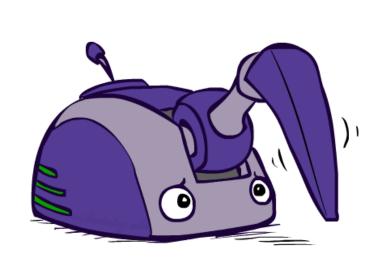
CSE 573: Artificial Intelligence

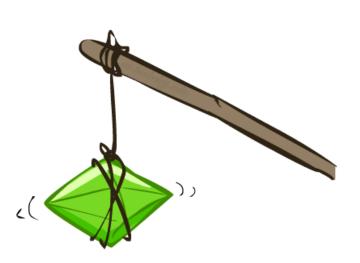
Hanna Hajishirzi Reinforcement Learning

slides adapted from Dan Klein, Pieter Abbeel ai.berkeley.edu And Dan Weld, Luke Zettlemoyer



Reinforcement Learning







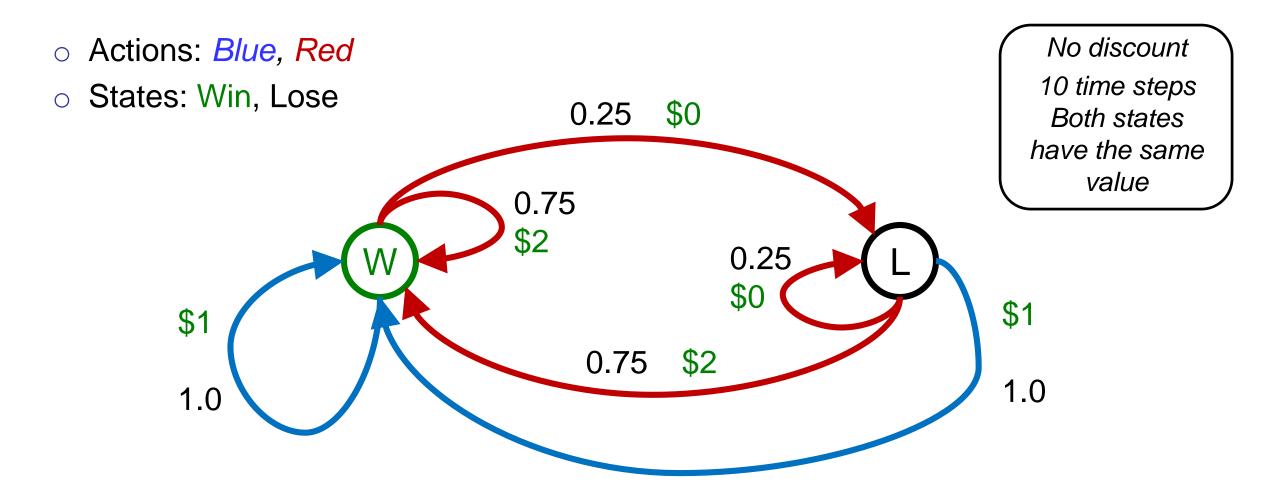
Double Bandits







Double-Bandit MDP

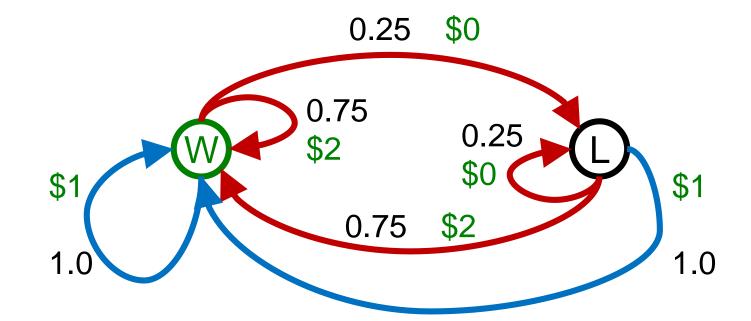


Offline Planning

- Solving MDPs is offline planning
 - You determine all quantities through computation
 - You need to know the details of the MDP
 - You do not actually play the game!

No discount 10 time steps





Let's Play!



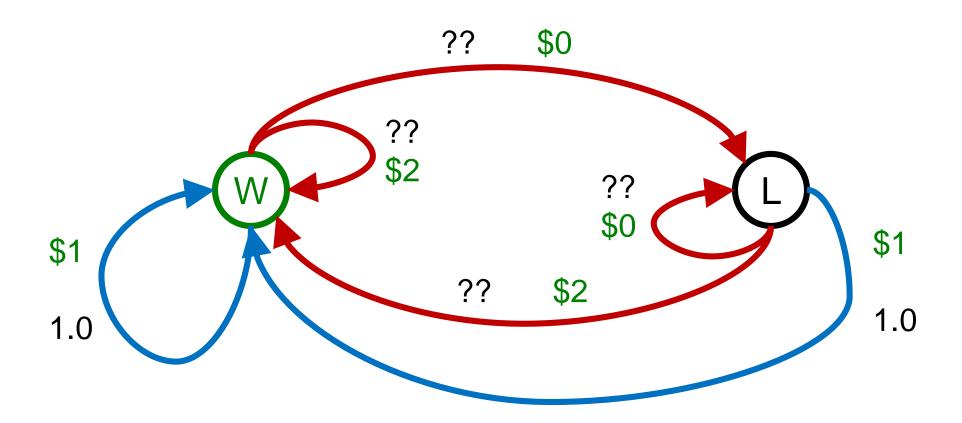


\$2 \$2 \$0 \$2 \$2

\$2 \$2 \$0 \$0 \$0

Online Planning

Rules changed! Red's win chance is different.



Let's Play!





\$0 \$0 \$2 \$0 \$0 \$2 \$2 \$0 \$0

What Just Happened?

That wasn't planning, it was learning!

- Specifically, reinforcement learning
- o There was an MDP, but you couldn't solve it with just computation
- You needed to actually act to figure it out



Important ideas in reinforcement learning that came up

- Exploration: you have to try unknown actions to get information
- o Exploitation: eventually, you have to use what you know
- Regret: even if you learn intelligently, you make mistakes
- Sampling: because of chance, you have to try things repeatedly
- o Difficulty: learning can be much harder than solving a known MDP

Reinforcement Learning

- Still assume a Markov decision process (MDP):
 - A set of states s ∈ S
 - A set of actions (per state) A
 - A model T(s,a,s')
 - A reward function R(s,a,s')
- Still looking for a policy $\pi(s)$

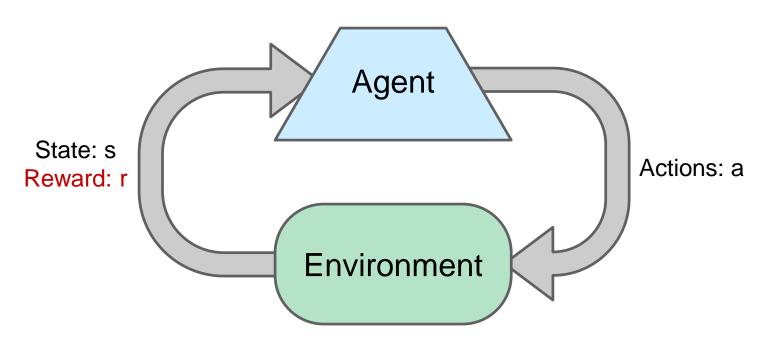






- New twist: don't know T or R
 - o I.e. we don't know which states are good or what the actions do
 - Must actually try actions and states out to learn

Reinforcement Learning



Basic idea:

- Receive feedback in the form of rewards
- Agent's utility is defined by the reward function
- Must (learn to) act so as to maximize expected rewards
- All learning is based on observed samples of outcomes!

