CSE 573: Artificial Intelligence
Winter 2019

Adversarial Search

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Based on slides from Dan Klein, Luke Zettlemoyer

Many slides over the course adapted from either Stuart Russell or Andrew Moore

Announcements

- PS2 is due Jan 30
- Final poster session date
  - New poll posted
  - Best time so far: Thu, March 14 12-2pm.

Outline

- Games
  - Review: Minimax search
  - α-β search
  - Evaluation functions
  - Expectimax search
  - Complex Games

Game Playing

- Many different kinds of games!

- Want algorithms for calculating a strategy (policy) which recommends a move in each state

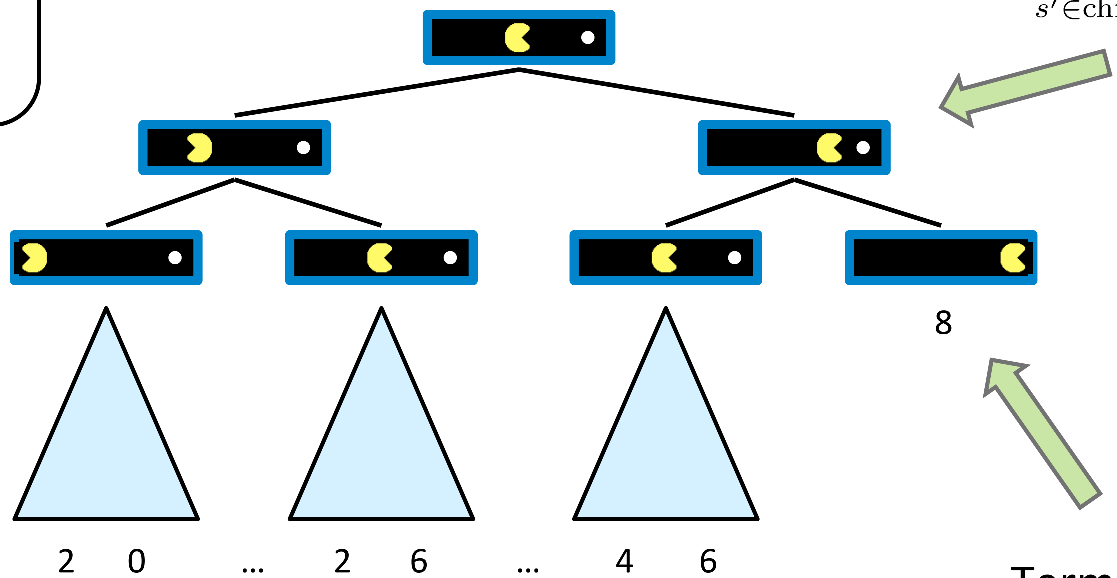
	deterministic	chance
perfect information	chess, checkers, go, othello	backgammon, monopoly
imperfect information	stratego	bridge, poker, scrabble, nuclear war

Deterministic Games

Value of a state:
The best achievable outcome (utility) from that state

Non-Terminal States:

$$V(s) = \max_{s' \in \text{children}(s)} V(s')$$



Terminal States:

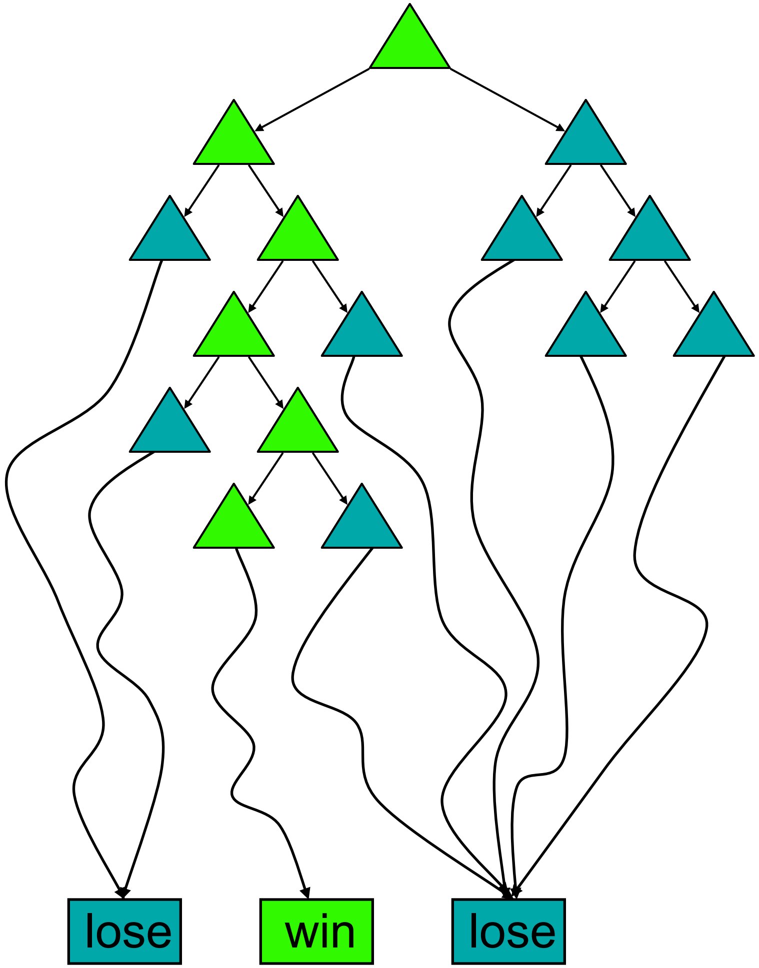
$$V(s) = \text{known}$$

- States: S (start at s_0)
- Players: P={1...N} (usually take turns)
- Actions: A (may depend on player / state)
- Transition Function: S x A → S
- Terminal Test: S → {t,f}
- Terminal Utilities: S x P → R

- Solution for a player is a policy: S → A

Deterministic Single-Player

- Deterministic, single player, perfect information:
  - Know the rules, action effects, winning states
  - E.g. Freecell, 8-Puzzle, Rubik's cube
- … it's just search!
- Slight reinterpretation:
  - Each node stores a value: the best outcome it can reach
  - This is the maximal outcome of its children (the max value)
  - Note that we don't have path sums as before (utilities at end)
- After search, can pick move that leads to best node



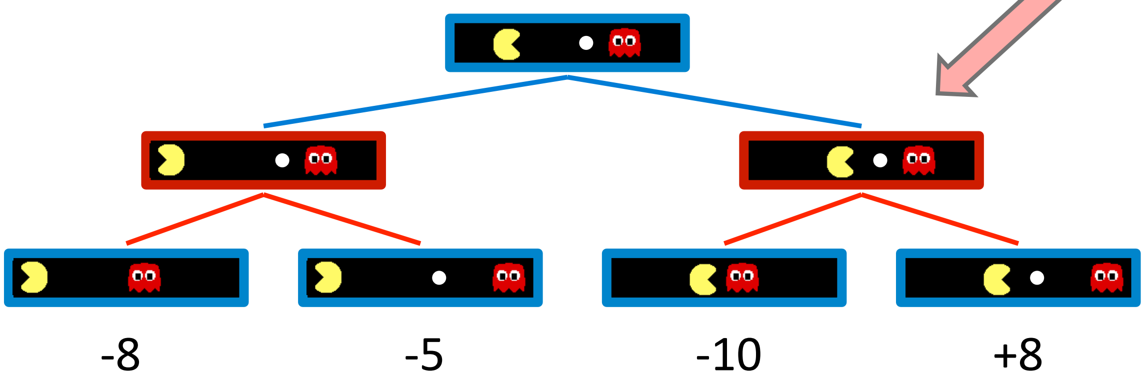
Adversarial Game Trees

States Under Agent's Control:

