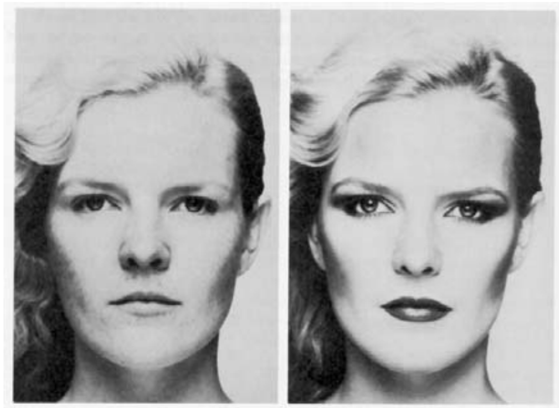
Photometric Stereo

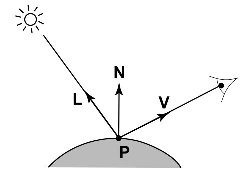


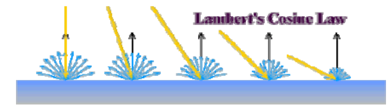
Merle Norman Cosmetics, Los Angeles

Readings

- R. Woodham, *Photometric Method for Determining Surface Orientation from Multiple Images*. Optical Engineering 19(1)139-144 (1980). (PDF)

Diffuse reflection





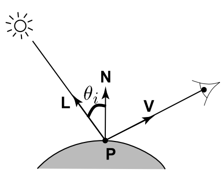
$$R_e = k_d \mathbf{N} \cdot \mathbf{L} R_i$$

image intensity of **P** → $I = k_d \mathbf{N} \cdot \mathbf{L}$

Simplifying assumptions

- $I = R_e$: camera response function f is the identity function:
  - can always achieve this in practice by solving for f and applying f^{-1} to each pixel in the image
- $R_i = 1$: light source intensity is 1
  - can achieve this by dividing each pixel in the image by R_i

Shape from shading



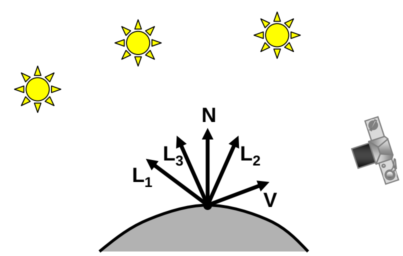
Suppose $k_d = 1$

$$\begin{aligned} I &= k_d \mathbf{N} \cdot \mathbf{L} \\ &= \mathbf{N} \cdot \mathbf{L} \\ &= cos\ \theta_i \end{aligned}$$

You can directly measure angle between normal and light source

- Not quite enough information to compute surface shape
- But can be if you add some additional info, for example
  - assume a few of the normals are known (e.g., along silhouette)
  - constraints on neighboring normals—“integrability”
  - smoothness
- Hard to get it to work well in practice
  - plus, how many real objects have constant albedo?

Photometric stereo



$$\begin{aligned} I_1 &= k_d \mathbf{N} \cdot \mathbf{L_1} \\ I_2 &= k_d \mathbf{N} \cdot \mathbf{L_2} \\ I_3 &= k_d \mathbf{N} \cdot \mathbf{L_3} \end{aligned}$$

Can write this as a matrix equation:

$$\left[\begin{array}{ccc} I_1 & I_2 & I_3 \end{array} \right] = k_d \mathbf{N}^T \left[\begin{array}{ccc} \mathbf{L_1} & \mathbf{L_2} & \mathbf{L_3} \end{array} \right]$$

Solving the equations

$$\underbrace{\begin{bmatrix} I_1 & I_2 & I_3 \end{bmatrix}}_{\substack{\mathbf{I} \\ 1\times 3}} = \underbrace{k_d \mathbf{N}^T}_{\substack{\mathbf{G} \\ 1\times 3}} \underbrace{\begin{bmatrix} \mathbf{L_1} & \mathbf{L_2} & \mathbf{L_3} \end{bmatrix}}_{\substack{\mathcal{L} \\ 3\times 3}}$$

$$\mathbf{G} = \mathbf{I}\mathbf{L}^{-1}$$

$$k_d = \|\mathbf{G}\|$$

$$\mathbf{N} = \frac{1}{k_d}\mathbf{G}$$

More than three lights

Get better results by using more lights

$$\begin{bmatrix} I_1 & \ldots & I_n \end{bmatrix} = k_d \mathbf{N}^T \begin{bmatrix} \mathbf{L_1} & \ldots & \mathbf{L_n} \end{bmatrix}$$

