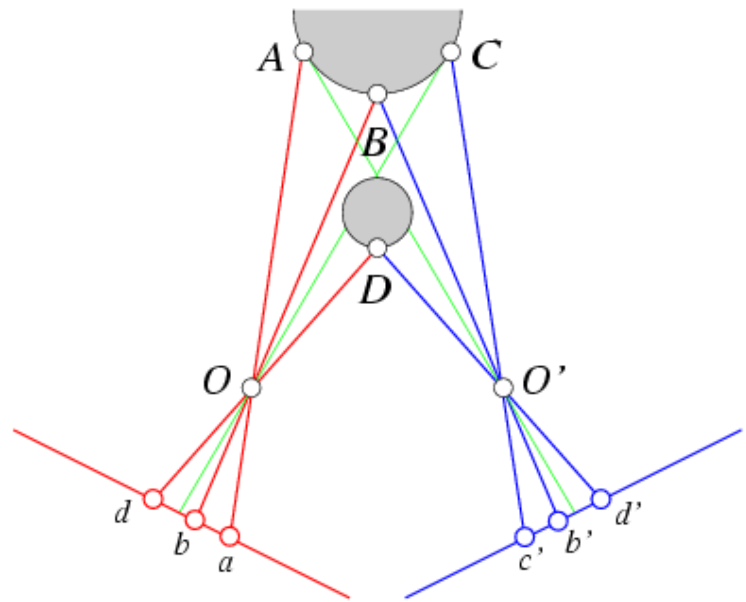


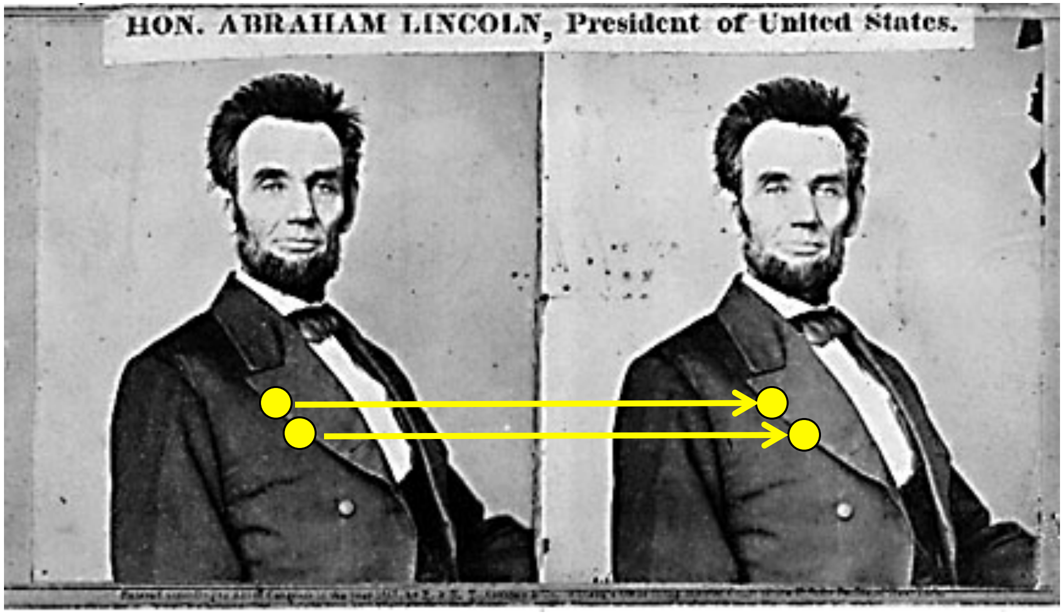


Ordering constraint doesn’t hold

Priors and constraints

- Uniqueness
  - For any point in one image, there should be at most one matching point in the other image
- Ordering
  - Corresponding points should be in the same order in both views
- Smoothness
  - We expect disparity values to change slowly (for the most part)

Stereo as energy minimization



What defines a good stereo correspondence?

