Today

- Agents that Plan Ahead
- Search Problems
- Uninformed Search Methods
  - Depth-First Search
  - Breadth-First Search
  - Uniform-Cost Search
Agents that Plan
Reflex Agents

- Reflex agents:
  - Choose action based on current percept (and maybe memory)
  - May have memory or a model of the world’s current state
  - Do not consider the future consequences of their actions
  - Consider how the world IS

- Can a reflex agent be rational?
Video of Demo Reflex Optimal
Video of Demo Reflex Odd
Planning Agents

- Planning agents:
  - Ask “what if”
  - Decisions based on (hypothesized) consequences of actions
  - Must have a model of how the world evolves in response to actions
  - Must formulate a goal (test)
  - Consider how the world WOULD BE

- Optimal vs. complete planning

- Planning vs. replanning
Video of Demo Replanning
Video of Demo Mastermind
Search Problems
Search Problems

- A search problem consists of:
  - A state space
  - A successor function (with actions, costs)
  - A start state and a goal test

- A solution is a sequence of actions (a plan) which transforms the start state to a goal state
Search: it is not just for agents

- Route Planning
- Hardware verification
- Planning optimal repair sequences

- Search: Modeling the world
Example: Traveling in Romania

- State space:
  - Cities
- Successor function:
  - Roads: Go to adjacent city with cost = distance
- Start state:
  - Arad
- Goal test:
  - Is state == Bucharest?
- Solution?
What’s in a State Space?

The world state includes every last detail of the environment.

A search state keeps only the details needed for planning (abstraction).

- **Problem: Pathing**
  - States: \((x,y)\) location
  - Actions: NSEW
  - Successor: update location only
  - Goal test: is \((x,y)\)=END

- **Problem: Eat-All-Dots**
  - States: \{\((x,y)\), dot booleans\}
  - Actions: NSEW
  - Successor: update location and possibly a dot boolean
  - Goal test: dots all false
This lecture is about search algorithms.
State Space Sizes?

- **World state:**
  - Agent positions: 120
  - Food count: 30
  - Ghost positions: 12
  - Agent facing: NSEW

- **How many**
  - World states?
    \[120 \times (2^{30}) \times (12^2) \times 4\]
  - States for pathing?
    120
  - States for eat-all-dots?
    \[120 \times (2^{30})\]
State Representation

- Real-world applications:
  - Requires approximations and heuristics
  - Need to design state representation so that search is feasible
    - Only focus on important aspects of the state
    - E.g., Use features to represent world states
Problem: eat all dots while keeping the ghosts perma-scared

What does the state space have to specify?

(agent position, dot booleans, power pellet booleans, remaining scared time)
State Space Graphs and Search Trees
State Space Graphs

- State space graph: A mathematical representation of a search problem
  - Nodes are (abstracted) world configurations
  - Arcs represent successors (action results)
  - The goal test is a set of goal nodes (maybe only one)

- In a state space graph, each state occurs only once!

- We can rarely build this full graph in memory (it’s too big), but it’s a useful idea
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A search tree:
- The start state is the root node
- Children correspond to successors
- Nodes show states, but correspond to PLANS that achieve those states
- For most problems, we can never actually build the whole tree

This is now / start
Possible futures
State Space Graphs vs. Search Trees

We construct both on demand – and we construct as little as possible.

Each NODE in the search tree is an entire PATH in the state space graph.
Nodes in state space graphs are problem states
- Represent an abstracted state of the world
- Have successors, can be goal / non-goal, have multiple predecessors

Nodes in search trees are plans
- Represent a plan (sequence of actions) which results in the node’s state
- Have a problem state and one parent, a path length, a depth & a cost
- The same problem state may be achieved by multiple search tree nodes
Consider this 4-state graph:

How big is its search tree (from $S$)?
State Space Graphs vs. Search Trees

Consider this 4-state graph:

Important: Lots of repeated structure in the search tree!

How big is its search tree (from S)?

Important: Lots of repeated structure in the search tree!
Tree Search
Searching with a Search Tree

- Search:
  - Expand out potential plans (tree nodes)
  - Maintain a fringe of partial plans under consideration
  - Try to expand as few tree nodes as possible
function Tree-Search( problem, strategy) returns a solution, or failure
initialize the search tree using the initial state of problem
loop do
    if there are no candidates for expansion then return failure
    choose a leaf node for expansion according to strategy
    if the node contains a goal state then return the corresponding solution
    else expand the node and add the resulting nodes to the search tree
end

- Important ideas:
  - Fringe
  - Expansion
  - Exploration strategy

- Main question: which fringe nodes to explore?
Example: Tree Search
Example: Tree Search
Search Algorithms

- Uninformed Search Methods
  - Depth-First Search
  - Breadth-First Search
  - Uniform-Cost Search

- Heuristic Search Methods
  - Best First / Greedy Search
  - A*
Depth-First Search
Depth-First Search

Strategy: expand a deepest node first

Implementation:
Fringe is a LIFO stack
Search Algorithm Properties
Search Algorithm Properties

- Complete: Guaranteed to find a solution if one exists?
- Optimal: Guaranteed to find the least cost path?
- Time complexity?
- Space complexity?