Example: Learning to Walk



Initial



A Learning Trial



After Learning [1K Trials]

Example: Toddler Robot



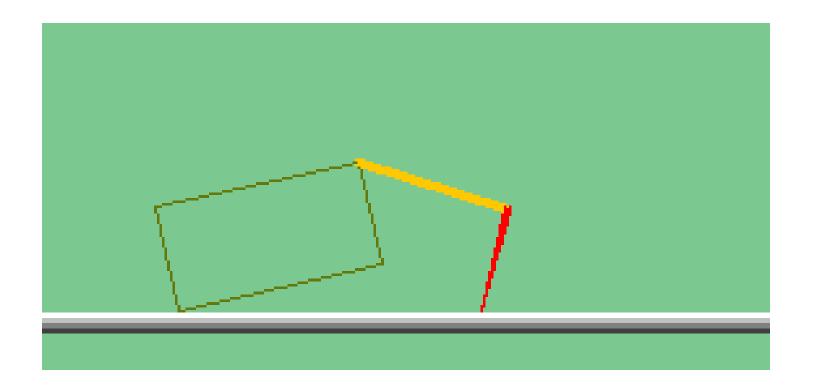
[Tedrake, Zhang and Seung, 2005]

Robotics Rubik Cube

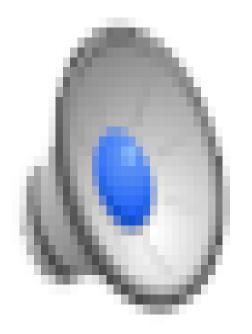
https://www.youtube.com/watch?v=x4O8pojMF0w



The Crawler!



Video of Demo Crawler Bot



Reinforcement Learning

- Still assume a Markov decision process (MDP):
 - A set of states s ∈ S
 - A set of actions (per state) A
 - A model T(s,a,s')
 - A reward function R(s,a,s')
- Still looking for a policy $\pi(s)$

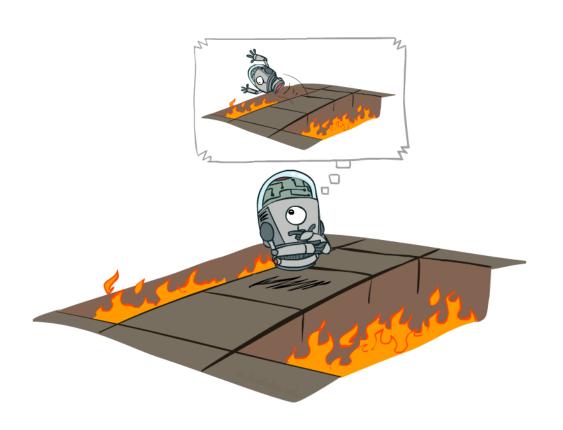






- New twist: don't know T or R
 - o I.e. we don't know which states are good or what the actions do
 - Must actually try actions and states out to learn

Offline (MDPs) vs. Online (RL)

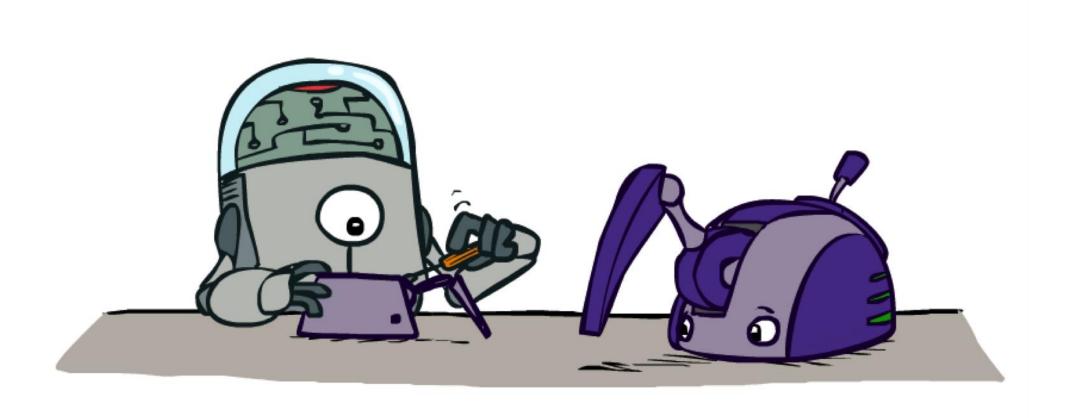




Offline Solution

Online Learning

Model-Based Learning



Model-Based Learning

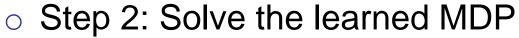
Model-Based Idea:

- Learn an approximate model based on experiences
- Solve for values as if the learned model were correct



Step 1: Learn empirical MDP model

- o Count outcomes s' for each s, a
- o Normalize to give an estimate $\hat{T}(s, a, s')$
- o Discover each $\hat{R}(s, a, s')$ when we experience (s, a, s')

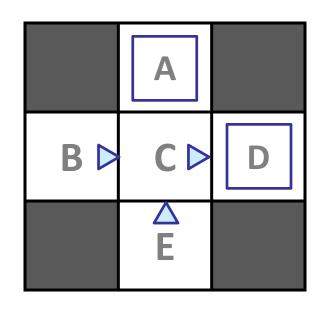


o For example, use value iteration, as before



Example: Model-Based Learning

Input Policy π



Assume: $\gamma = 1$

Observed Episodes (Training)

Episode 1

B, east, C, -1 C, east, D, -1 D, exit, x, +10

Episode 2

B, east, C, -1 C, east, D, -1 D, exit, x, +10

Learned Model

$$\widehat{T}(s, a, s')$$

T(B, east, C) = 1.00 T(C, east, D) = 0.75 T(C, east, A) = 0.25

Episode 3

E, north, C, -1 C, east, D, -1 D, exit, x, +10

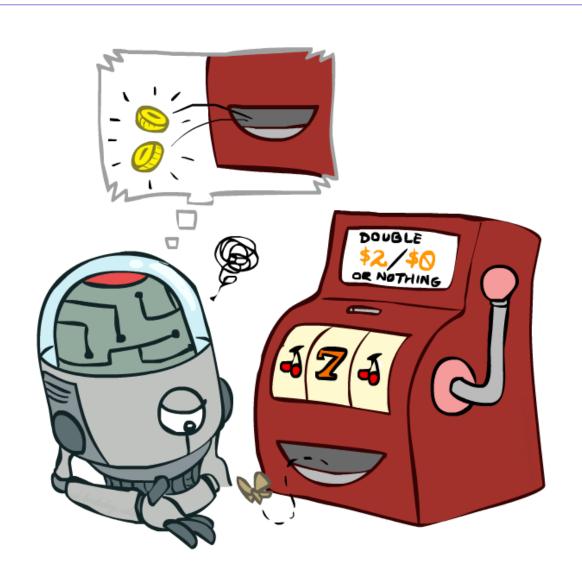
Episode 4

E, north, C, -1 C, east, A, -1 A, exit, x, -10

$$\hat{R}(s, a, s')$$

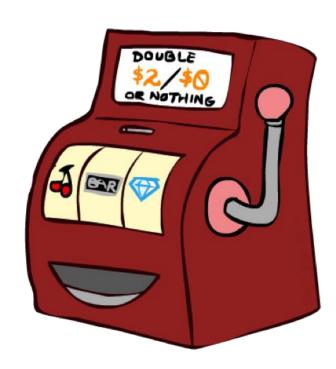
R(B, east, C) = -1 R(C, east, D) = -1 R(D, exit, x) = +10

Model-Free Learning



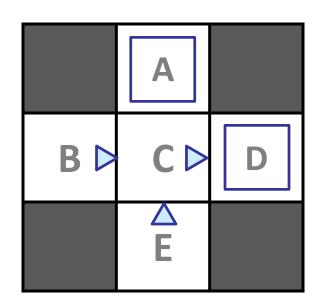
Direct Evaluation

- Goal: Compute values for each state under π
- Idea: Average together observed sample values
 - \circ Act according to π
 - Every time you visit a state, write down what the sum of discounted rewards turned out to be
 - Average those samples
- This is called direct evaluation



Example: Direct Evaluation

Input Policy π



Assume: $\gamma = 1$

Observed Episodes (Training)

Episode 1

B, east, C, -1 C, east, D, -1 D, exit, x, +10

Episode 2

B, east, C, -1 C, east, D, -1 D, exit, x, +10

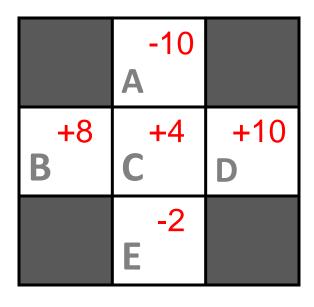
Episode 3

E, north, C, -1 C, east, D, -1 D, exit, x, +10

Episode 4

E, north, C, -1 C, east, A, -1 A, exit, x, -10

Output Values

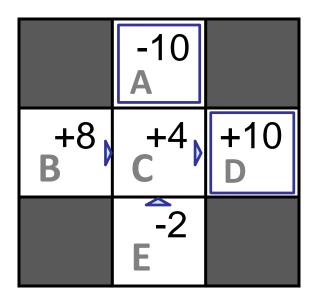


If B and E both go to C under this policy, how can their values be different?