$$V(s) = \max_{s' \in \text{successors}(s)} V(s')$$

States Under Opponent's Control:

$$V(s') = \min_{s \in \text{successors}(s')} V(s)$$

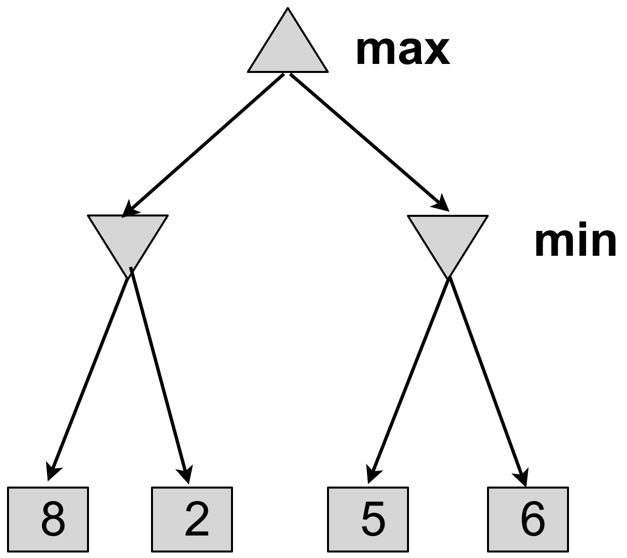


Terminal States:

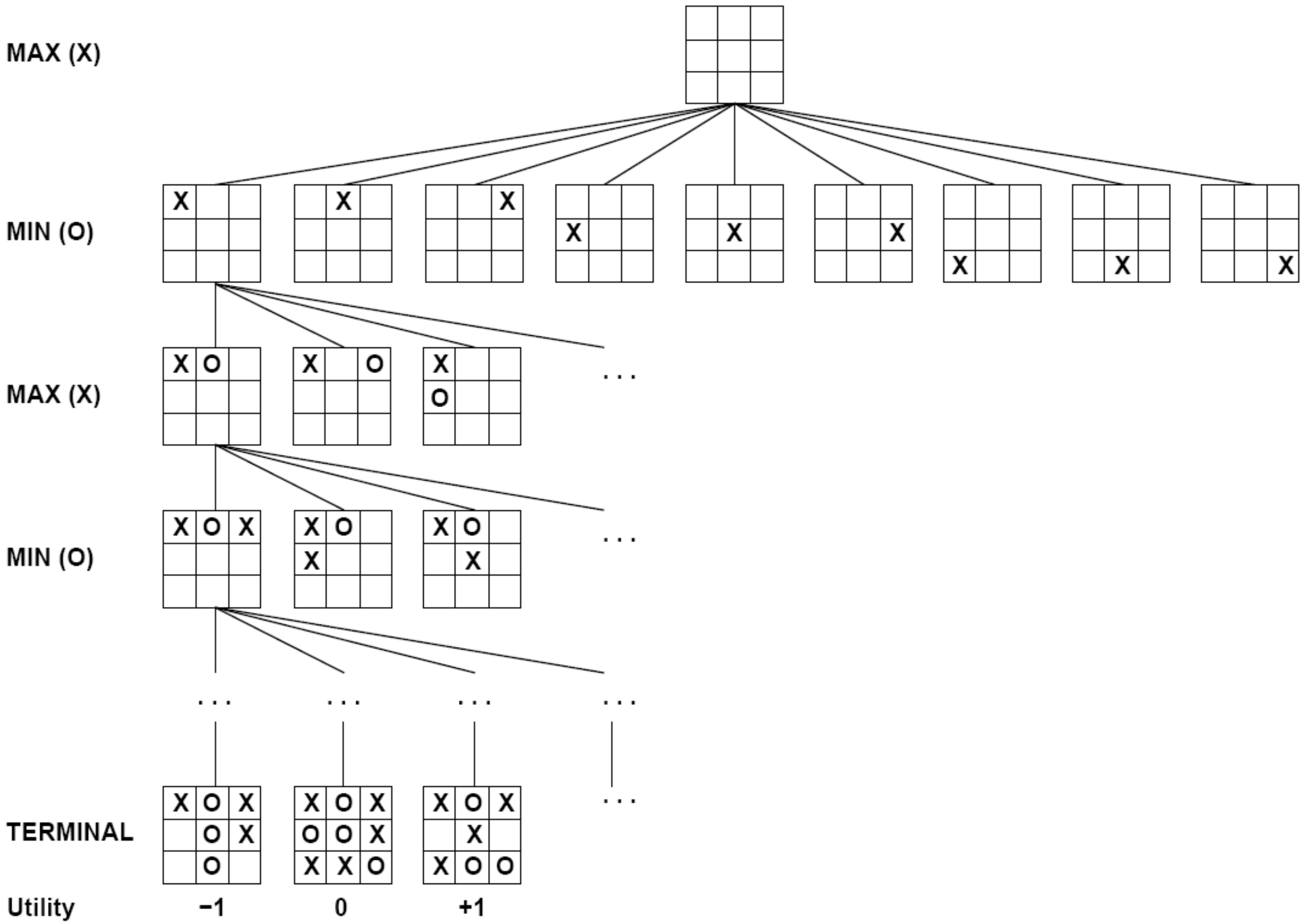
$$V(s) = \text{known}$$

Deterministic Two-Player

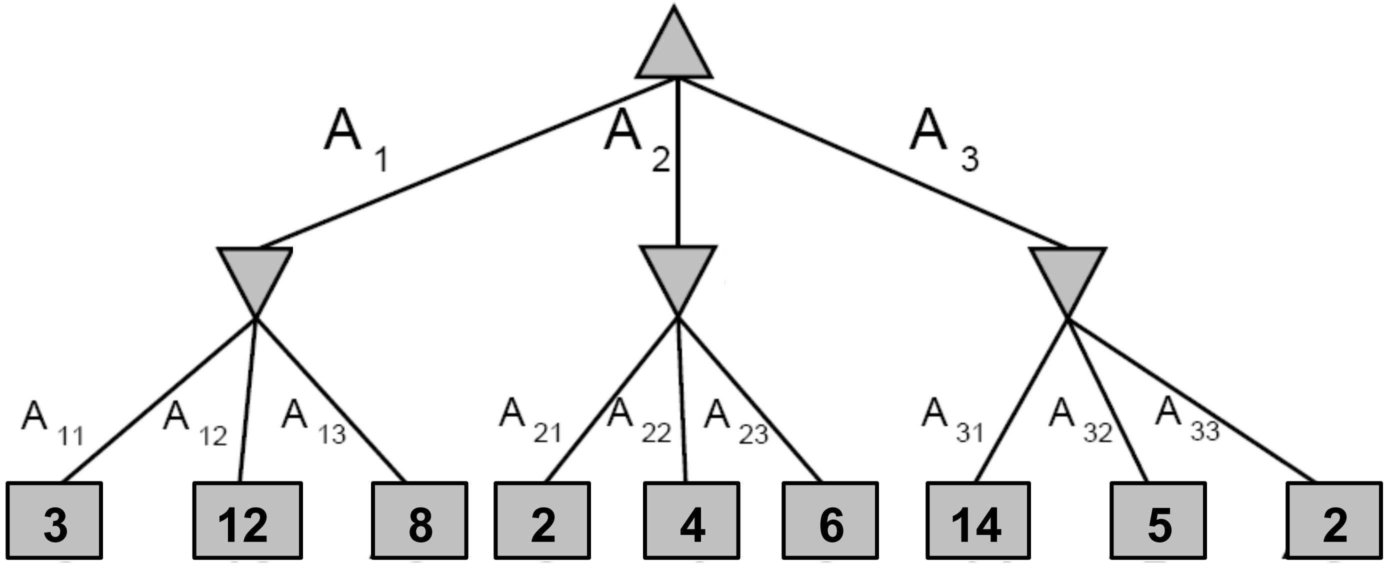
- E.g. tic-tac-toe, chess, checkers
- Zero-sum games
  - Agents have opposite utilities
  - One player maximizes result
  - The other minimizes result
- **Minimax search**
  - A state-space search tree
  - Players alternate
  - Choose move to position with highest minimax value = best achievable utility against best play



