Lucas-Kanade Motion Estimation

Thanks to Steve Seitz, Simon Baker, Takeo Kanade, and anyone else who helped develop these slides.

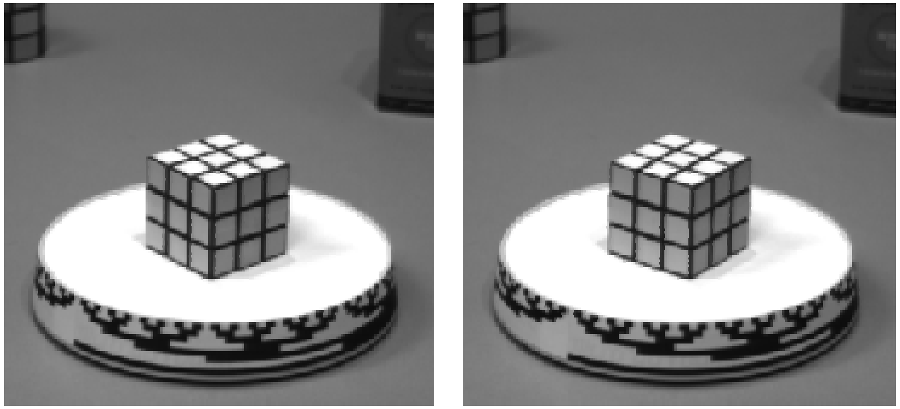
Why estimate motion?

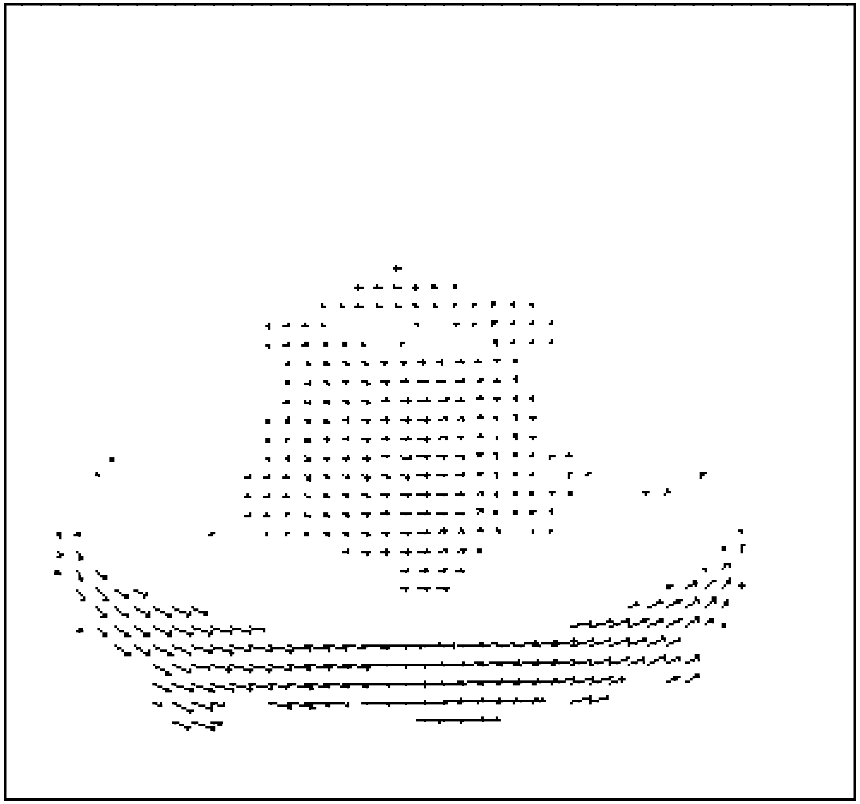
We live in a 4-D world

Wide applications

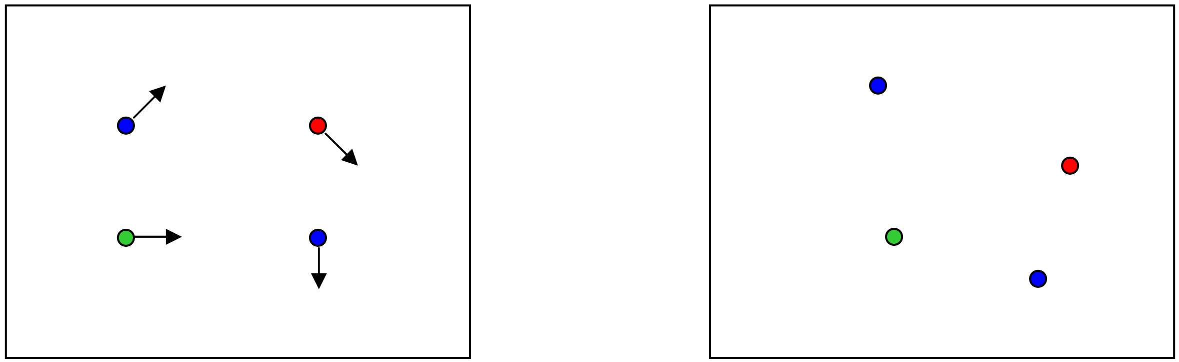
- Object Tracking
- Camera Stabilization
- Image Mosaics
- 3D Shape Reconstruction (SFM)
- Special Effects (Match Move)

Optical flow





Problem definition: optical flow



$H(x, y)$ $I(x, y)$

How to estimate pixel motion from image H to image I?

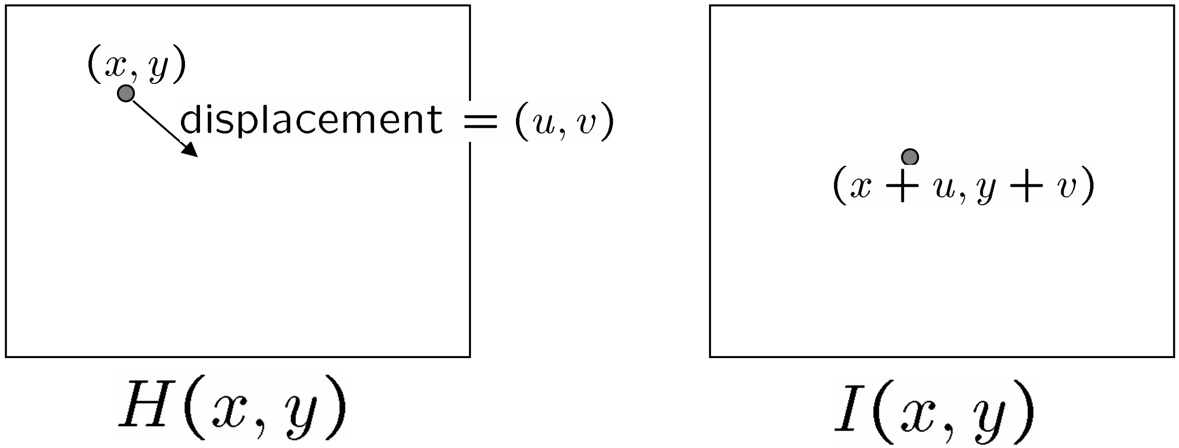
- Solve pixel correspondence problem
  - given a pixel in H, look for nearby pixels of the same color in I

Key assumptions

- **color constancy:** a point in H looks the same in I
  - For grayscale images, this is **brightness constancy**
- **small motion**: points do not move very far

This is called the **optical flow** problem

Optical flow constraints (grayscale images)



Let's look at these constraints more closely

- brightness constancy: Q: what's the equation?

$$H(x, y) = I(x+u, y+v)$$

- small motion: (u and v are less than 1 pixel)
  - suppose we take the Taylor series expansion of I:

$$I(x+u, y+v) = I(x, y) + \frac{\partial I}{\partial x}u + \frac{\partial I}{\partial y}v + \text{higher order terms}$$

$$\approx I(x, y) + \frac{\partial I}{\partial x}u + \frac{\partial I}{\partial y}v$$

Optical flow equation

Combining these two equations

$$0 = I(x+u, y+v) - H(x,y)$$

$$\approx I(x,y) + I_x u + I_y v - H(x,y)$$

$$\approx (I(x,y) - H(x,y)) + I_x u + I_y v$$

$$\approx I_t + I_x u + I_y v$$

$$\approx I_t + \nabla I \cdot [u \;\; v]$$

shorthand: $I_x = \frac{\partial I}{\partial x}$

The x-component of the gradient vector.

What is I_t ? The time derivative of the image at (x,y)

How do we calculate it?

Optical flow equation

$$0 = I_t + \nabla I \cdot [u \;\; v]$$

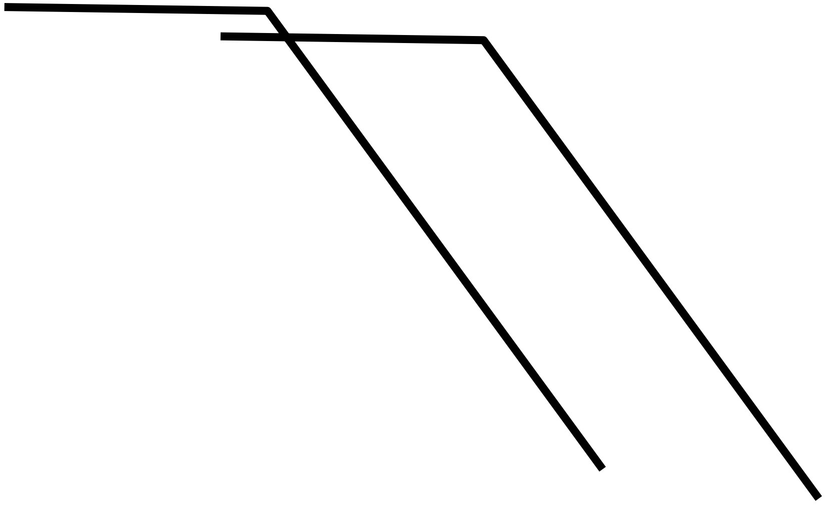
Q: how many unknowns and equations per pixel?

