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## Motion estimation

**Computer Vision**  
CSE576, Spring 2005  
Richard Szeliski

## Why estimate visual motion?

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Visual Motion can be annoying

- Camera instabilities, jitter
- Measure it; remove it (stabilize)

Visual Motion indicates dynamics in the scene

- Moving objects, behavior
- Track objects and analyze trajectories

Visual Motion reveals spatial layout

- Motion parallax

## Today's lecture

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Motion estimation

- image warping (skip: see handout)
- patch-based motion (optic flow)
- parametric (global) motion
- application: image morphing
- advanced: layered motion models

## Readings

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- Bergen *et al.* *Hierarchical model-based motion estimation*. ECCV'92, pp. 237–252.
- Szeliski, R. *Image Alignment and Stitching: A Tutorial*, MSR-TR-2004-92, Sec. 3.4 & 3.5.
- Shi, J. and Tomasi, C. (1994). Good features to track. In CVPR'94, pp. 593–600.
- Baker, S. and Matthews, I. (2004). Lucas-kanade 20 years on: A unifying framework. IJCV, 56(3), 221–255.

# Image Warping

## Image Warping

image filtering: change *range* of image

$$g(x) = h(f(x))$$

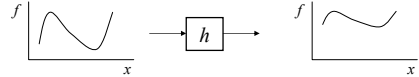
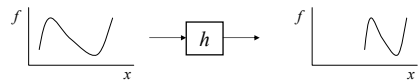


image warping: change *domain* of image

$$g(x) = f(h(x))$$



## Image Warping

image filtering: change *range* of image

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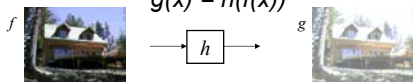
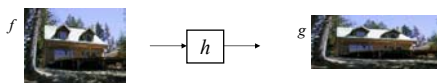


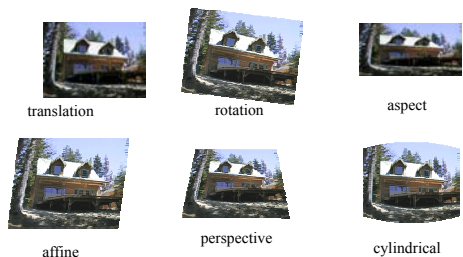
image warping: change *domain* of image

$$g(x) = f(h(x))$$



## Parametric (global) warping

Examples of parametric warps:



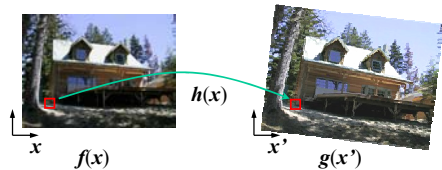
## 2D coordinate transformations

translation:  $\mathbf{x}' = \mathbf{x} + \mathbf{t}$        $\mathbf{x} = (x, y)$   
 rotation:  $\mathbf{x}' = \mathbf{R} \mathbf{x} + \mathbf{t}$   
 similarity:  $\mathbf{x}' = s \mathbf{R} \mathbf{x} + \mathbf{t}$   
 affine:  $\mathbf{x}' = \mathbf{A} \mathbf{x} + \mathbf{t}$   
 perspective:  $\underline{\mathbf{x}}' \cong \mathbf{H} \underline{\mathbf{x}}$        $\underline{\mathbf{x}} = (x, y, 1)$   
 ( $\underline{\mathbf{x}}$  is a *homogeneous* coordinate)

These all form a nested *group* (closed w/ inv.)

## Image Warping

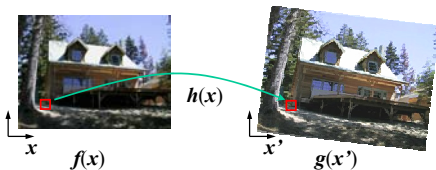
Given a coordinate transform  $\mathbf{x}' = \mathbf{h}(\mathbf{x})$  and a source image  $\mathbf{f}(\mathbf{x})$ , how do we compute a transformed image  $\mathbf{g}(\mathbf{x}') = \mathbf{f}(\mathbf{h}(\mathbf{x}))$ ?