Tic-tac-toe Game Tree



Minimax Example



Minimax Search

function MAX-VALUE(*state*) **returns** *a utility value*
  if TERMINAL-TEST(*state*) **then return** UTILITY(*state*)
  $v \leftarrow -\infty$
  for $a,\ s$ in SUCCESSORS(*state*) **do** $v \leftarrow \text{MAX}(v, \text{MIN-VALUE}(s))$
  return v

---

function MIN-VALUE(*state*) **returns** *a utility value*
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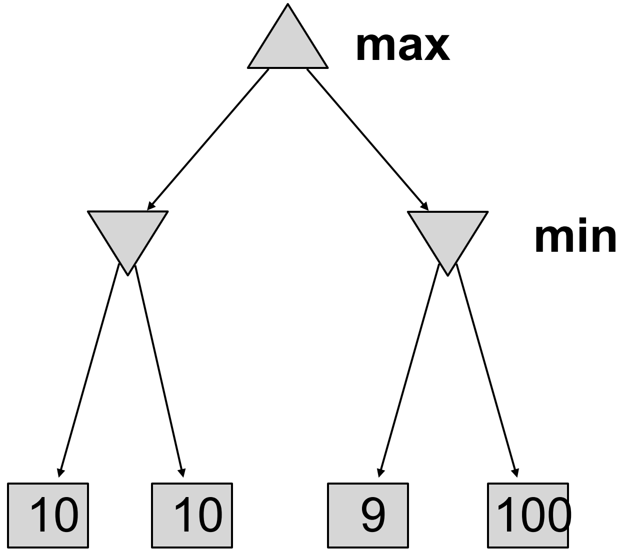
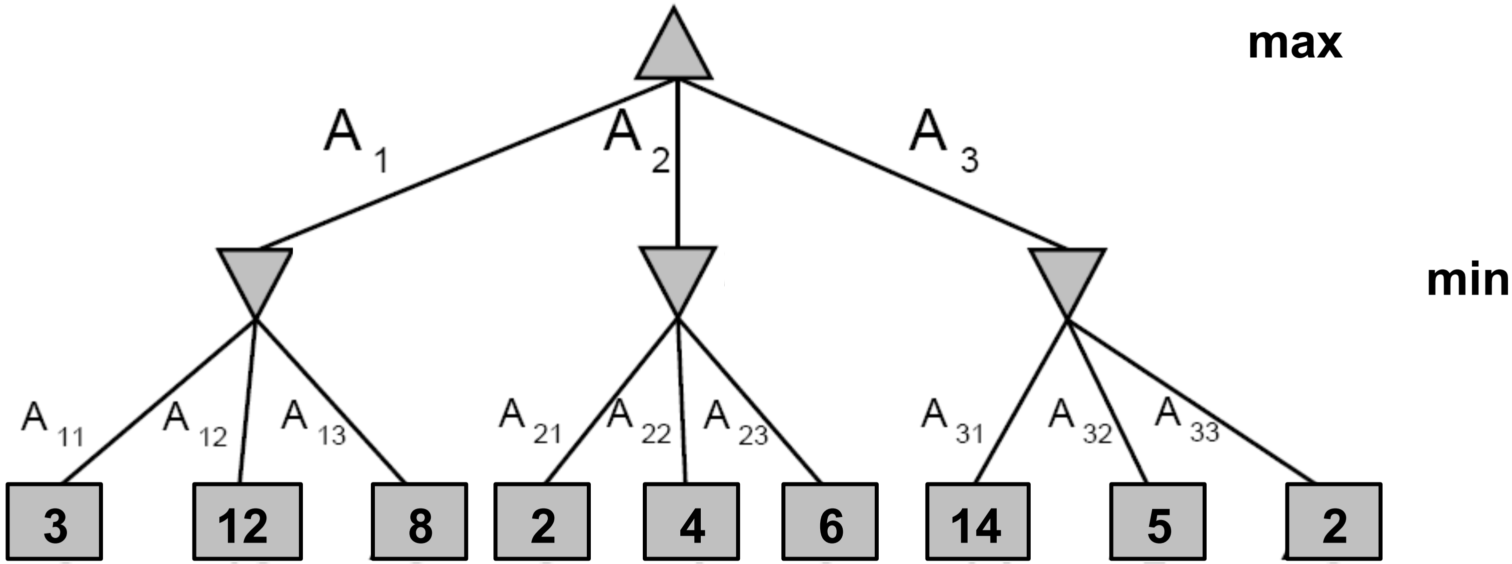
  $v \leftarrow \infty$
  for $a,\ s$ in SUCCESSORS(*state*) **do** $v \leftarrow \text{MIN}(v, \text{MAX-VALUE}(s))$
  return v

Minimax Properties

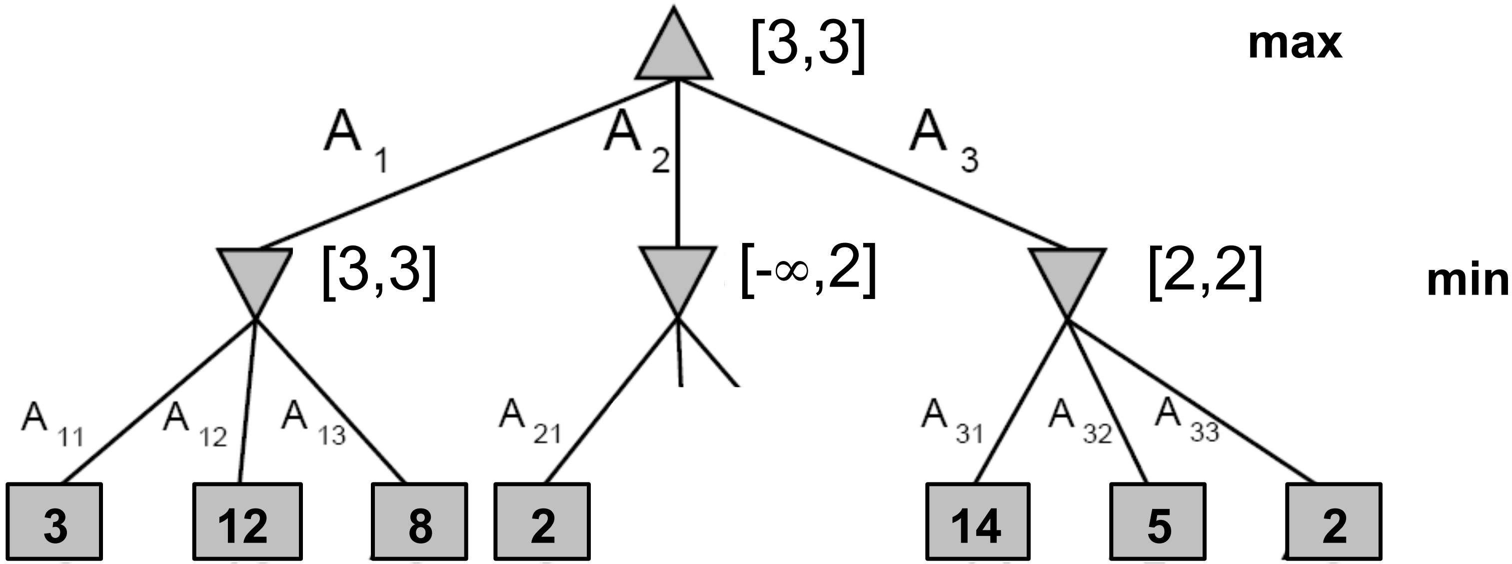
- Optimal?
  - Yes, against perfect player. Otherwise?
- Time complexity?
  - $O(b^m)$
- Space complexity?
  - $O(bm)$
- For chess, $b \approx 35$, $m \approx 100$
  - Exact solution is completely infeasible
  - But, do we need to explore the whole tree?



