UNIVERSITY of WASHINGTON

Introduction to Reinforcement Learning

Jiafei Duan, 24 February 2026

CSE 571

Used Materials

- Acknowledgement: Some of the material and slides for the lecture were borrowed from Deep RL Bootcamp at UC Berkeley, 10-703 course at CMU by Katerina Fragkiadaki and Ruslan Salakhutdinov, whom originated from Rich Sutton and David Silver class and from Paul Laing CMU Lecture 9.1 and 9.2. Some advance material is also borrowed from Stanford CS234 and CIS 522 UPenn. Some animation are from MUTALINFORMATION YouTube.

Things to cover today:

- Introduction and motivation to RL
- Markov Decision Processes (MDPs)
- Solving known MDPs using value and policy iteration
- Temporal Difference & Q-Learning
- Tabular Q-Learning & Deep Q-Learning
- Policy Gradient Methods
- PPO

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What is Reinforcement Learning?

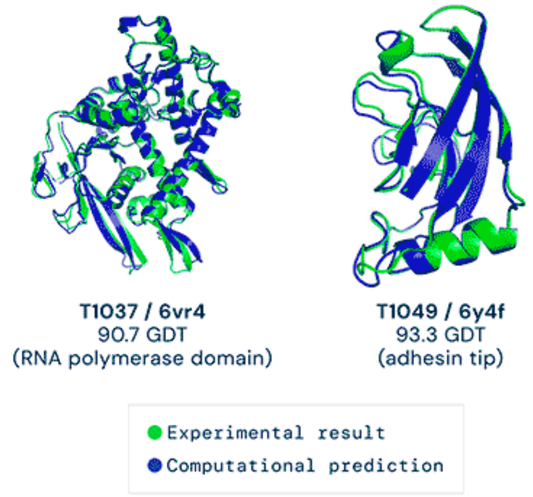
- Learning through experience/data to make good decisions under uncertainty.
- Essential part of intelligence.
- Builds strongly from theory and ideas starting in the 1950s with Richard Bellman.
- A number of impressive successes in the last decade.

Thorndike’s Puzzle Box (Law of effect, 1898)