## Forward Warping

Send each pixel  $\mathbf{f}(\mathbf{x})$  to its corresponding location  $\mathbf{x}' = \mathbf{h}(\mathbf{x})$  in  $\mathbf{g}(\mathbf{x}')$

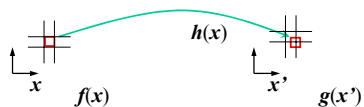
- What if pixel lands “between” two pixels?



## Forward Warping

Send each pixel  $\mathbf{f}(\mathbf{x})$  to its corresponding location  $\mathbf{x}' = \mathbf{h}(\mathbf{x})$  in  $\mathbf{g}(\mathbf{x}')$

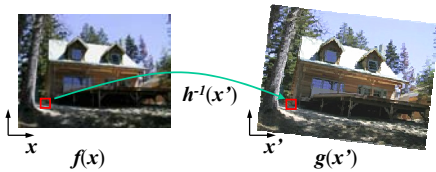
- What if pixel lands “between” two pixels?
- Answer: add “contribution” to several pixels, normalize later (*splatting*)



## Inverse Warping

Get each pixel  $g(x')$  from its corresponding location  $x = h^{-1}(x')$  in  $f(x)$

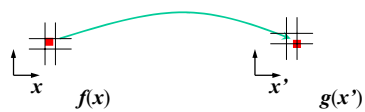
- What if pixel comes from “between” two pixels?



## Inverse Warping

Get each pixel  $g(x')$  from its corresponding location  $x = h^{-1}(x')$  in  $f(x)$

- What if pixel comes from “between” two pixels?
- Answer: *resample* color value from *interpolated (prefiltered)* source image



## Interpolation

Possible interpolation filters:

- nearest neighbor
- bilinear
- bicubic (interpolating)
- sinc / FIR

Needed to prevent “jaggies” and “texture crawl” (see [demo](#))

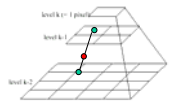


## Prefiltering

Essential for *downsampling (decimation)* to prevent *aliasing*

MIP-mapping [Williams'83]:

1. build pyramid (but what decimation filter?):
  - block averaging
  - Burt & Adelson (5-tap binomial)
  - 7-tap wavelet-based filter (better)
2. *trilinear* interpolation
  - bilinear within each 2 adjacent levels
  - linear blend *between* levels (determined by pixel size)



## Prefiltering

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Essential for *downsampling (decimation)* to prevent *aliasing*

Other possibilities:

- summed area tables
- elliptically weighted Gaussians (EWA) [Heckbert'86]

## Patch-based motion estimation

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## Classes of Techniques

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### **Feature-based methods**

- Extract visual features (corners, textured areas) and track them over multiple frames
- Sparse motion fields, but possibly robust tracking
- Suitable especially when image motion is large (10-s of pixels)

### **Direct-methods**

- Directly recover image motion from spatio-temporal image brightness variations
- Global motion parameters directly recovered without an intermediate feature motion calculation
- Dense motion fields, but more sensitive to appearance variations
- Suitable for video and when image motion is small (< 10 pixels)

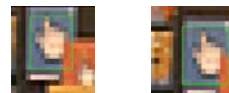
## Patch matching (revisited)

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How do we determine correspondences?

- *block matching* or *SSD* (sum squared differences)

$$E(x, y; d) = \sum_{(x', y') \in N(x, y)} [I_L(x'+d, y') - I_R(x', y')]^2$$



## The Brightness Constraint

Brightness Constancy Equation:

$$J(x, y) \approx I(x+u(x, y), y+v(x, y))$$

Or, equivalently, minimize :

$$E(u, v) = (J(x, y) - I(x+u, y+v))^2$$

Linearizing (assuming small  $(u, v)$ )  
using Taylor series expansion:

$$J(x, y) \approx I(x, y) + I_x(x, y) \cdot u(x, y) + I_y(x, y) \cdot v(x, y)$$

## The Brightness Constraint

Brightness Constancy Equation:

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Or, equivalently, minimize :