Can we do better?

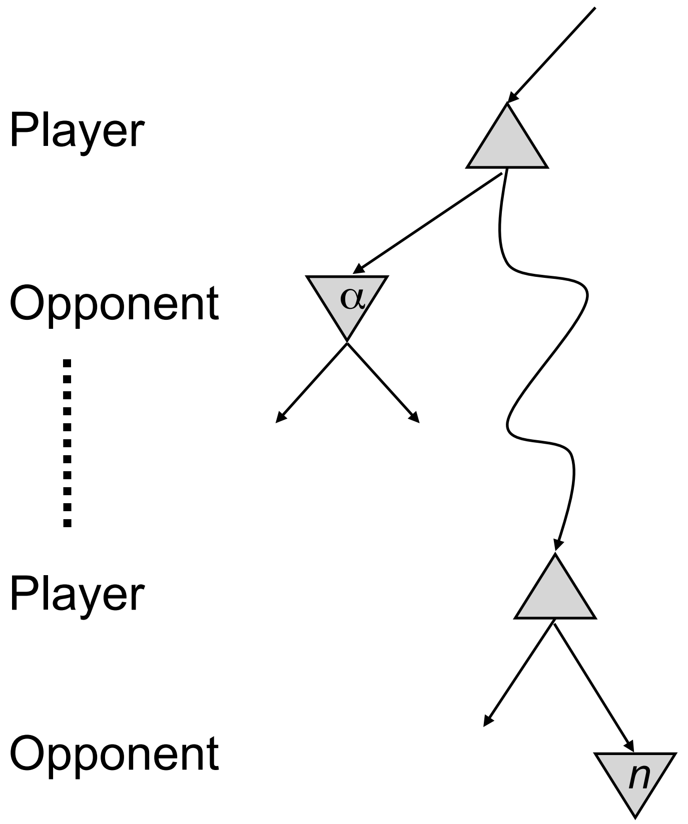


α-β Pruning Example

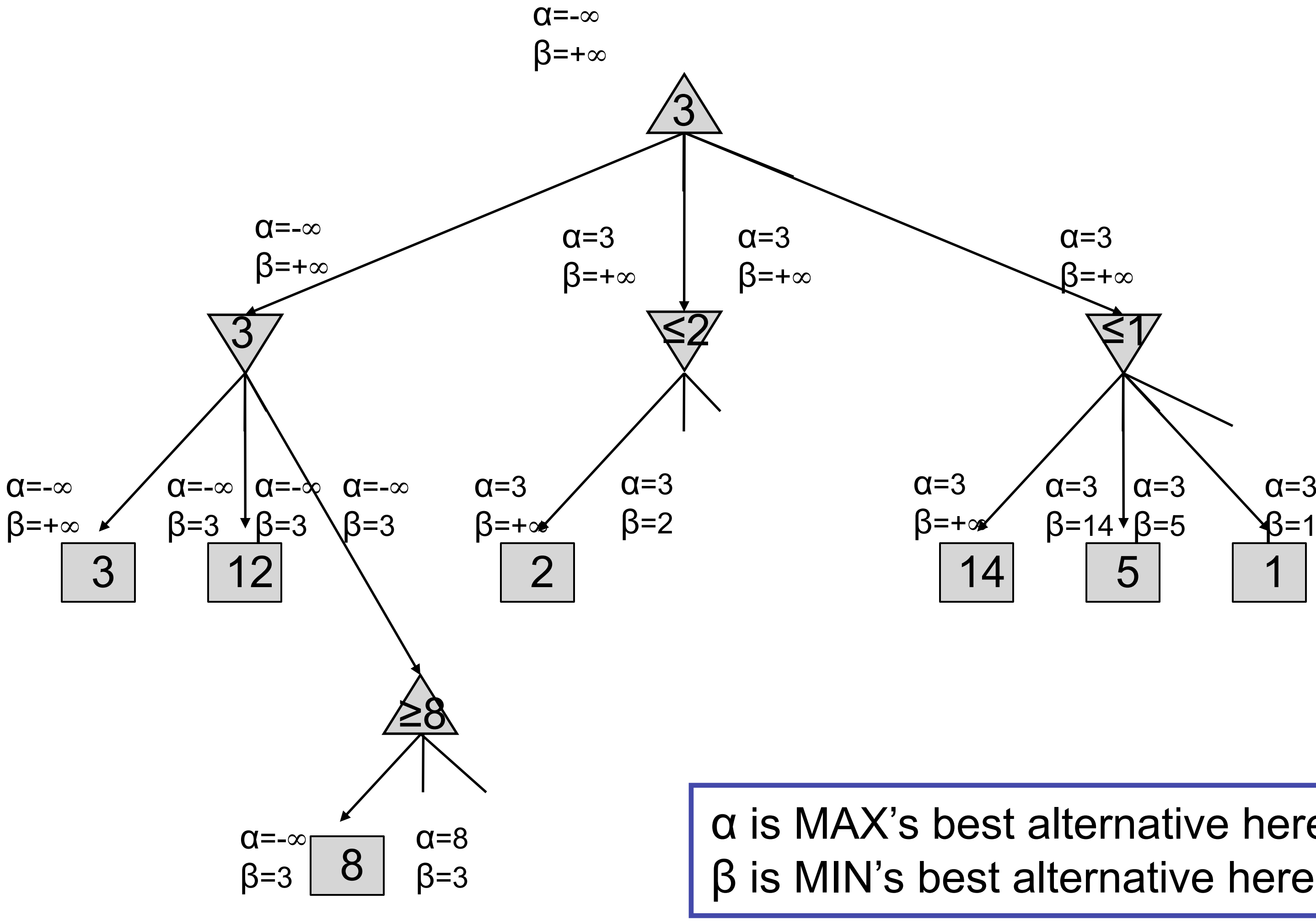


α-β Pruning

- General configuration
  - α is the best value that MAX can get at any choice point along the current path
  - If *n* becomes worse than α, MAX will avoid it, so can stop considering *n*'s other children
  - Define β similarly for MIN: value of the best (lowest value) choice along the current path for MIN



