Supervised Learning (contd)
Decision Trees

Mausam

(based on slides by UW-AI faculty)

Decision Trees

http://www.sfgate.com/blogs/images/sfgate/sgreen/2007/09/05/2240773250x321.jpg

Example data for learning the concept "Good day for tennis"

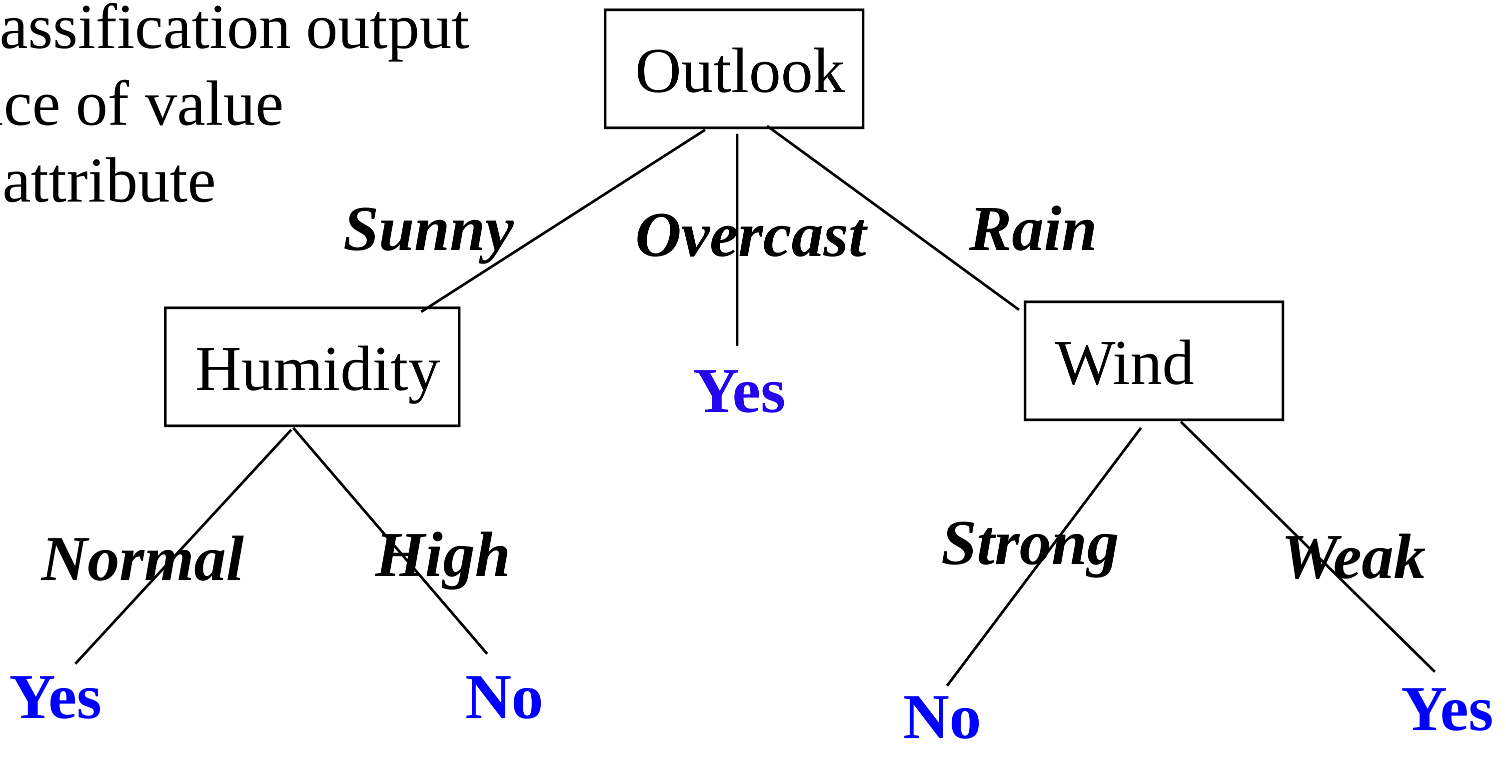
Day	Outlook	Humid	Wind	**PlayTennis**?
d1	s	h	w	n
d2	s	h	s	n
d3	o	h	w	y
d4	r	h	w	y
d5	r	n	w	y
d6	r	n	s	y
d7	o	n	s	y
d8	s	h	w	n
d9	s	n	w	y
d10	r	n	w	y
d11	s	n	s	y
d12	o	h	s	y
d13	o	n	w	y
d14	r	h	s	n

- Outlook = sunny, overcast, rain
- Humidity = high, normal
- Wind = weak, strong

A Decision Tree for the Same Data

Decision Tree for "PlayTennis?"

Leaves = classification output
Arcs = choice of value
for parent attribute

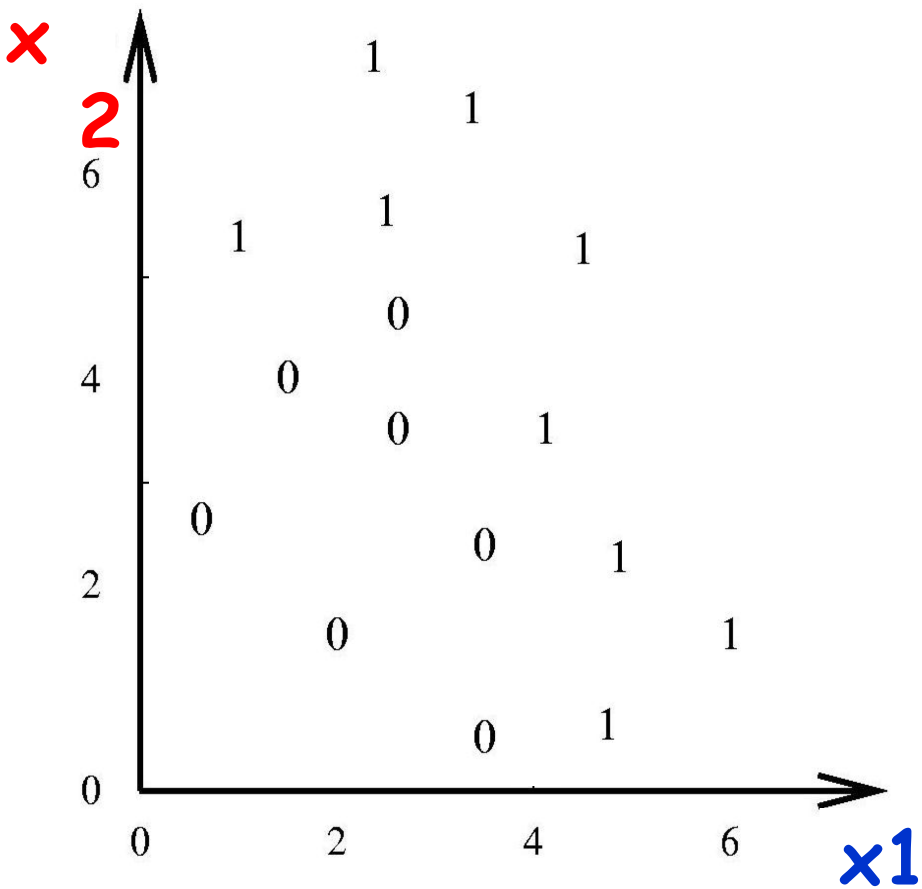


Decision tree is equivalent to logic in disjunctive normal form

PlayTennis ⇔ (Sunny ∧ Normal) ∨ Overcast ∨ (Rain ∧ Weak)