- Cartoon of search tree:
  - $b$ is the branching factor
  - $m$ is the maximum depth
  - Solutions at various depths

- Number of nodes in entire tree?
  - $1 + b + b^2 + \ldots + b^m = O(b^m)$
Depth-First Search (DFS) Properties

- What nodes DFS expand?
  - Some left prefix of the tree.
  - Could process the whole tree!
  - If m is finite, takes time $O(b^m)$

- How much space does the fringe take?
  - Only has siblings on path to root, so $O(bm)$

- Is it complete?
  - $m$ could be infinite, so only if we prevent cycles (more later)

- Is it optimal?
  - No, it finds the “leftmost” solution, regardless of depth or cost
Breadth-First Search
Breadth-First Search

Strategy: expand a shallowest node first

Implementation: Fringe is a FIFO queue
Breadth-First Search (BFS) Properties

- **What nodes does BFS expand?**
  - Processes all nodes above shallowest solution
  - Let depth of shallowest solution be $s$
  - Search takes time $O(b^s)$

- **How much space does the fringe take?**
  - Has roughly the last tier, so $O(b^s)$

- **Is it complete?**
  - $s$ must be finite if a solution exists, so yes!

- **Is it optimal?**
  - Only if costs are all 1 (more on costs later)
**BFS**

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Complete</th>
<th>Optimal</th>
<th>Time</th>
<th>Space</th>
</tr>
</thead>
<tbody>
<tr>
<td>DFS w/ Path Checking</td>
<td>Y</td>
<td>N</td>
<td>$O(b^m)$</td>
<td>$O(bm)$</td>
</tr>
<tr>
<td>BFS</td>
<td>Y</td>
<td>$Y^*$</td>
<td>$O(b^d)$</td>
<td>$O(b^d)$</td>
</tr>
</tbody>
</table>

Diagram:
- **d tiers**
  - 1 node
  - $b$ nodes
  - $b^2$ nodes
  - $b^d$ nodes
  - $b^m$ nodes
Quiz: DFS vs BFS
Video of Demo Maze Water DFS/BFS (part 1)
Video of Demo Maze Water DFS/BFS (part 2)
DFS vs BFS

- When will BFS outperform DFS?
- When will DFS outperform BFS?
Iterative Deepening

- Idea: get DFS’s space advantage with BFS’s time / shallow-solution advantages
  - Run a DFS with depth limit 1. If no solution...
  - Run a DFS with depth limit 2. If no solution...
  - Run a DFS with depth limit 3. ..... 

- Isn’t that wastefully redundant?
  - Generally most work happens in the lowest level searched, so not so bad!
BFS finds the shortest path in terms of number of actions. It does not find the least-cost path. We will now cover a similar algorithm which does find the least-cost path.
Uniform Cost Search
Uniform Cost Search

Strategy: expand a cheapest node first:
Fringe is a priority queue (priority: cumulative cost)
Uniform Cost Search (UCS) Properties

- What nodes does UCS expand?
  - Processes all nodes with cost less than cheapest solution!
  - If that solution costs $C^*$ and arcs cost at least $\varepsilon$, then the “effective depth” is roughly $C^*/\varepsilon$
  - Takes time $O(b^{C^*/\varepsilon})$ (exponential in effective depth)

- How much space does the fringe take?
  - Has roughly the last tier, so $O(b^{C^*/\varepsilon})$

- Is it complete?
  - Assuming best solution has a finite cost and minimum arc cost is positive, yes!

- Is it optimal?
  - Yes! (Proof next lecture via A*)
Uniform Cost Issues

- Remember: UCS explores increasing cost contours

- The good: UCS is complete and optimal!

- The bad:
  - Explores options in every “direction”
  - No information about goal location

- We’ll fix that soon!
Video of Demo Empty UCS
Video of Demo Maze with Deep/Shallow Water --- DFS, BFS, or UCS? (part 1)
Video of Demo Maze with Deep/Shallow Water --- DFS, BFS, or UCS? (part 2)
Video of Demo Maze with Deep/Shallow Water --- DFS, BFS, or UCS? (part 3)
The One Queue

- All these search algorithms are the same except for fringe strategies
  - Conceptually, all fringes are priority queues (i.e. collections of nodes with attached priorities)
  - Practically, for DFS and BFS, you can avoid the $\log(n)$ overhead from an actual priority queue, by using stacks and queues
  - Can even code one implementation that takes a variable queuing object
Search and Models

- Search operates over models of the world
  - The agent doesn’t actually try all the plans out in the real world!
  - Planning is all “in simulation”
- Your search is only as good as your models…
To Do:

- Try python practice (PS0)
  - Won’t be graded
- Will release PS1 soon
  - Start ASAP
  - Submission: Canvas

Website:
- Do readings for search algorithms
- Try this search visualization tool
  - http://qiao.github.io/PathFinding.js/visual/