1. Match quality
   – Want each pixel to find a good match in the other image
2. Smoothness
   – If two pixels are adjacent, they should (usually) move about the same amount

Stereo as energy minimization

Better objective function

$$E(d) = \underbrace{E_d(d)}_{\text{match cost}} + \lambda \underbrace{E_s(d)}_{\text{smoothness cost}}$$

Want each pixel to find a good match in the other image

Adjacent pixels should (usually) move about the same amount

Stereo as energy minimization

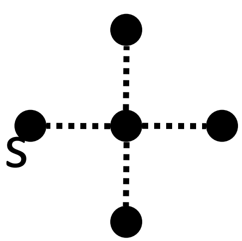
$$E(d) = E_d(d) + \lambda E_s(d)$$

match cost: $$E_d(d) = \sum_{(x,y) \in I} C(x, y, d(x,y))$$

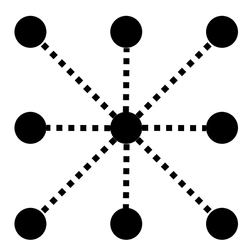
smoothness cost: $$E_s(d) = \sum_{(p,q) \in \mathcal{E}} V(d_p, d_q)$$

$$C(x, y, d(x,y)) =$$ SSD distance between windows $I(x, y)$ and $J(x, y + d(x,y))$

$\mathcal{E}$: set of neighboring pixels



4-connected neighborhood



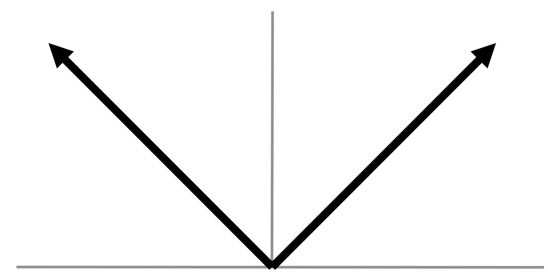
8-connected neighborhood

Smoothness cost

$$E_s(d) = \sum_{(p,q)\in\mathcal{E}} V(d_p, d_q)$$

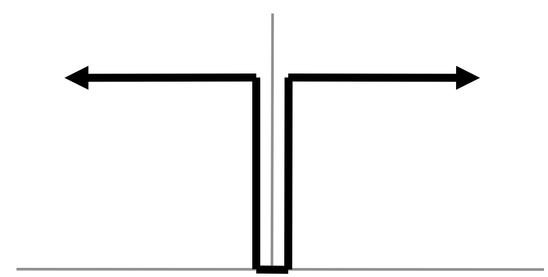
$$V(d_p, d_q) = |d_p - d_q|$$

L_1 distance



$$V(d_p, d_q) = \begin{cases} 0 & \text{if } d_p = d_q \\ 1 & \text{if } d_p \neq d_q \end{cases}$$

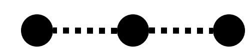
"Potts model"