Problems with Direct Evaluation

- What's good about direct evaluation?
 - o It's easy to understand
 - o It doesn't require any knowledge of T, R
 - It eventually computes the correct average values, using just sample transitions
- What bad about it?
 - o It wastes information about state connections
 - Each state must be learned separately
 - So, it takes a long time to learn

Output Values



If B and E both go to C under this policy, how can their values be different?

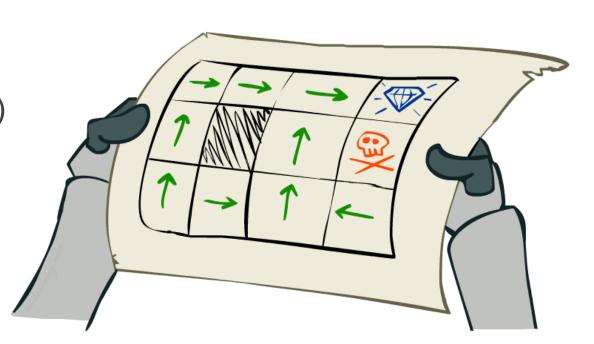
Passive Reinforcement Learning

Simplified task: policy evaluation

- o Input: a fixed policy $\pi(s)$
- You don't know the transitions T(s,a,s')
- You don't know the rewards R(s,a,s')
- Goal: learn the state values

o In this case:

- Learner is "along for the ride"
- No choice about what actions to take
- Just execute the policy and learn from experience
- This is NOT offline planning! You actually take actions in the world.



Why Not Use Policy Evaluation?

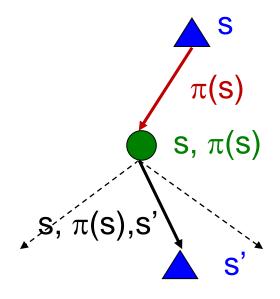
- Simplified Bellman updates calculate V for a fixed policy:
 - Each round, replace V with a one-step-look-ahead layer over V

$$V_0^{\pi}(s) = 0$$

$$V_{k+1}^{\pi}(s) \leftarrow \sum_{s'} T(s,\pi(s),s')[R(s,\pi(s),s') + \gamma V_k^{\pi}(s')] \qquad \text{s, $\pi(s),s'$}$$
 This approach fully exploited the connections between the states Unfortunately, we need T and R to do it!



Unfortunately, we need T and R to do it!



- Key question: how can we do this update to V without knowing T and R?
 - o In other words, how to we take a weighted average without knowing the weights?

Sample-Based Policy Evaluation?

We want to improve our estimate of V by computing these averages:

$$V_{k+1}^{\pi}(s) \leftarrow \sum_{s'} T(s, \pi(s), s') [R(s, \pi(s), s') + \gamma V_k^{\pi}(s')]$$

o Idea: Take samples of outcomes s' (by doing the action!) and

average

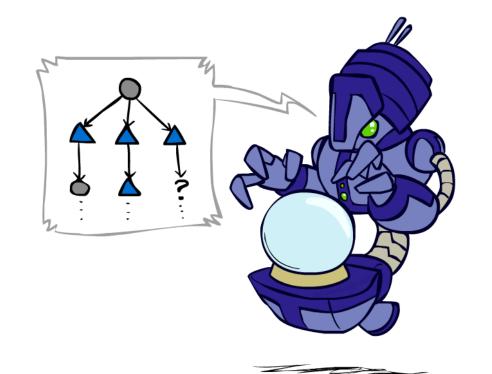
$$sample_1 = R(s, \pi(s), s'_1) + \gamma V_k^{\pi}(s'_1)$$

$$sample_2 = R(s, \pi(s), s'_2) + \gamma V_k^{\pi}(s'_2)$$

$$\dots$$

$$sample_n = R(s, \pi(s), s'_n) + \gamma V_k^{\pi}(s'_n)$$

$$V_{k+1}^{\pi}(s) \leftarrow \frac{1}{n} \sum_{i} sample_i$$



Temporal Difference Learning

- Big idea: learn from every experience!
 - Update V(s) each time we experience a transition (s, a, s', r)
 - Likely outcomes s' will contribute updates more often

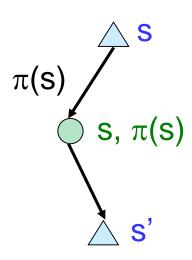


- Policy still fixed, still doing evaluation!
- Move values toward value of whatever successor occurs: running average

Sample of V(s):
$$sample = R(s, \pi(s), s') + \gamma V^{\pi}(s')$$

Update to V(s):
$$V^{\pi}(s) \leftarrow (1 - \alpha)V^{\pi}(s) + (\alpha)sample$$

Same update:
$$V^{\pi}(s) \leftarrow V^{\pi}(s) + \alpha(sample - V^{\pi}(s))$$

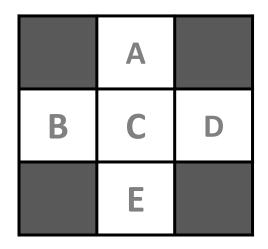


Exponential Moving Average

- Exponential moving average
 - o The running interpolation update: $\bar{x}_n = (1 \alpha) \cdot \bar{x}_{n-1} + \alpha \cdot x_n$
 - Makes recent samples more important
 - Forgets about the past (distant past values were wrong anyway)
- Decreasing learning rate (alpha) can give converging averages

Example: Temporal Difference Learning

States

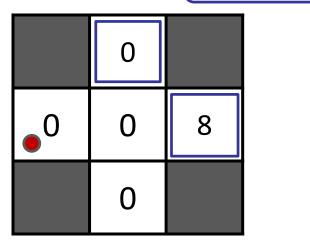


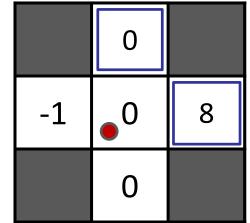
Assume: $\gamma = 1$, $\alpha = 1/2$

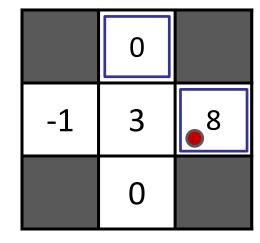
Observed Transitions

B, east, C, -2

C, east, D, -2







$$V^{\pi}(s) \leftarrow (1 - \alpha)V^{\pi}(s) + \alpha \left[R(s, \pi(s), s') + \gamma V^{\pi}(s') \right]$$

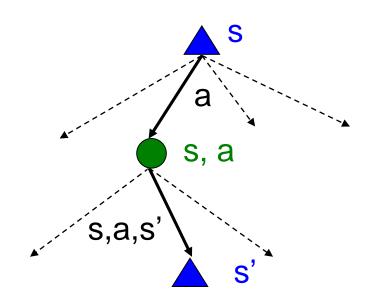
Problems with TD Value Learning

- TD value leaning is a model-free way to do policy evaluation, mimicking Bellman updates with running sample averages
- However, if we want to turn values into a (new) policy, we're sunk:

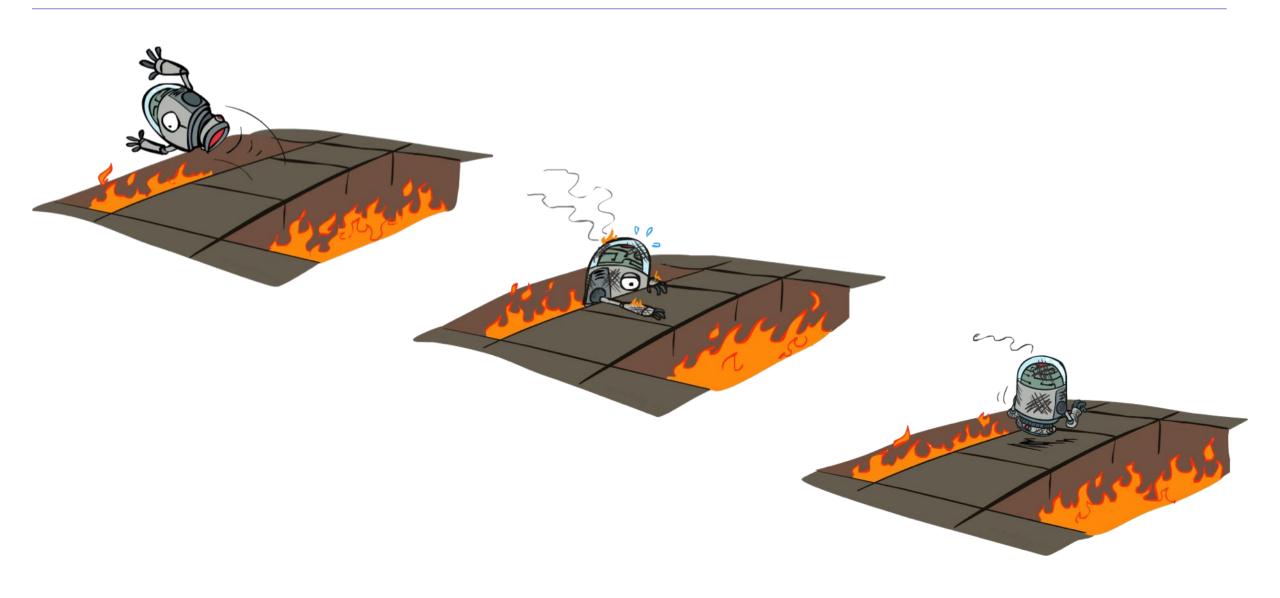
$$\pi(s) = \arg\max_{a} Q(s, a)$$

$$Q(s,a) = \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma V(s') \right]$$

- Idea: learn Q-values, not values
- Makes action selection model-free too!



Active Reinforcement Learning



Active Reinforcement Learning

- Full reinforcement learning: optimal policies (like value iteration)
 - You don't know the transitions T(s,a,s')
 - You don't know the rewards R(s,a,s')
 - You choose the actions now
 - Goal: learn the optimal policy / values



o In this case:

- o Learner makes choices!
- o Fundamental tradeoff: exploration vs. exploitation
- This is NOT offline planning! You actually take actions in the world and find out what happens...

Detour: Q-Value Iteration

- Value iteration: find successive (depth-limited) values
 - Start with $V_0(s) = 0$, which we know is right
 - Given V_k, calculate the depth k+1 values for all states:

$$V_{k+1}(s) \leftarrow \max_{a} \sum_{s'} T(s, a, s') \left[R(s, a, s') + \gamma V_k(s') \right]$$

- But Q-values are more useful, so compute them instead
 - Start with $Q_0(s,a) = 0$, which we know is right
 - Given Q_k, calculate the depth k+1 q-values for all q-states:

$$Q_{k+1}(s,a) \leftarrow \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma \max_{a'} Q_k(s',a') \right]$$

Q-Learning

Q-Learning: sample-based Q-value iteration

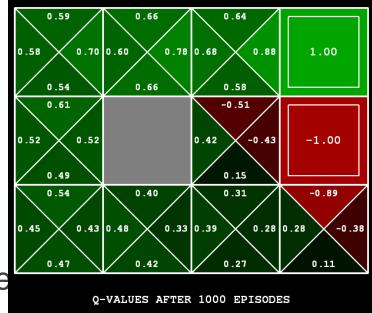
$$Q_{k+1}(s, a) \leftarrow \sum_{s'} T(s, a, s') \left[R(s, a, s') + \gamma \max_{a'} Q_k(s', a') \right]$$

- Learn Q(s,a) values as you go
 - Receive a sample (s,a,s',r)
 - \circ Consider your old estimateQ(s,a)
 - Consider your new sample estimate:

$$sample = R(s, a, s') + \gamma \max_{a'} Q(s', a')$$
 no longer policy evaluation!

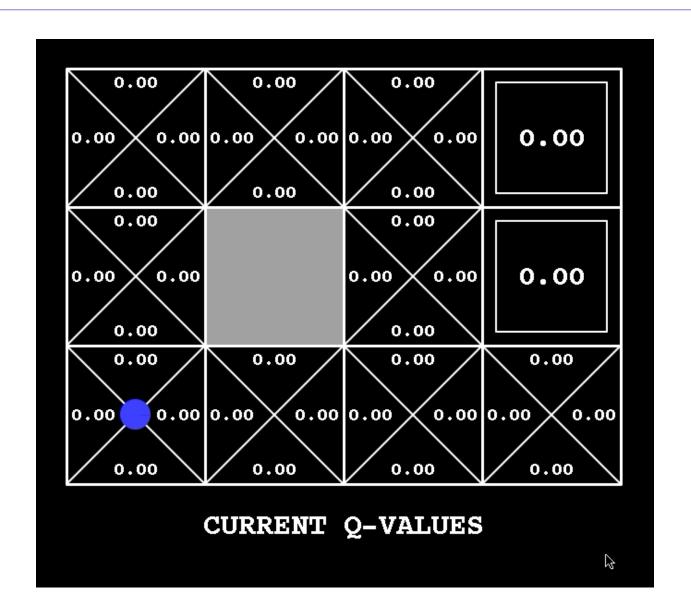
Incorporate the new estimate into a running average

$$Q(s,a) \leftarrow (1-\alpha)Q(s,a) + (\alpha) [sample]$$

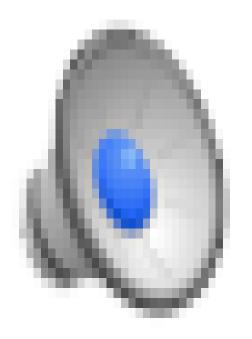


[Demo: Q-learning – gridworld (L10D2)] [Demo: Q-learning – crawler (L10D3)]

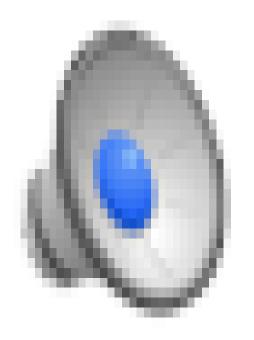
Q-Learning Demo



Video of Demo Q-Learning -- Gridworld

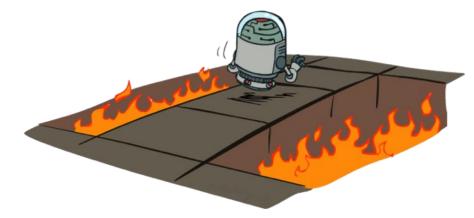


Video of Demo Q-Learning -- Crawler



Q-Learning: act according to current optimal (and also explore...)

- Full reinforcement learning: optimal policies (like value iteration)
 - You don't know the transitions T(s,a,s')
 - You don't know the rewards R(s,a,s')
 - You choose the actions now
 - Goal: learn the optimal policy / values

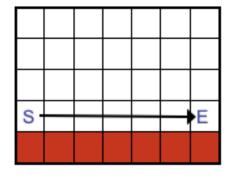


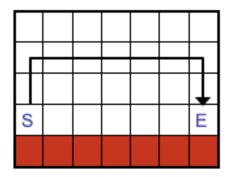
o In this case:

- o Learner makes choices!
- o Fundamental tradeoff: exploration vs. exploitation
- This is NOT offline planning! You actually take actions in the world and find out what happens...