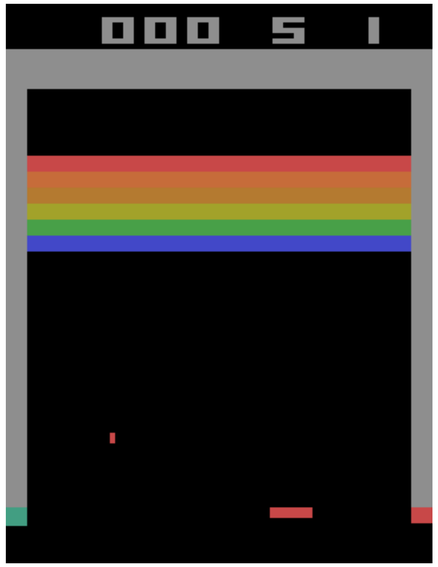
Motivation for RL

Self-driving

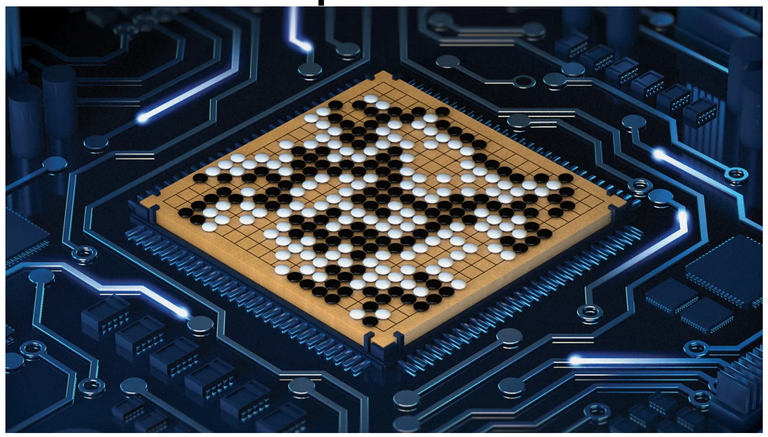
AlphaFold



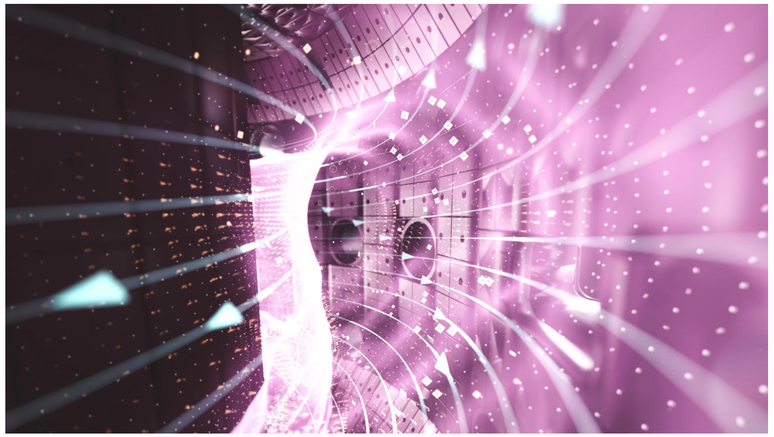
Atari



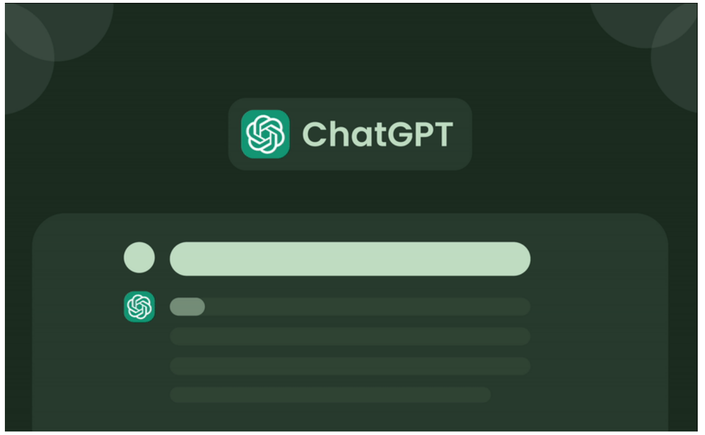
AlphaGo



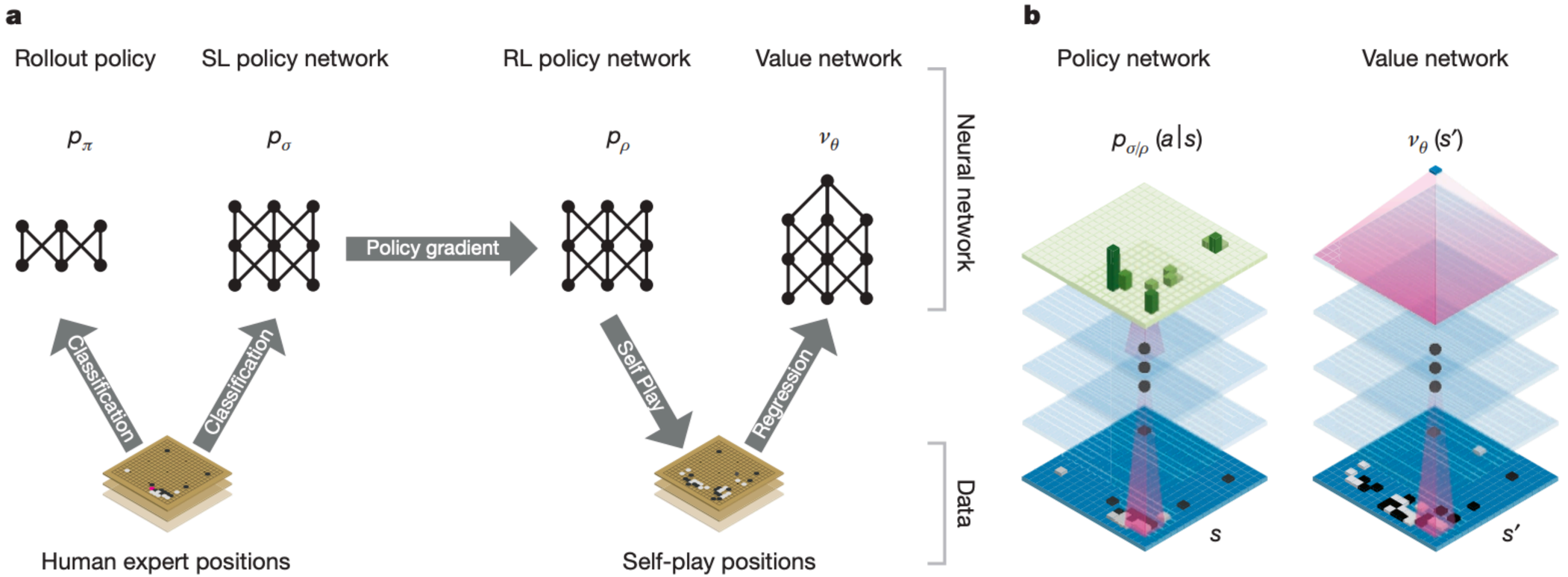
Plasma control



Chatbots



Beyond human performance on Go



[Mastering the game of Go with deep neural networks and tree search, Silver et al. Nature 2017]

ChatGPT with RL

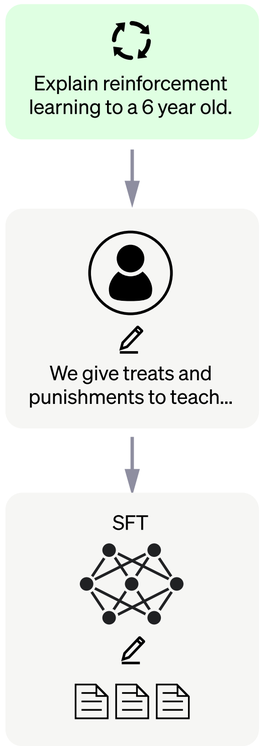
Step 1

Collect demonstration data and train a supervised policy.

A prompt is sampled from our prompt dataset.

A labeler demonstrates the desired output behavior.

This data is used to fine-tune GPT-3.5 with supervised learning.



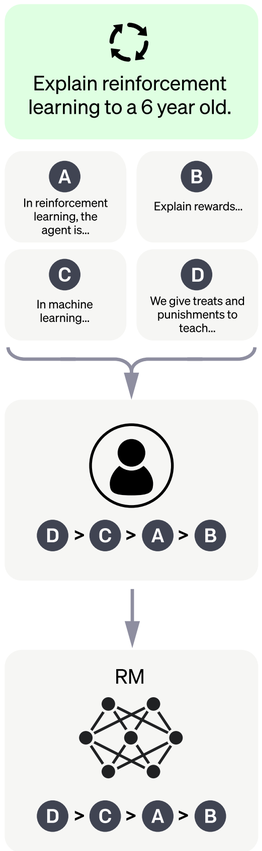
Step 2

Collect comparison data and train a reward model.

A prompt and several model outputs are sampled.

A labeler ranks the outputs from best to worst.

This data is used to train our reward model.



Step 3

Optimize a policy against the reward model using the PPO reinforcement learning algorithm.

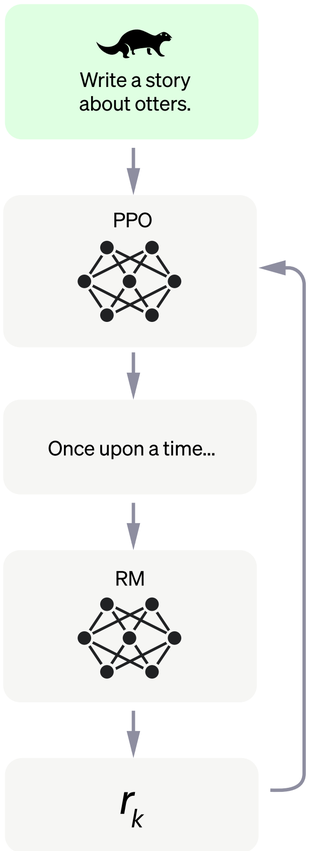
A new prompt is sampled from the dataset.

The PPO model is initialized from the supervised policy.

The policy generates an output.