Dynamic programming

$$E(d) = E_d(d) + \lambda E_s(d)$$

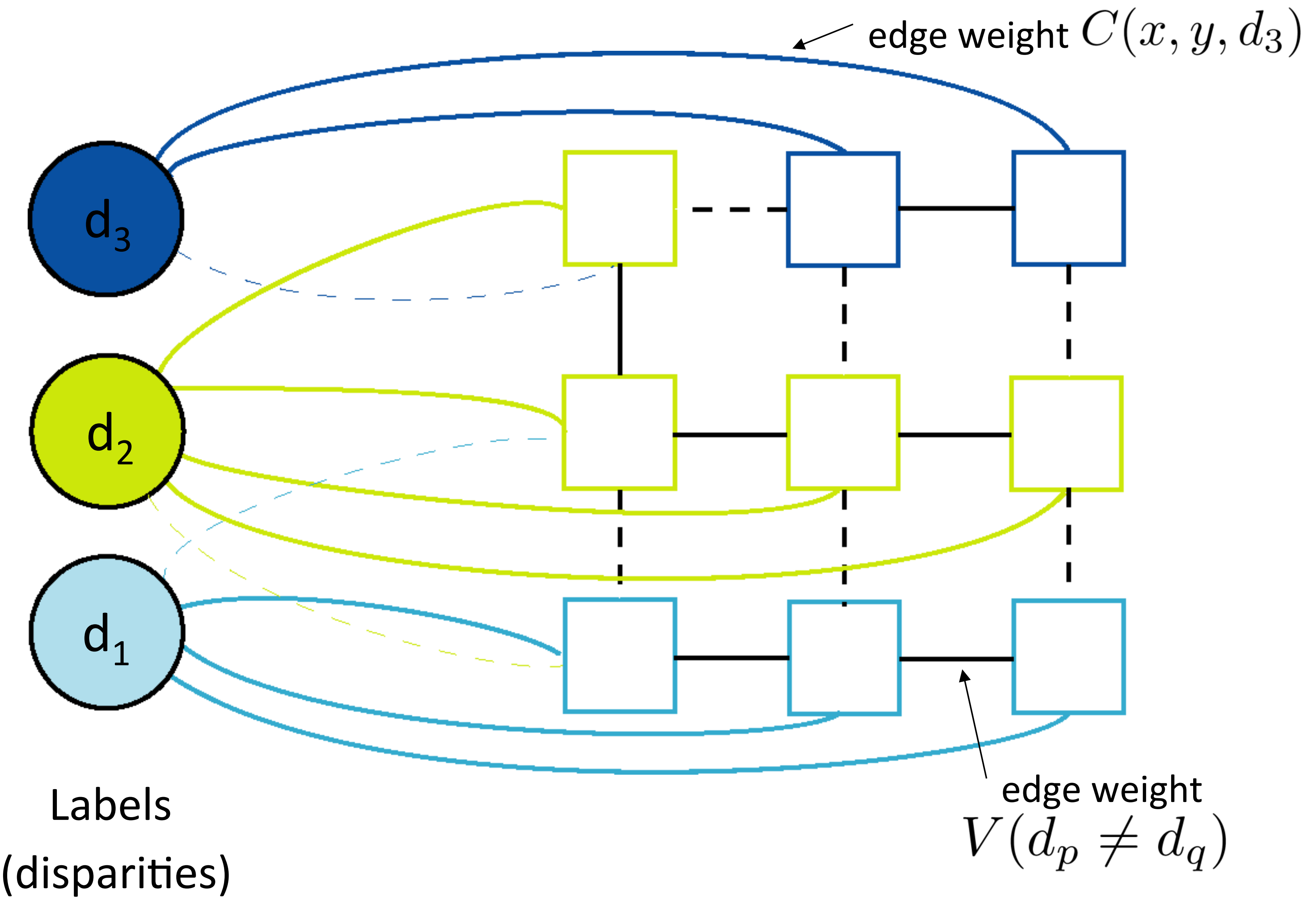
Can minimize this independently per scanline using dynamic programming (DP)



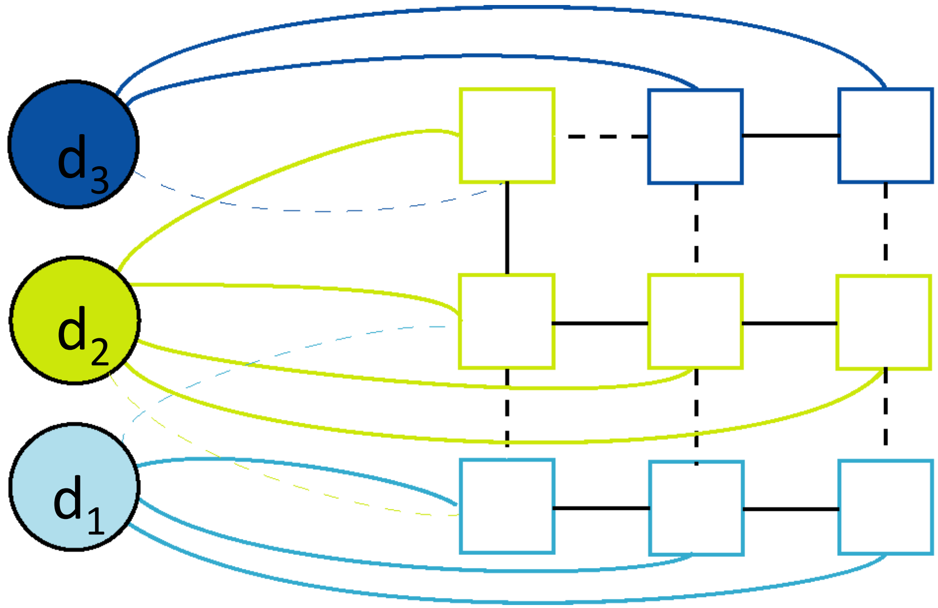
$D(x, y, d)$: minimum cost of solution such that *d*(*x*,*y*) = *d*