1 equation, but 2 unknowns (u and v)

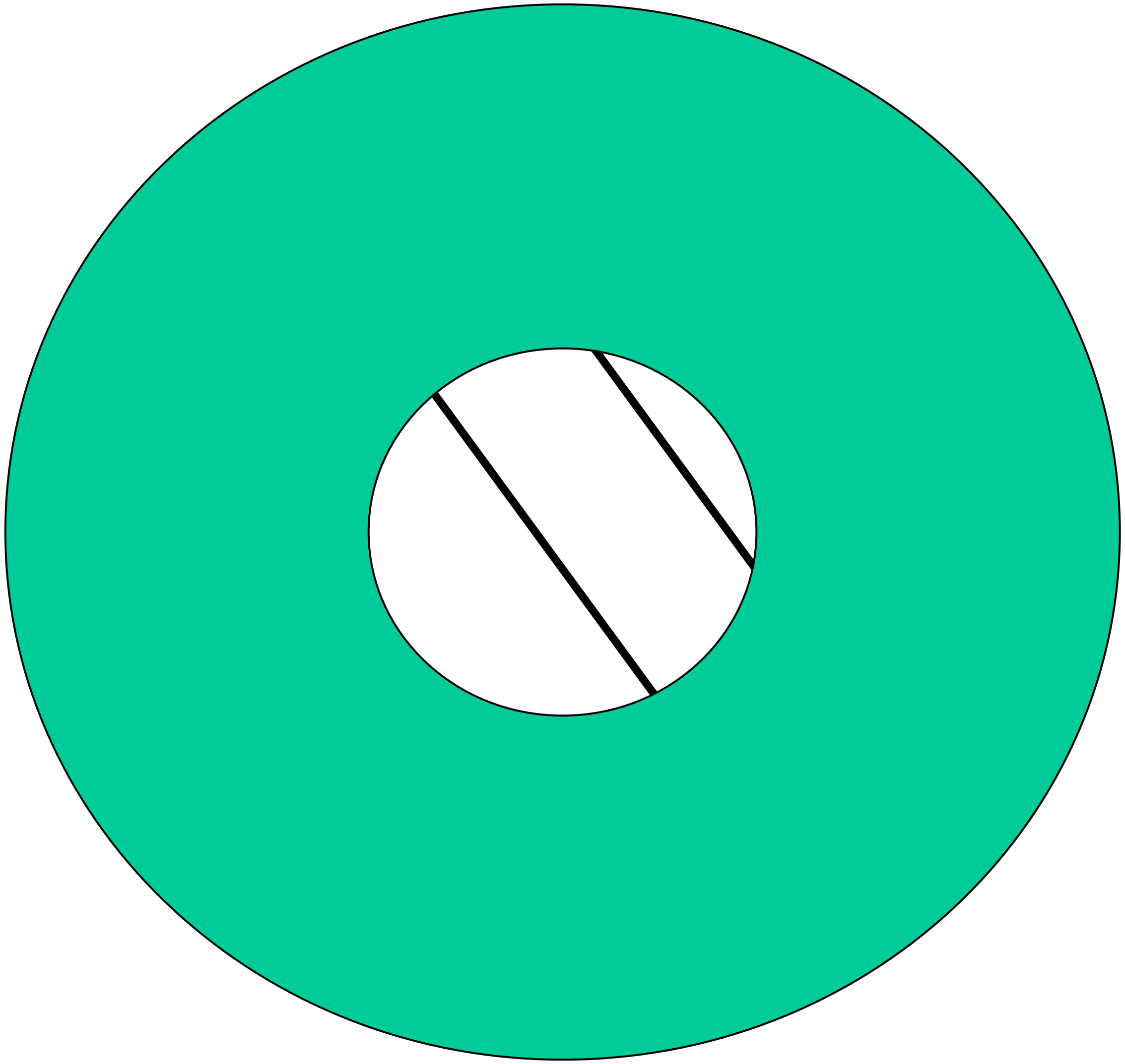
Intuitively, what does this constraint mean?

- The component of the flow in the gradient direction is determined
- The component of the flow parallel to an edge is unknown

Aperture problem



Aperture problem



Solving the aperture problem

Basic idea: assume motion field is smooth

Lukas & Kanade: assume locally constant motion

- pretend the pixel's neighbors have the same (u,v)
  - If we use a 5x5 window, that gives us 25 equations per pixel!

$$0 = I_t(\mathbf{p_i}) + \nabla I(\mathbf{p_i}) \cdot [u \;\; v]$$

Many other methods exist. Here's an overview:

- Barron, J.L., Fleet, D.J., and Beauchemin, S, Performance of optical flow techniques**,** *International Journal of Computer Vision*, 12(1):43-77, 1994.

Lukas-Kanade flow

How to get more equations for a pixel?

- Basic idea: impose additional constraints
  - most common is to assume that the flow field is smooth locally
  - one method: pretend the pixel's neighbors have the same (u,v)
    - If we use a 5x5 window, that gives us 25 equations per pixel!

$$0 = I_t(\mathbf{p_i}) + \nabla I(\mathbf{p_i}) \cdot [u \;\; v]$$

$$\underset{25\times 2}{\underbrace{\begin{bmatrix} I_x(\mathbf{p_1}) & I_y(\mathbf{p_1}) \\ I_x(\mathbf{p_2}) & I_y(\mathbf{p_2}) \\ \vdots & \vdots \\ I_x(\mathbf{p_{25}}) & I_y(\mathbf{p_{25}}) \end{bmatrix}}_{A}} \underset{2\times 1}{\underbrace{\begin{bmatrix} u \\ v \end{bmatrix}}_{d}} = - \underset{25\times 1}{\underbrace{\begin{bmatrix} I_t(\mathbf{p_1}) \\ I_t(\mathbf{p_2}) \\ \vdots \\ I_t(\mathbf{p_{25}}) \end{bmatrix}}_{b}}$$

RGB version

How to get more equations for a pixel?

- Basic idea: impose additional constraints
  - most common is to assume that the flow field is smooth locally
  - one method: pretend the pixel's neighbors have the same (u,v)
    - » If we use a 5x5 window, that gives us 25*3 equations per pixel!

$$0 = I_t(\mathbf{p_i})[0,1,2] + \nabla I(\mathbf{p_i})[0,1,2] \cdot [u \;\; v]$$

$$\underbrace{\begin{bmatrix} I_x(\mathbf{p_1})[0] & I_y(\mathbf{p_1})[0] \\ I_x(\mathbf{p_1})[1] & I_y(\mathbf{p_1})[1] \\ I_x(\mathbf{p_1})[2] & I_y(\mathbf{p_1})[2] \\ \vdots & \vdots \\ I_x(\mathbf{p_{25}})[0] & I_y(\mathbf{p_{25}})[0] \\ I_x(\mathbf{p_{25}})[1] & I_y(\mathbf{p_{25}})[1] \\ I_x(\mathbf{p_{25}})[2] & I_y(\mathbf{p_{25}})[2] \end{bmatrix}}_{\substack{A \\ 75\times 2}} \underbrace{\begin{bmatrix} u \\ v \end{bmatrix}}_{\substack{d \\ 2\times 1}} = - \underbrace{\begin{bmatrix} I_t(\mathbf{p_1})[0] \\ I_t(\mathbf{p_1})[1] \\ I_t(\mathbf{p_1})[2] \\ \vdots \\ I_t(\mathbf{p_{25}})[0] \\ I_t(\mathbf{p_{25}})[1] \\ I_t(\mathbf{p_{25}})[2] \end{bmatrix}}_{\substack{b \\ 75\times 1}}$$

Lukas-Kanade flow

Prob: we have more equations than unknowns

$$\underset{25\times2}{A}\ \underset{2\times1}{d} = \underset{25\times1}{b} \longrightarrow \text{minimize } \|Ad - b\|^2$$

Solution: solve least squares problem

- minimum least squares solution given by solution (in d) of:

$$\underset{2\times2}{(A^T A)}\ \underset{2\times1}{d} = \underset{2\times1}{A^T b}$$

$$\underbrace{\begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_x I_y & \sum I_y I_y \end{bmatrix}}_{A^T A} \begin{bmatrix} u \\ v \end{bmatrix} = - \underbrace{\begin{bmatrix} \sum I_x I_t \\ \sum I_y I_t \end{bmatrix}}_{A^T b}$$

- The summations are over all pixels in the K x K window
- This technique was first proposed by Lukas & Kanade (1981)

Conditions for solvability

- Optimal (u, v) satisfies Lucas-Kanade equation

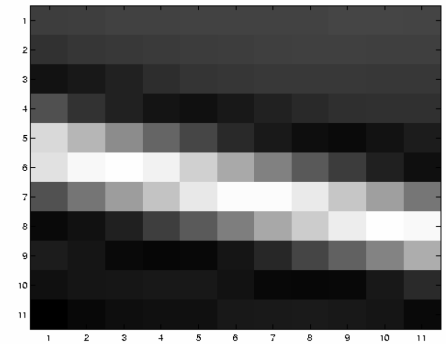
$$\begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_x I_y & \sum I_y I_y \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = - \begin{bmatrix} \sum I_x I_t \\ \sum I_y I_t \end{bmatrix}$$

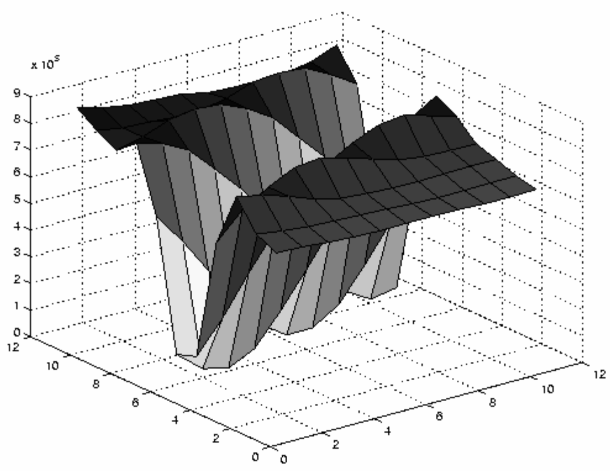
$$A^T A \qquad\qquad\qquad\qquad A^T b$$

When is This Solvable?