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**Rederive this on the board**

Linearizing (assuming small  $(u, v)$ )  
using Taylor series expansion:

$$J(x, y) \approx I(x, y) + I_x(x, y) \cdot u(x, y) + I_y(x, y) \cdot v(x, y)$$

## Gradient Constraint (or the Optical Flow Constraint)

$$E(u, v) = (I_x \cdot u + I_y \cdot v + I_t)^2$$

**Minimizing:**  $\frac{\partial E}{\partial u} = \frac{\partial E}{\partial v} = 0$

$$I_x(I_x u + I_y v + I_t) = 0$$

$$I_y(I_x u + I_y v + I_t) = 0$$

**In general**  $I_x, I_y \neq 0$

**Hence,**  $I_x \cdot u + I_y \cdot v + I_t \approx 0$

## Patch Translation [Lucas-Kanade]

Assume a single velocity for all pixels within an image patch

$$E(u, v) = \sum_{x, y \in \Omega} (I_x(x, y)u + I_y(x, y)v + I_t)^2$$

Minimizing

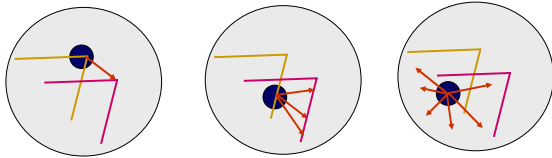
$$\begin{bmatrix} \sum I_x^2 & \sum I_x I_y \\ \sum I_x I_y & \sum I_y^2 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = - \begin{bmatrix} \sum I_x I_t \\ \sum I_y I_t \end{bmatrix}$$

$$\left( \sum \nabla I \nabla I^T \right) \vec{U} = - \sum \nabla I I_t$$

LHS: sum of the 2x2 outer product of the gradient vector

## Local Patch Analysis

How *certain* are the motion estimates?



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25

## The Aperture Problem

$$\text{Let } M = \sum (\nabla I)(\nabla I)^T \quad \text{and} \quad b = \begin{bmatrix} -\sum I_x I_t \\ -\sum I_y I_t \end{bmatrix}$$

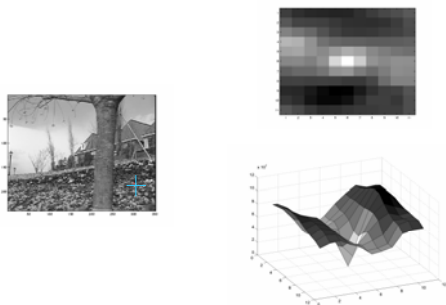
- Algorithm: At each pixel compute  $U$  by solving  $MU=b$
- $M$  is singular if all gradient vectors point in the same direction
  - e.g., along an edge
  - of course, trivially singular if the summation is over a single pixel or there is no texture
  - i.e., only *normal flow* is available (aperture problem)
- Corners and textured areas are OK

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26

## SSD Surface – Textured area

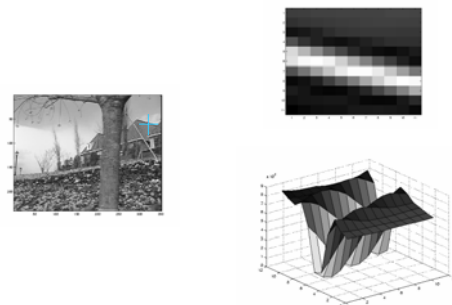


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27

## SSD Surface -- Edge

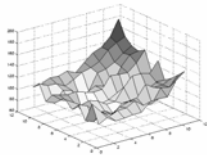
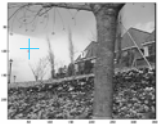


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28

## SSD – homogeneous area



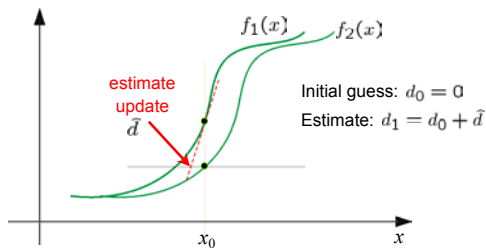
## Iterative Refinement

Estimate velocity at each pixel using one iteration of Lucas and Kanade estimation  
 Warp one image toward the other using the estimated flow field