Example: Decision Tree for Continuous Valued Features and Discrete Output

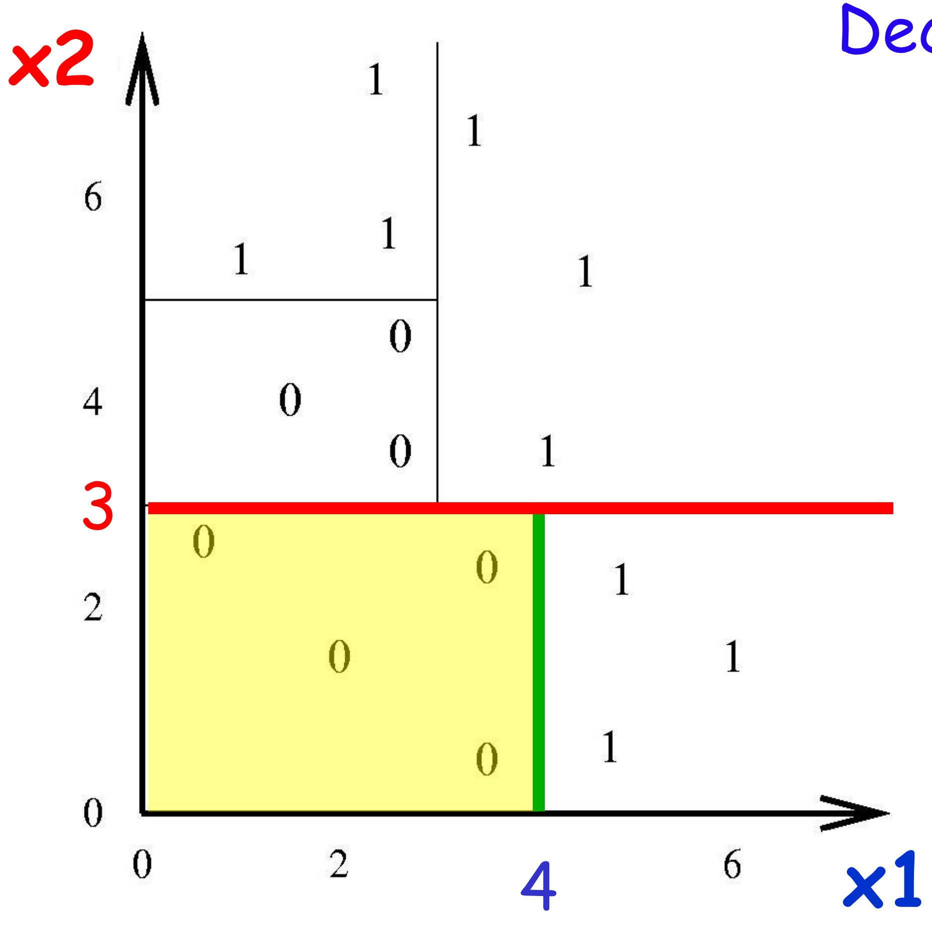
Input real number attributes (x1,x2), Classification output: 0 or 1



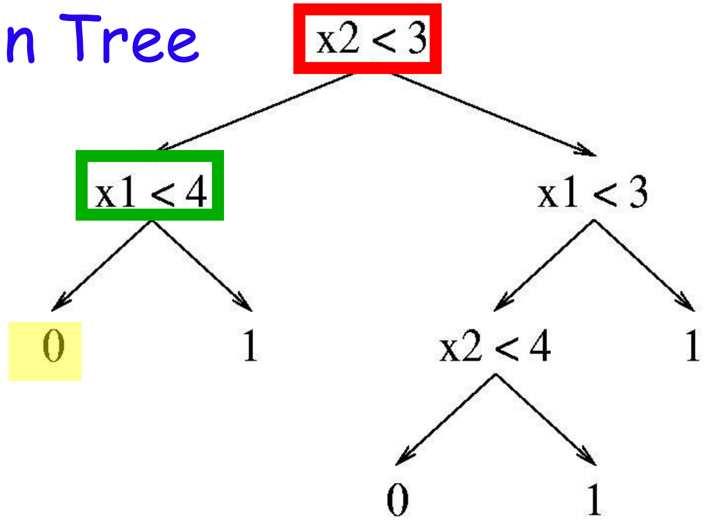
How do we branch using attribute values x1 and x2 to partition the space correctly?

Example: Classification of Continuous Valued Inputs

Decision trees divide the feature space into axis-parallel rectangles, and label each rectangle with one of the K classes.



Decision Tree

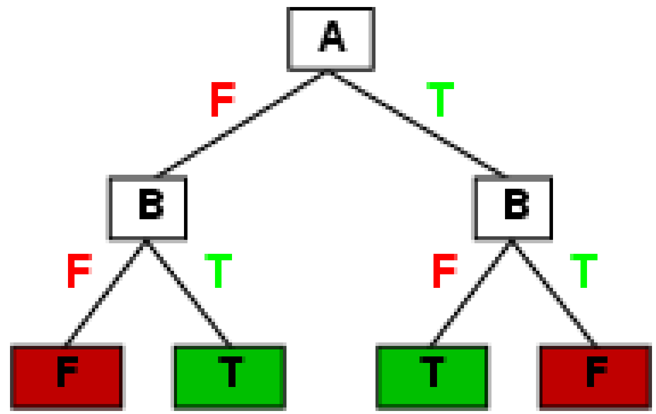


Expressiveness of Decision Trees

Decision trees can express any function of the input attributes.

E.g., for Boolean functions, truth table row = path to leaf:

A	B	A xor B
F	F	F
F	T	T
T	F	T
T	T	F



Trivially, there is a consistent decision tree for any training set with one path to leaf for each example

- But most likely won't generalize to new examples

Prefer to find more compact decision trees

Learning Decision Trees

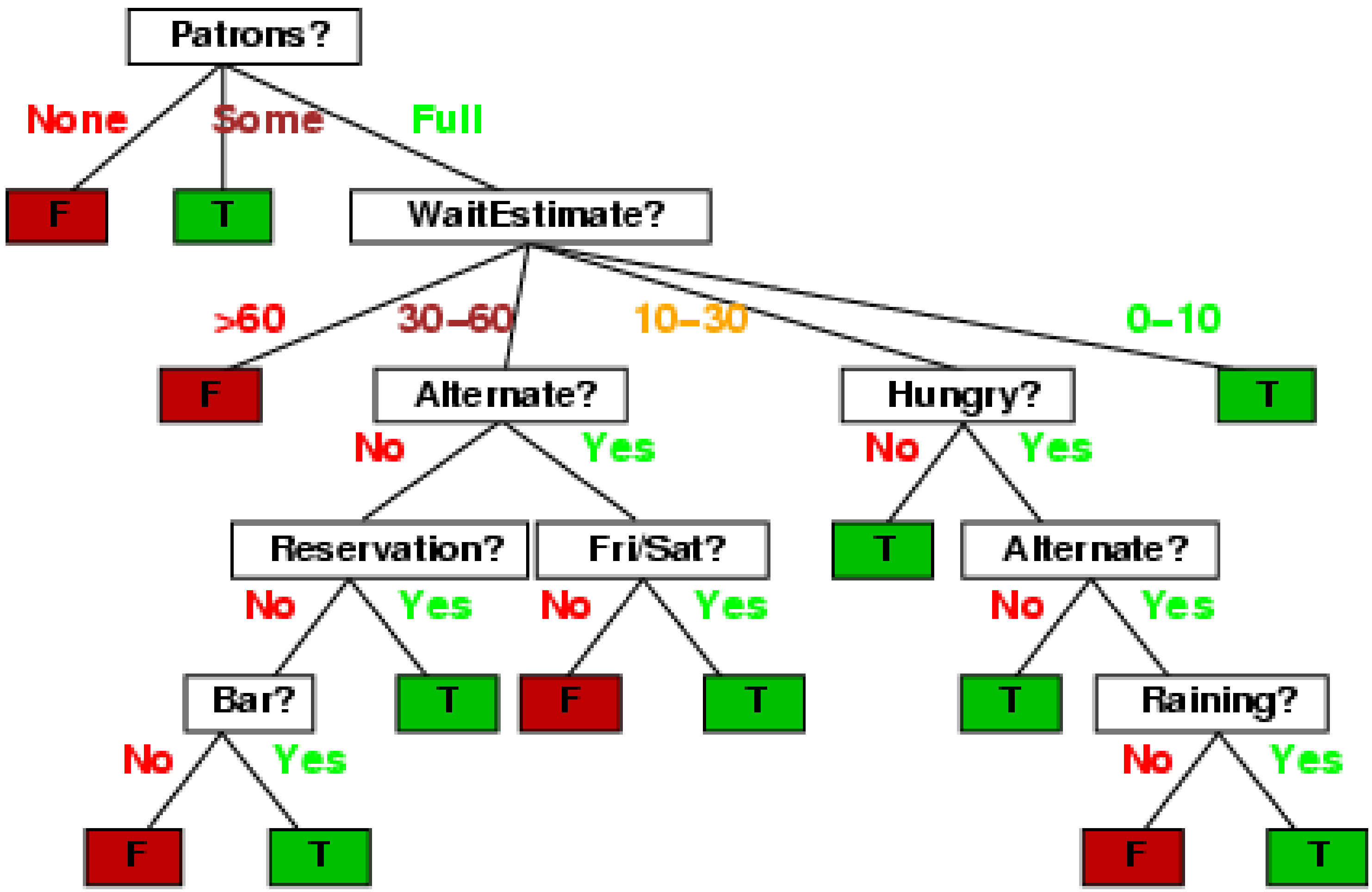
Example: When should I wait for a table at a restaurant?

