Convolution

A **convolution** operation is a cross-correlation where the filter is flipped both horizontally and vertically before being applied to the image:

$$G[i,j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} H[u,v] F[i-u, j-v]$$

It is written:

$$G = H \star F$$

Suppose H is a Gaussian or mean kernel. How does convolution differ from cross-correlation?

Continuous filtering

We can also apply *continuous* filters to *continuous* images.

In the case of cross correlation: $g = h \otimes f$

$$g(x,y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h(u,v) f(x+u, y+v)\, du dv$$

In the case of convolution: $g = h \star f$

$$g(x,y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h(u,v) f(x-u, y-v)\, du dv$$

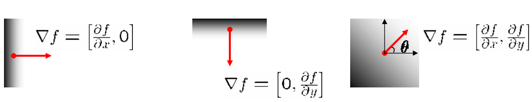
Note that the image and filter are infinite.

Image gradient

The gradient of an image:

$$\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$$

The gradient points in the direction of most rapid change in intensity



The gradient direction is given by:

$$\theta = \tan^{-1}\left(\frac{\partial f}{\partial y} / \frac{\partial f}{\partial x}\right)$$

- how does this relate to the direction of the edge?

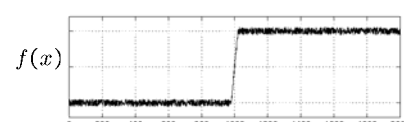
The *edge strength* is given by the gradient magnitude

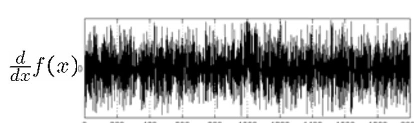
$$\|\nabla f\| = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}$$

Effects of noise

Consider a single row or column of the image

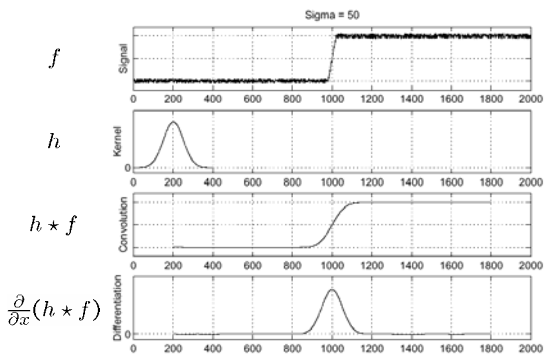
- Plotting intensity as a function of position gives a signal





Where is the edge?

Solution: smooth first

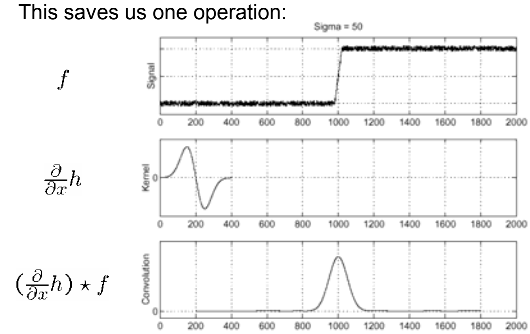


Where is the edge? Look for peaks in $\frac{\partial}{\partial x}(h \star f)$

Derivative theorem of convolution

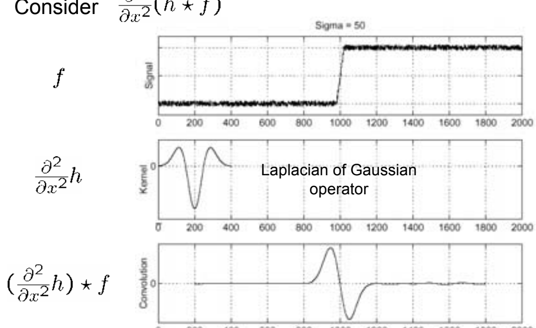
$$\frac{\partial}{\partial x}(h \star f) = (\frac{\partial}{\partial x}h) \star f$$

This saves us one operation:



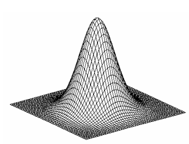
Laplacian of Gaussian

Consider $\frac{\partial^2}{\partial x^2}(h \star f)$



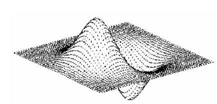
Where is the edge? Zero-crossings of bottom graph

2D edge detection filters



Gaussian

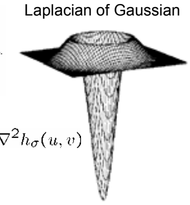
$h_\sigma(u,v) = \frac{1}{2\pi\sigma^2} e^{-\frac{u^2+v^2}{2\sigma^2}}$



derivative of Gaussian

$\frac{\partial}{\partial x} h_\sigma(u,v)$

Laplacian of Gaussian



$\nabla^2 h_\sigma(u,v)$

∇^2 is the **Laplacian** operator:

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

filter demo

Edge detection by subtraction

original

Edge detection by subtraction

smoothed (5x5 Gaussian)

Edge detection by subtraction

Why does this work?

smoothed – original

(scaled by 4, offset +128)

filter demo

Gaussian - image filter