*(easier said than done)*

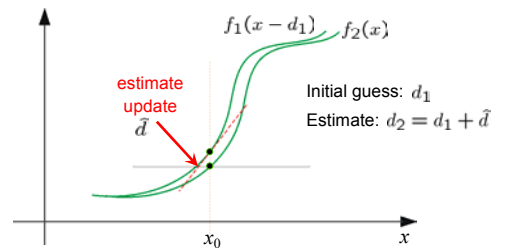
Refine estimate by repeating the process

## Optical Flow: Iterative Estimation



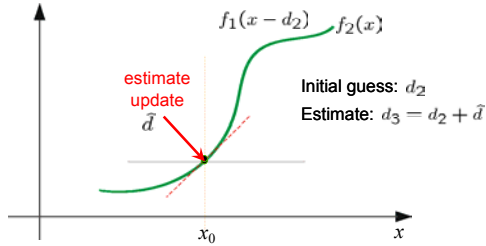
(using  $d$  for displacement here instead of  $u$ )

## Optical Flow: Iterative Estimation

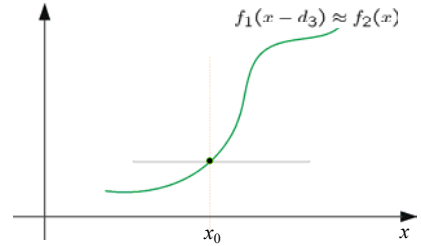




## Optical Flow: Iterative Estimation



## Optical Flow: Iterative Estimation



## Optical Flow: Iterative Estimation

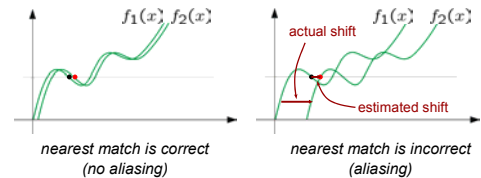
### Some Implementation Issues:

- Warping is not easy (ensure that errors in warping are smaller than the estimate refinement)
- Warp one image, take derivatives of the other so you don't need to re-compute the gradient after each iteration.
- Often useful to low-pass filter the images before motion estimation (for better derivative estimation, and linear approximations to image intensity)

## Optical Flow: Aliasing

Temporal aliasing causes ambiguities in optical flow because images can have many pixels with the same intensity.

I.e., how do we know which 'correspondence' is correct?



To overcome aliasing: coarse-to-fine estimation.

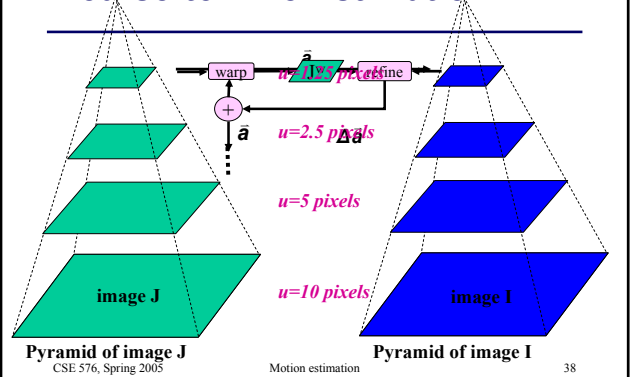
## Limits of the gradient method

Fails when intensity structure in window is poor  
 Fails when the displacement is large (typical operating range is motion of 1 pixel)

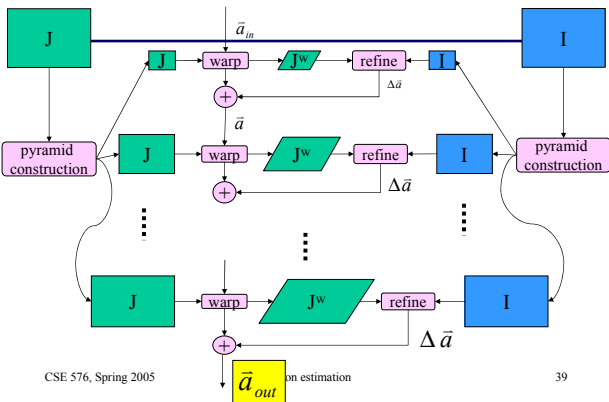
*Linearization of brightness is suitable only for small displacements*