Attributes (features) relevant to *Wait?* decision:

1. Alternate: is there an alternative restaurant nearby?
2. Bar: is there a comfortable bar area to wait in?
3. Fri/Sat: is today Friday or Saturday?
4. Hungry: are we hungry?
5. Patrons: number of people in the restaurant (None, Some, Full)
6. Price: price range ($, $$, $$$)
7. Raining: is it raining outside?
8. Reservation: have we made a reservation?
9. Type: kind of restaurant (French, Italian, Thai, Burger)
10. WaitEstimate: estimated waiting time (0-10, 10-30, 30-60, >60)

Example Decision tree

A decision tree for *Wait?* based on personal "rules of thumb":



Input Data for Learning

Past examples when I did/did not wait for a table:

Example	Attributes										Target
	Alt	*Bar*	*Fri*	*Hun*	*Pat*	*Price*	*Rain*	*Res*	*Type*	*Est*	*Wait*
X_1	T	F	F	T	Some	$$$	F	T	French	0–10	T
X_2	T	F	F	T	Full	$	F	F	Thai	30–60	F
X_3	F	T	F	F	Some	$	F	F	Burger	0–10	T
X_4	T	F	T	T	Full	$	F	F	Thai	10–30	T
X_5	T	F	T	F	Full	$$$	F	T	French	>60	F
X_6	F	T	F	T	Some	$$	T	T	Italian	0–10	T
X_7	F	T	F	F	None	$	T	F	Burger	0–10	F
X_8	F	F	F	T	Some	$$	T	T	Thai	0–10	T
X_9	F	T	T	F	Full	$	T	F	Burger	>60	F
X_{10}	T	T	T	T	Full	$$$	F	T	Italian	10–30	F
X_{11}	F	F	F	F	None	$	F	F	Thai	0–10	F
X_{12}	T	T	T	T	Full	$	F	F	Burger	30–60	T

Classification of examples is positive (T) or negative (F)

Decision Tree Learning

Aim: find a small tree consistent with training examples

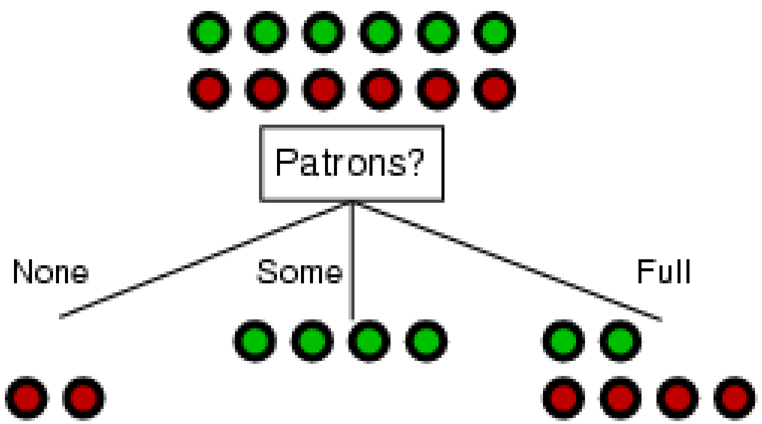
Idea: (recursively) choose "most significant" attribute as root of (sub)tree

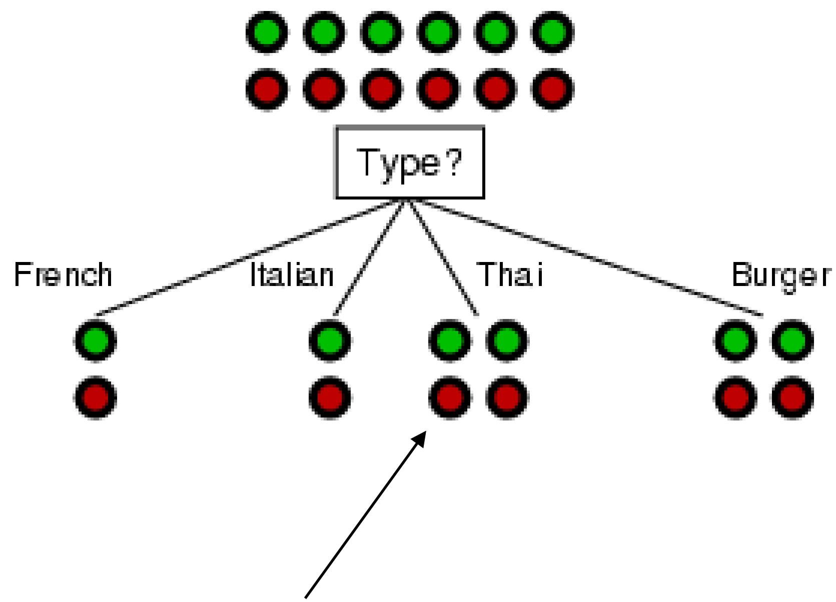
```
function DTL(examples, attributes, default) returns a decision tree
    if examples is empty then return default
    else if all examples have the same classification then return the classification
    else if attributes is empty then return MODE(examples)
    else
        best ← CHOOSE-ATTRIBUTE(attributes, examples)
        tree ← a new decision tree with root test best
        for each value v_i of best do
            examples_i ← {elements of examples with best = v_i}
            subtree ← DTL(examples_i, attributes − best, MODE(examples))
            add a branch to tree with label v_i and subtree subtree
        return tree
```

Choosing an attribute to split on

Idea: a good attribute should reduce uncertainty

- E.g., splits the examples into subsets that are (ideally) "all positive" or "all negative"





Patrons? is a better choice

For *Type?*, to wait or not to wait is still at 50%

How do we quantify uncertainty?

http://a.espncdn.com/media/ten/2006/0306/photo/g_mcenroe_195.jpg

Using information theory to quantify uncertainty

Entropy measures the amount of uncertainty in a probability distribution

Entropy (or Information Content) of an answer to a question with possible answers $v_1, \ldots, v_n$:

$$I(P(v_1), \ldots, P(v_n)) = \Sigma_{i=1} -P(v_i) \log_2 P(v_i)$$

Using information theory

Imagine we have p examples with Wait = True (positive) and n examples with Wait = false (negative).

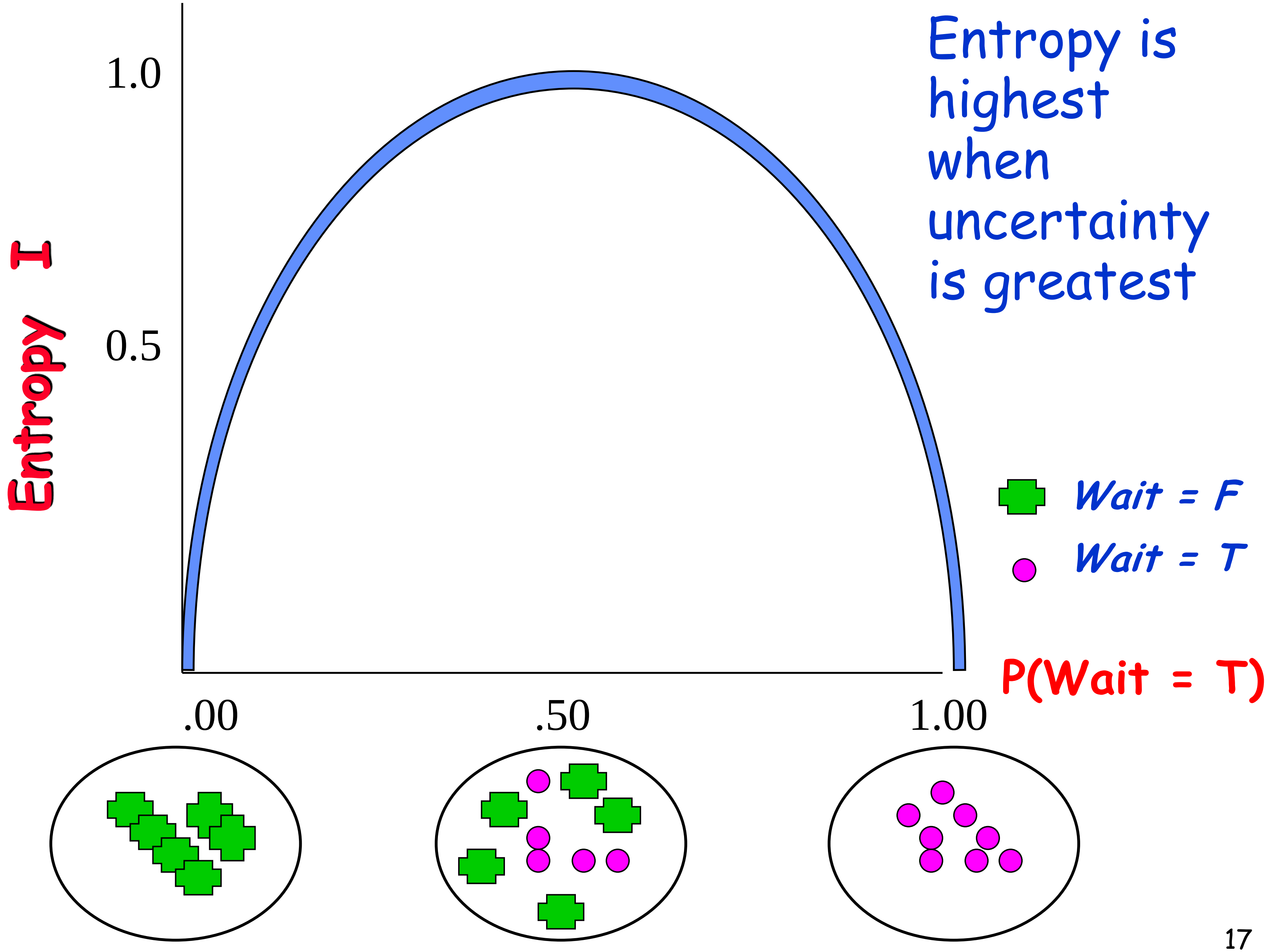
Our best estimate of the probabilities of Wait = true or false is given by:

$$P(true) \approx p / p + n$$

$$p(false) \approx n / p + n$$

Hence the entropy of Wait is given by:

$$I(\frac{p}{p+n}, \frac{n}{p+n}) = -\frac{p}{p+n}\log_2 \frac{p}{p+n} - \frac{n}{p+n}\log_2 \frac{n}{p+n}$$