- **$A^T A$** should be invertible
- **$A^T A$** should not be too small due to noise
  - eigenvalues λ_1 and λ_2 of **$A^T A$** should not be too small
- **$A^T A$** should be well-conditioned
  - λ_1 / λ_2 should not be too large (λ_1 = larger eigenvalue)

Edges cause problems



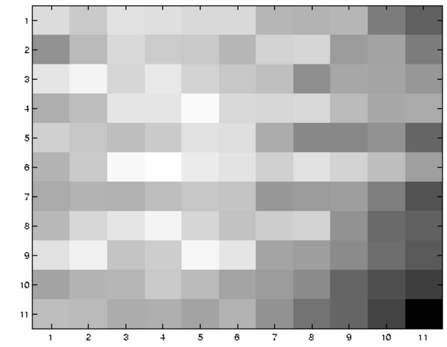


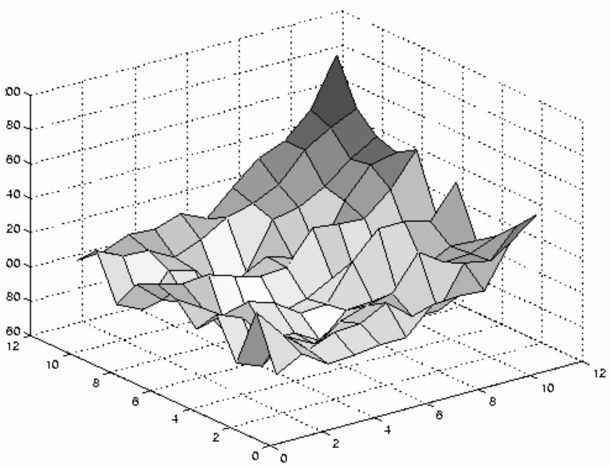
$$\sum \nabla I (\nabla I)^T$$

– large gradients, all the same

– large λ_1, small λ_2

Low texture regions don't work



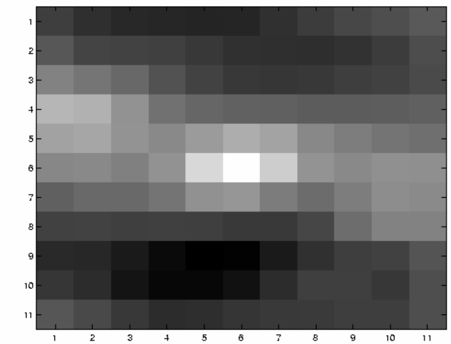


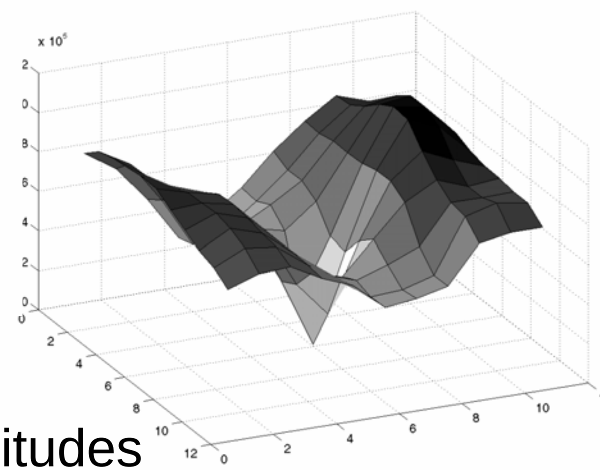
$$\sum \nabla I (\nabla I)^T$$

– gradients have small magnitude

– small λ_1, small λ_2

High textured region work best





$$\sum \nabla I (\nabla I)^T$$

– gradients are different, large magnitudes

– large λ_1, large λ_2

Errors in Lukas-Kanade

What are the potential causes of errors in this procedure?

- Suppose A^TA is easily invertible
- Suppose there is not much noise in the image

When our assumptions are violated

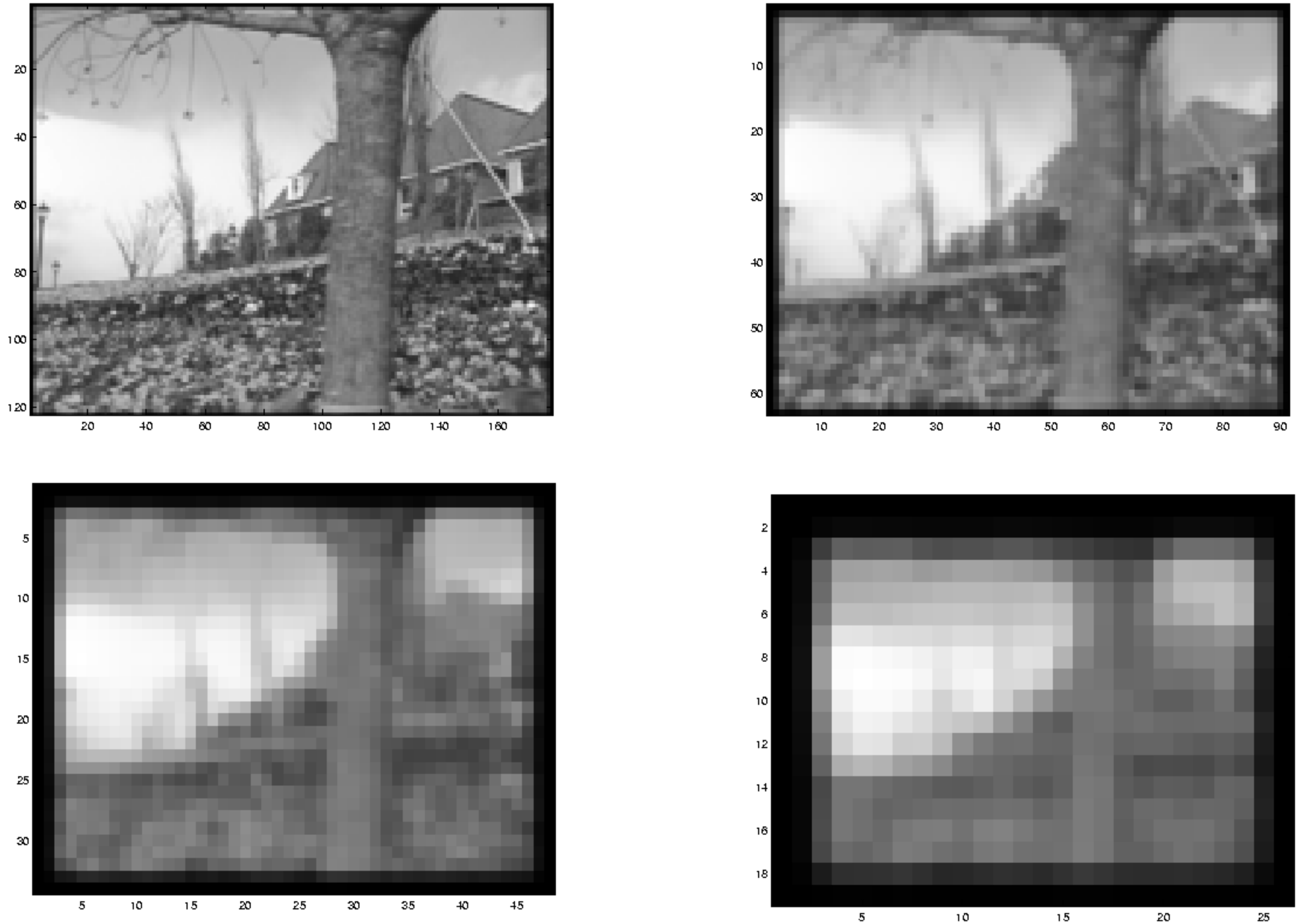
- Brightness constancy is **not** satisfied
- The motion is **not** small
- A point does **not** move like its neighbors
  - window size is too large
  - what is the ideal window size?