Also, brightness is not strictly constant in images  
*actually less problematic than it appears, since we can pre-filter images to make them look similar*

## Coarse-to-Fine Estimation



## Coarse-to-Fine Estimation



## Parametric motion estimation

## Global (parametric) motion models

### 2D Models:

Affine

Quadratic

Planar projective transform (Homography)

### 3D Models:

Instantaneous camera motion models

Homography+epipole

Plane+Parallax

## Motion models

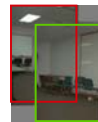


Translation

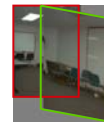
Affine

Perspective

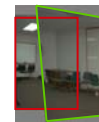
3D rotation



2 unknowns



6 unknowns



8 unknowns



3 unknowns

## Example: Affine Motion

$u(x, y) = a_1 + a_2x + a_3y$  Substituting into the B.C. Equation:

$v(x, y) = a_4 + a_5x + a_6y$

$$I_x(a_1 + a_2x + a_3y) + I_y(a_4 + a_5x + a_6y) + I_t \approx 0$$

Each pixel provides 1 linear constraint in 6 *global* unknowns

**Least Square Minimization (over all pixels):**

$$Err(\vec{a}) = \sum [I_x(a_1 + a_2x + a_3y) + I_y(a_4 + a_5x + a_6y) + I_t]^2$$

## Other 2D Motion Models

**Quadratic** – instantaneous approximation to planar motion

$$\begin{aligned} u &= q_1 + q_2x + q_3y + q_7x^2 + q_8xy \\ v &= q_4 + q_5x + q_6y + q_7xy + q_8y^2 \end{aligned}$$

**Projective** – exact planar motion

$$\begin{aligned} x' &= \frac{h_1 + h_2x + h_3y}{h_7 + h_8x + h_9y} \\ y' &= \frac{h_4 + h_5x + h_6y}{h_7 + h_8x + h_9y} \end{aligned}$$

and

$$u = x' - x, \quad v = y' - y$$

## 3D Motion Models

### Instantaneous camera motion:

Global parameters:  $\Omega_x, \Omega_y, \Omega_z, T_x, T_y, T_z$

Local Parameter:  $Z(x, y)$

$$u = -xy\Omega_x + (1+x^2)\Omega_y - y\Omega_z + (T_x - T_z x)/Z$$

$$v = -(1+y^2)\Omega_x + xy\Omega_y - x\Omega_z + (T_y - T_z y)/Z$$

### Homography+Epipole

Global parameters:  $h_1, \dots, h_6, t_1, t_2, t_3$

Local Parameter:  $\gamma(x, y)$

$$x' = \frac{h_1 x + h_2 y + h_3 + \gamma t_1}{h_7 x + h_8 y + h_9 + \gamma t_3}$$

$$y' = \frac{h_4 x + h_5 y + h_6 + \gamma t_2}{h_7 x + h_8 y + h_9 + \gamma t_3}$$

and :  $u = x' - x, \quad v = y' - y$

### Residual Planar Parallax Motion

Global parameters:  $t_1, t_2, t_3$

Local Parameter:  $\gamma(x, y)$

$$u = x' - x = \frac{\gamma}{1 + \gamma t_3} (t_3 x - t_1)$$

$$v = y' - y = \frac{\gamma}{1 + \gamma t_3} (t_3 y - t_2)$$

45

## Patch matching (revisited)

How do we determine correspondences?