Q-Learning Properties

- Amazing result: Q-learning converges to optimal policy -even if you're acting suboptimally!
- This is called off-policy learning





- o Caveats:
 - You have to explore enough
 - You have to eventually make the learning rate small enough
 - ... but not decrease it too quickly
 - o Basically, in the limit, it doesn't matter how you select actions



Discussion: Model-Based vs Model-Free RL

Model-Based vs. Model Free

Active vs. Passive

Recap: Reinforcement Learning

- Still assume a Markov decision process (MDP):
 - A set of states s ∈ S
 - A set of actions (per state) A
 - A model T(s,a,s')
 - A reward function R(s,a,s')
- Still looking for a policy $\pi(s)$







- New twist: don't know T or R
 - o l.e. we don't know which states are good or what the actions do
 - Must actually try actions and states out to learn
- Big Idea: Compute all averages over T using sample outcomes

The Story So Far: MDPs and RL

Known MDP: Offline Solution

Goal Technique

Compute V^* , Q^* , π^* Value / policy iteration

Evaluate a fixed policy π Policy evaluation

Unknown MDP: Model-Based

Goal Technique

Compute V*, Q*, π * VI/PI on approx. MDP

Evaluate a fixed policy π PE on approx. MDP

Unknown MDP: Model-Free

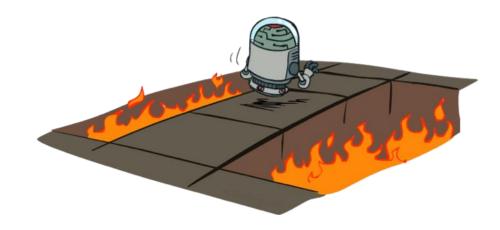
Goal Technique

Compute V*, Q*, π * Q-learning

Evaluate a fixed policy π Value Learning

Model-Free Learning

- act according to current optimal (based on Q-Values)
- but also explore...



Q-Learning

Q-Learning: sample-based Q-value iteration

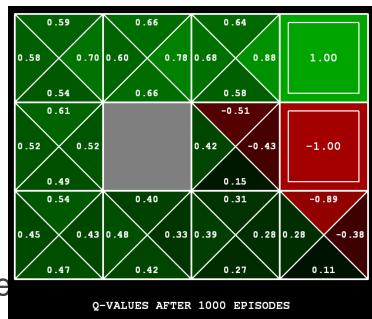
$$Q_{k+1}(s, a) \leftarrow \sum_{s'} T(s, a, s') \left[R(s, a, s') + \gamma \max_{a'} Q_k(s', a') \right]$$

- Learn Q(s,a) values as you go
 - Receive a sample (s,a,s',r)
 - \circ Consider your old estimateQ(s,a)
 - Consider your new sample estimate:

$$sample = R(s, a, s') + \gamma \max_{a'} Q(s', a')$$
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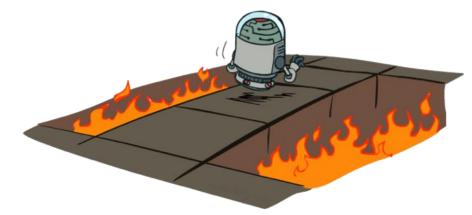
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Q-Learning: act according to current optimal (and also explore...)

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 - You don't know the transitions T(s,a,s')
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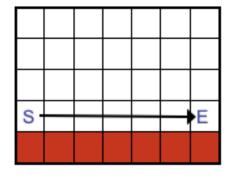


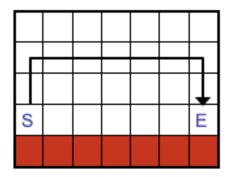
o In this case:

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- o Fundamental tradeoff: exploration vs. exploitation
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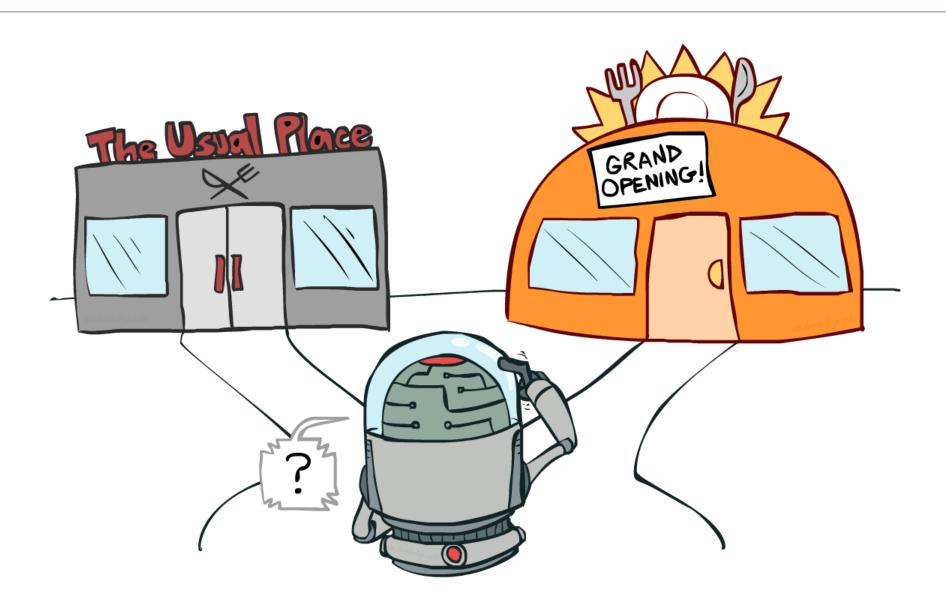




- o Caveats:
 - You have to explore enough
 - You have to eventually make the learning rate small enough
 - ... but not decrease it too quickly
 - o Basically, in the limit, it doesn't matter how you select actions



Exploration vs. Exploitation



How to Explore?

- Several schemes for forcing exploration
 - Simplest: random actions (ε-greedy)
 - Every time step, flip a coin
 - With (small) probability ε, act randomly
 - With (large) probability 1-ε, act on current policy
 - o Problems with random actions?
 - You do eventually explore the space, but keep thrashing around once learning is done
 - One solution: lower ε over time
 - Another solution: exploration functions



Exploration Functions

O When to explore?

- Random actions: explore a fixed amount
- Better idea: explore areas whose badness is not (yet) established, eventually stop exploring

Exploration function

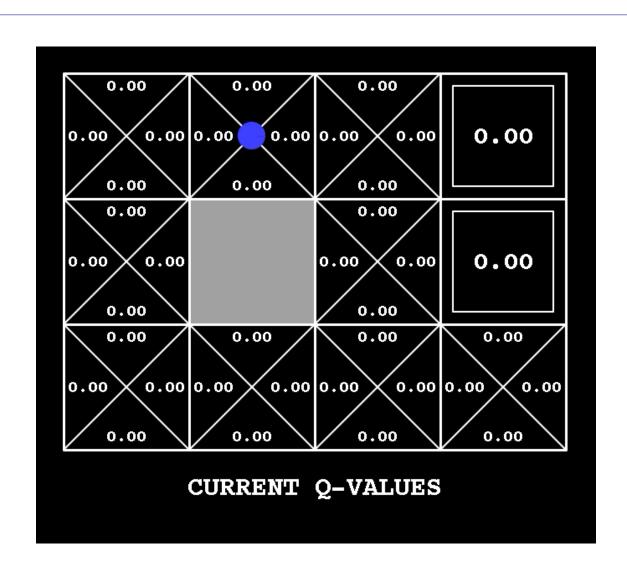
 \circ Takes a value estimate u and a visit count n, and returns an optimistic utility, e.g. f(u,n) = u + k/n



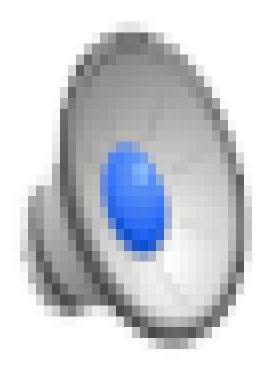
Regular Q-Update:
$$Q(s, a) \leftarrow_{\alpha} R(s, a, s') + \gamma \max_{a'} Q(s', a')$$