$$D(x, y, d) = C(x, y, d) + \min_{d'} \{D(x-1, y, d') + \lambda |d - d'|\}$$

Energy minimization via graph cuts



Energy minimization via graph cuts



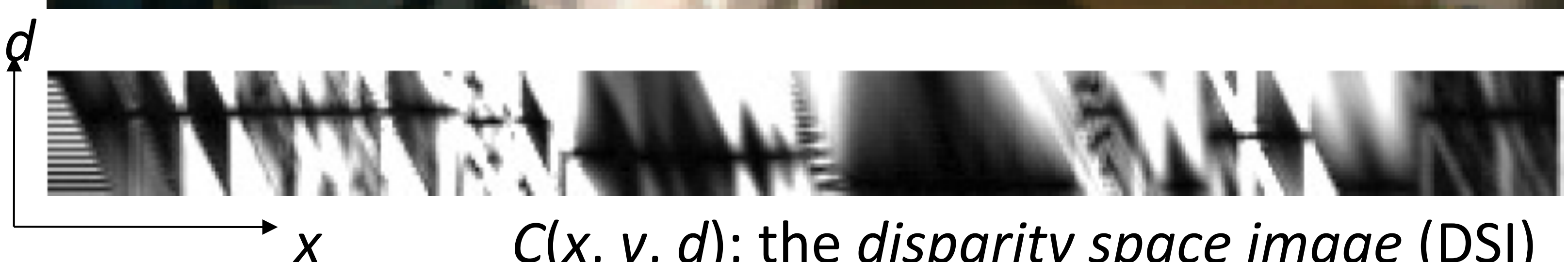
- Graph Cut
  - Delete enough edges so that
    - each pixel is connected to exactly one label node
  - Cost of a cut: sum of deleted edge weights
  - Finding min cost cut equivalent to finding global minimum of energy function

Stereo as energy minimization

$I(x, y)$

$J(x, y)$

y = 141



$C(x, y, d)$; the *disparity space image* (DSI)

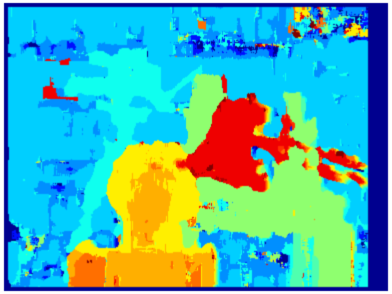
Stereo as energy minimization

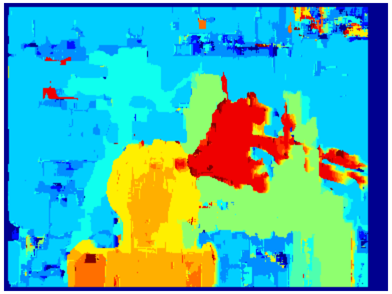
Simple pixel / window matching: choose the minimum of each column in the DSI independently:

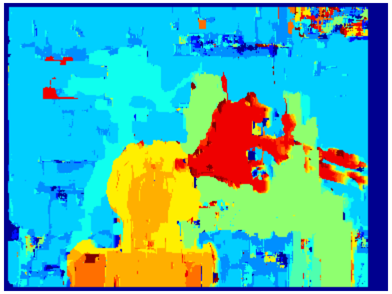
$$d(x, y) = \arg\min_{d'} \; C(x, y, d')$$

Matching windows

Similarity Measure	Formula
Sum of Absolute Differences (SAD)	$\sum_{(i,j)\in W} \lvert I_1(i,j) - I_2(x+i,y+j)\rvert$
Sum of Squared Differences (SSD)	$\sum_{(i,j)\in W} \left(I_1(i,j) - I_2(x+i,y+j)\right)^2$
Zero-mean SAD	$\sum_{(i,j)\in W} \lvert I_1(i,j) - \bar{I}_1(i,j) - I_2(x+i,y+j) + \bar{I}_2(x+i,y+j)\rvert$
Locally scaled SAD	$\sum_{(i,j)\in W} \lvert I_1(i,j) - \frac{\bar{I}_1(i,j)}{\bar{I}_2(x+i,y+j)} I_2(x+i,y+j)\rvert$
Normalized Cross Correlation (NCC)	$\frac{\sum_{(i,j)\in W} I_1(i,j).I_2(x+i,y+j)}{\sqrt[2]{\sum_{(i,j)\in W} I_1^2(i,j).\sum_{(i,j)\in W} I_2^2(x+i,y+j)}}$


