

CSE 573 Spring 2014

# Assignment 1: Problem Solving

Due: April 25, 11:00PM  
Turn in via Catalyst Dropbox

## Search

**Problem 1.** (20 points) You have two jugs measuring 4 gallons and 7 gallons respectively, and a water faucet. You can fill the jugs up or empty them out from one to another or onto the ground. Filling a full jug or emptying an empty jug is not allowed. Initially both jugs are empty, and your goal is to measure out exactly 2 gallons. There can be multiple solutions, in which the one using least amount of water from the faucet is optimal.

(a) (6 points) Give the state description, initial state, goal test, actions and cost function.

(b) (6 points) Draw a state diagram to show the steps for finding a solution by breadth first search.

(c) (2 points) Is the above solution (b) optimal? Briefly explain.

(d) (6 points) You will use A\* search with a poor heuristic that estimates 0 cost from any state to a goal state. Given the actions are considered in the same order for successors as in (b), is it possible that this naive A\* search will expand more nodes than (b)? Explain.

*Note: Assume you can detect duplicate states and will never expand a state twice. For (b) you may omit some edges to two trivial states (i.e. all empty and all full) to make the state diagram clearer.*

**Problem 2.** (20 points)  $n$  vehicles occupy squares  $(1, 1)$  through  $(n, 1)$  (i.e. the bottom row) of an  $n \times n$  grid. The vehicles must be moved to the top row but in reverse order; so the vehicle  $i$  that starts in  $(i, 1)$  must end up in  $(n - i + 1, n)$ . On each time step, every one of  $n$  vehicles can move one square up, down, left, or right, or stay put; but if a vehicle stays put, other adjacent vehicle (but not more than one) can hop over it. Two vehicles cannot occupy the same square.

(a) (4 points) Calculate the size of the state space as a function of  $n$ .

(b) (4 points) Calculate the branching factor as a function of  $n$ .

(c) (6 points) Suppose that vehicle  $i$  is at  $(x_i, y_i)$ . Write a nontrivial admissible heuristic  $h_i$  for the number of moves it will require to get to its goal location  $(n - i + 1, n)$ , assuming no other vehicles are on the grid.

(d) (6 points) Which of the following heuristics are admissible for the problem of moving all  $n$  vehicles to their destinations? Explain.

(i)  $\sum_{i=1}^n h_i$

(ii)  $\max(h_1, \dots, h_n)$

(iii)  $\min(h_1, \dots, h_n)$

## Constraint Satisfaction Problem

**Problem 3.** (30 points) Seven people meet at a European conference, and they want to talk to each other. However, there is no common language for them all, so they decide to divide themselves into two groups with 3 people and 4 people each such that within a group any two people can talk in a common language. Every person can speak up to three languages:

A speaks English, Italian and Spanish.

B speaks English, German and Italian.

C speaks English and Russian.

D speaks English, French and Italian.

E speaks German, Italian and Russian.

F speaks French and Spanish.

G speaks English and German.

(a) (6 points) Formulate the above problem as a constraint satisfaction problem (CSP) by giving variables, domains and constraints. (Hint: carefully define your constraints such that setting a variable will force some other variables to be set by forward checking and constraint propagation.)