Modified Q-Update:
$$Q(s, a) \leftarrow_{\alpha} R(s, a, s') + \gamma \max_{a'} f(Q(s', a'), N(s', a'))$$

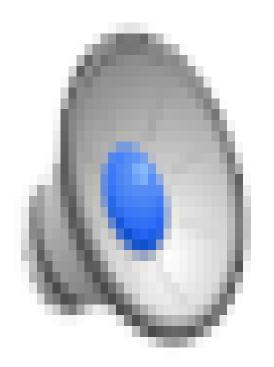
Q-Learn Epsilon Greedy



Video of Demo Q-learning – Epsilon-Greedy – Crawler

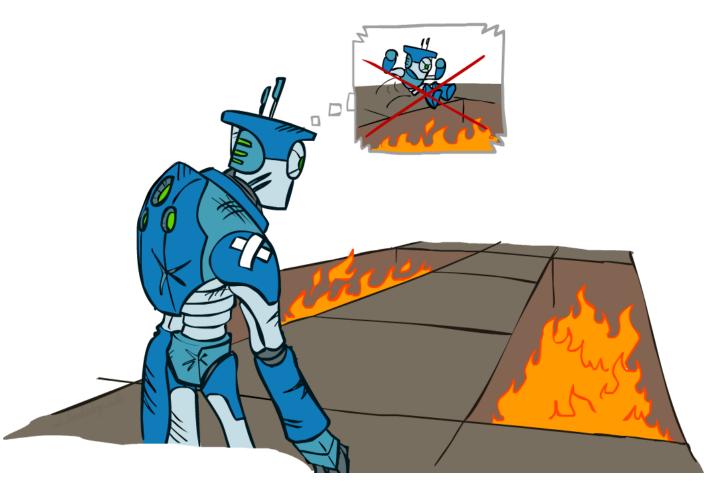


Video of Demo Q-learning – Exploration Function – Crawler

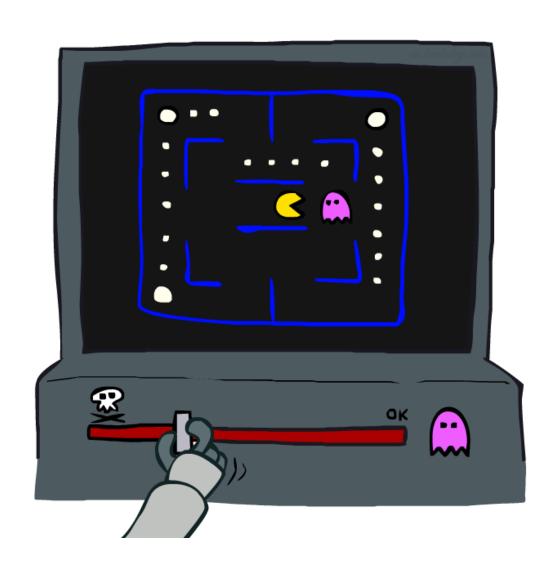


Regret

- Even if you learn the optimal policy you still make mistakes along the way!
- Regret is a measure of your total mistake cost: the difference between your (expected) rewards and optimal (expected) rewards
- Minimizing regret goes beyond learning to be optimal – it requires optimally learning to be optimal
- Example: random exploration and exploration functions both end up optimal, but random exploration has higher regret

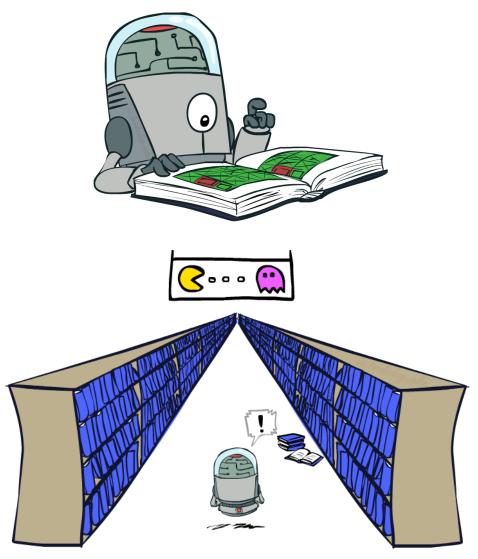


Approximate Q-Learning

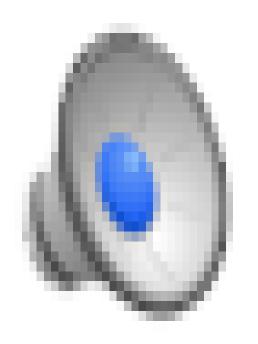


Generalizing Across States

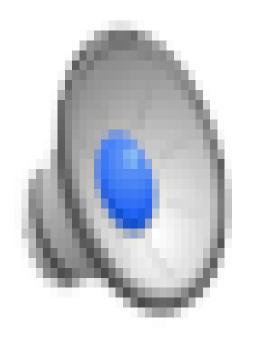
- Basic Q-Learning keeps a table of all q-values
- In realistic situations, we cannot possibly learn about every single state!
 - Too many states to visit them all in training
 - Too many states to hold the q-tables in memory
- Instead, we want to generalize:
 - Learn about some small number of training states from experience
 - Generalize that experience to new, similar situations
 - This is a fundamental idea in machine learning, and we'll see it over and over again



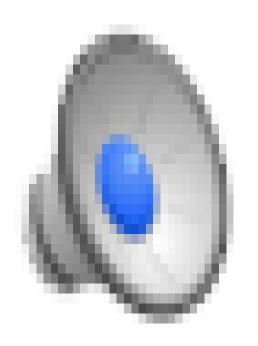
Video of Demo Q-Learning Pacman – Tiny – Watch All



Video of Demo Q-Learning Pacman – Tiny – Silent Train



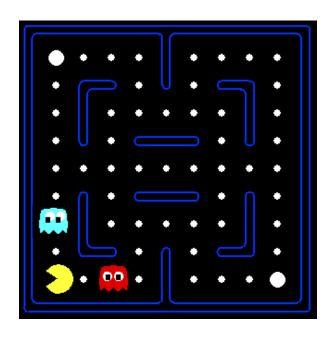
Video of Demo Q-Learning Pacman – Tricky – Watch All

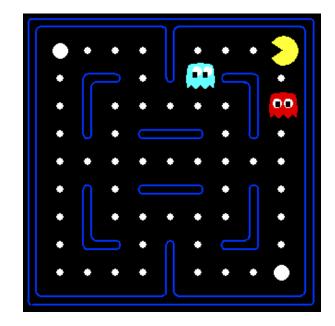


Example: Pacman

Let's say we discover through experience that this state is bad: In naïve q-learning, we know nothing about this state:

Or even this one!

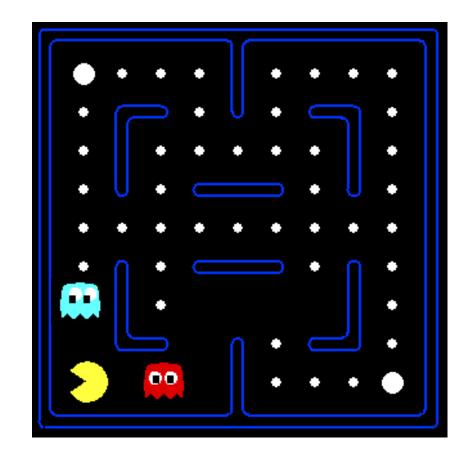






Feature-Based Representations

- Solution: describe a state using a vector of features (properties)
 - Features are functions from states to real numbers (often 0/1) that capture important properties of the state
 - o Example features:
 - Distance to closest ghost
 - Distance to closest dot
 - Number of ghosts
 - \circ 1 / (dist to dot)²
 - Is Pacman in a tunnel? (0/1)
 - o etc.
 - o Is it the exact state on this slide?
 - Can also describe a q-state (s, a) with features (e.g. action moves closer to food)



Linear Value Functions

 Using a feature representation, we can write a q function (or value function) for any state using a few weights:

$$V(s) = w_1 f_1(s) + w_2 f_2(s) + \dots + w_n f_n(s)$$
$$Q(s, a) = w_1 f_1(s, a) + w_2 f_2(s, a) + \dots + w_n f_n(s, a)$$

- Advantage: our experience is summed up in a few powerful numbers
- Disadvantage: states may share features but actually be very different in value!