SAD


SSD

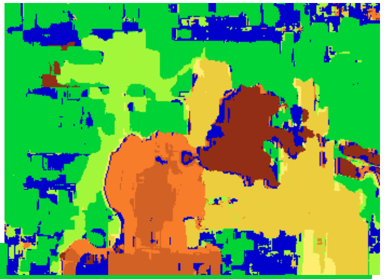

NCC

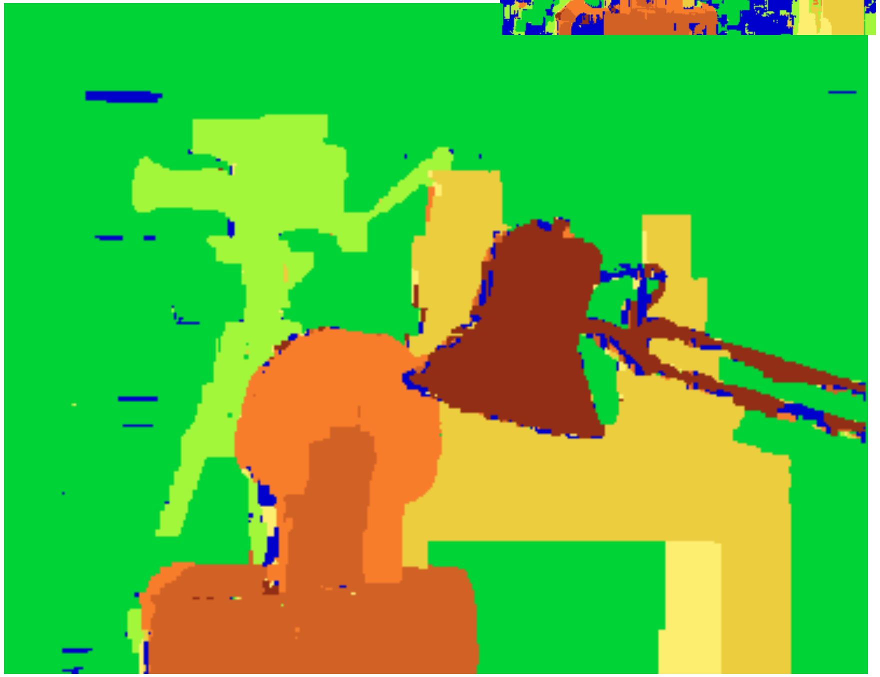

Ground truth

http://siddhantahuja.wordpress.com/category/stereo-vision/

Before & After

Before





Graph cuts

Ground truth

Y. Boykov, O. Veksler, and R. Zabih,
Fast Approximate Energy Minimization via Graph Cuts, PAMI 2001

For the latest and greatest: http://www.middlebury.edu/stereo/

Real-time stereo

Nomad robot searches for meteorites in Antartica
http://www.frc.ri.cmu.edu/projects/meteorobot/index.html

Used for robot navigation (and other tasks)

- Several software-based real-time stereo techniques have been developed (most based on simple discrete search)

Why does stereo fail?

Fronto-Parallel Surfaces: Depth is constant within the region of local support

Why does stereo fail?

Monotonic Ordering - Points along an epipolar scanline appear in the same order in both stereo images
Occlusion – All points are visible in each image

Why does stereo fail?

Image Brightness Constancy: Assuming Lambertian surfaces, the brightness of corresponding points in stereo images are the same.

Why does stereo fail?

Match Uniqueness: For every point in one stereo image, there is at most one corresponding point in the other image.