- *block matching* or *SSD* (sum squared differences)

$$E(x, y; d) = \sum_{(x', y') \in N(x, y)} [I_L(x' + d, y') - I_R(x', y')]^2$$



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46

## Correlation and SSD

For larger displacements, do template matching

- Define a small area around a pixel as the template
- Match the template against each pixel within a search area in next image.
- Use a match measure such as correlation, normalized correlation, or sum-of-squares difference
- Choose the maximum (or minimum) as the match
- Sub-pixel estimate (Lucas-Kanade)

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47

## Discrete Search vs. Gradient Based

Consider image  $I$  translated by  $u_0, v_0$

$$I_0(x, y) = I(x, y)$$

$$I_1(x + u_0, y + v_0) = I(x, y) + \eta_1(x, y)$$

$$E(u, v) = \sum_{x, y} (I(x, y) - I_1(x + u, y + v))^2$$

$$= \sum_{x, y} (I(x, y) - I(x - u_0 + u, y - v_0 + v) - \eta_1(x, y))^2$$

The discrete search method simply searches for the best estimate. The gradient method linearizes the intensity function and solves for the estimate

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48

## Shi-Tomasi feature tracker

1. Find good features (min eigenvalue of  $2 \times 2$  Hessian)
2. Use Lucas-Kanade to track with pure translation
3. Use affine registration with first feature patch
4. Terminate tracks whose dissimilarity gets too large
5. Start new tracks when needed

## Tracking results



Figure 1: Three frame details from Woody Allen's *Manhattan*. The details are from the 1st, 11th, and 21st frames of a subsequence from the movie.



Figure 2: The traffic sign windows from frames 1, 6, 11, 16, 21 as tracked (top), and warped by the computed deformation matrices (bottom).

## Tracking - dissimilarity

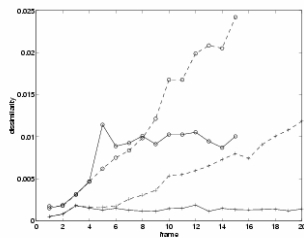


Figure 3: Pure translation (dashed) and affine motion (solid) dissimilarity measures for the window sequence of figure 1 (pluses) and 4 (circles).

## Tracking results

Figure 13: Labels of some of the features in figure 11.

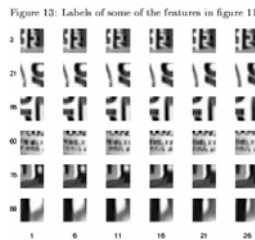


Figure 14: Six sample features through six sample frames.

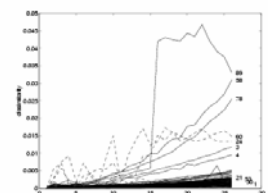


Figure 15: Affine motion dissimilarity for the features in figure 11. Notice the good discrimination between good and bad features. Dashed plots indicate aliasing (see text).

Features 24 and 60 deserve a special discussion, and

## Correlation Window Size

Small windows lead to more false matches

Large windows are better this way, but...

- Neighboring flow vectors will be more correlated (since the template windows have more in common)
- Flow resolution also lower (same reason)
- More expensive to compute

Small windows are good for local search:  
more detailed and less smooth (noisy?)

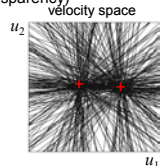
Large windows good for global search:  
less detailed and smoother

## Robust Estimation

Noise distributions are often non-Gaussian, having much heavier tails. Noise samples from the tails are called outliers.

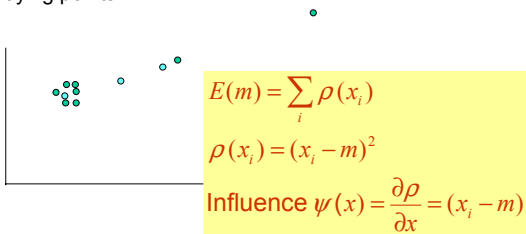
Sources of outliers (multiple motions):

- specularities / highlights
- jpeg artifacts / interlacing / motion blur
- multiple motions (occlusion boundaries, transparency)



## Robust Estimation

Standard Least Squares Estimation allows too much influence for outlying points



## Robust Estimation


$$E_d(u_s, v_s) = \sum \rho(I_x u_s + I_y v_s + I_t) \quad \text{Robust gradient constraint}$$

$$E_d(u_s, v_s) = \sum \rho(I(x, y) - J(x + u_s, y + v_s)) \quad \text{Robust SSD}$$

