Please work on this problem set individually. (Subsequent problem sets may allow group discussion). If any problem doesn’t contain enough information for you to answer it, feel free to make any assumptions necessary to find the answer, but state the assumptions clearly. You will be graded on your choice of assumptions as well as the clarity of your written answers.

1. Let T be an acyclic, connected, undirected graph (i.e., a tree) whose edges are uniformly of unit length. The **diameter** of T is the maximum distance between any two nodes in T.
   a) How might you use breadth-first search (BFS) to find the diameter of T? Do you need to run BFS from different initial, starting nodes?
   b) What is the time complexity of your algorithm (in \(n\), the number of nodes in T and \(d\), the diameter and any other variables you deem relevant).
   c) Can you devise an algorithm which is linear in \(n\) (or prove that such an algorithm is impossible)?

2. The figure below shows a problem-space graph, where A is the initial state and G denotes the goal. Edges are labeled with their true cost. We have a heuristic function, \(f()\), written in the standard form: \(f(n) = g(n) + h(n)\) where \(g(n)\) is the cost to get from A to n and \(h(n)\) is an estimate of the remaining distance to G.
   a) In the graph below, is \(f()\) admissible? Why or why not?
   b) Is \(f()\) monotonic? Why or why not?
   c) Suppose we use IDA* to search the graph and that states having the same \(f\) values are visited in alphabetical order. In what order does the algorithm consider \(f\)-limits and visit states? Fill out the table below:
3. Prove that if a heuristic is monotonic, then it must be admissible (or provide a counter example).

4. Prove that if a heuristic is admissible, then it must be monotonic (or provide a counter example).

5. Recall that with the n-queens problem, we seek to put n queens on an n x n chess board.
   
a) Let n=4 can consider the state space of possible board positions. Suppose that the initial state has no queens on it and a goal state has 4 peaceful queens (i.e. none are attacking each other). Operators add one queen to the board. The rules say that one can never put more than four queens on the board. What is the total size of the complete state space (i.e., including states where some queens are attacking each other)?

   b) By observing the fact that one can never put two queens on the same column (without violating peacefulness) one can come up with a simpler representation of state (e.g. encoding one which takes less memory if encoded as a datastructure). Describe such a representation. How many 4-queens states are possible in this representation (again, some states may not be peaceful)?

   c) Can you shrink the state space even more by eliminating symmetry? Explain?

6. Do problem 4.14 from R&N. And be sure that your answer for part (a) describes the offline problem they are presenting formally as a search problem. (What is a belief state in this case?)