Stereo reconstruction pipeline

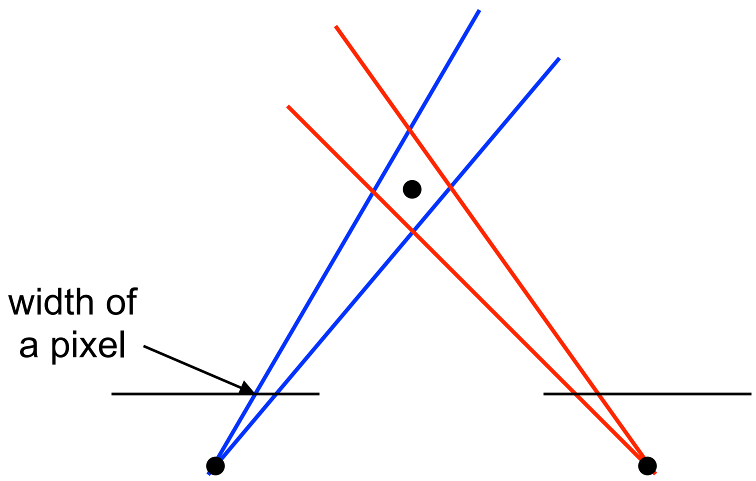
Steps

- Calibrate cameras
- Rectify images
- Compute disparity
- Estimate depth

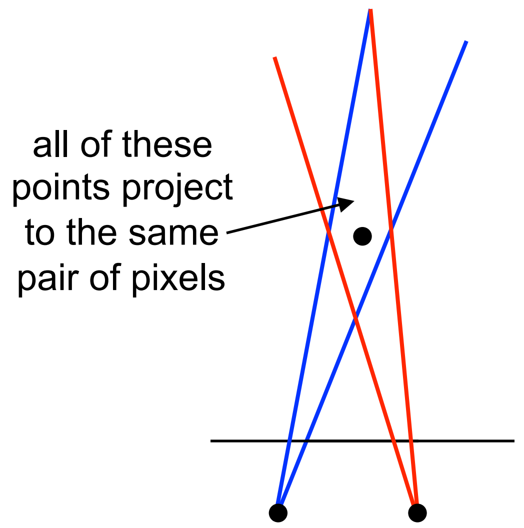
What will cause errors?

- Camera calibration errors
- Poor image resolution
- Occlusions
- Violations of brightness constancy (specular reflections)
- Large motions
- Low-contrast image regions