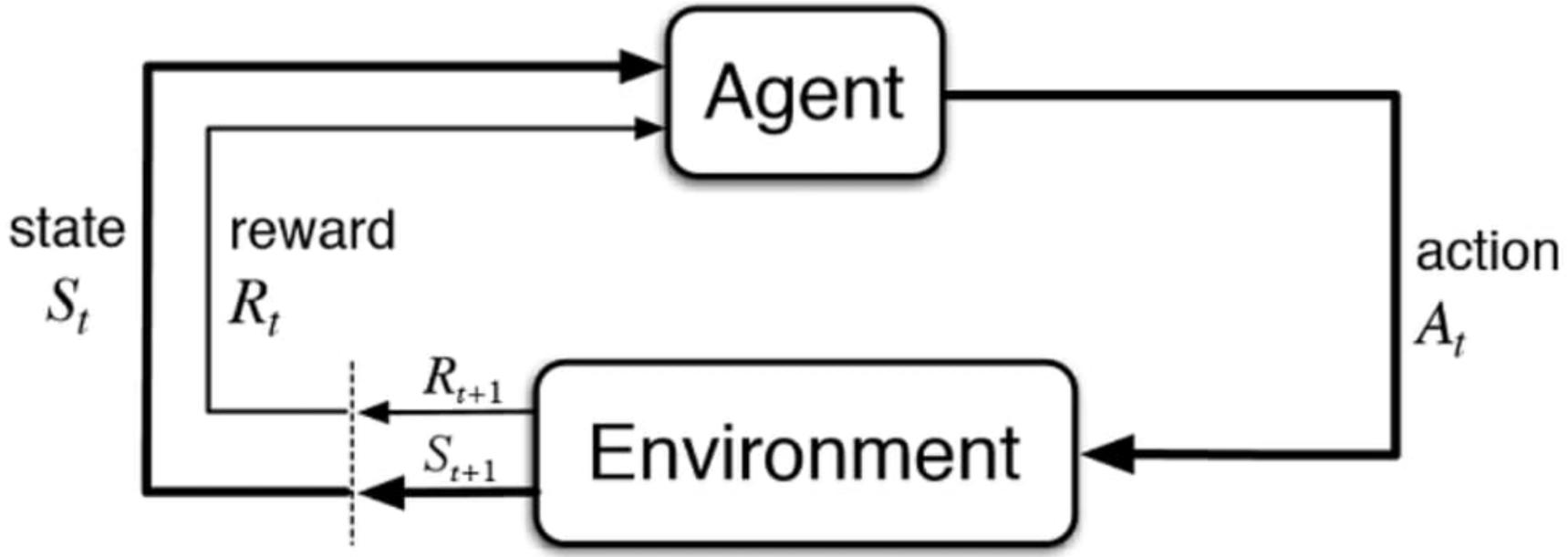
The reward model calculates a reward for the output.

The reward is used to update the policy using PPO.



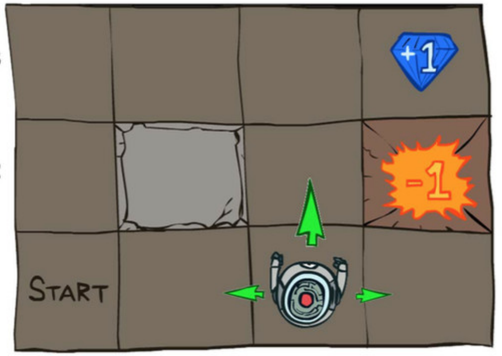
[ChatGPT blogpost]

Reinforcement Learning



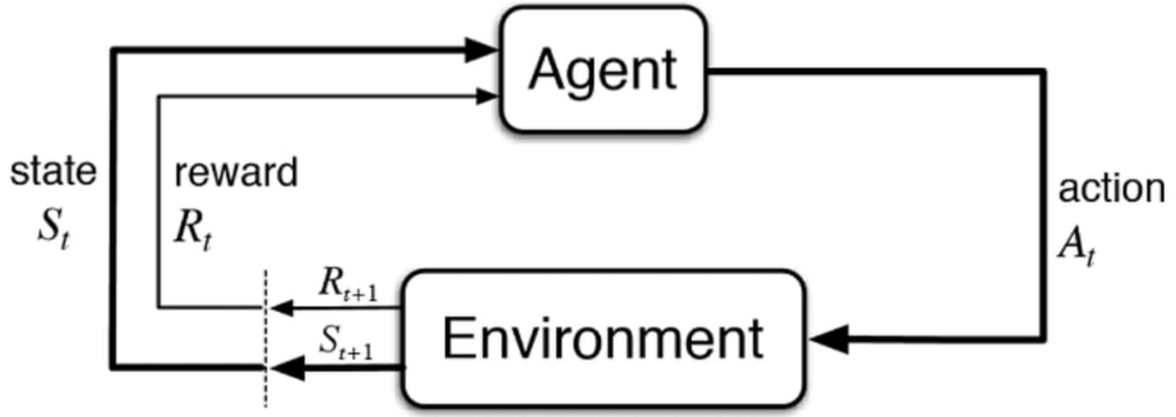
$$s_0, a_0, r_0, s_1, a_1, r_1, s_2, a_2, r_2 \ldots$$

Markov Decision Process (MDPs)



An MDP consist of 7 terms:

- Set of states, S
- Set of actions, A
- Transition function P(s' | s, a)
- Reward function R(s, a, s')
- Start state s_0
- Discount factor γ
- Horizon H