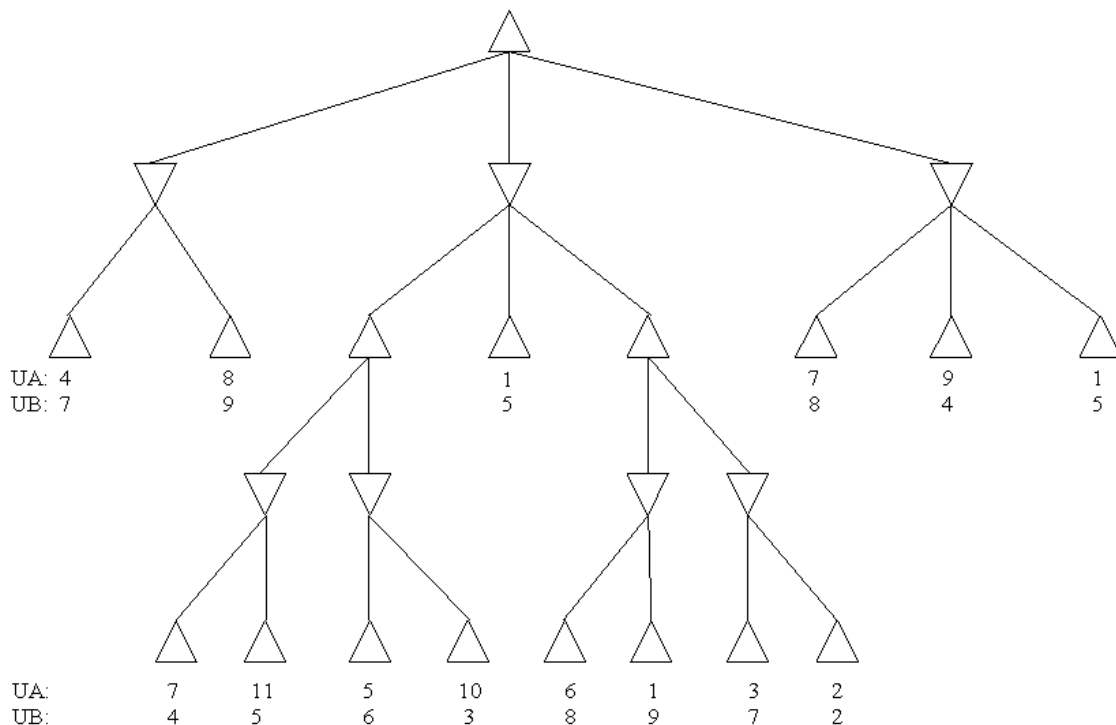
(b) (8 points) Use the uninformed backtracking algorithm, and start from A to G. Show the steps leading to the second backtracking point. Give a rough estimate of the number of backtrackings needed for the problem to be solved in this way.

(c) (8 points) Use forward checking + constraint propagation (that is, when an unassigned variable has only one domain value left, it is set and propagated; the propagation process is repeated until nothing more can be propagated.), and start from A to G. Show the steps leading to a solution.

(d) (8 points) Show that with forward checking + constraint propagation, you can select a variable to start and find a solution without backtracking. What kind of heuristic will likely give you such a start variable?

## Game Playing

**Problem 4.** (30 points) Consider a variation of mini-max search where two players view utility values differently. Assume A (the max player) and B (the min player) have their own utility functions  $U_A$  and  $U_B$ , which do not agree on all leaves (terminal states) of the search tree. As shown in the figure, each leaf is labeled with both utility values by  $U_A$  and  $U_B$ . A's goal is to maximize its outcome measured by  $U_A$ , while B's goal is to minimize its outcome measured by  $U_B$ . As a rational player, A (or B) always assumes the opponent will make the strongest move, based on all the information available to itself. Therefore, A (or B) plays against the worst case to itself, taking into account all its information.



Mark the leaf that will be reached by alternate moves of A and B for the following cases (a) – (d), always assuming that A knows  $U_A$  and B knows  $U_B$ . For each case, you may briefly show your reasoning process for partial credits in case your result is incorrect.

(a) (6 points) A does not know  $U_B$ , and B does not know  $U_A$ .

(b) (6 points) A knows  $U_B$ .

(c) (6 points) A knows  $U_B$ , and B knows  $U_A$ .

(d) (6 points) A knows  $U_B$ , B knows  $U_A$ , and A knows that B knows  $U_A$ .

(e) (6 points) Is it always true that the more A knows the better utility value A can get? Is it always true that if A knows more than in (a), A can do at least no worse than in (a)? Give some intuitive explanation of the above. ("always true" means not restricted by (a)-(d).)