How do we estimate the Second Derivative?

- Laplacian Filter: $\nabla^2 f = \partial^2 f / \partial x^2 + \partial^2 f / \partial y^2$

0	1	0
1	-4	1
0	1	0

- Standard mask implementation
- Derivation: In 1D, the first derivative can be computed with mask [-1 0 1]
- The 1D second derivative is [1 -2 1]
- The Laplacian mask estimates the 2D second derivative.

How did you get those masks?

1D function f(x)

[f(-1) f(0) f(1)] pixel values

f(0)-f(-1) f(1)-f(0) first difference

(f(1)-f(0))-(f(0)-f(-1)) second difference

1f(-1)**-2**f(0)+**1**f(1) simplify

[1 -2 1] mask

and in 2D

	f(0,1)	
f(-1,0)	f(0,0)	f(1,0)
	f(0,-1)	

$\partial f/\partial x(1/2) = f(1,0)-f(0,0)$

$\partial f/\partial x(-1/2) = f(0,0)-f(-1,0)$

$\partial f/\partial x^2 = f(1,0)-2f(0,0)+f(-1,0)$

$\partial f/\partial y^2 = f(0,1)-2f(0,0)+f(0,-1)$

$$\nabla^2 f = \partial f/\partial x^2 + \partial f/\partial y^2$$
$$= 1f(1,0) -4f(0,0)+1f((-1,0)+1f(0,1)+1f(0,-1)$$

0	1	0
1	-4	1
0	1	0