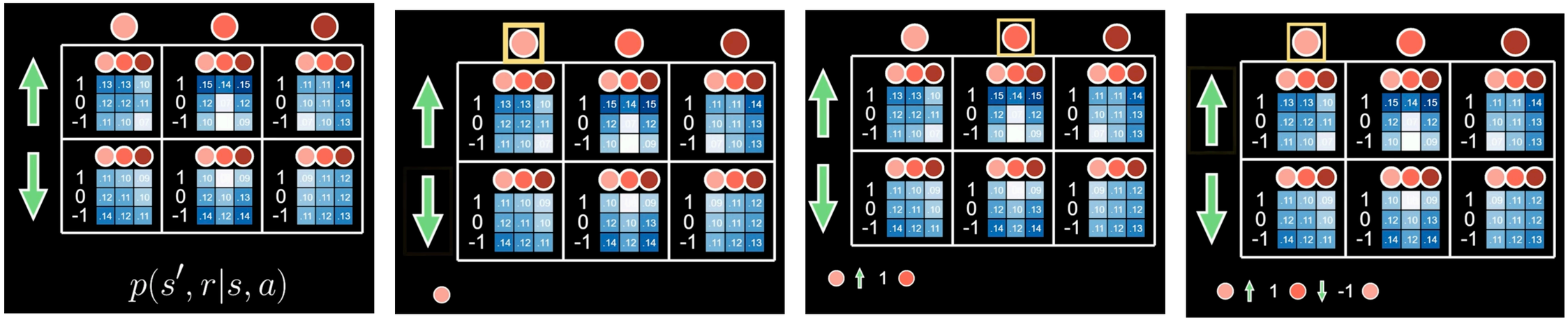


$$s_0, a_0, r_0, s_1, a_1, r_1, s_2, a_2, r_2 \ldots$$

Andrey Andreyevich Markov (1856-1922)

Russian mathematician known for his work on stochastic processes.

Example MDP



Policy: $\pi(a \mid s) = \frac{1}{2}$

Starting Distribution: $\pi(a \mid s) = p_o(s) = \frac{1}{3}$

Markov assumption + fully observable

A state should summarize all past information and have the Markov property.

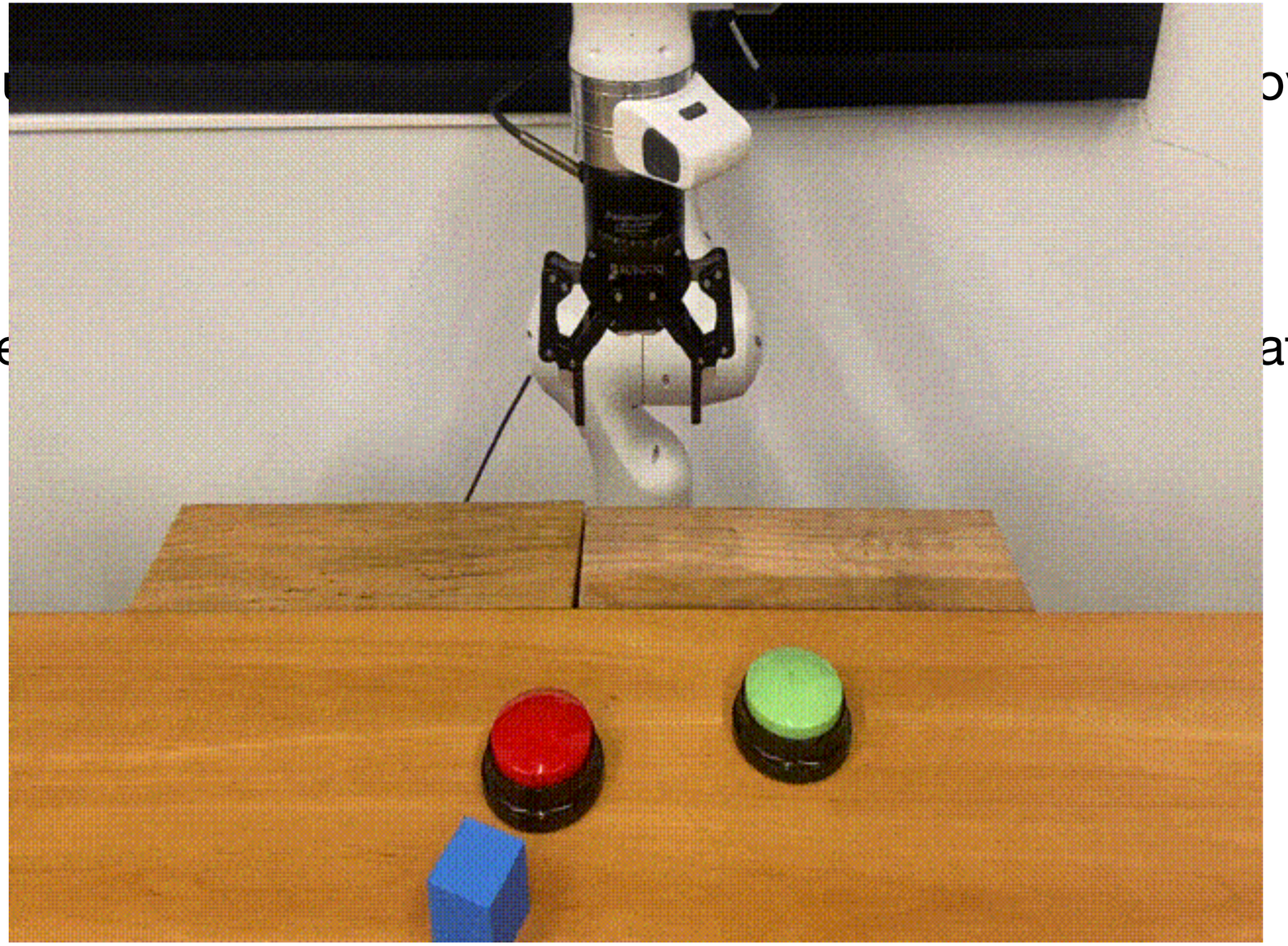
$$P(S_{t+1} \mid S_t, S_{t-1}, S_{t-2}, \dots) = P(S_{t+1} \mid S_t)$$

This means the current state S_t contains all necessary information to predict the next state. You don't need the full history.

Markov assumption + fully observable

A state shou ov property.

This me ation to



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Suppose your state is s_t = current RGB image

Now imagine a pedestrian stepping into the road.

Is this Markov?

What is the goal? Return or Utility.

We aim to maximize the total discounted reward:

$$G_t = R_{t+1} + \gamma R_{t+2} + \cdots = \sum_{k=0}^{\infty} \gamma^k R_{t+k+1} \qquad \gamma : [0 - 1]$$

Discount factor

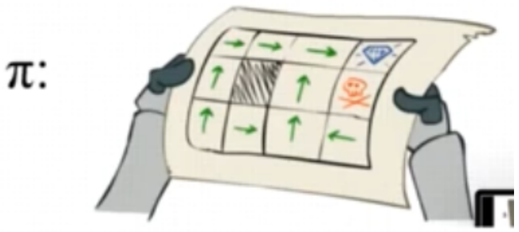
γ close to 0 leads to “myopic” evaluation.