Revisiting the small motion assumption

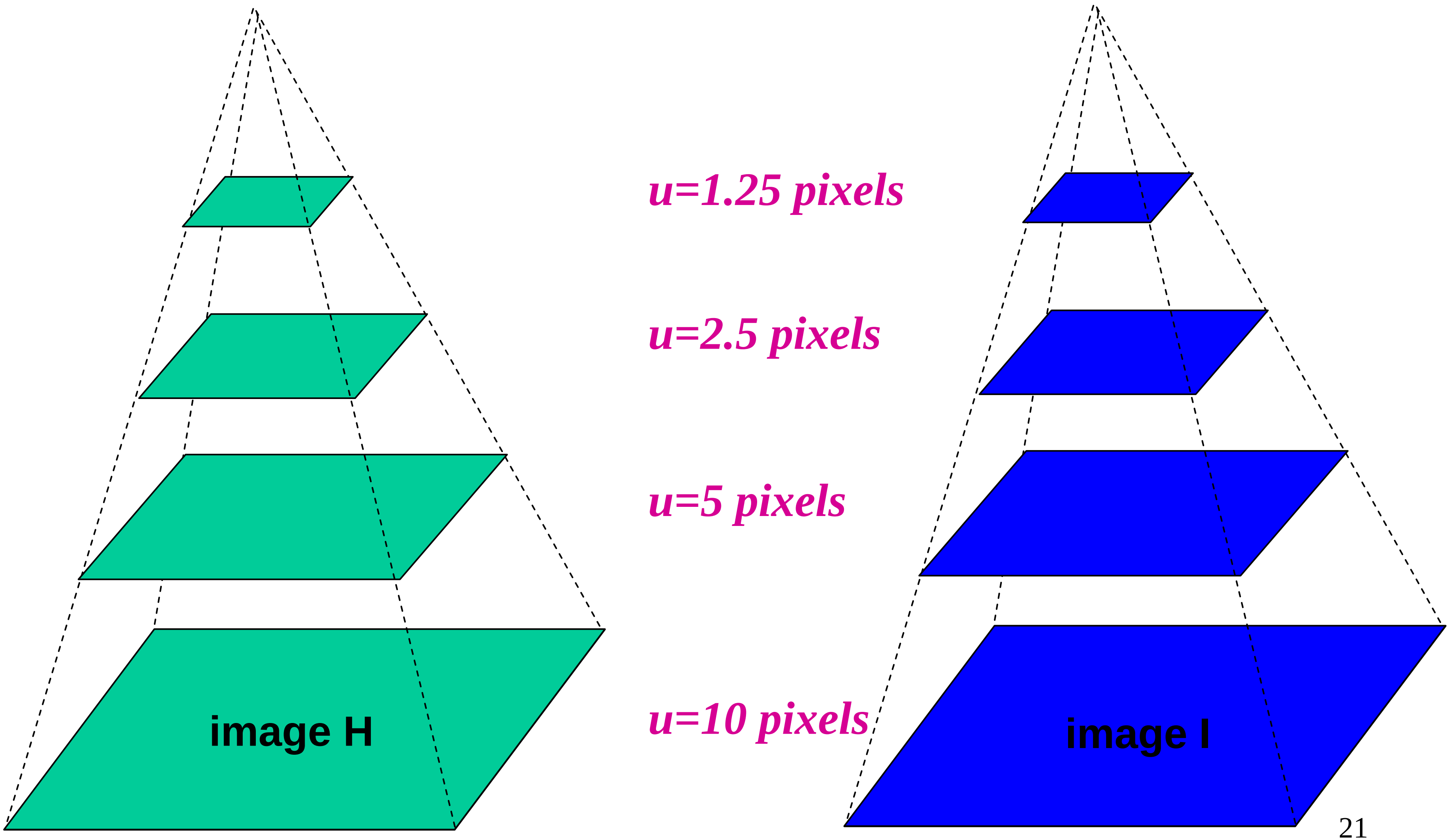
Is this motion small enough?

- Probably not—it's much larger than one pixel (2nd order terms dominate)
- How might we solve this problem?

Reduce the resolution!

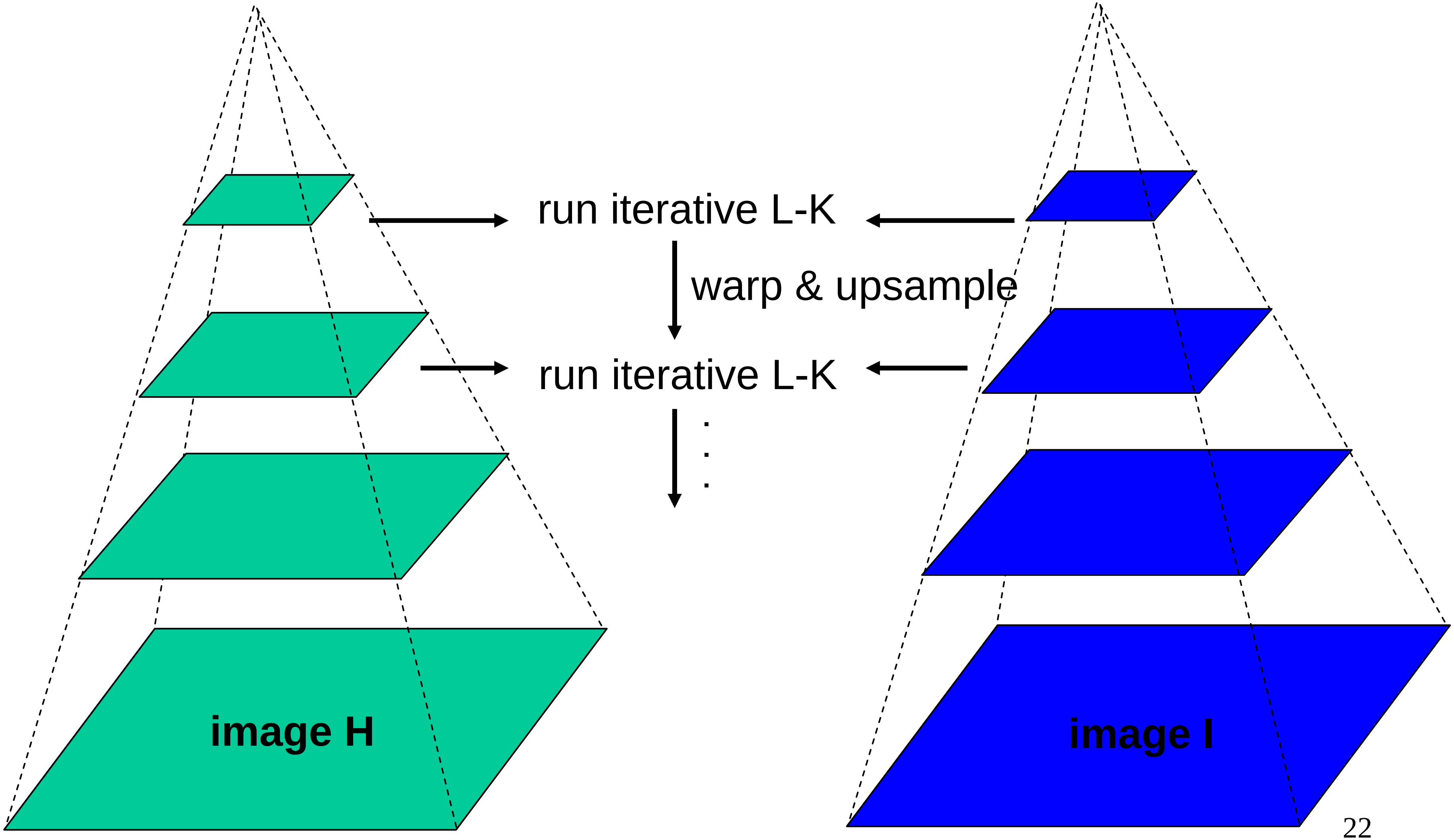


Coarse-to-fine optical flow estimation



Gaussian pyramid of image H **Gaussian pyramid of image I**

Coarse-to-fine optical flow estimation



Gaussian pyramid of image H

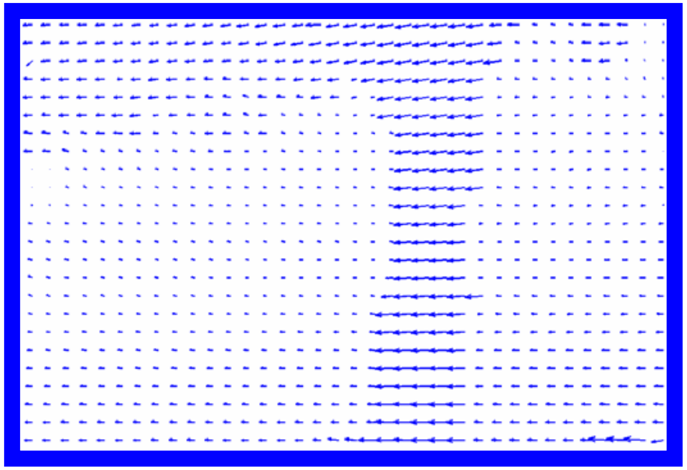
Gaussian pyramid of image I

A Few Details

- Top Level
  - Apply L-K to get a flow field representing the flow from the first frame to the second frame.
  - Apply this flow field to warp the first frame toward the second frame.
  - Rerun L-K on the new warped image to get a flow field from it to the second frame.
  - Repeat till convergence.
- Next Level
  - Upsample the flow field to the next level as the first guess of the flow at that level.
  - Apply this flow field to warp the first frame toward the second frame.
  - Rerun L-K and warping till convergence as above.
- Etc.

The Flower Garden Video

What should the optical flow be?



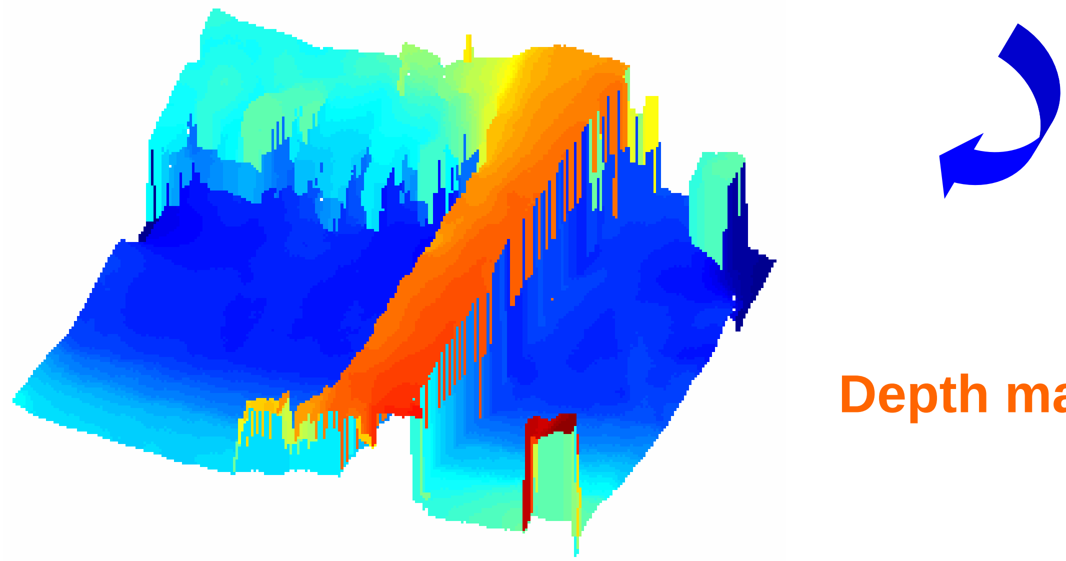
Robust Visual Motion Analysis:
Piecewise-Smooth Optical Flow

