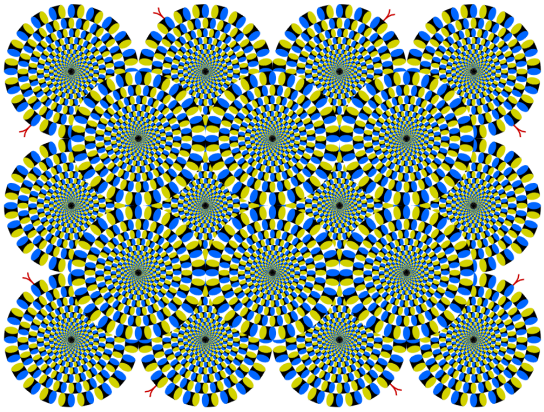
Motion Estimation

http://www.sandlotscience.com/Distortions/Breathing_Square.htm

http://www.sandlotscience.com/Ambiguous/Barberpole_Illusion.htm

Today's Readings

- Trucco & Verri, 8.3 – 8.4 (skip 8.3.3, read only top half of p. 199)
- Newton's method Wikpedia page



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Why estimate motion?

Lots of uses

- Track object behavior
- Correct for camera jitter (stabilization)
- Align images (mosaics)
- 3D shape reconstruction
- Special effects
- Video slow motion
- Video super-resolution

Motion estimation

Input: sequence of images
Output: point correspondence

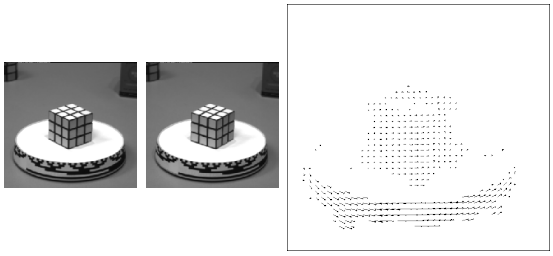
Feature tracking

- we've seen this already (e.g., SIFT)
- can modify this to be more efficient

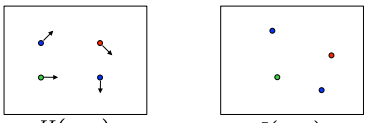
Pixel tracking: "Optical Flow"

- today's lecture

Optical flow



Problem definition: optical flow



$H(x,y)$ $I(x,y)$

How to estimate pixel motion from image H to image I?

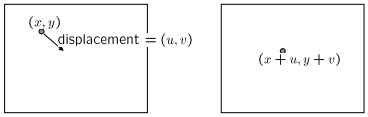
- Solve pixel correspondence problem
  - given a pixel in H, look for nearby pixels of the same color in I

Key assumptions

- color constancy: a point in H looks the same in I
  - For grayscale images, this is brightness constancy
- small motion: points do not move very far

This is called the optical flow problem

Optical flow constraints (grayscale images)



$H(x,y)$ $I(x,y)$

Let's look at these constraints more closely

- brightness constancy: Q: what's the equation?

- small motion: (u and v are less than 1 pixel)
  - suppose we take the Taylor series expansion of I:

$$I(x+u,y+v) = I(x,y)+\frac{\partial I}{\partial x}u+\frac{\partial I}{\partial y}v+\text{higher order terms}$$
$$\approx I(x,y)+\frac{\partial I}{\partial x}u+\frac{\partial I}{\partial y}v$$

Optical flow equation

Combining these two equations

shorthand: $I_x = \frac{\partial I}{\partial x}$

$$\begin{aligned} 0 &= I(x+u,y+v) - H(x,y) \\ &\approx I(x,y) + I_x u + I_y v - H(x,y) \\ &\approx (I(x,y) - H(x,y)) + I_x u + I_y v \\ &\approx I_t + I_x u + I_y v \\ &\approx I_t + \nabla I \cdot [u\ v] \end{aligned}$$

In the limit as u and v go to zero, this becomes exact

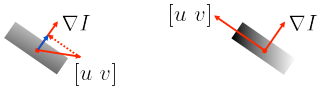
$$0 = I_t + \nabla I \cdot [\tfrac{\partial x}{\partial t}\ \tfrac{\partial y}{\partial t}]$$

Optical flow equation

$$0 = I_t + \nabla I \cdot [u\ v]$$

Q: how many unknowns and equations per pixel?

Intuitively, what does this constraint mean?