## Robust Estimation

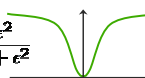
Problem: Least-squares estimators penalize deviations between data & model with quadratic error  $f^n$  (extremely sensitive to outliers)

error penalty function                      influence function

$$\rho(\epsilon) = \epsilon^2 \quad \psi(\epsilon) = \frac{\partial \rho(\epsilon)}{\partial \epsilon} = 2\epsilon$$


Redescending error functions (e.g., Geman-McClure) help to reduce the influence of outlying measurements.

error penalty function                      influence function

$$\rho(\epsilon; s) = \frac{\epsilon^2}{s + \epsilon^2} \quad \psi(\epsilon; s) = \frac{2\epsilon s}{(s + \epsilon^2)^2}$$


Motion estimation

## Image Morphing



## Image Warping – non-parametric

Specify more detailed warp function

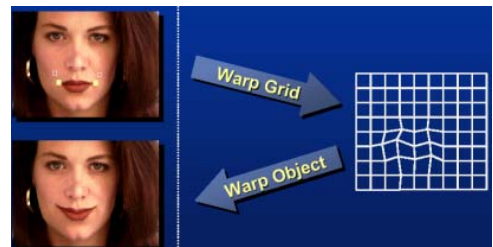


Examples:

- splines
- triangles
- optical flow (per-pixel motion)

## Image Warping – non-parametric

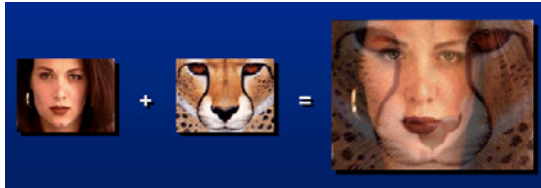
Move control points to specify spline warp



## Image Morphing

How can we *in-between* two images?

1. Cross-dissolve

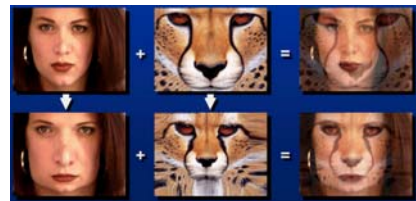


(all examples from [Gomes *et al.*'99])

## Image Morphing

How can we *in-between* two images?

2. Warp then cross-dissolve = *morph*



## Warp specification

How can we specify the warp?

1. Specify corresponding *points*
  - *interpolate* to a complete warping function

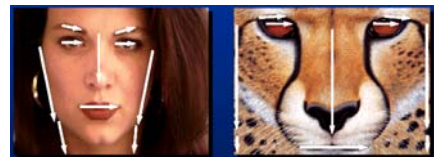


- Nielson, *Scattered Data Modeling*, IEEE CG&A'93]

## Warp specification

How can we specify the warp?

2. Specify corresponding *vectors*
  - *interpolate* to a complete warping function

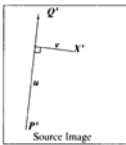
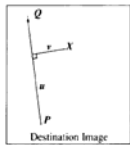




## Warp specification

How can we specify the warp?

- Specify corresponding vectors
  - interpolate* [Beier & Neely, SIGGRAPH'92]



For each pixel  $X$  in the destination  
 $DSUM = (0,0)$   
 $weightsum = 0$   
For each line  $P_i Q_i$   
calculate  $u, v$  based on  $P_i, Q_i$   
calculate  $X'_i$  based on  $u, v$  and  $P'_i, Q'_i$   
calculate displacement  $D_i = X'_i - X_i$  for this line  
 $dist =$  shortest distance from  $X$  to  $P_i, Q_i$   
 $weight = (length)^2 / (a + dist)^2$   
 $DSUM += D_i * weight$   
 $weightsum += weight$   
 $X' = X + DSUM / weightsum$   
 $destinationImage(X) = sourceImage(X')$

## Warp specification

How can we specify the warp?