Ming Ye
Electrical Engineering
University of Washington

Structure From Motion

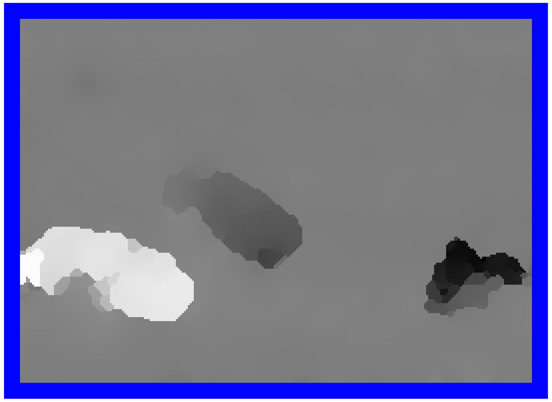
Rigid scene + camera translation

Estimated horizontal motion



Depth map

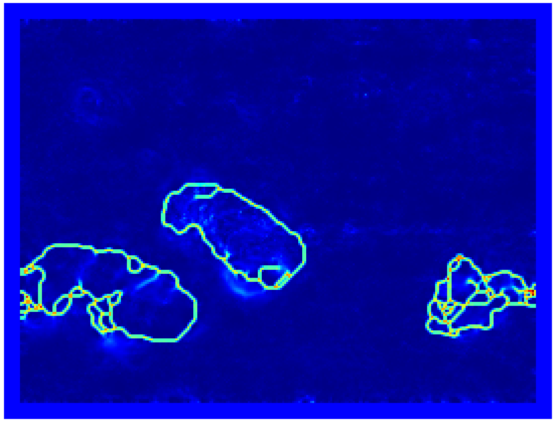
Scene Dynamics Understanding



Brighter pixels => larger speeds.

Estimated horizontal motion

- Surveillance
- Event analysis
- Video compression



Motion boundaries are smooth.

Motion smoothness

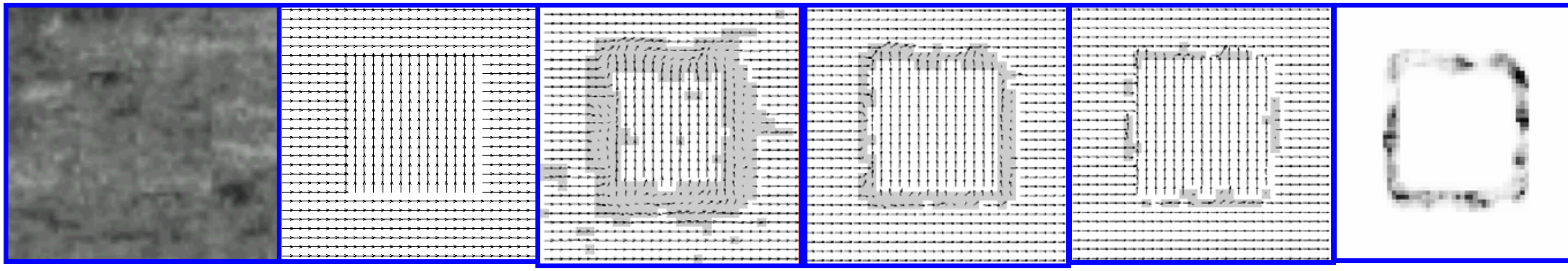
Target Detection and Tracking

A tiny airplane --- only observable by its distinct motion

Tracking results

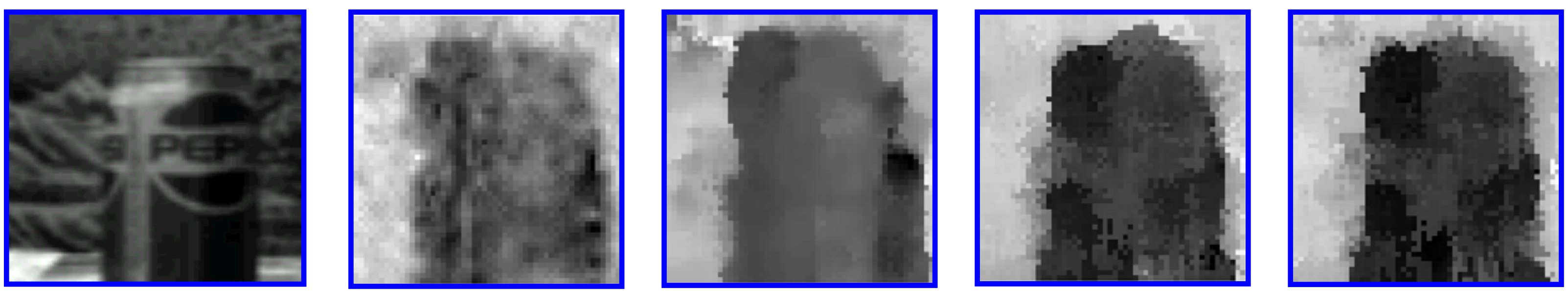
Results from Prior Methods:

LS = Least Squares, LS-R = Robust Least Squares, R = new robust method



Sampled by2: True LS-LS LS-R R-R Confidence

LS = Least Squares, LS-R = Robust Least Squares, R = new robust method



Horizontal flow: M-OFC LS-LMedS LS-R R-R

M-OFC = solving the optical flow constraint using the M-Estimator

LMedS = Least Median of Squares

Estimating Piecewise-Smooth Optical Flow with Global Matching and Graduated Optimization

A Bayesian Approach

Problem Statement

Assuming only brightness conservation and piecewise-smooth motion, find the optical flow to best describe the intensity change in three frames.

Approach: Matching-Based Global Optimization

- **Step 1. Robust local gradient-based method for high-quality initial flow estimate.**
- **Step 2. Global gradient-based method to improve the flow-field coherence.**
- **Step 3. Global matching that minimizes energy by a greedy approach.**

Global Energy Design

Global energy

V is the optical flow field.

$$E = \sum_{\text{all sites s}} E_B(V_s) + E_S(V_s)$$

V_s is the optical flow at pixel s.

E_B is the brightness conservation.

Matching error

- Warping error

$$E_B(V_s) = \rho(e_W(V_s), \sigma_{B_s})$$

$$e_W(V_s) = \min(|I^-(V_s) - I_s|, |I^+(V_s) - I_s|)$$

Smoothness error

I^- and I^+ are prev & next frame; $I^-(V_S)$ is the warped intensity in prev frame.

E_S is the flow smoothness error in a neighborhood about pixel s.

$$E_S(V_i) = \frac{1}{8} \sum_{n \in N_s^8} \rho(|V_s - V_n|, \sigma_{S_s})$$

Error function: $\rho(x,\sigma) = \dfrac{x^2}{\sigma^2 + x^2}$

Step 1: Gradient-Based Local Regression

- **A crude flow estimate is assumed available (and has been compensated for)**
- **A robust gradient-based local regression is used to compute the incremental flow ΔV.**
- **The dominant translational motion in the neighborhood of each pixel is computed by solving a set of flow equations using a least-median-of-squares criterion.**

Step 2: Gradient-Based Global Optimization

- **The coherence of ΔV using a gradient-based global optimization method.**

- **The energy to minimize is given by**

$$E(\Delta V) = \sum_{\text{all sites } s} \{\rho(e_B(\Delta V_s), \sigma_{B_s}) + \frac{1}{8}\sum_{n \in N_s^8} \rho(|V_s + \Delta V_s - V_n - \Delta V_n|, \sigma_{S_s})\}$$

where e_B is the residual of the OFC,
V_s is the ith vector of the initial flow, and
the sigmas are parameters.

Step 3: Global Matching

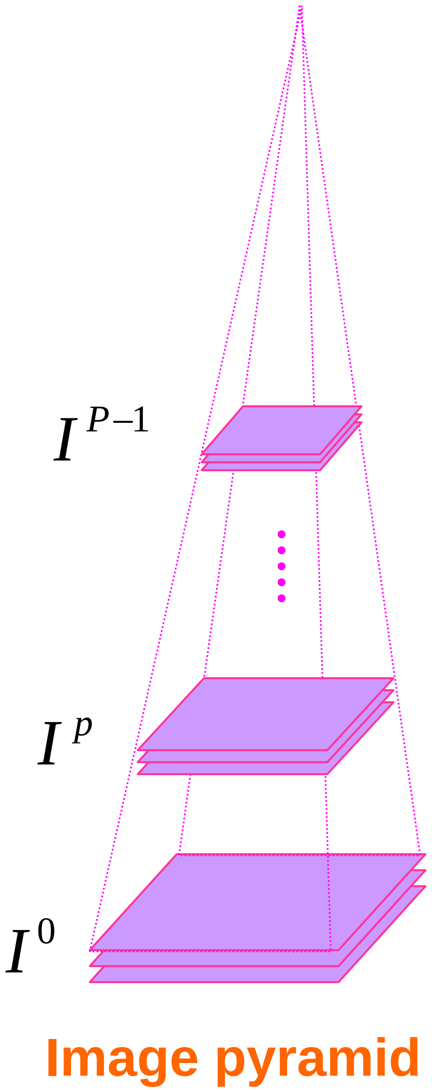
- **The new flow estimate still exhibits gross errors at motion boundaries and other places with poor gradient estimates.**