Alpha-Beta Pruning Example



α is MAX's best alternative here or above
β is MIN's best alternative here or above

Min-Max Implementation

```
def max-val(state        ):
    if leaf?(state), return U(state)
    initialize v = -∞
    for each c in children(state):
        v = max(v, min-val(c        ))


    return v
```

```
def min-val(state        ):
    if leaf?(state), return U(state)
    initialize v = +∞
    for each c in children(state):
        v = min(v, max-val(c        ))


    return v
```

Alpha-Beta implementation

α: MAX's best option on path to root
β: MIN's best option on path to root

```
def max-val(state, α, β):
    if leaf?(state), return U(state)
    initialize v = -∞
    for each c in children(state):
        v = max(v, min-val(c, α, β))


    return v
```

```
def min-val(state , α, β):
    if leaf?(state), return U(state)
    initialize v = +∞
    for each c in children(state):
        v = min(v, max-val(c, α, β))


    return v
```

Alpha-Beta Implementation

α: MAX's best option on path to root
β: MIN's best option on path to root

```
def max-val(state, α, β):
    if leaf?(state), return U(state)
    initialize v = -∞
    for each c in children(state):
        v = max(v, min-val(c, α, β))
        if v ≥ β return v
        α = max(α, v)
    return v
```

```
def min-val(state, α, β):
    if leaf?(state), return U(state)
    initialize v = +∞
    for each c in children(state):
        v = min(v, max-val(c, α, β))
        if v ≤ α return v
        β = min(β, v)
    return v
```

Slide adapted from Dan Klein & Pieter Abbeel - ai.berkeley.edu

Alpha-Beta Pruning Properties

- This pruning has no effect on final result at the root

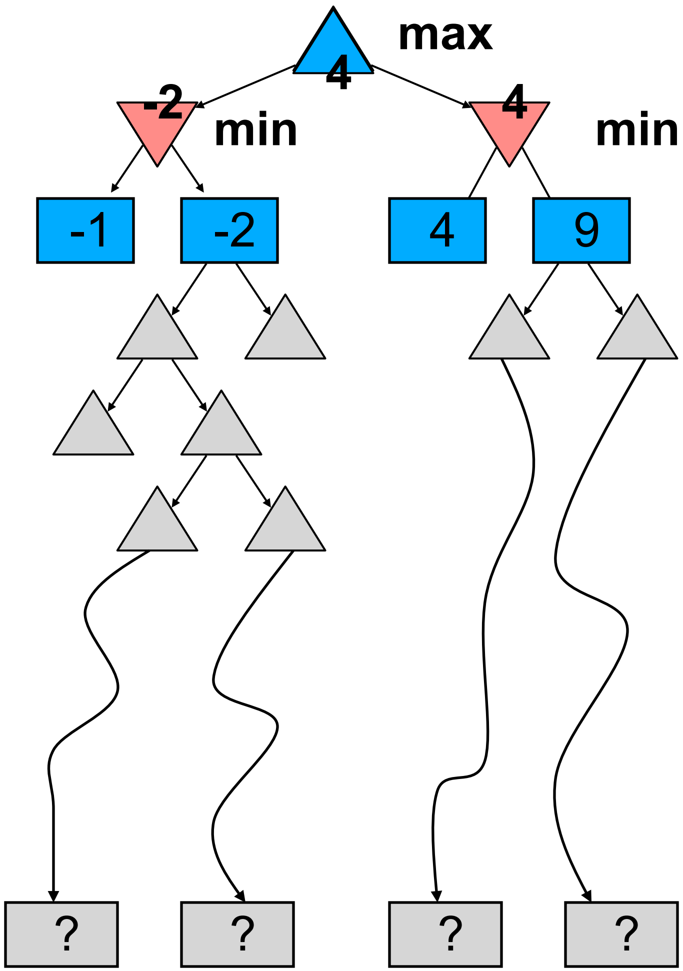
- Values of intermediate nodes might be wrong!
  - but, they are bounds

- Good child ordering improves effectiveness of pruning

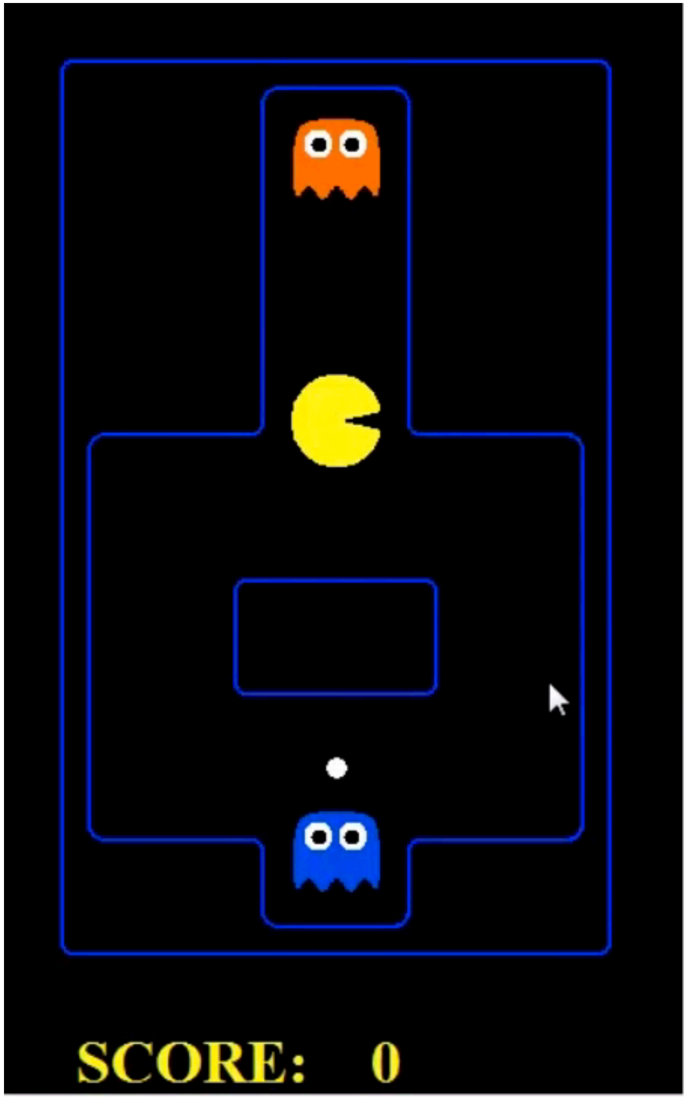
- With “perfect ordering”:
  - Time complexity drops to $O(b^{m/2})$
  - Doubles solvable depth!
  - Full search of, e.g. chess, is still hopeless…

Resource Limits

- Cannot search to leaves
- Depth-limited search
  - Instead, search a limited depth of tree
  - Replace terminal utilities with an eval function for non-terminal positions
  - e.g., α-β reaches about depth 8 – decent chess program
- Guarantee of optimal play is gone
- Evaluation function matters
  - It works better when we have a greater depth look ahead