Least squares solution:

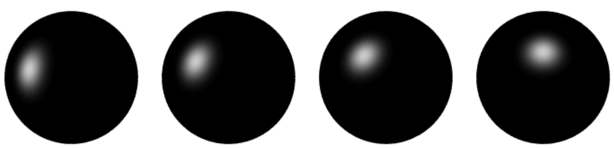
$$\begin{aligned} \mathbf{I} &= \mathbf{GL} \\ \mathbf{IL^T} &= \mathbf{GLL^T} \\ \mathbf{G} &= (\mathbf{IL^T})(\mathbf{LL^T})^{-1} \end{aligned}$$

Solve for N, k_d as before

What's the size of $\mathbf{LL^T}$?

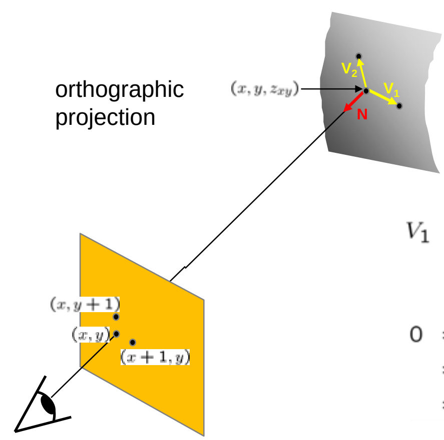
Computing light source directions

Trick: place a chrome sphere in the scene



- the location of the highlight tells you where the light source is

Depth from normals



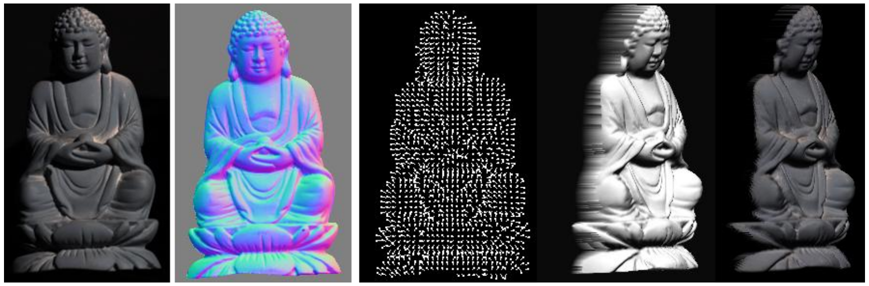
$$\begin{aligned} V_1 &= (x+1, y, z_{x+1,y}) - (x, y, z_{xy}) \\ &= (1, 0, z_{x+1,y} - z_{xy}) \end{aligned}$$

$$\begin{aligned} 0 &= N \cdot V_1 \\ &= (n_x, n_y, n_z) \cdot (1, 0, z_{x+1,y} - z_{xy}) \\ &= n_x + n_z(z_{x+1,y} - z_{xy}) \end{aligned}$$

Get a similar equation for $\mathbf{V_2}$

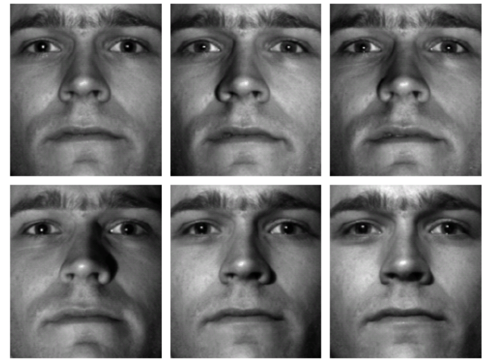
- Each normal gives us two linear constraints on z
- compute z values by solving a matrix equation

Results…



Input (1 of 12) | Normals | Normals | Shaded rendering | Textured rendering

Results…



from Athos Georghiades
http://cvc.yale.edu/people/Athos.html

Limitations

Big problems

- doesn't work for shiny things, semi-translucent things
- shadows, inter-reflections

Smaller problems

- camera and lights have to be distant
- calibration requirements
  - measure light source directions, intensities
  - camera response function

Newer work addresses some of these issues

Some pointers for further reading:

- Zickler, Belhumeur, and Kriegman, "*Helmholtz Stereopsis: Exploiting Reciprocity for Surface Reconstruction*." IJCV, Vol. 49 No. 2/3, pp 215-227.
- Hertzmann & Seitz, "*Example-Based Photometric Stereo: Shape Reconstruction with General, Varying BRDFs*." IEEE Trans. PAMI 2005