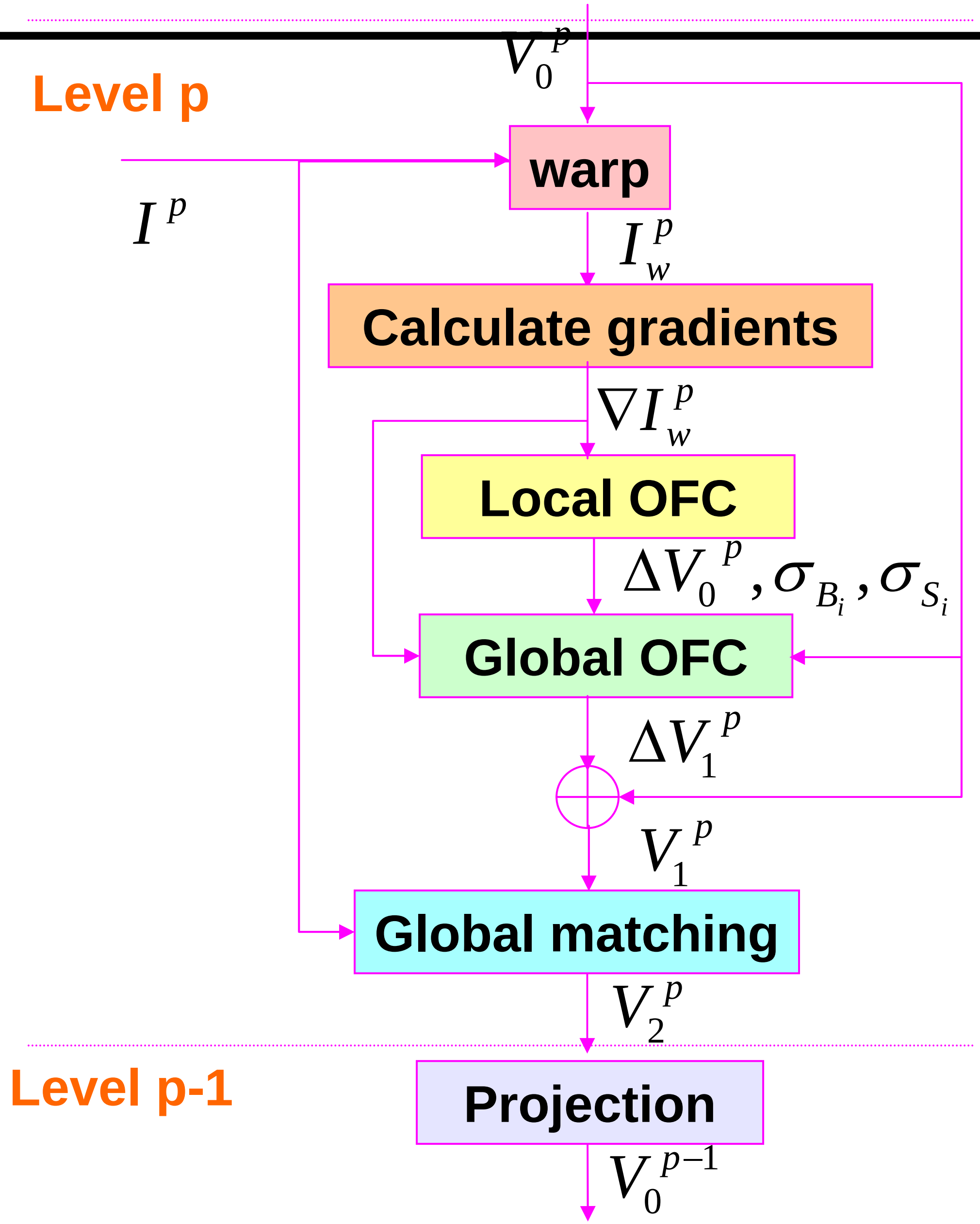
- **This error is reduced by solving the matching-based formulation equation through greedy propagation.**

- **The energy is calculated for all pixels.**

- **Then each pixel is visited, examining whether a trial estimate from the candidates in its neighborhood is better (lower energy). If so, this becomes the new estimate for that pixel. This is repeated iteratively.**

Overall Algorithm





Advantages

Best of Everything

- Local OFC
  - High-quality initial flow estimates
  - Robust local scale estimates
- Global OFC
  - Improve flow smoothness
- Global Matching
  - The optimal formulation
  - Correct errors caused by poor gradient quality and hierarchical process

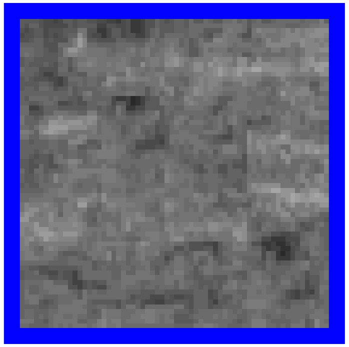
Results: fast convergence, high accuracy, simultaneous motion boundary detection

Experiments

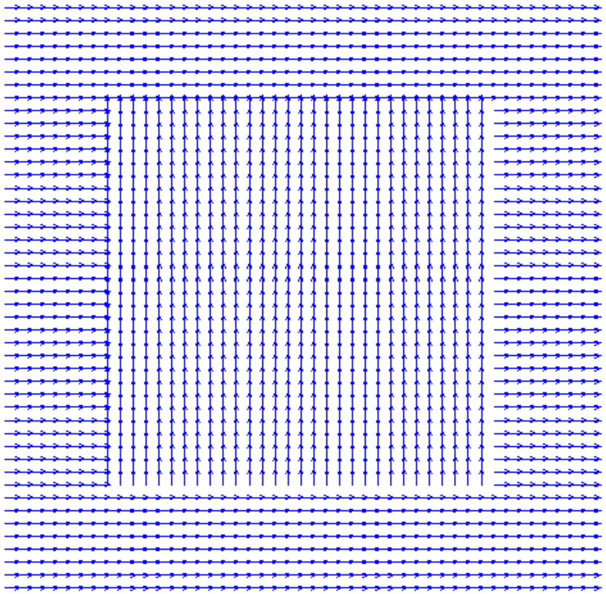
- **Experiments were run on several standard test videos.**
- **Estimates of optical flow were made for the middle frame of every three.**
- **The results were compared with the Black and Anandan algorithm.**

TS: Translating Squares

Homebrew, ideal setting, test performance upper bound

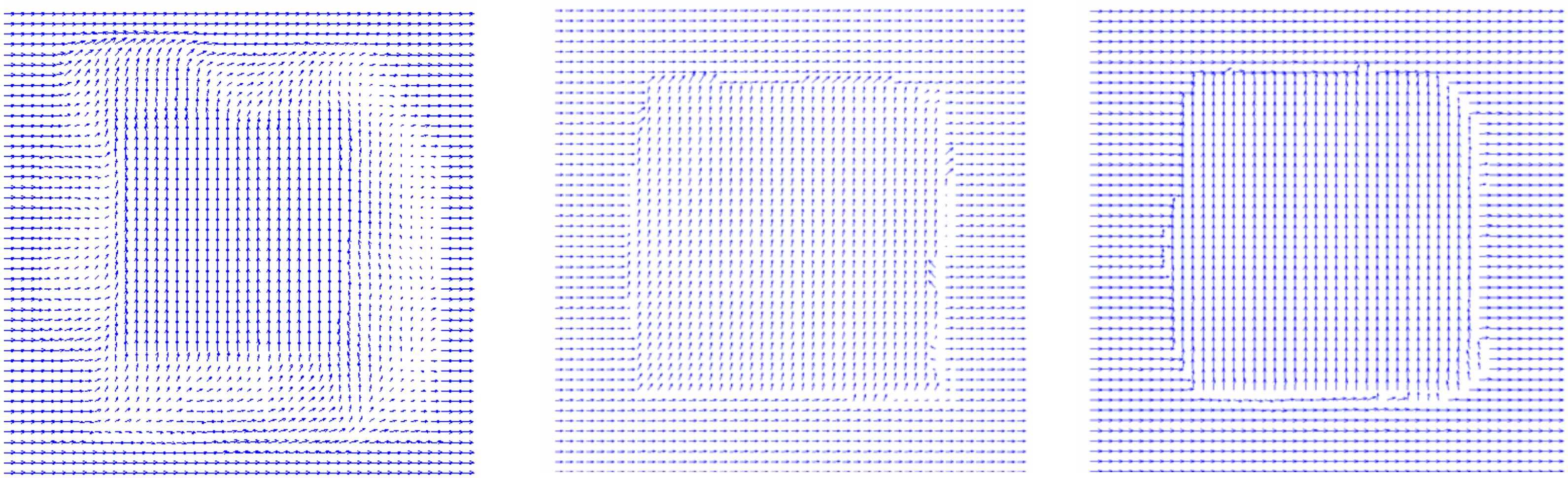


64x64, 1pixel/frame



Groundtruth (cropped),
Our estimate looks the same

TS: Flow Estimate Plots



LS   BA   S1 (S2 is close)

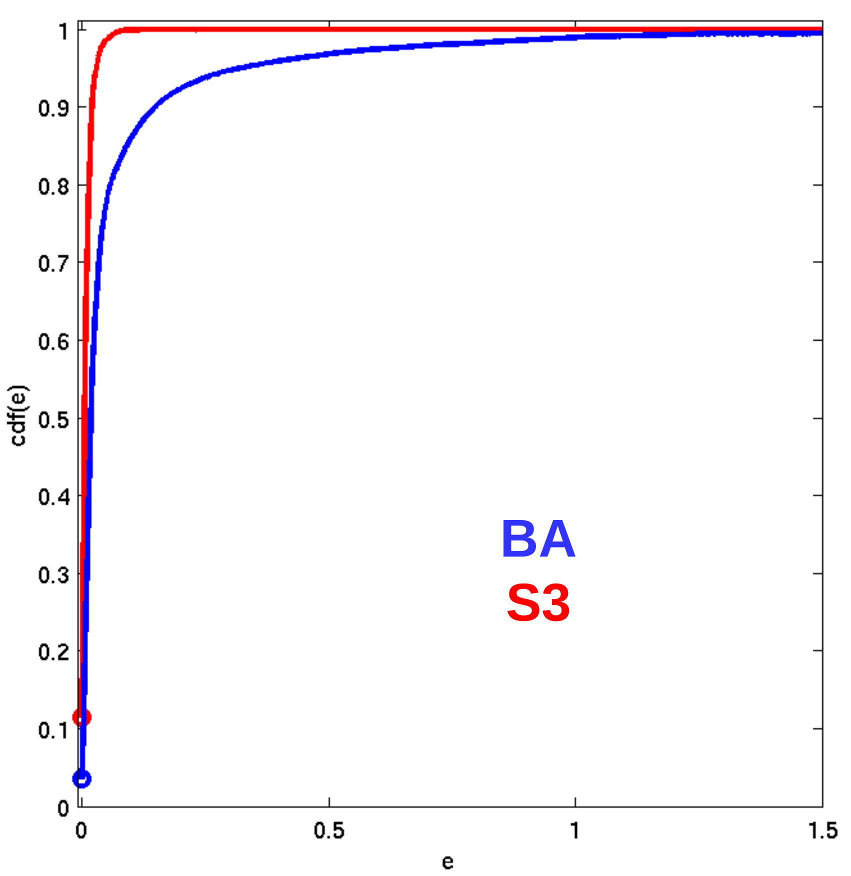
S3 looks the same as the groundtruth.