γ close to 1 leads to “far-sighted” evaluation.

What is the solution? A Policy

A policy is a distribution over actions given states

$$\pi(a \mid s) = P(A_t = a \mid S_t = s)$$



- A policy fully defines the behavior of an agent
- The policy is stationary (time-independent)
- During learning, the agent changes its policy as a result of experience.

A deterministic policy is used when agent always chooses exactly one action for each state instead of sampling from a distribution.

$$a = \pi_\theta(s)$$

Sequential Decision Making-Problem examples

Action: Muscle contractions

Observation: sight, smell

Rewards: food

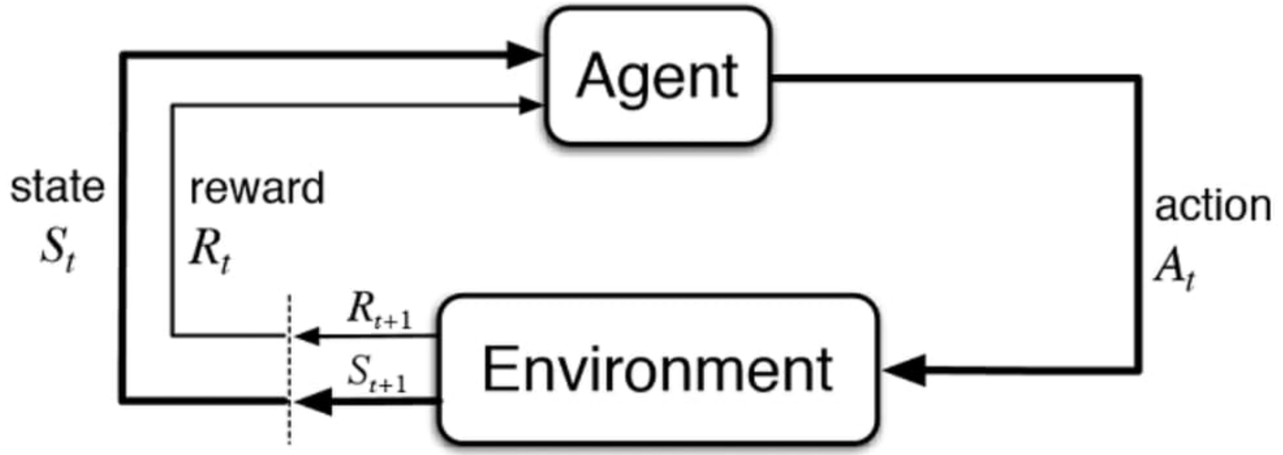
Actions: Motor current or torque

Observations: Sensory feedback

Rewards: Task success metrics

Learning the optimal policy to maximize return

MDP



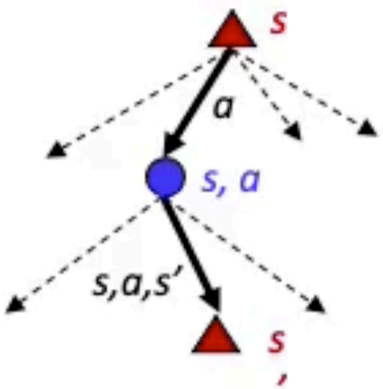
Return

Goal

How to solve MDPs: State Value Functions of Policies.

When given a MDP (S, A, P, R, γ):

Value of a state s under policy π:



$V^{\pi}(s) =$ Expected utility starting in s and acting according to π

$$V^{\pi}(s) = \mathbb{E}\left(\sum_{t=0}^{\infty} \gamma^t r_{t+1} \mid S_0 = s\right)$$ Sequence of rewards generated by following π

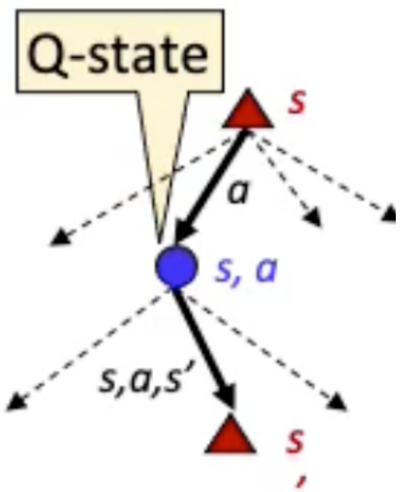
$V^{*}(s) =$ Expected utility starting in s and acting optimally

$$V^{\pi^*}(s) = \mathbb{E}\left(\sum_{t=0}^{\infty} \gamma^t r_{t+1} \mid S_0 = s\right)$$ Rewards generated by following π^*