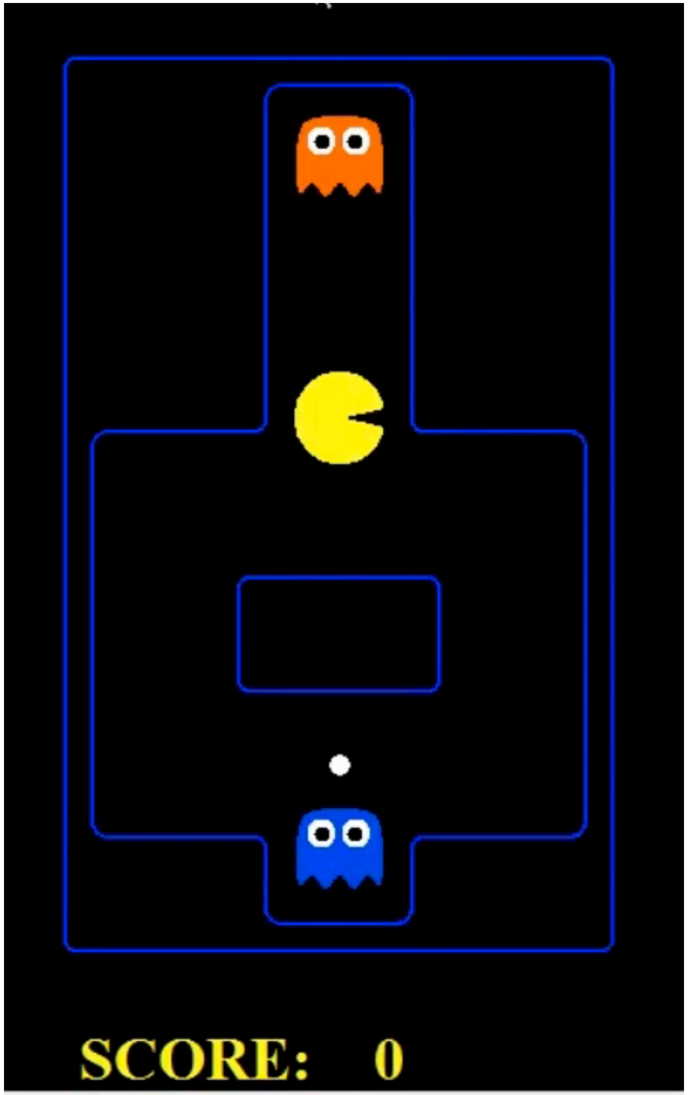


Depth Matters



depth 2

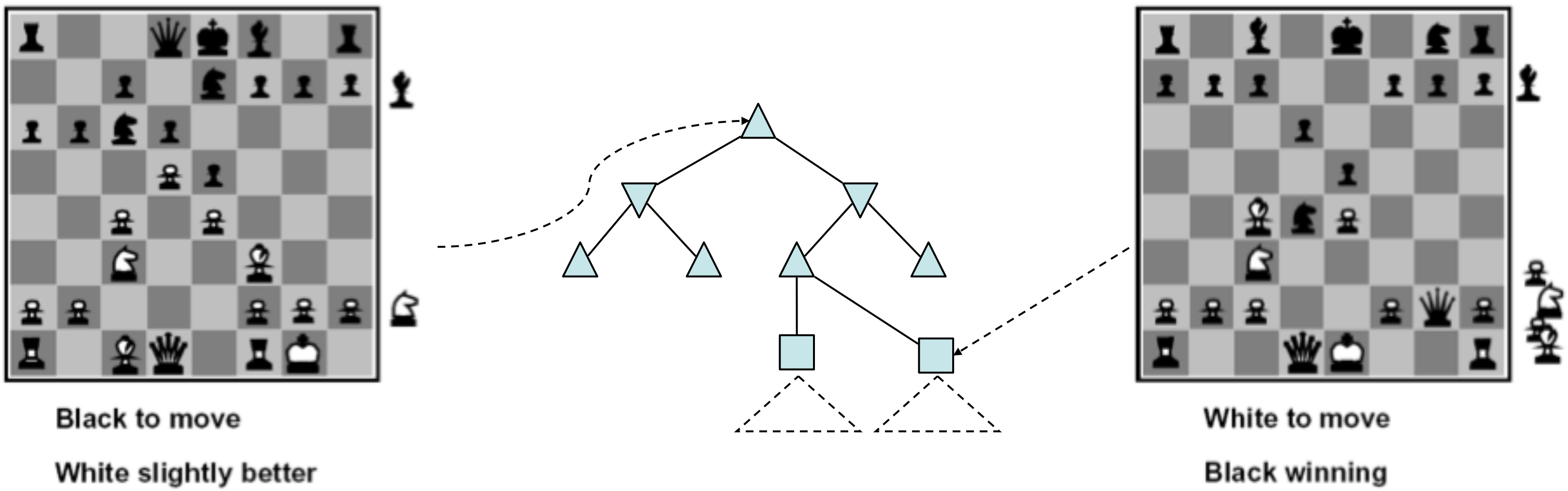
Depth Matters



depth 10

Evaluation Functions

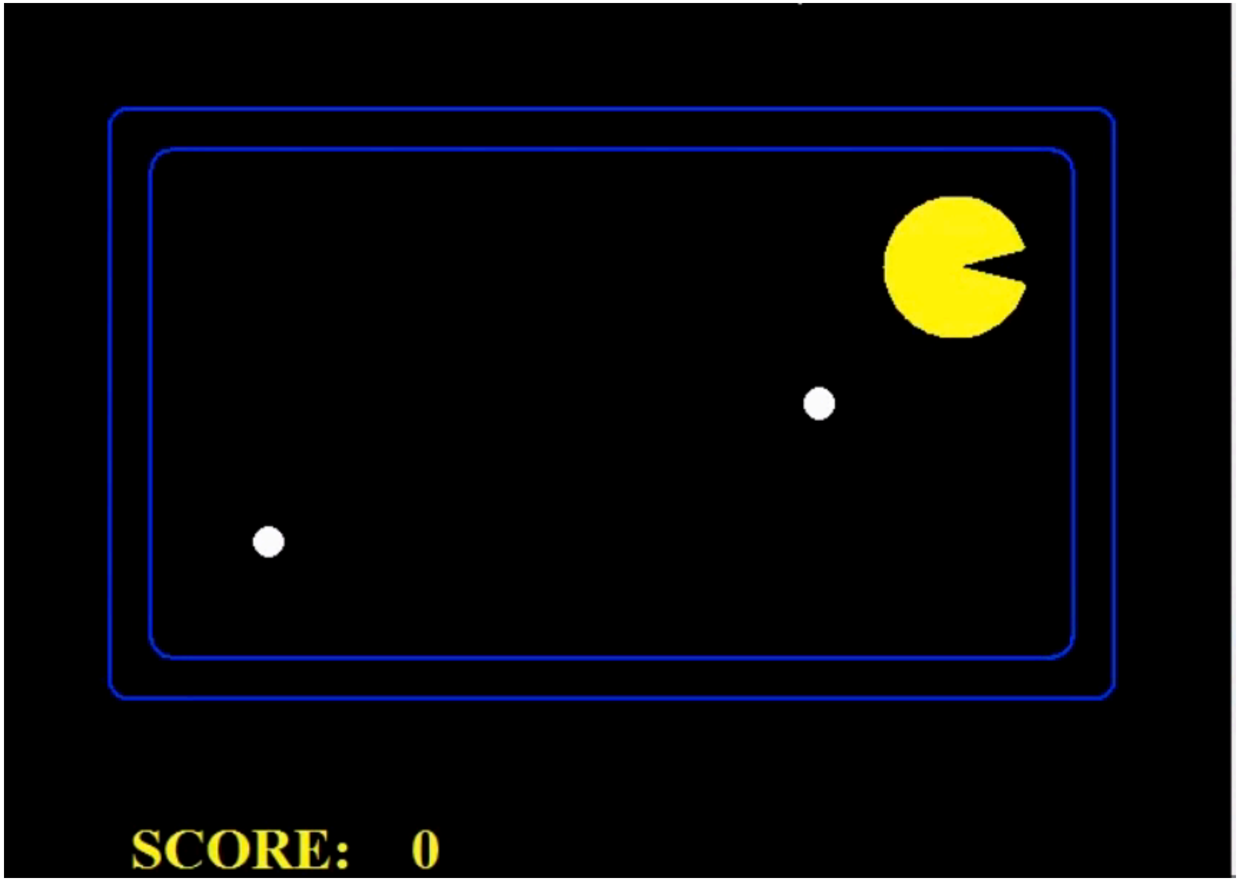
- Function which scores non-terminals



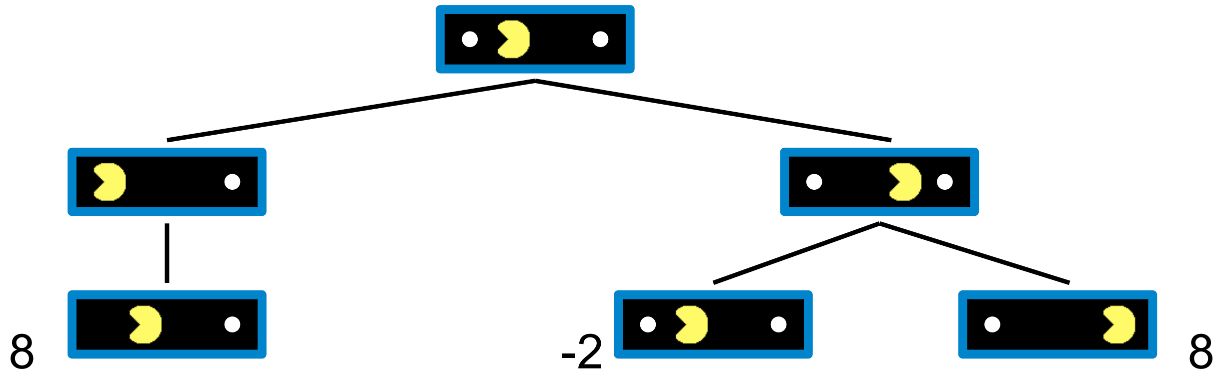
$$Eval(s) = w_1 f_1(s) + w_2 f_2(s) + \ldots + w_n f_n(s)$$

- Ideal function: returns the utility of the position
- In practice: typically weighted linear sum of features:
  - e.g. $f_1(s)$ = (num white queens – num black queens), etc.

Bad Evaluation Function

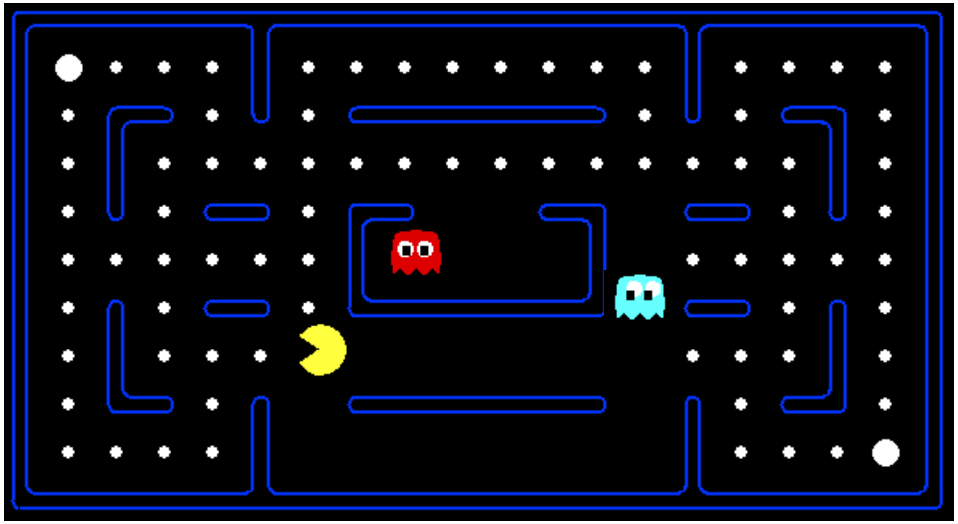


Why Pacman Starves



- He knows his score will go up by eating the dot now
- He knows his score will go up just as much by eating the dot later on
- There are no point-scoring opportunities after eating the dot
- Therefore, waiting seems just as good as eating

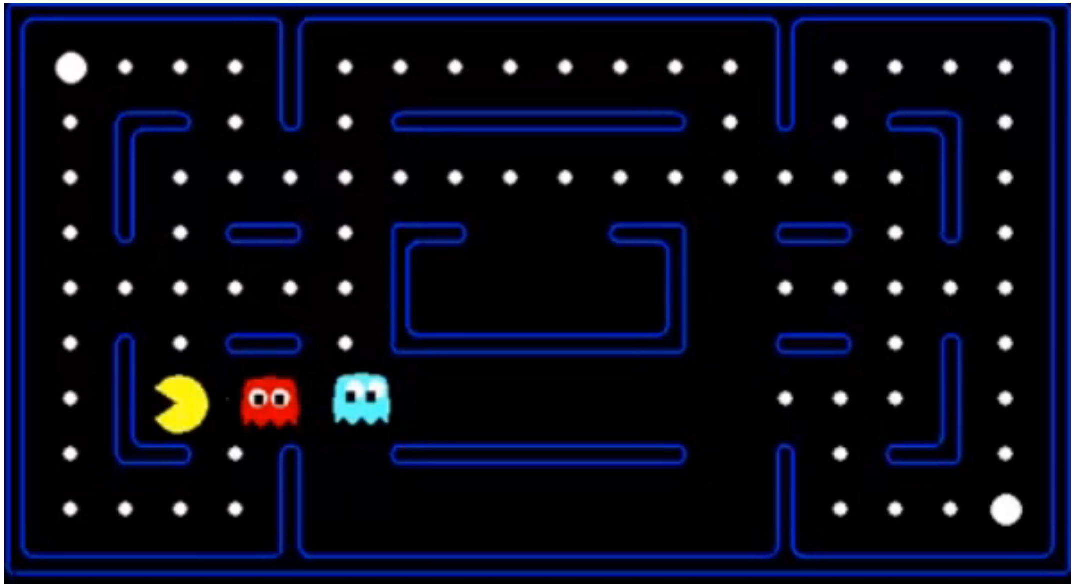
Evaluation for Pacman



What features would be good for Pacman?