- Specify corresponding *spline control points*
  - interpolate* to a complete warping function



## Final Morph Result



## Layered Scene Representations

## Motion representations

How can we describe this scene?



## Block-based motion prediction

Break image up into square blocks

Estimate translation for each block

Use this to predict next frame, code difference (MPEG-2)



## Layered motion

Break image sequence up into "layers":



Describe each layer's motion

## Layered motion

Advantages:

- can represent occlusions / disocclusions
- each layer's motion can be smooth
- video segmentation for semantic processing

Difficulties:

- how do we determine the correct number?
- how do we assign pixels?
- how do we model the motion?

## Layers for video summarization



Frame 0

Frame 50

Frame 80



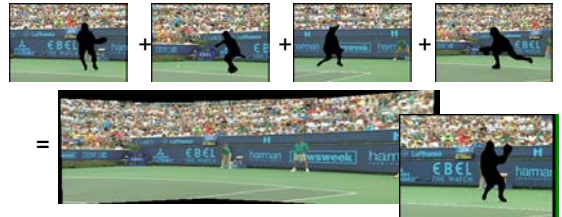
Background scene (players removed)

Motion estimation

Complete synopsis of the video

## Background modeling (MPEG-4)

Convert masked images into a background sprite for layered video coding



## What are layers?

[Wang & Adelson, 1994]

- intensities
- alphas
- velocities



Intensity map

Alpha map

Velocity map



Intensity map

Alpha map

Velocity map



Frame 1

Frame 2

Frame 3

## How do we composite them?

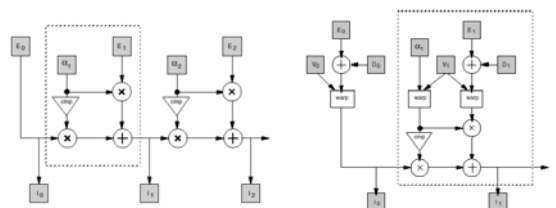
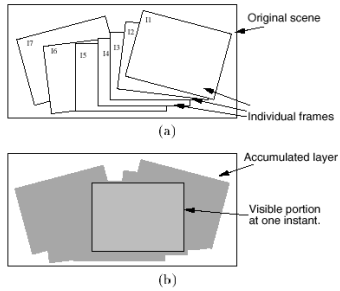


Figure 3: A flow chart for compositing a series of layers. The box labeled "cmp" generates the complement of alpha,  $(1 - \alpha)$ .

Figure 4: A flow chart for compositing that incorporates velocity maps,  $V$ , and delta maps,  $D$ .

## How do we form them?



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Motion estimation

77

## How do we form them?

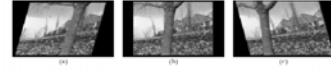


Figure 7: (a) Frame 9 warped with an affine transformation to align the Bowerbird region with that of frame 18. (b) Original frame 18 used as reference. (c) Frame 10 warped with an affine transformation to align the Bowerbird region with that of frame 18.

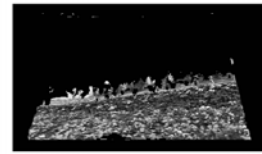


Figure 8: An accumulation of the Bowerbird. Image intensities are obtained from a temporal median operation on the median-composited images. Only the regions belonging to the Bowerbird are accumulated in this image. Non-occluded regions are naturally reinforced by accumulating data over many frames.

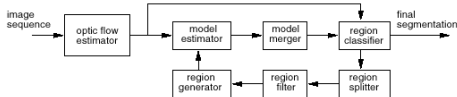
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78

## How do we estimate the layers?

1. compute coarse-to-fine flow
2. estimate affine motion in blocks (regression)
3. cluster with *k-means*
4. assign pixels to best fitting affine region
5. re-estimate affine motions in each region...



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79

## Layer synthesis

For each layer:

- stabilize the sequence with the affine motion
- compute median value at each pixel

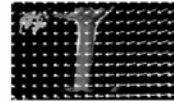
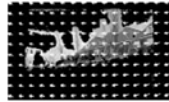
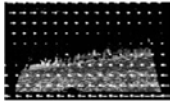
Determine occlusion relationships

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80

## Results



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