Slide adapted from CIS 522

How to solve MDPs: Action Value Function of Policies

It is also helpful to define action-value functions



Q-value of taking action a in state s then following policy π

$Q^{\pi}(s, a) =$ Expected utility taking a in s and then following π

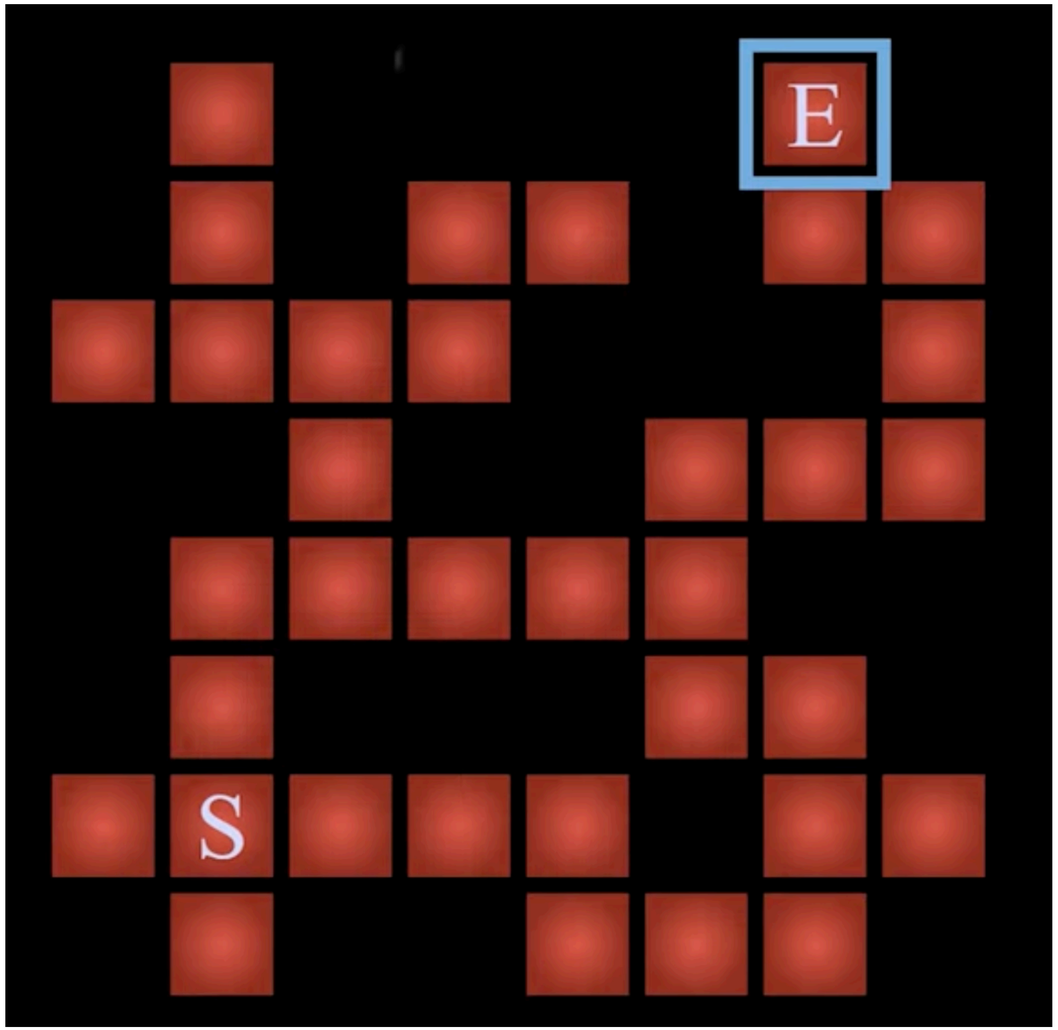
$$Q^{\pi}(s, a) = \mathbb{E}\left(\sum_{t=0}^{\infty} \gamma^t r_{t+1} \mid S_0 = s, A_0 = a \right)$$

Optimal Q-value of taking action a in state s : $Q^*(s, a) = Q^{\pi^*}(s, a)$

π^* can be greedily determined from Q^* : $\pi^*(s) = \arg\max_a Q^*(s, a)$

Slide adapted from CIS 522

Example of State Value Function



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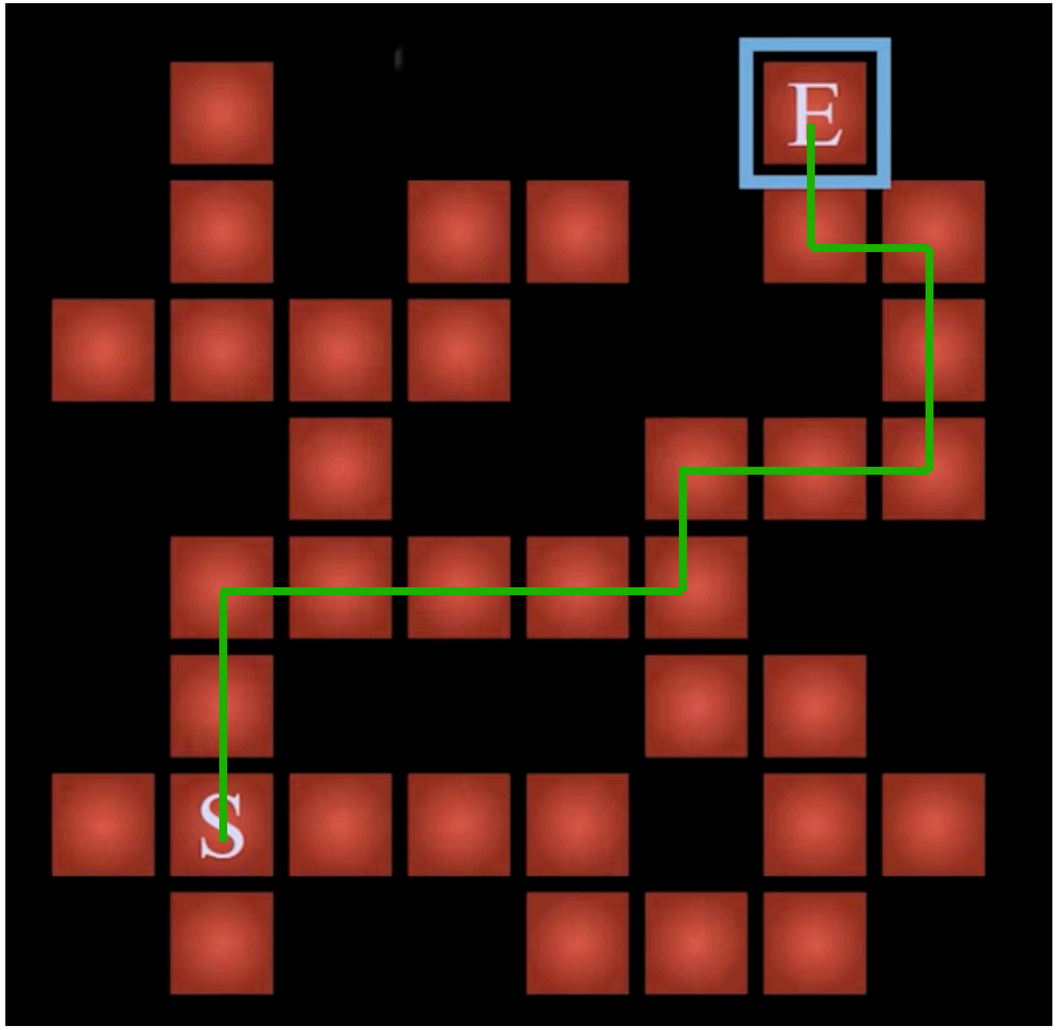