- S1, S2, S3: results from our Step I, II, III (final)

TT: Translating Tree

150x150 (Barron 94)

	$e_{\angle}(^{\circ})$	$e_{\lvert\bullet\rvert}(\text{pix})$	$\bar{e}(\text{pix})$
BA	2.60	0.128	0.0724
S3	0.248	0.0167	0.00984

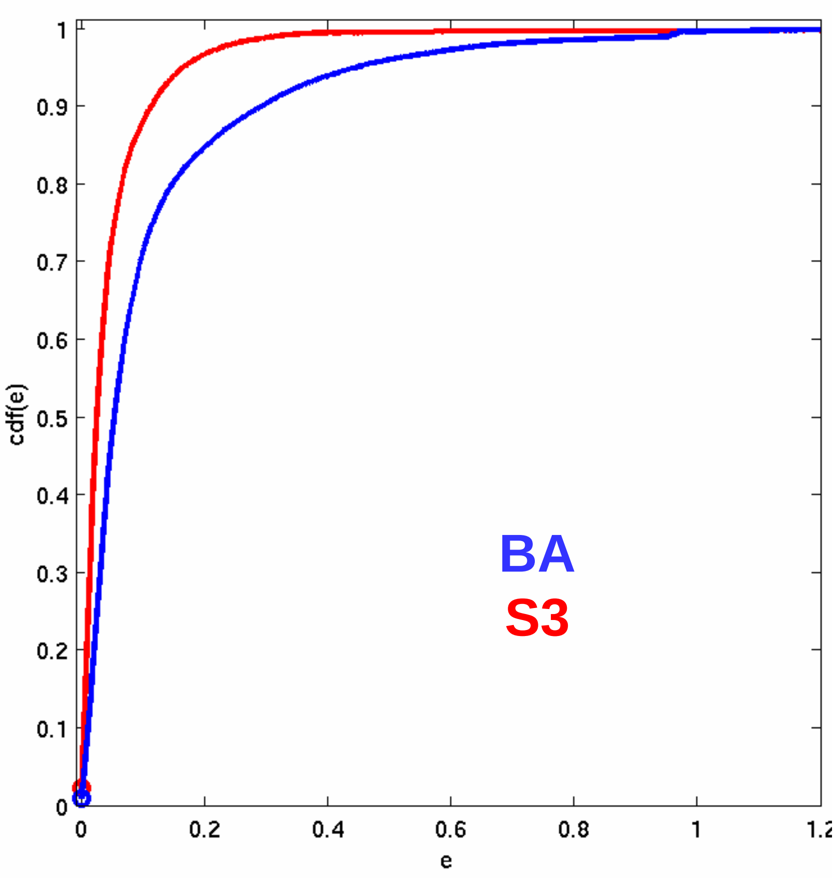


e: error in pixels, cdf: culmulative distribution function for all pixels

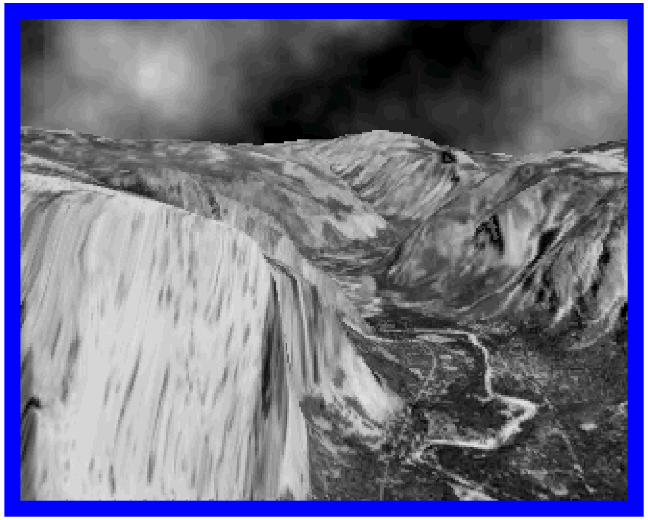
DT: Diverging Tree

150x150 (Barron 94)

	$e_{\angle}(^{\circ})$	$e_{\lvert\bullet\rvert}(\text{pix})$	$\bar{e}(\text{pix})$
BA	**6.36**	**0.182**	**0.114**
S3	**2.60**	**0.0813**	**0.0507**

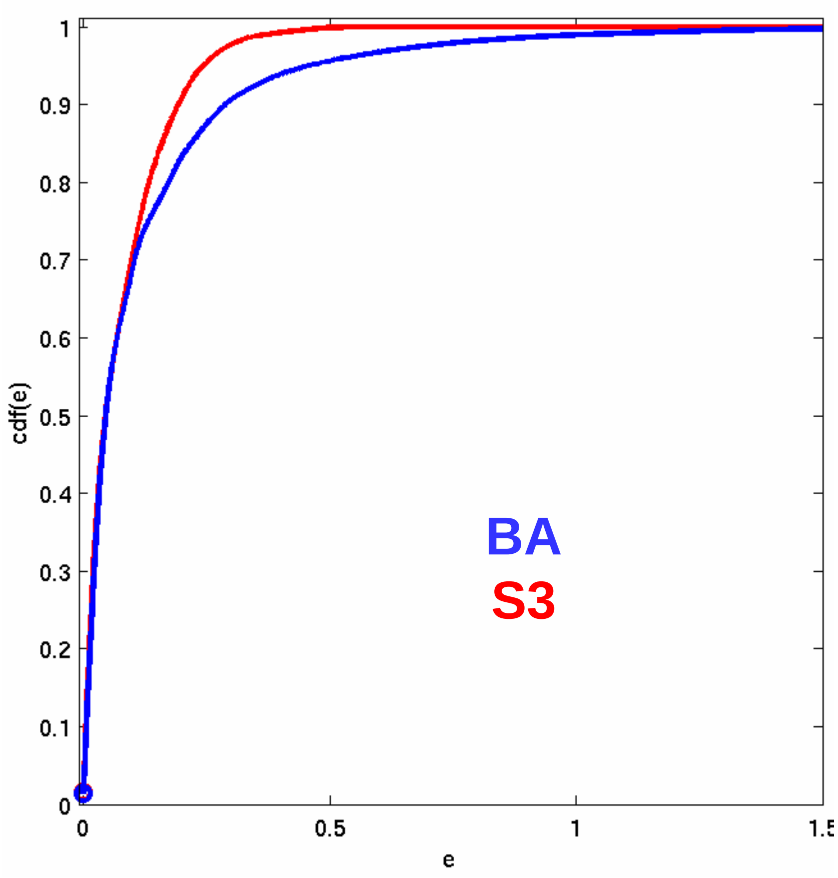


YOS: Yosemite Fly-Through



316x252 (Barron, cloud excluded)

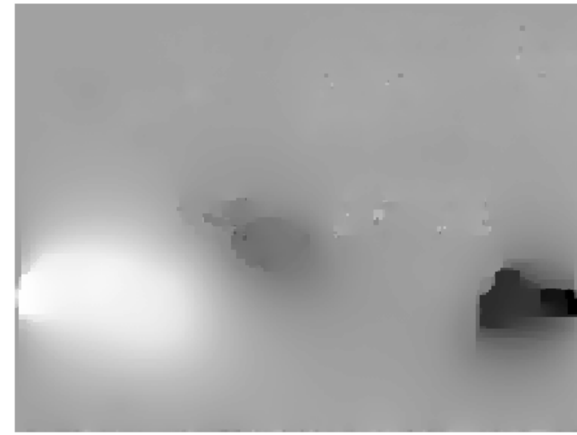
	$e_{\angle}(^{\circ})$	$e_{\lvert\bullet\rvert}(\text{pix})$	$\bar{e}(\text{pix})$
BA	**2.71**	**0.185**	**0.118**
S3	**1.92**	**0.120**	**0.0776**