Entropy I
1.0
0.5
Entropy is highest when uncertainty is greatest
Wait = F
Wait = T
P(Wait = T)
.00
.50
1.00

Choosing an attribute to split on

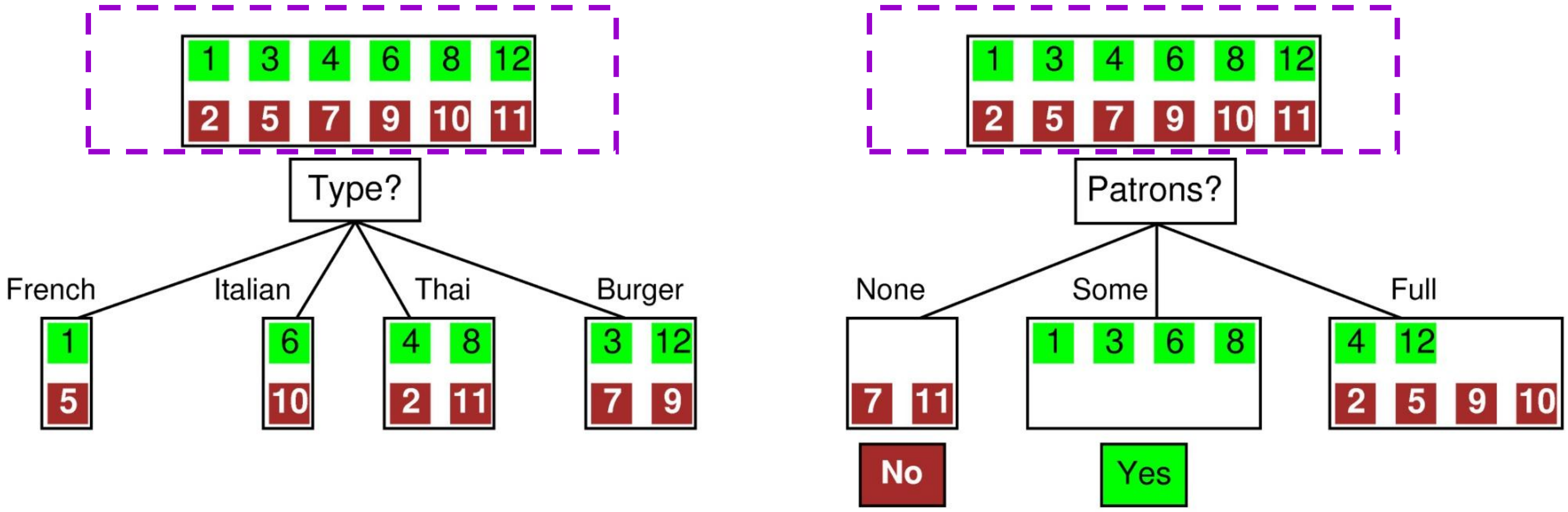
Idea: a good attribute should reduce uncertainty and result in "gain in information"

How much information do we gain if we disclose the value of some attribute?

Answer:

uncertainty before – uncertainty after

Back at the Restaurant



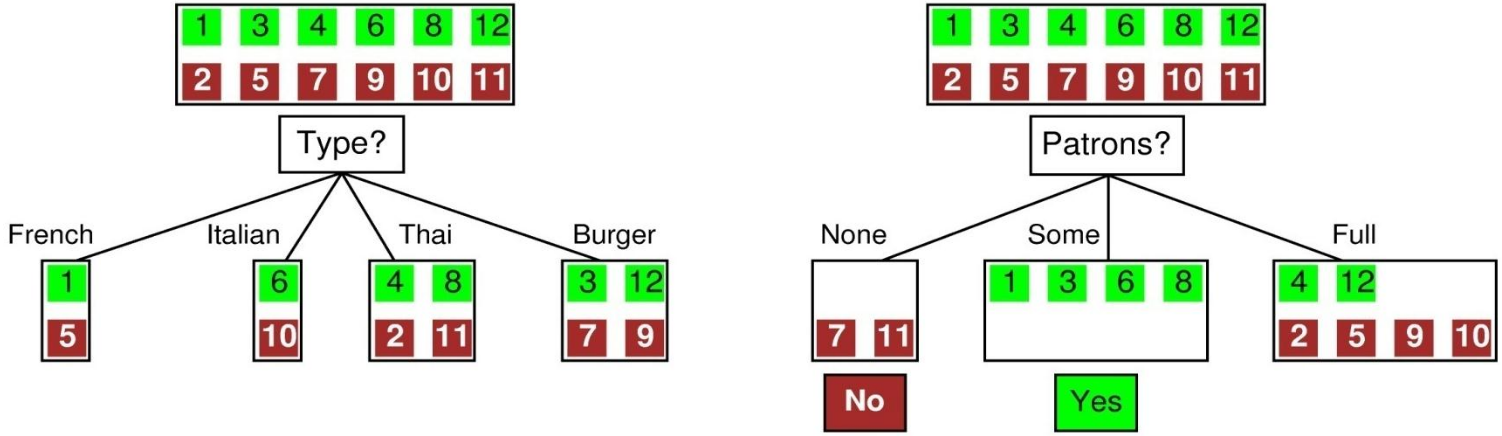
Before choosing an attribute:

Entropy = - 6/12 log(6/12) – 6/12 log(6/12)

= - log(1/2) = log(2) = 1 bit

There is “1 bit of information to be discovered”

Back at the Restaurant



If we choose Type: Go along branch "French": we have entropy = 1 bit; similarly for the others.

Information gain = 1-1 = 0 along any branch

If we choose Patrons:

In branch "None" and "Some", entropy = 0

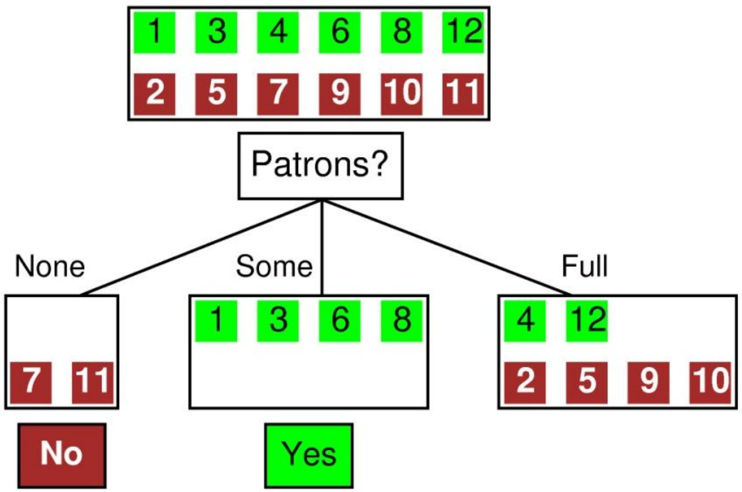
For "Full", entropy = -2/6 log(2/6)-4/6 log(4/6) = 0.92

Info gain = (1-0) or (1-0.92) bits > 0 in both cases

So choosing Patrons gains more information!

Entropy across branches

- How do we combine entropy of different branches?
- Answer: Compute average entropy
- Weight entropies according to probabilities of branches
  - 2/12 times we enter "None", so weight for "None" = 1/6
  - "Some" has weight: 4/12 = 1/3
  - "Full" has weight 6/12 = ½



$$AvgEntropy = \sum_{i=1}^{n} \frac{p_i + n_i}{p + n} Entropy\left(\frac{p_i}{p_i + n_i}, \frac{n_i}{p_i + n_i}\right)$$

weight for each branch

entropy for each branch

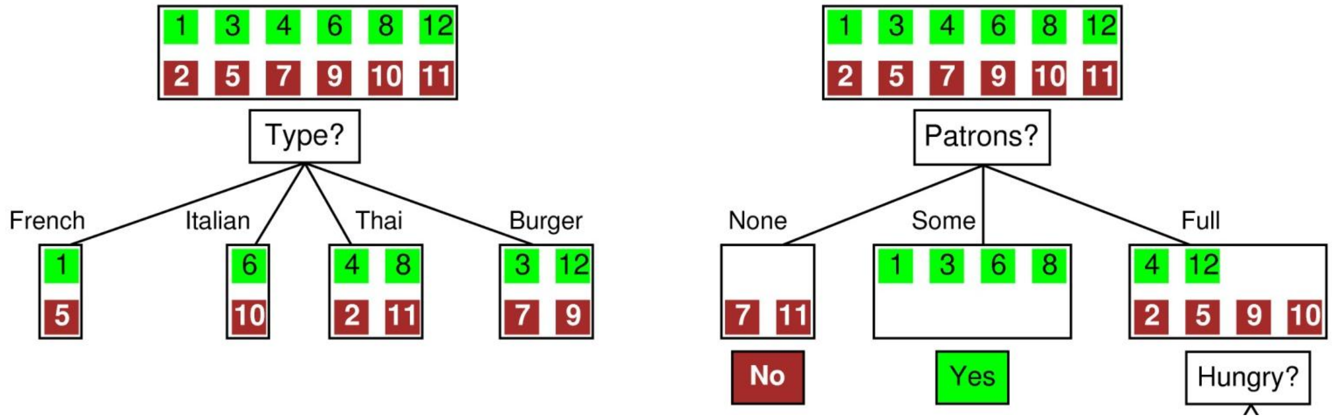
Information gain

Information Gain (IG) or reduction in entropy from using attribute A:

IG(A) = Entropy before – AvgEntropy after choosing A

Choose the attribute with the largest IG

Information gain in our example



$$IG(Patrons) = 1 - [\frac{2}{12} I(0,1) + \frac{4}{12} I(1,0) + \frac{6}{12} I(\frac{2}{6}, \frac{4}{6})] = .541 \text{ bits}$$

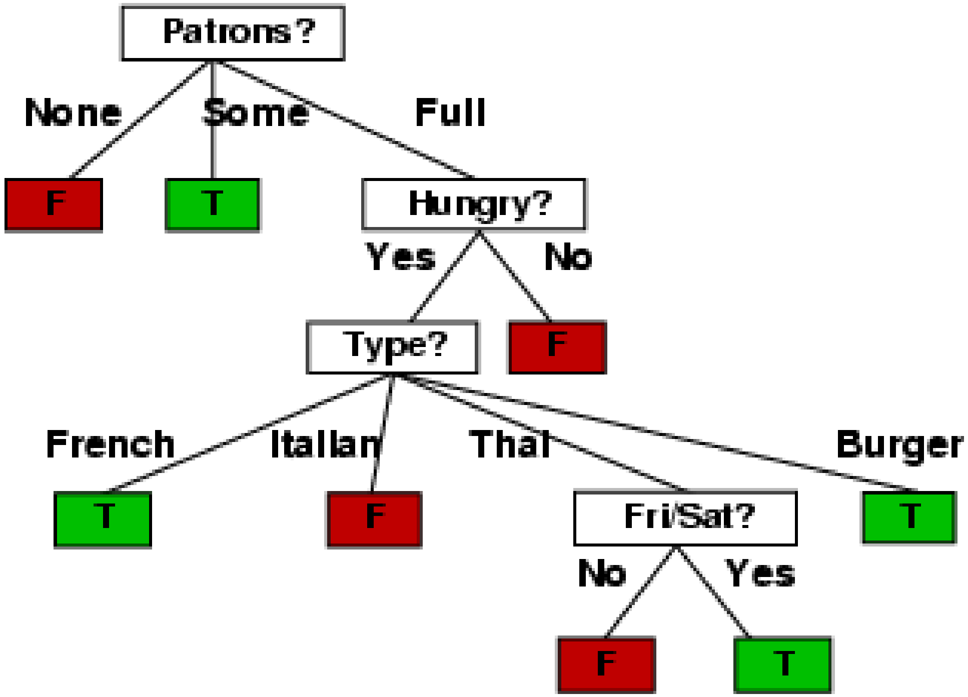
$$IG(Type) = 1 - [\frac{2}{12} I(\frac{1}{2}, \frac{1}{2}) + \frac{2}{12} I(\frac{1}{2}, \frac{1}{2}) + \frac{4}{12} I(\frac{2}{4}, \frac{2}{4}) + \frac{4}{12} I(\frac{2}{4}, \frac{2}{4})] = 0 \text{ bits}$$

Patrons has the highest IG of all attributes

⇒ DTL algorithm chooses *Patrons* as the root

Should I stay or should I go? Learned Decision Tree

Decision tree learned from the 12 examples:



Substantially simpler than “rules-of-thumb” tree

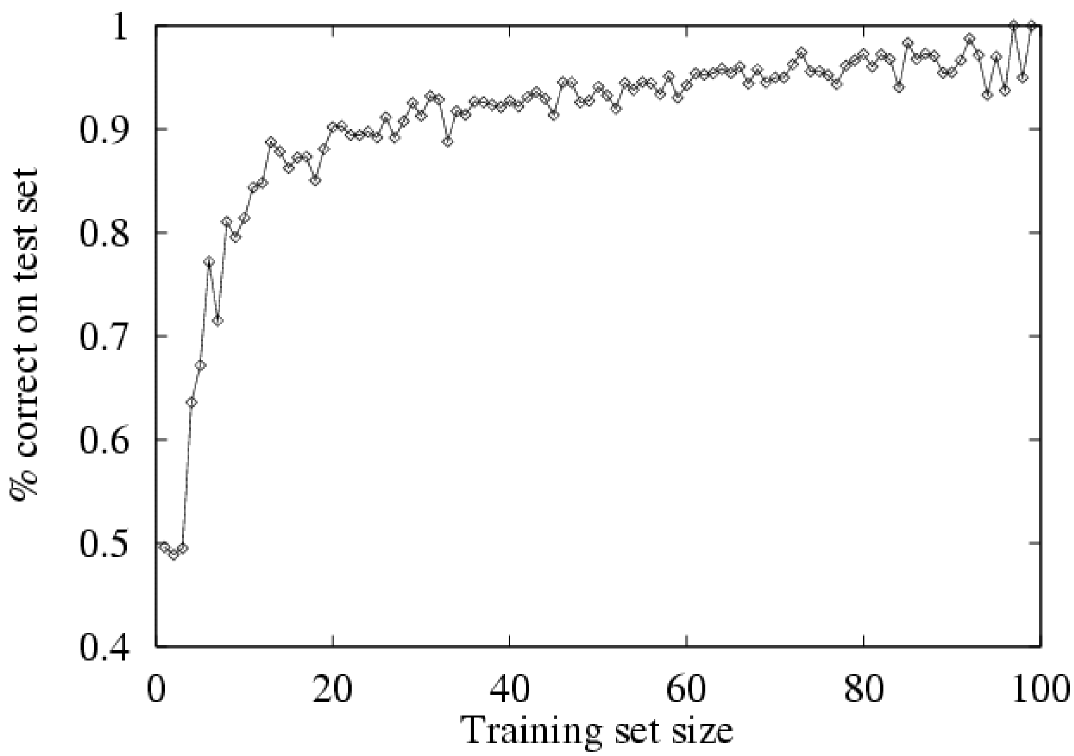
- more complex hypothesis not justified by small amount of data

Performance Evaluation

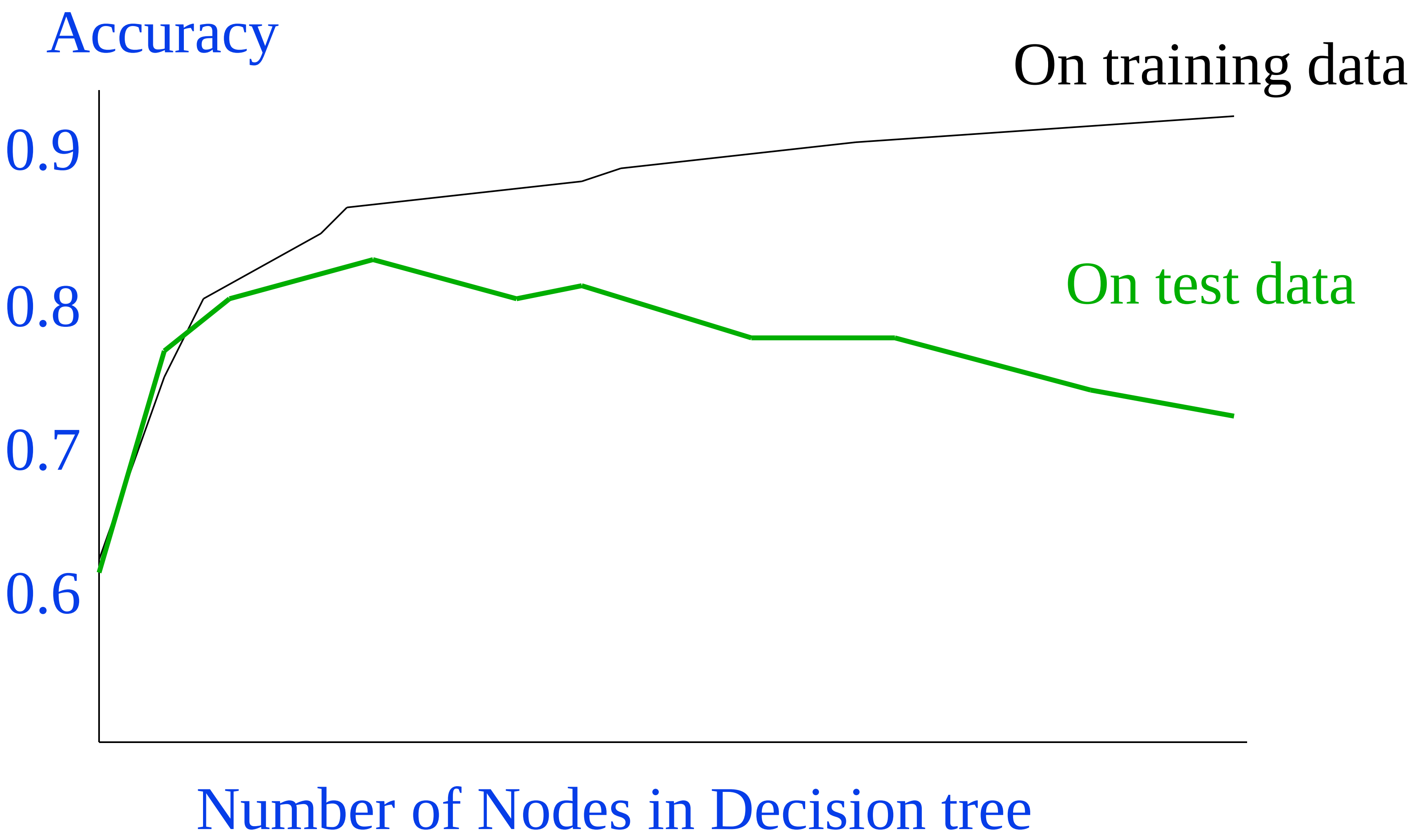
How do we know that the learned tree $h \approx f$?