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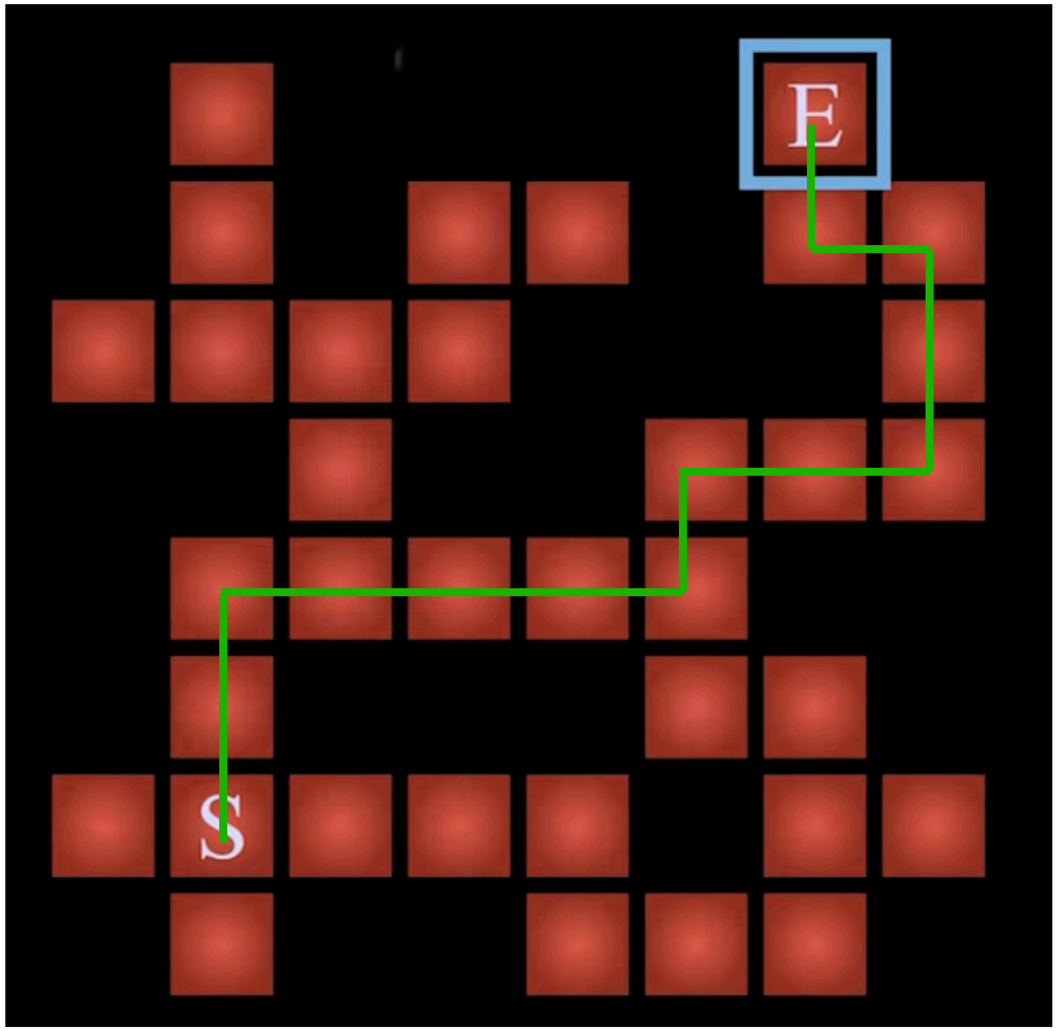
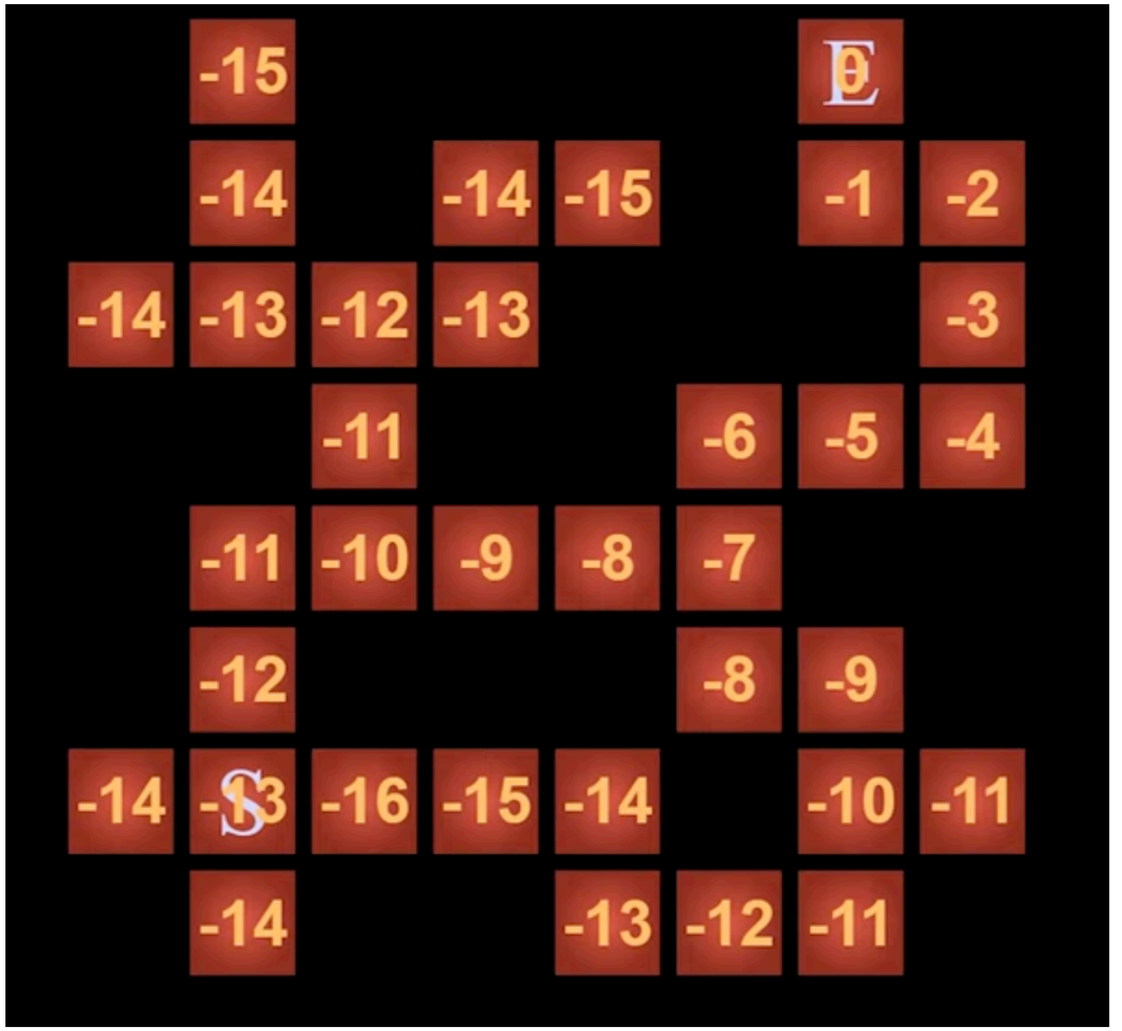
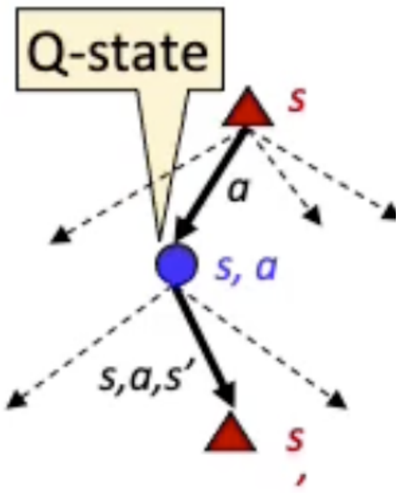