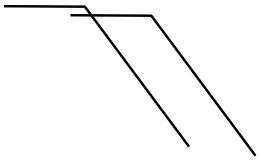


Example on the board

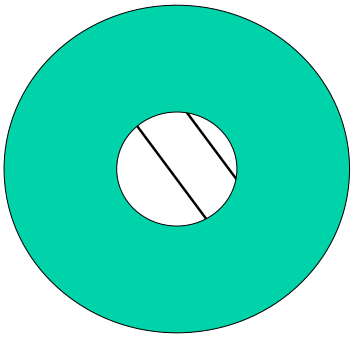
- The component of the flow in the gradient direction is determined
- The component of the flow parallel to an edge is unknown

This explains the Barber Pole illusion
http://www.sandlotscience.com/Ambiguous/Barberpole_Illusion.htm

Aperture problem



Aperture problem



Solving the aperture problem

How to get more equations for a pixel?

- Basic idea: impose additional constraints
  - most common is to assume that the flow field is smooth locally
  - one method: pretend the pixel's neighbors have the same (u,v)
    - » If we use a 5x5 window, that gives us 25 equations per pixel!

$$0 = I_t(\mathbf{p}_i) + \nabla I(\mathbf{p}_i) \cdot [u\ v]$$

$$\underset{25\times2}{\begin{bmatrix} I_x(\mathbf{p}_1) & I_y(\mathbf{p}_1) \\ I_x(\mathbf{p}_2) & I_y(\mathbf{p}_2) \\ \vdots & \vdots \\ I_x(\mathbf{p}_{25}) & I_y(\mathbf{p}_{25}) \end{bmatrix}} \underset{2\times1}{\begin{bmatrix} u \\ v \end{bmatrix}} = - \underset{25\times1}{\begin{bmatrix} I_t(\mathbf{p}_1) \\ I_t(\mathbf{p}_2) \\ \vdots \\ I_t(\mathbf{p}_{25}) \end{bmatrix}}$$

A d b

Lucas-Kanade flow

Prob: we have more equations than unknowns

$$\underset{25\text{x}2}{A}\ \underset{2\text{x}1}{d} = \underset{25\text{x}1}{b} \longrightarrow \text{minimize } \|Ad - b\|^2$$

Solution: solve least squares problem

- minimum least squares solution given by solution (in d) of:

$$\underset{2\text{x}2}{(A^T A)}\ \underset{2\text{x}1}{d} = \underset{2\text{x}1}{A^T b}$$

$$\begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_x I_y & \sum I_y I_y \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = - \begin{bmatrix} \sum I_x I_t \\ \sum I_y I_t \end{bmatrix}$$

$A^T A$ $\qquad$ $A^T b$

- The summations are over all pixels in the K x K window
- This technique was first proposed by Lucas & Kanade (1981)

Conditions for solvability

- Optimal (u, v) satisfies Lucas-Kanade equation

$$\begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_x I_y & \sum I_y I_y \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = - \begin{bmatrix} \sum I_x I_t \\ \sum I_y I_t \end{bmatrix}$$

$A^T A$ $\qquad$ $A^T b$

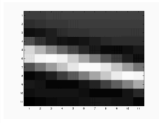
When is This Solvable?

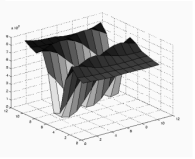
- A^TA should be invertible
- A^TA should not be too small due to noise
  - eigenvalues λ_1 and λ_2 of A^TA should not be too small
- A^TA should be well-conditioned
  - λ_1 / λ_2 should not be too large (λ_1 = larger eigenvalue)

Does this look familiar?

- A^TA is the Harris matrix

Edge

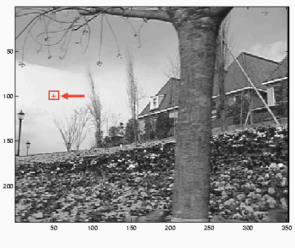


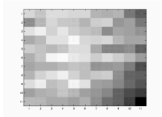


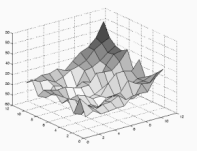
$$\sum \nabla I (\nabla I)^T$$

- large gradients, all the same
- large λ_1, small λ_2

Low Texture Region



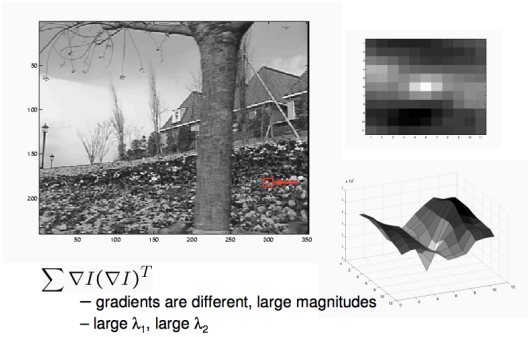




$$\sum \nabla I (\nabla I)^T$$

- gradients have small magnitude
- small λ_1, small λ_2

High Texture Region



$\sum \nabla I (\nabla I)^T$

– gradients are different, large magnitudes

– large λ_1, large λ_2

Observation

This is a two image problem BUT

- Can measure sensitivity by just looking at one of the images!
- This tells us which pixels are easy to track, which are hard
  - very useful later on when we do feature tracking...