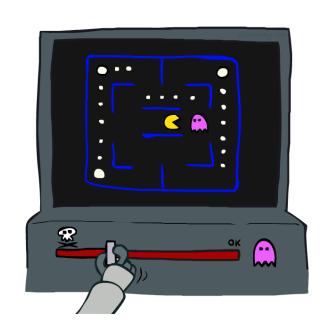
Approximate Q-Learning

$$Q(s,a) = w_1 f_1(s,a) + w_2 f_2(s,a) + \dots + w_n f_n(s,a)$$

Q-learning with linear Q-functions:

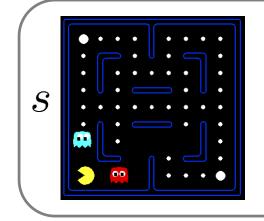
$$\begin{aligned} & \text{transition } = (s, a, r, s') \\ & \text{difference} = \left[r + \gamma \max_{a'} Q(s', a')\right] - Q(s, a) \\ & Q(s, a) \leftarrow Q(s, a) + \alpha \text{ [difference]} \end{aligned} \quad & \text{Exact Q's} \\ & w_i \leftarrow w_i + \alpha \text{ [difference]} f_i(s, a) \quad & \text{Approximate Q's} \end{aligned}$$

- Intuitive interpretation:
 - Adjust weights of active features
 - E.g., if something unexpectedly bad happens, blame the features that were on: disprefer all states with that state's features
- Formal justification: online least squares



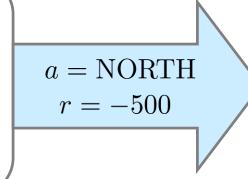
Example: Q-Pacman

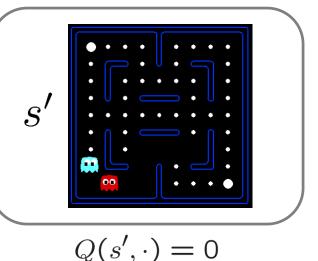
$$Q(s,a) = 4.0f_{DOT}(s,a) - 1.0f_{GST}(s,a)$$



 $f_{DOT}(s, NORTH) = 0.5$

 $f_{GST}(s, NORTH) = 1.0$





$$Q(s, NORTH) = +1$$

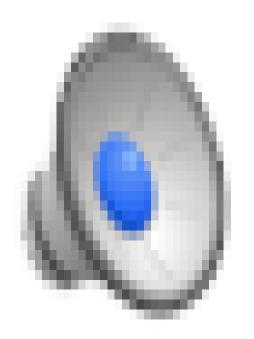
 $r + \gamma \max_{a'} Q(s', a') = -500 + 0$

difference = -501
$$w_{DOT} \leftarrow 4.0 + \alpha \, [-501] \, 0.5$$

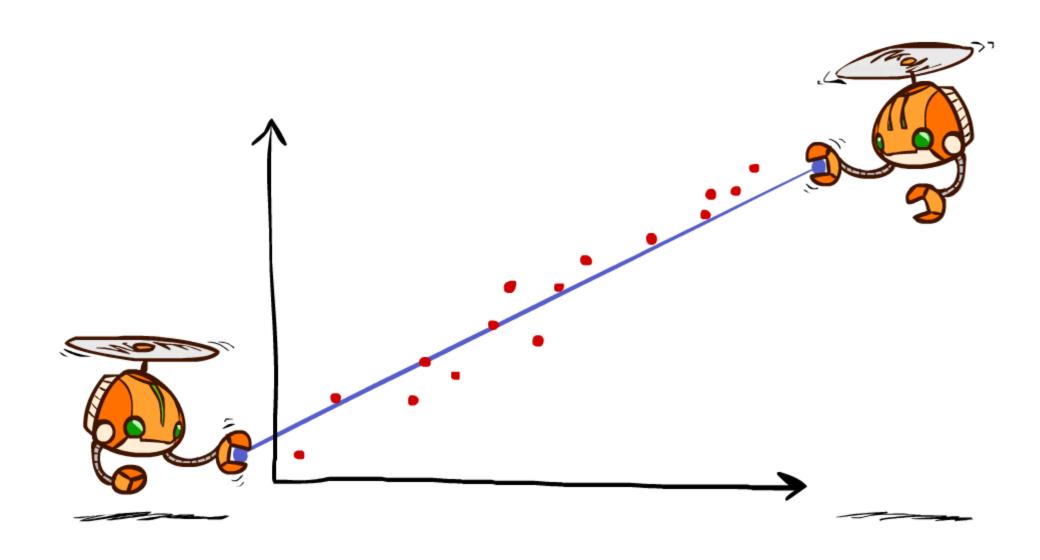
$$w_{GST} \leftarrow -1.0 + \alpha \, [-501] \, 1.0$$

$$Q(s, a) = 3.0 f_{DOT}(s, a) - 3.0 f_{GST}(s, a)$$

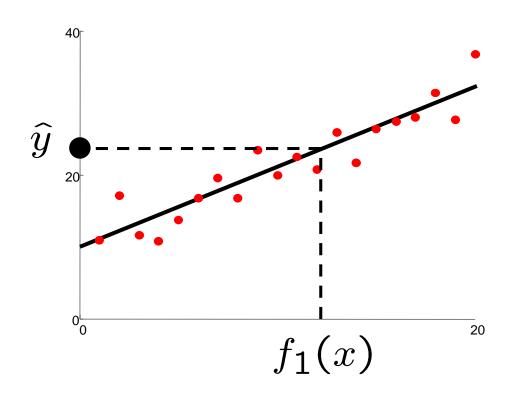
Video of Demo Approximate Q-Learning -- Pacman

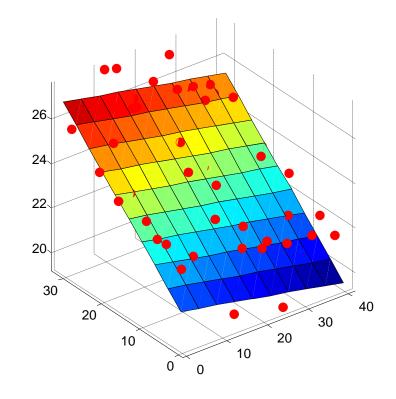


Q-Learning and Least Squares



Linear Approximation: Regression





Prediction:

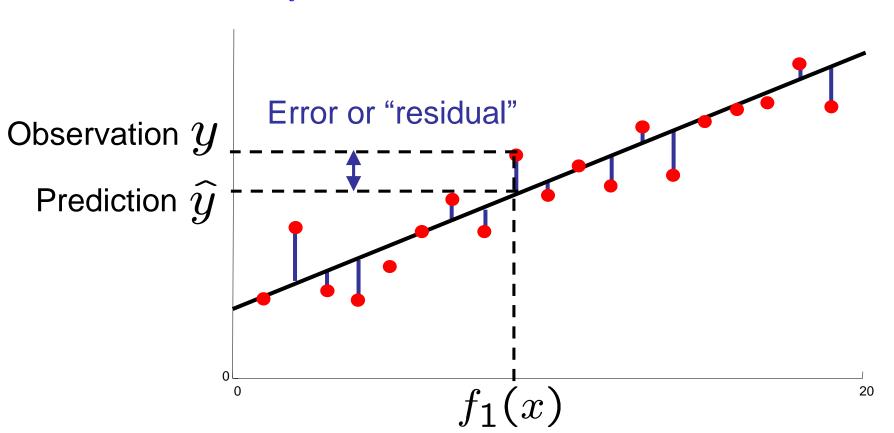
$$\hat{y} = w_0 + w_1 f_1(x)$$

Prediction:

$$\hat{y}_i = w_0 + w_1 f_1(x) + w_2 f_2(x)$$

Optimization: Least Squares

total error =
$$\sum_{i} (y_i - \hat{y}_i)^2 = \sum_{i} \left(y_i - \sum_{k} w_k f_k(x_i) \right)^2$$