Choosing the stereo baseline



Large Baseline

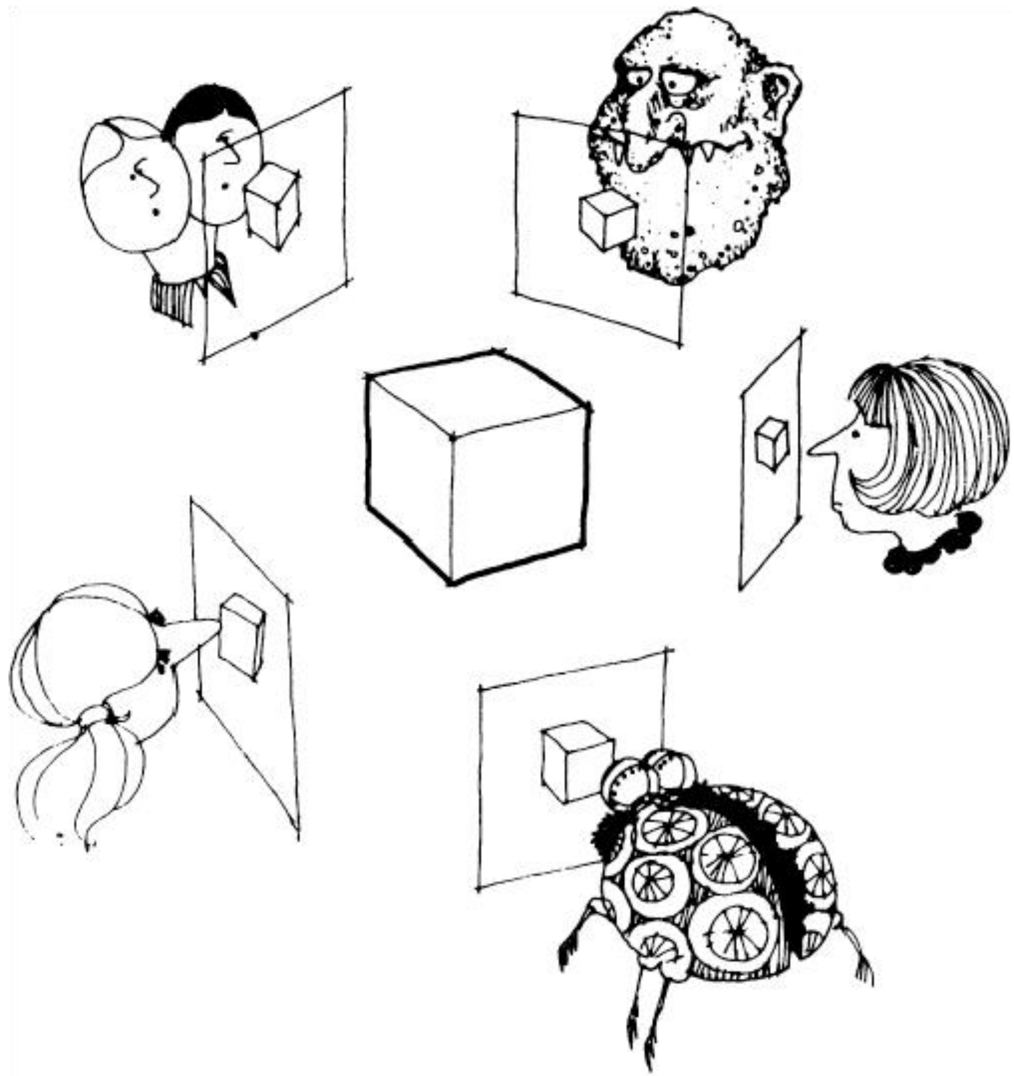


Small Baseline

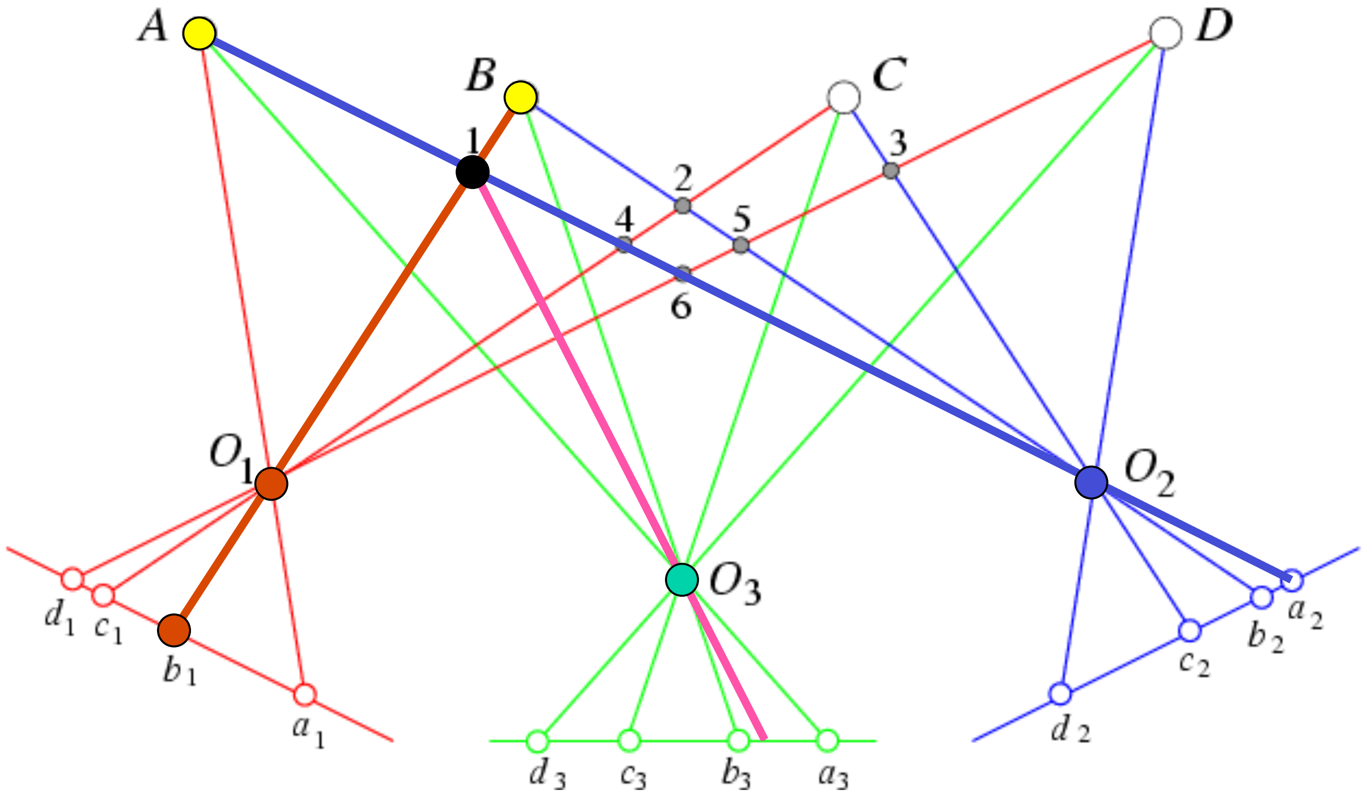
What' s the optimal baseline?

- Too small: large depth error
- Too large: difficult search problem

Multi-view stereo ?