Answer: Try h on a new test set of examples

Learning curve = % correct on test set as a function of training set size



Overfitting



Overfitting

Consider error of hypothesis h over

- training data: $error_{train}(h)$
- entire distribution $\mathcal{D}$ of data: $error_{\mathcal{D}}(h)$

Hypothesis $h \in H$ **overfits** training data if there is an alternative hypothesis $h' \in H$ such that

$$error_{train}(h) < error_{train}(h')$$

and

$$error_{\mathcal{D}}(h) > error_{\mathcal{D}}(h')$$

Rule #2 of Machine Learning

The *best* hypothesis almost never achieves 100% accuracy on the training data.

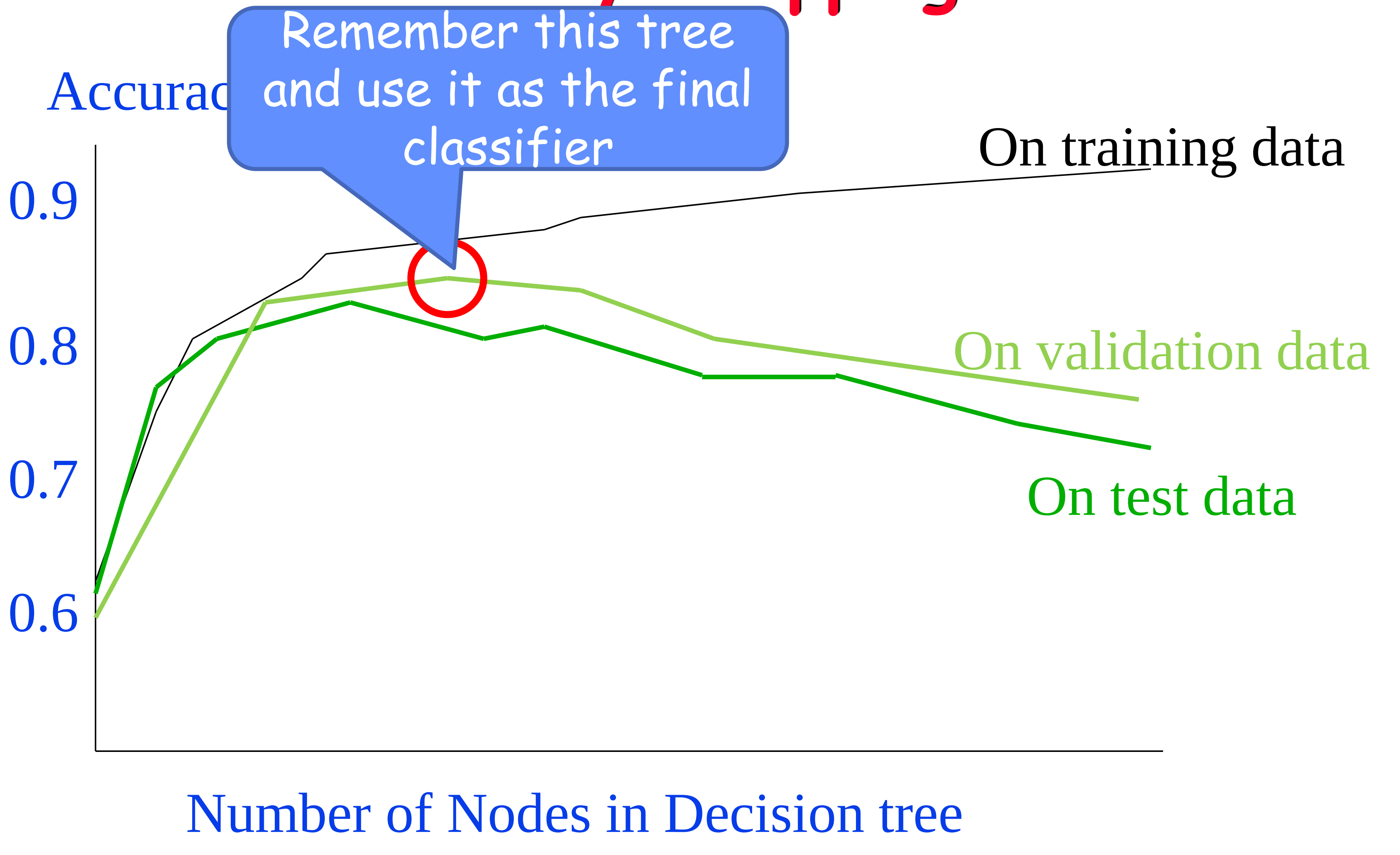
(Rule #1 was: you can't learn anything without inductive bias)

Avoiding overfitting

- Stop growing when data split not statistically significant
- Grow full tree and then prune

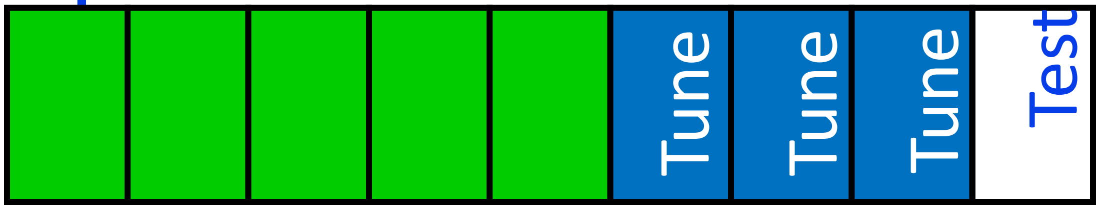
- How to select best tree?
  - Measure performance over the training data
  - Measure performance over separate validation set
  - Add complexity penalty to performance measure