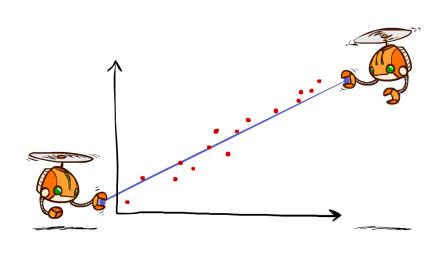
Minimizing Error

Imagine we had only one point x, with features f(x), target value y, and weights w:

$$\operatorname{error}(w) = \frac{1}{2} \left(y - \sum_{k} w_{k} f_{k}(x) \right)^{2}$$

$$\frac{\partial \operatorname{error}(w)}{\partial w_{m}} = -\left(y - \sum_{k} w_{k} f_{k}(x) \right) f_{m}(x)$$

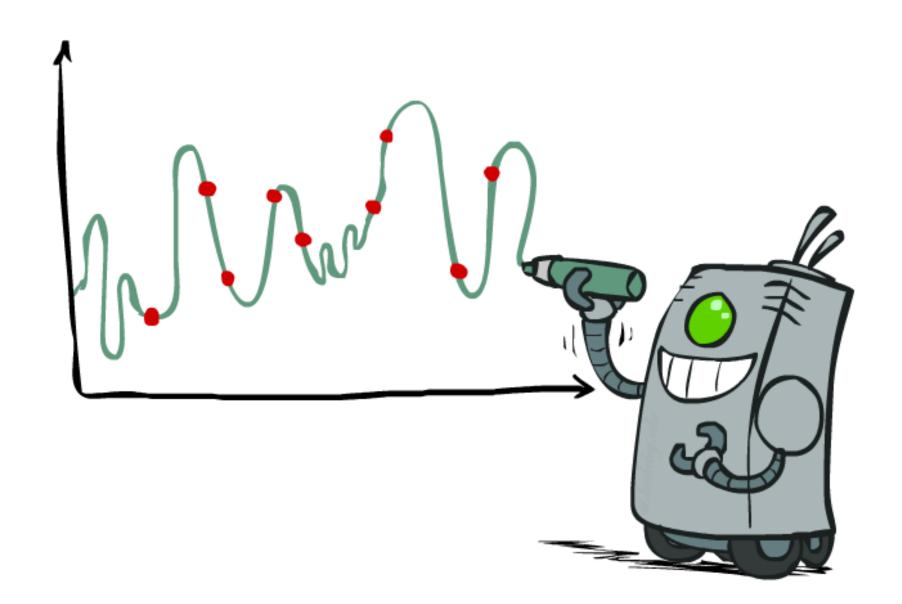
$$w_{m} \leftarrow w_{m} + \alpha \left(y - \sum_{k} w_{k} f_{k}(x) \right) f_{m}(x)$$



Approximate q update explained:

$$w_m \leftarrow w_m + \alpha \left[r + \gamma \max_a Q(s', a') - Q(s, a) \right] f_m(s, a)$$
"target" "prediction"

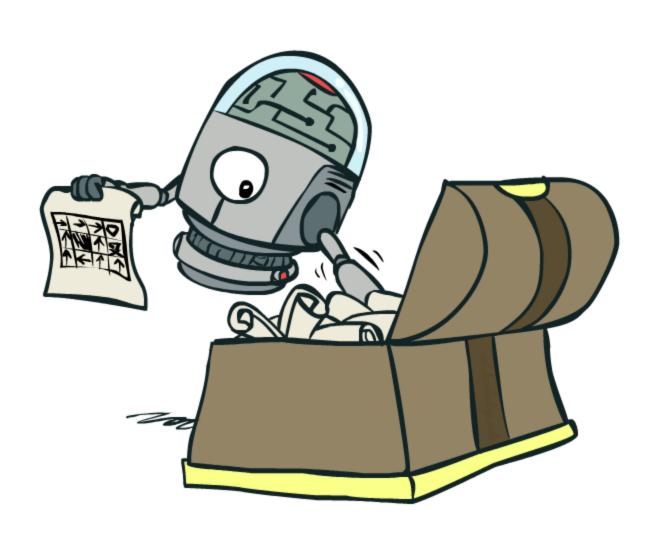
Overfitting: Why Limiting Capacity Can Help



New in Model-Free RL Playing Atari Games



Policy Search



Policy Search

- Problem: often the feature-based policies that work well (win games, maximize utilities) aren't the ones that approximate V / Q best
 - E.g. your value functions from project 2 were probably horrible estimates of future rewards, but they still produced good decisions
 - Q-learning's priority: get Q-values close (modeling)
 - Action selection priority: get ordering of Q-values right (prediction)
- Solution: learn policies that maximize rewards, not the values that predict them
- Policy search: start with an ok solution (e.g. Q-learning) then fine-tune by hill climbing on feature weights

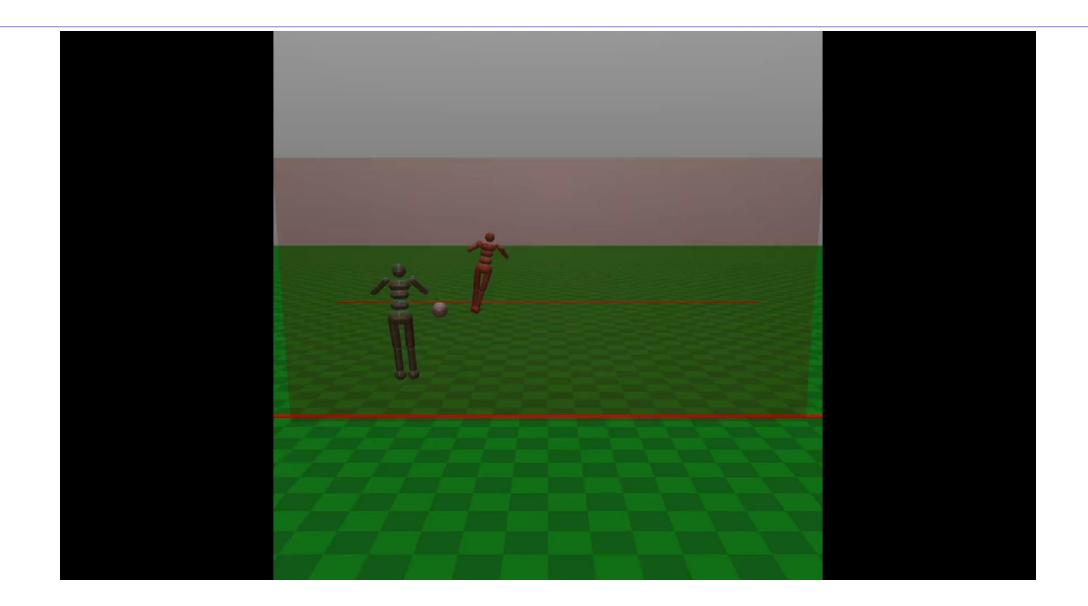
Policy Search

Simplest policy search:

- Start with an initial linear value function or Q-function
- Nudge each feature weight up and down and see if your policy is better than before

o Problems:

- o How do we tell the policy got better?
- Need to run many sample episodes!
- o If there are a lot of features, this can be impractical
- Better methods exploit lookahead structure, sample wisely, change multiple parameters...



Summary: MDPs and RL

Known MDP: Offline Solution

Goal Technique

Compute V*, Q*, π * Value / policy iteration

Evaluate a fixed policy π Policy evaluation

Unknown MDP: Model-Based

*use features

Goal to generalize Technique

Compute V*, Q*, π * VI/PI on approx. MDP

Evaluate a fixed policy π PE on approx. MDP

Unknown MDP: Model-Free

*use features

Goal to generalize Technique

Compute V*, Q*, π * Q-learning

Evaluate a fixed policy π Value Learning

Conclusion

- We've seen how AI methods can solve problems in:
 - o Search
 - o Games
 - Markov Decision Problems
 - Reinforcement Learning
- Next up: Uncertainty and Learning!