Errors in Lucas-Kanade

What are the potential causes of errors in this procedure?

- Suppose A^TA is easily invertible
- Suppose there is not much noise in the image

When our assumptions are violated

- Brightness constancy is not satisfied
- The motion is not small
- A point does not move like its neighbors

– window size is too large

Improving accuracy

Recall our small motion assumption

$$0 = I(x+u, y+v) - H(x,y)$$
$$\approx I(x,y) + I_x u + I_y v - H(x,y)$$

This is not exact

- To do better, we need to add higher order terms back in:

$$= I(x,y) + I_x u + I_y v + \text{higher order terms} - H(x,y)$$

This is a polynomial root finding problem

- Can solve using Newton's method
  - Also known as Newton-Raphson method
- Approach so far does one iteration of Newton's method
  - Better results are obtained via more iterations

1D case on board

Iterative Refinement

Iterative Lucas-Kanade Algorithm

1. Estimate velocity at each pixel by solving Lucas-Kanade equations
2. Warp H towards I using the estimated flow field

- use image warping techniques

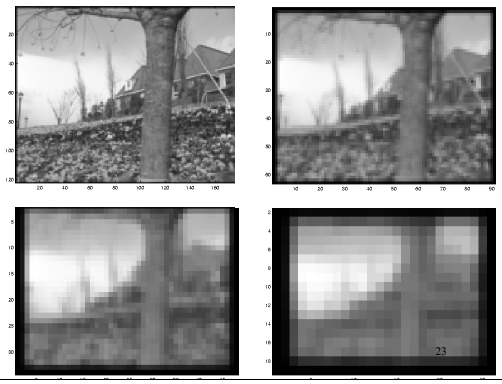
3. Repeat until convergence

Revisiting the small motion assumption

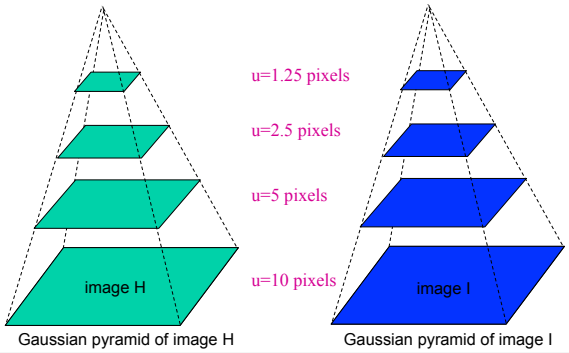
Is this motion small enough?

- Probably not—it's much larger than one pixel (2nd order terms dominate)
- How might we solve this problem?

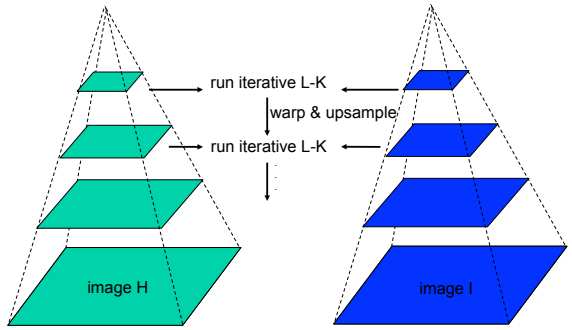
Reduce the resolution!



Coarse-to-fine optical flow estimation



Coarse-to-fine optical flow estimation



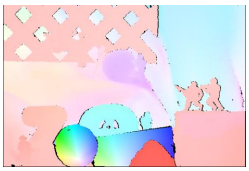
Flow quality evaluation

Flow quality evaluation

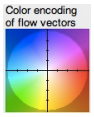
Flow quality evaluation

Middlebury flow page

- http://vision.middlebury.edu/flow/



Ground Truth



Flow quality evaluation

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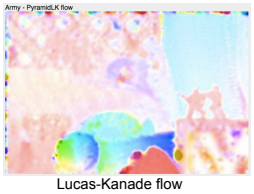
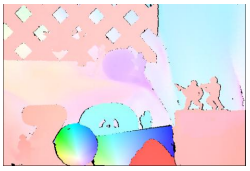
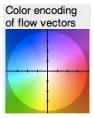


Lucas-Kanade flow



Ground Truth



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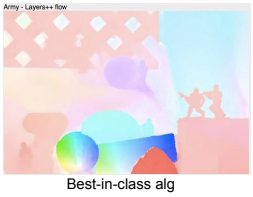
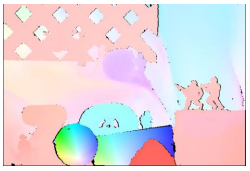


Best-in-class alg



Ground Truth