Early Stopping



Reduced Error Pruning

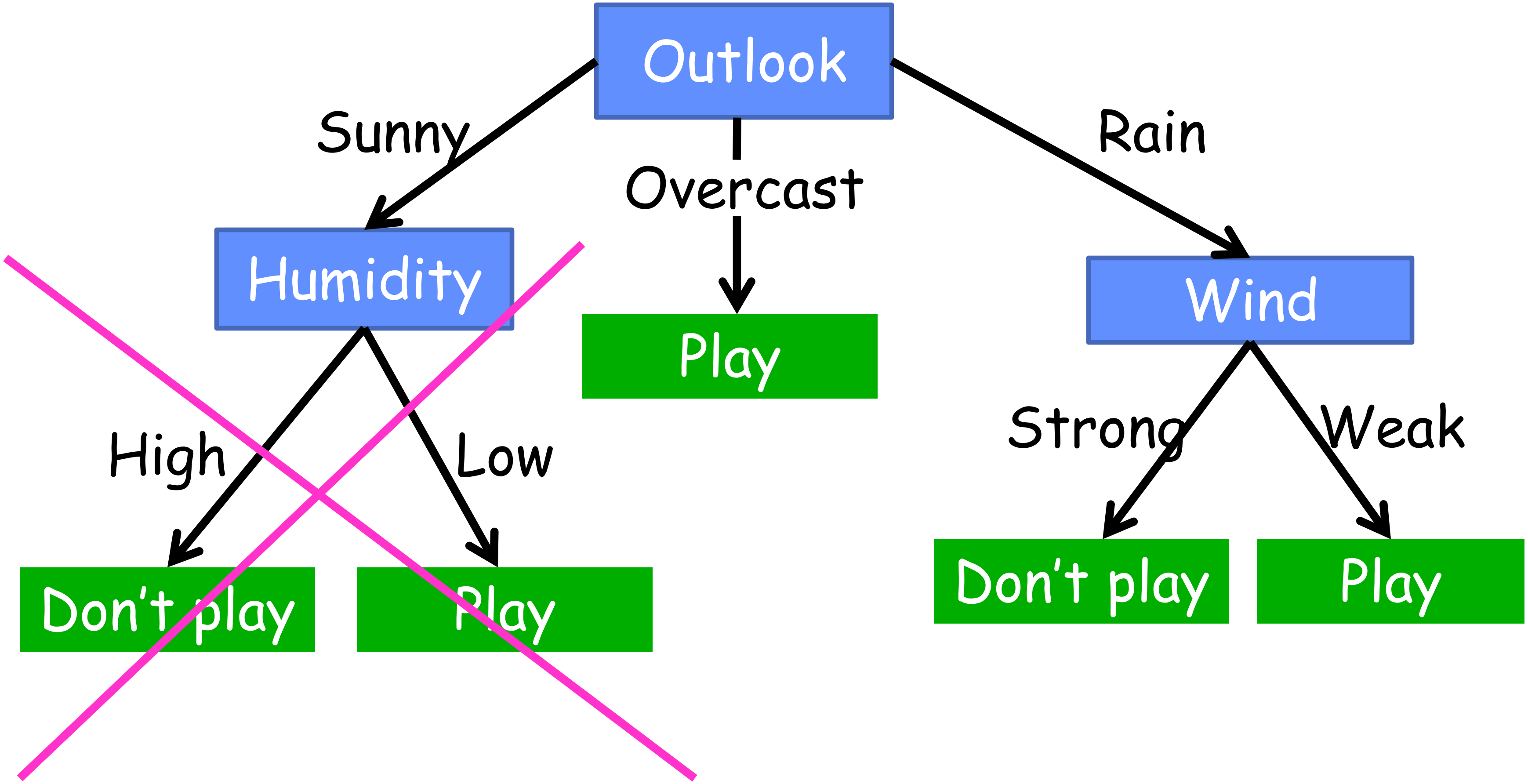
Split data into train and validation set



Repeat until pruning is harmful

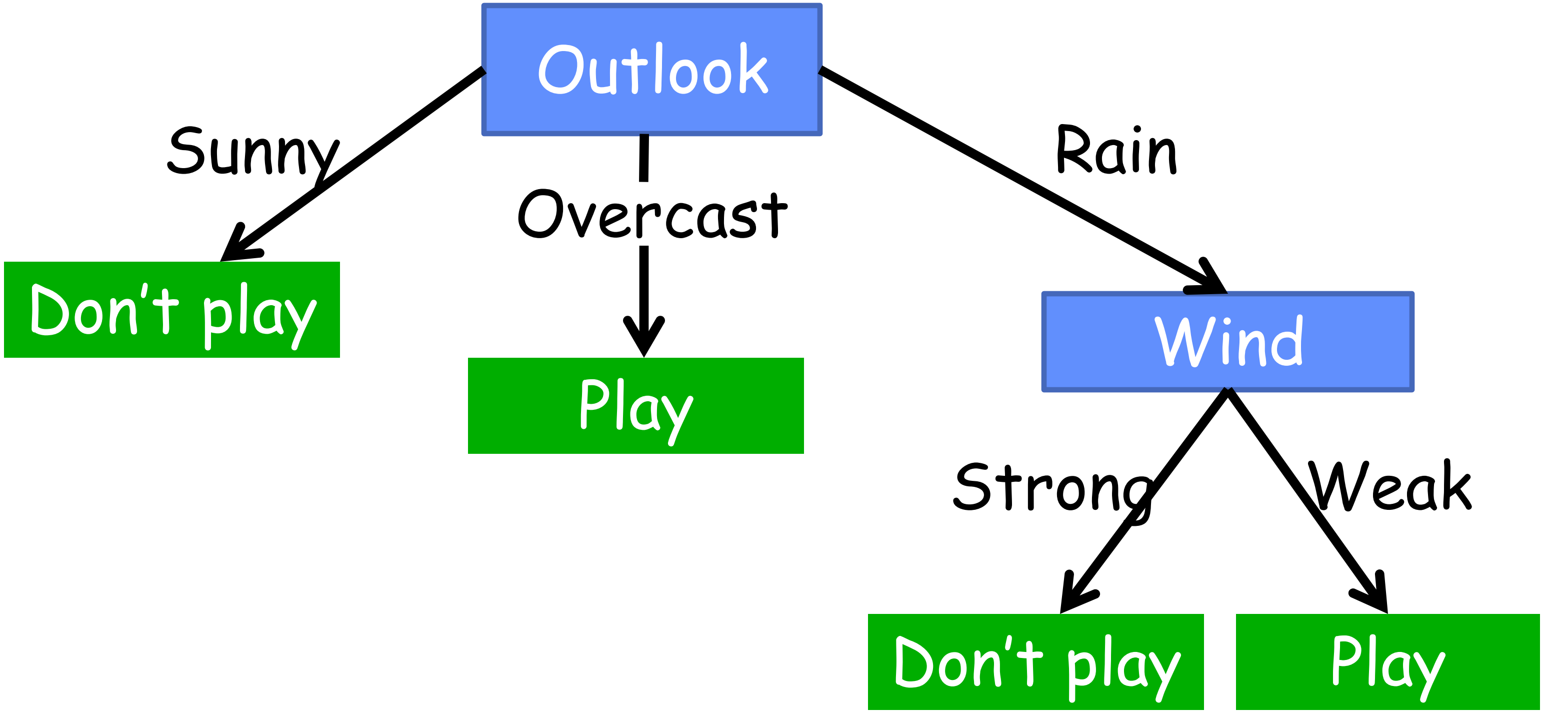
- Remove each subtree and replace it with majority class and evaluate on validation set
- Remove subtree that leads to largest gain in accuracy