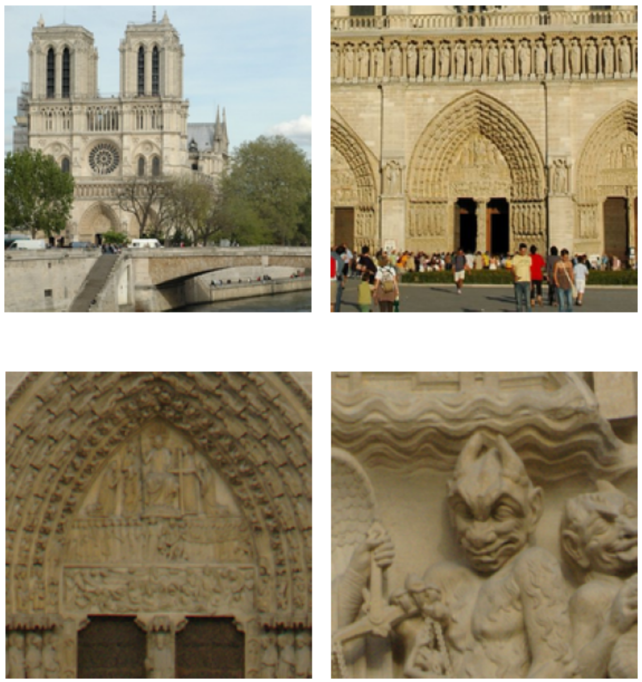


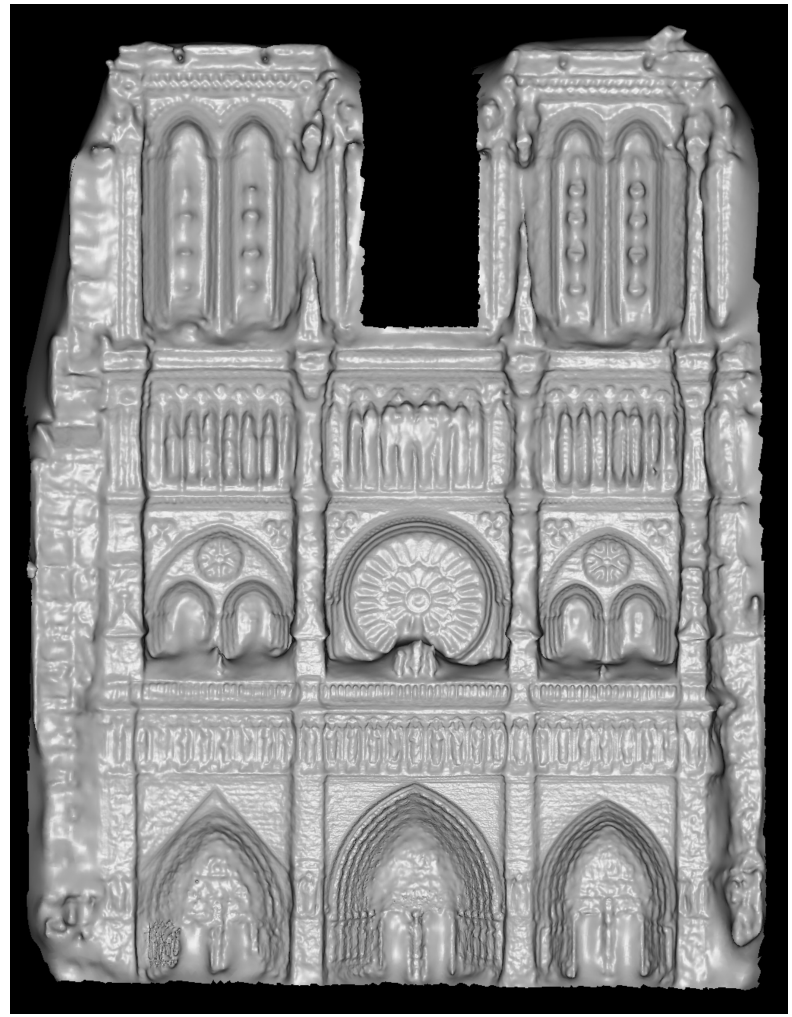
Beyond two-view stereo



The third view can be used for verification

Using more than two images





Multi-View Stereo for Community Photo Collections
M. Goesele, N. Snavely, B. Curless, H. Hoppe, S. Seitz
Proceedings of ICCV 2007,