$$Eval(s) = w_1 f_1(s) + w_2 f_2(s) + \ldots + w_n f_n(s)$$

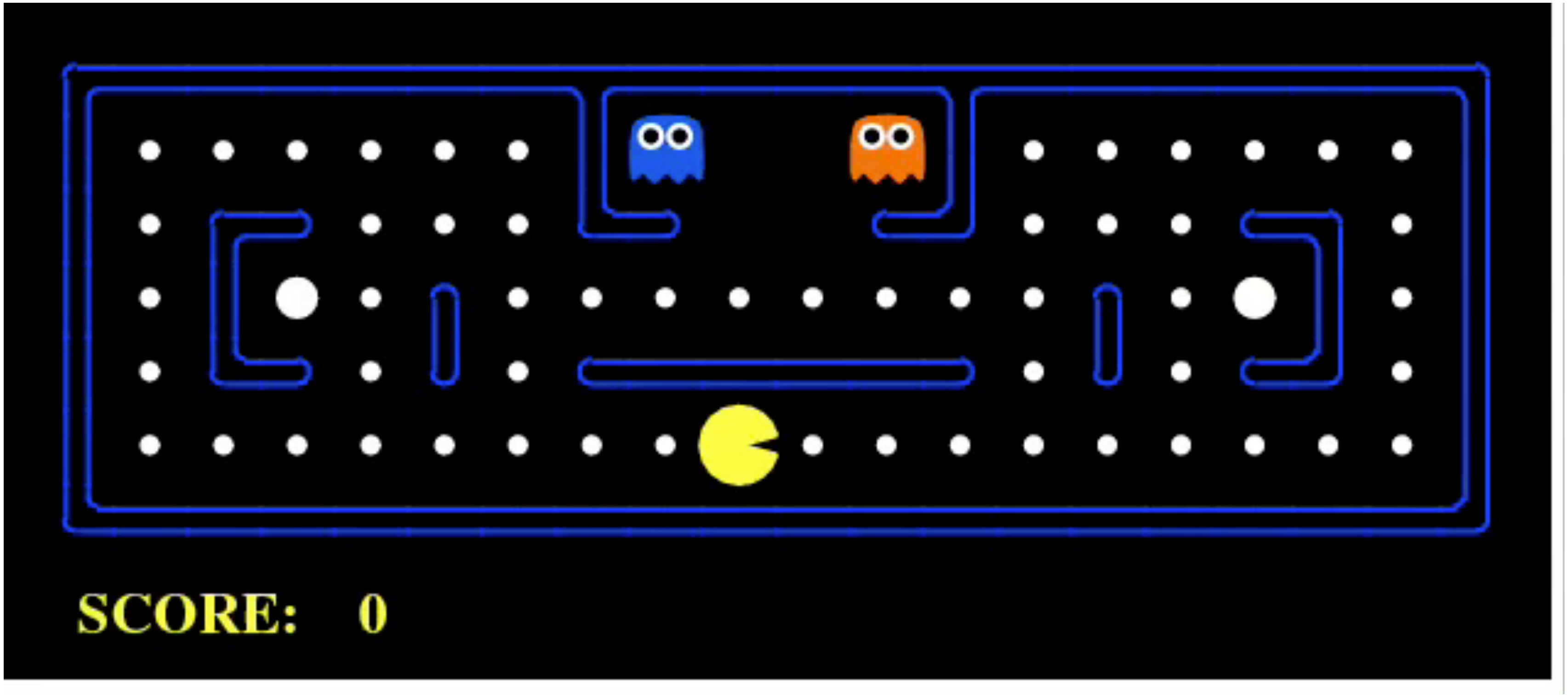
Evaluation Function



Evaluation Function

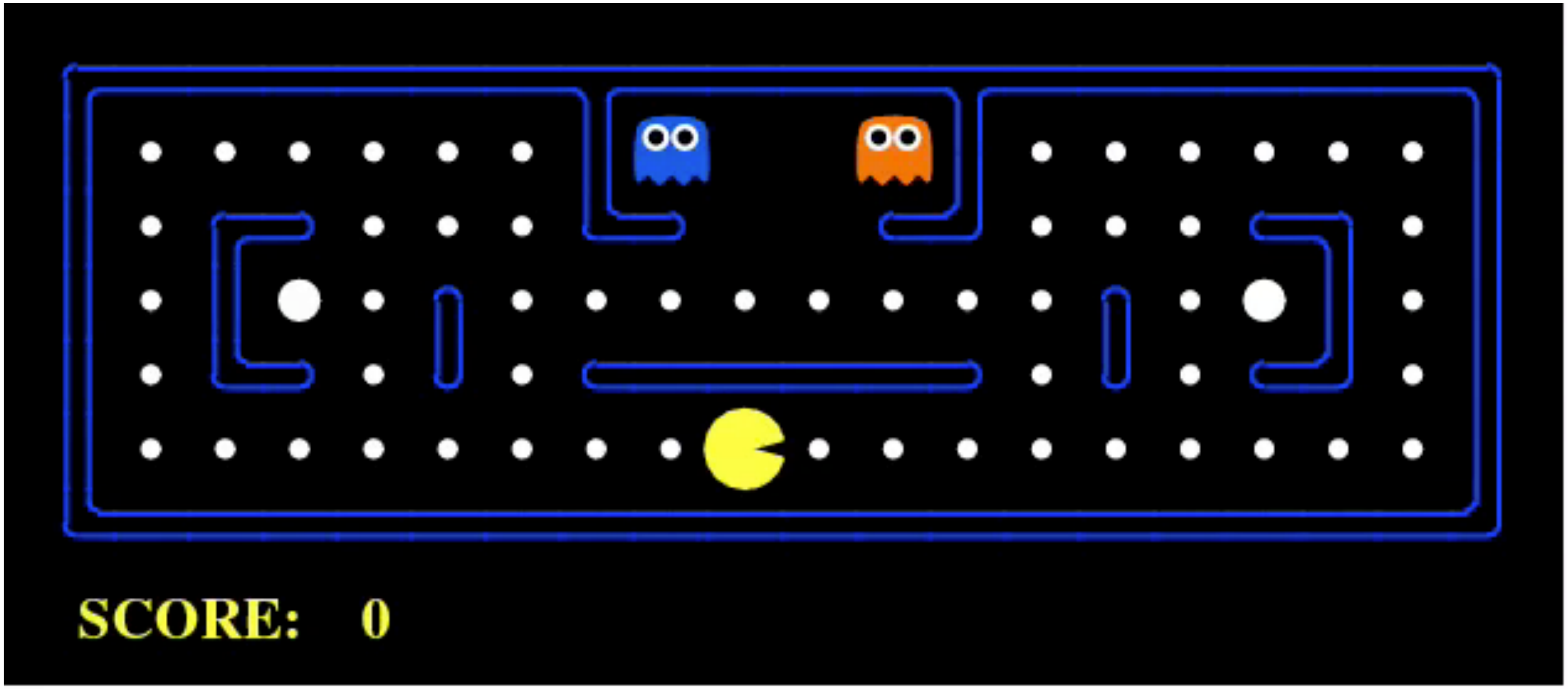
Which algorithm?

α-β, depth 4, simple eval fun



Which algorithm?

α-β, depth 4, better eval fun



Depth Matters

- Evaluation functions are always imperfect
- The deeper in the tree the evaluation function is buried, the less the quality of the evaluation function matters

Synergies between alpha-beta and evaluation function

- Alpha-Beta: amount of pruning depends on expansion ordering
  - Evaluation function can provide guidance to expand most promising nodes first

- Alpha-beta:
  - Similar for roles of mini-max swapped
  - Value at a min-node will only keep going down
  - Once value of min-node lower than better option for max along path to root, can prune
  - Hence, IF evaluation function provides upper-bound on value at min-node, and upper-bound already lower than better option for max along path to root THEN can prune