Reduced Error Pruning Example



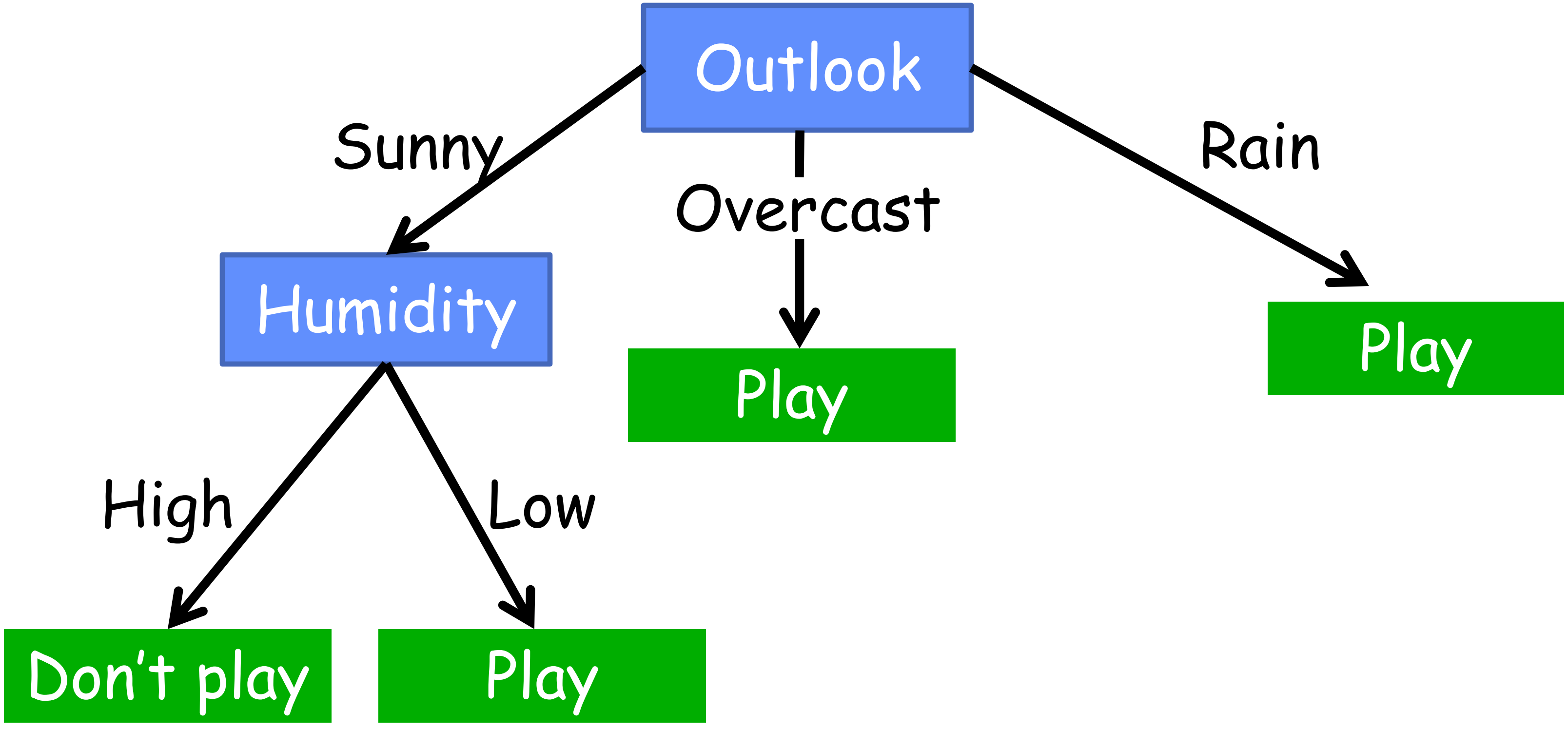
Validation set accuracy = 0.75

Reduced Error Pruning Example



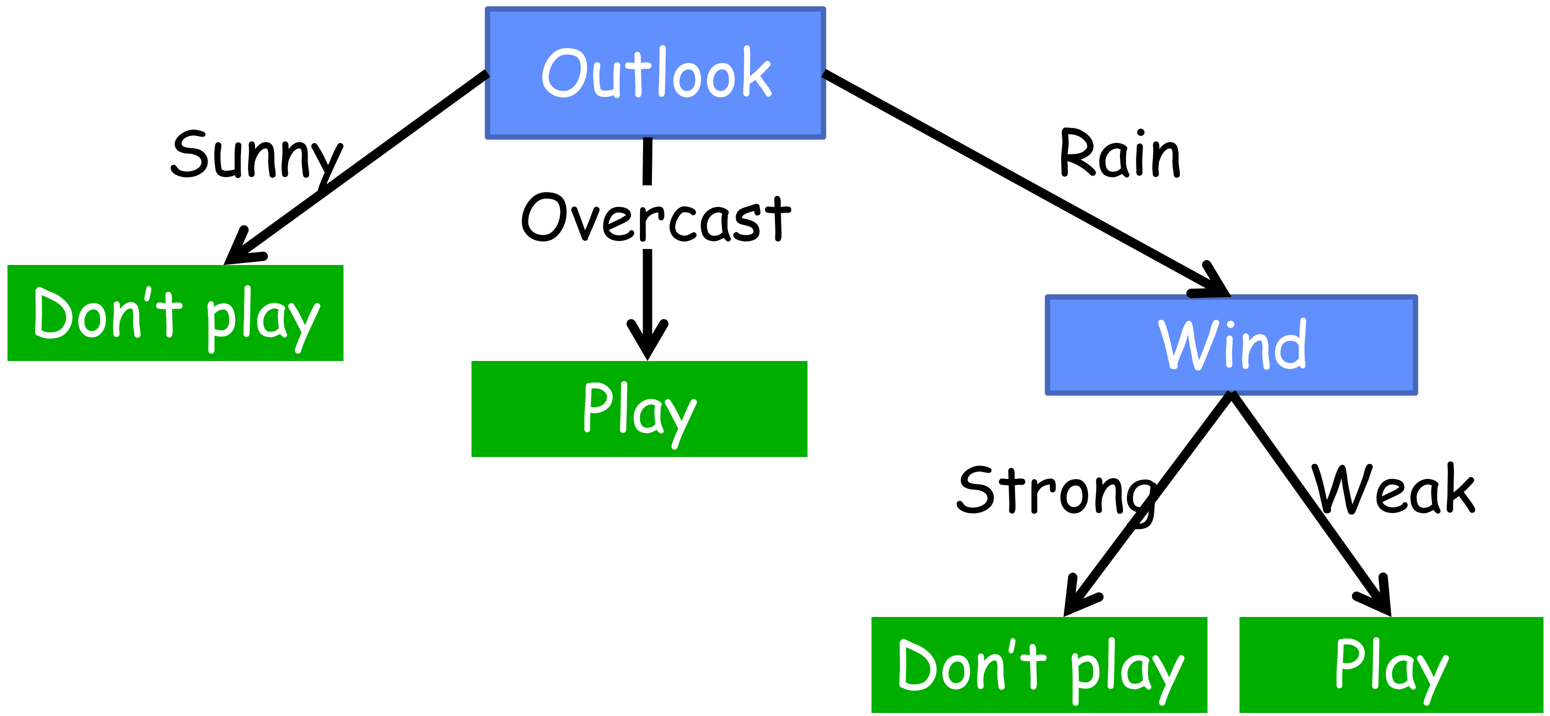
Validation set accuracy = 0.80

Reduced Error Pruning Example



Validation set accuracy = 0.70

Reduced Error Pruning Example

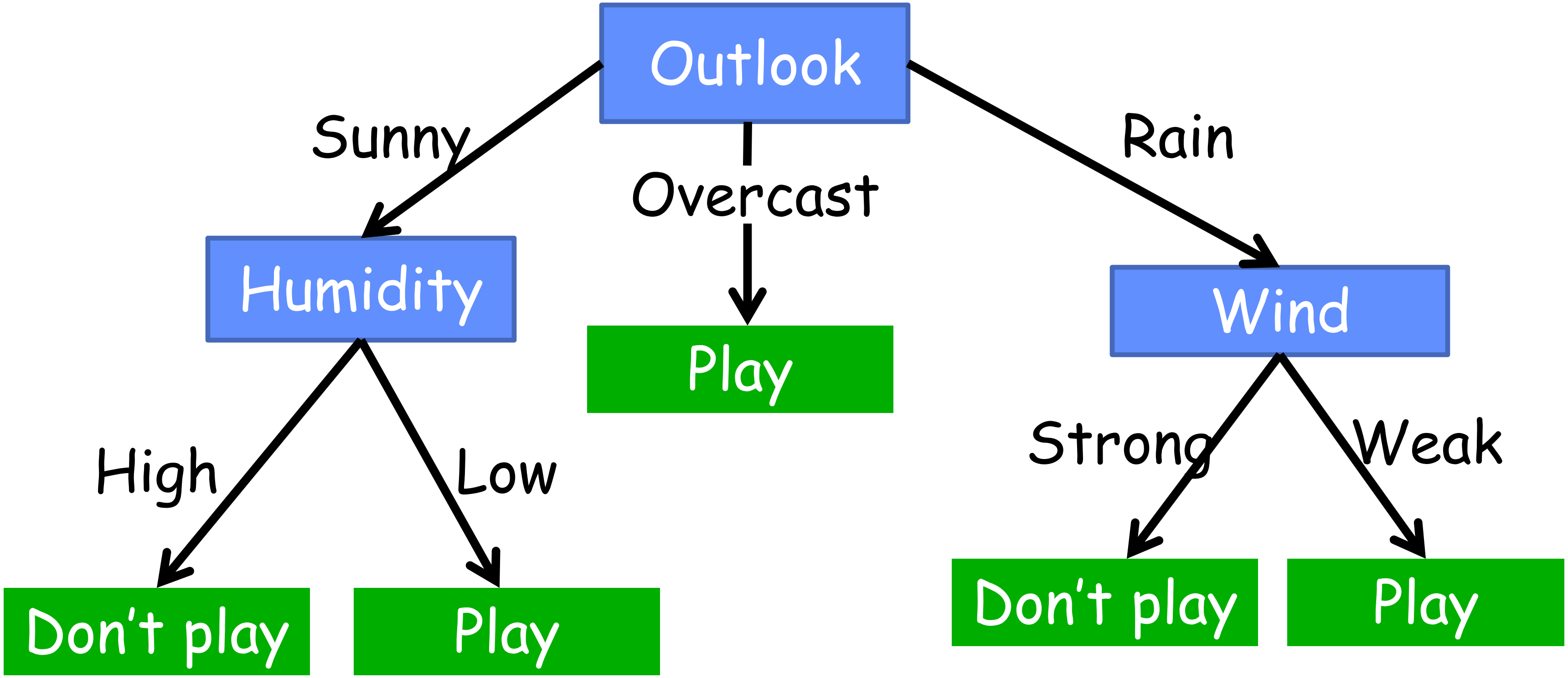


Use this as final tree

Post Rule Pruning

- Split data into train and validation set
- Prune each rule independently
  - Remove each pre-condition and evaluate accuracy
  - Pick pre-condition that leads to largest improvement in accuracy
- Note: ways to do this using training data and statistical tests

Conversion to Rule



Outlook = Sunny $\wedge$ Humidity = High $\Rightarrow$ Don't play

Outlook = Sunny $\wedge$ Humidity = Low $\Rightarrow$ Play

Outlook = Overcast $\Rightarrow$ Play

Scaling Up

- ID3 and C4.5 assume data fits in main memory

  (ok for 100,000s examples)

- SPRINT, SLIQ: multiple sequential scans of data

  (ok for millions of examples)

- VFDT: at most one sequential scan

  (ok for billions of examples)

Decision Trees - Strengths

Very Popular Technique

Fast

Useful when

- Target Function is discrete
- Concepts are likely to be disjunctions
- Attributes may be noisy

Decision Trees – Weaknesses

Less useful for continuous outputs

Can have difficulty with continuous input features as well…

- E.g., what if your target concept is a circle in the x1, x2 plane?
  - Hard to represent with decision trees…
  - Very simple with instance-based methods we'll discuss later…