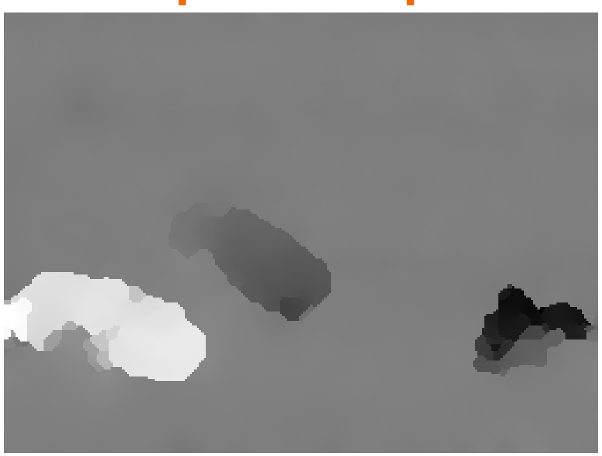
TAXI: Hamburg Taxi

256x190, (Barron 94)
max speed 3.0 pix/frame

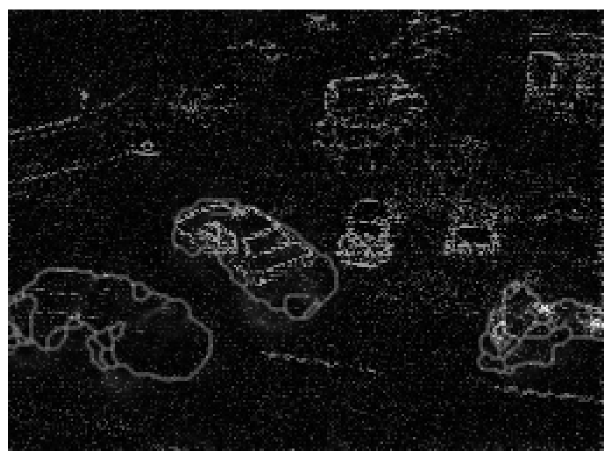
LMS



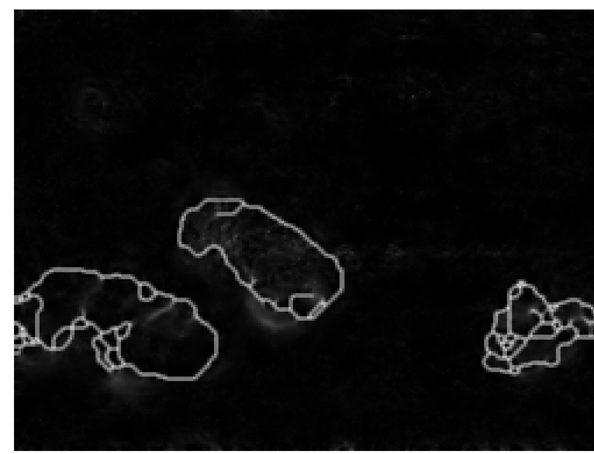
BA



Ours



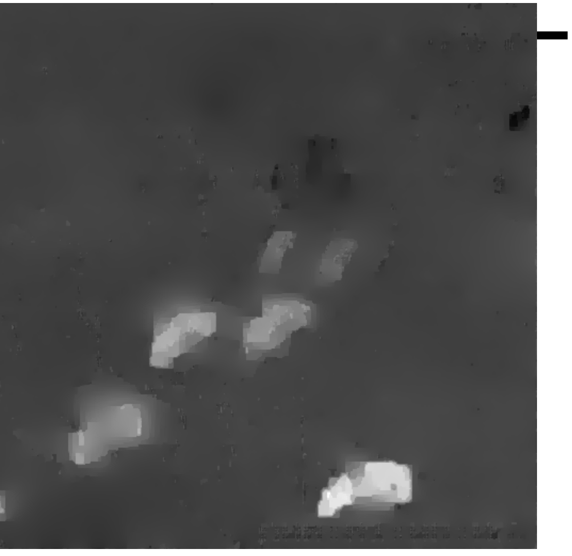
Error map



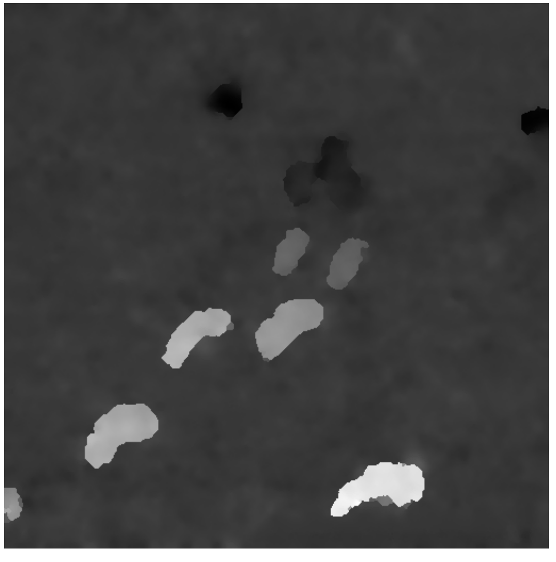
Smoothness error

Traffic

512x512
(Nagel)
max speed:
6.0 pix/frame

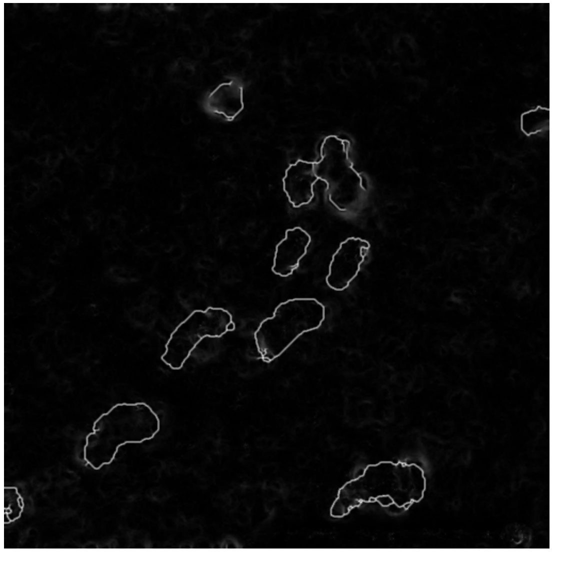


BA



Ours

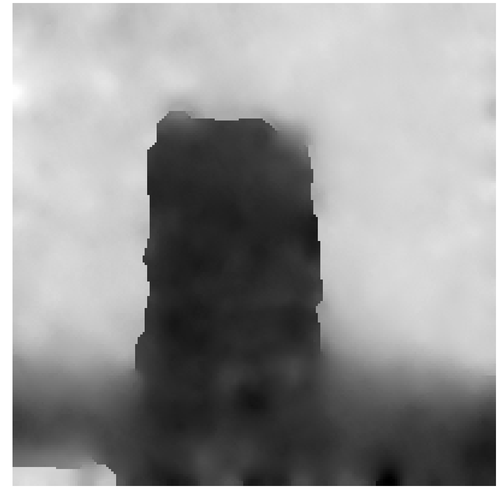
Error map



Smoothness error

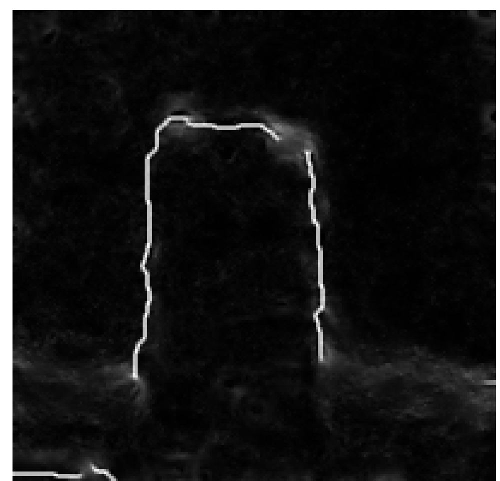
Pepsi Can

201x201 (Black) Max speed: 2pix/frame



Ours

BA

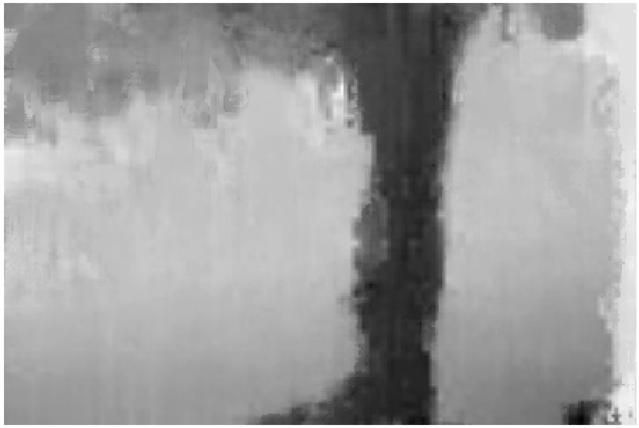


Smoothness error

FG: Flower Garden

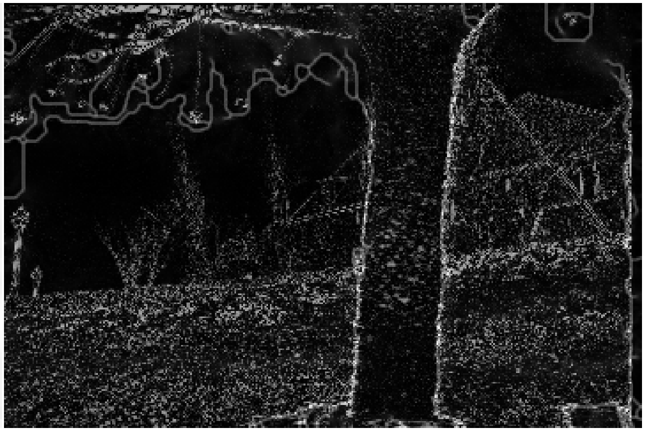
360x240 (Black)
Max speed: 7pix/frame

BA

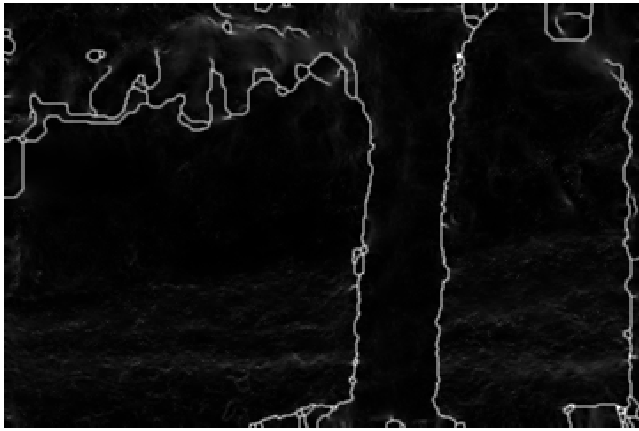


LMS

Ours



Error map



Smoothness error

Contributions (1/2)

Formulation

- More complete design, minimal parameter tuning
  - Adaptive local scales
  - Strength of two error terms automatically balanced
- 3-frame matching to avoid visibility problems

Solution: 3-step optimization

- Robust initial estimates and scales
- Model parameter self-learning
- Inherit merits of 3 methods and overcome shortcomings

Contributions (2/2)

Results

- High accuracy
- Fast convergence
- By product: motion boundaries

Significance

- Foundation for higher-level (model-based) visual motion analysis
- Methodology applicable to other low-level vision problems