If agent get state value function of the optimal policy, can agent solve this?

Solving MDPs: Bellman Equations

The Bellman equations connect values functions at consecutive time steps:



$$V^*(s) = \max_{a \in A} Q^*(s, a)$$

Optimal value of s is what we get by picking the optimal action

$$Q^*(s, a) = \sum_{s' \in S} P(s' \mid s, a)\left[r(s, a, s') + \gamma V^*(s')\right]$$

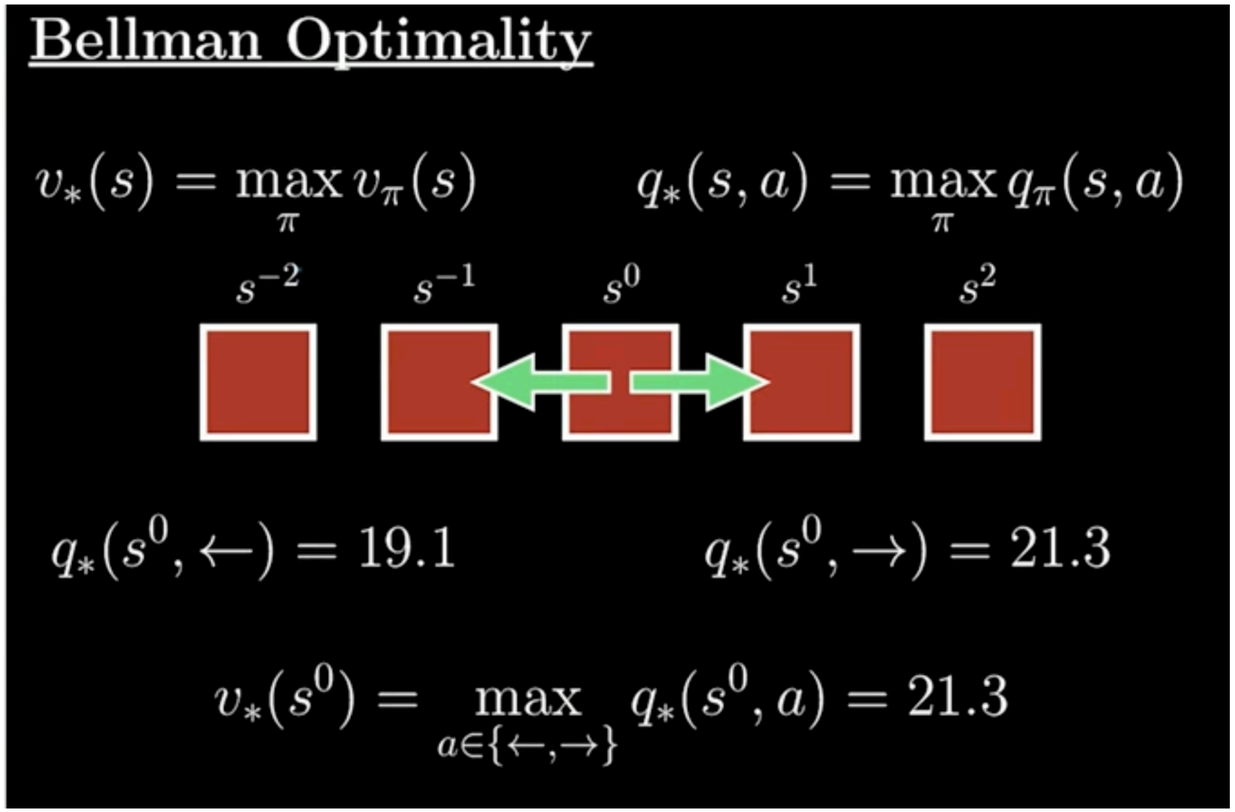
Expected value over successor state s'

Current reward + discounted future reward

$$V^*(s) = \max_{a \in A}\left(\sum_{s' \in S} P(s' \mid s, a)\left[r(s, a, s') + \gamma V^*(s')\right)\right.$$

Slide adapted from CIS 522

Example Bellman Optimality



* Because there may be multiple actions which achieve the highest action-value, the choose highest action value rule cannot differentiate among them. This means any policy that assigns non-zero probabilities only to these actions is an optimal policy.

Solving MDPs with known P and R: Value Iteration

Bellman equation give us a recursive definition of the optimal value:

$$V^*(s) = \max_{a \in A} \sum_{s' \in S} P(s' \mid s, a) \Big[R(s, a, s') + \gamma V^*(s') \Big]$$

Idea: solve iteratively via dynamic programming (DP)

Start with V_0 (s) = 0 for all states s

Iterate the Bellman update until convergence:

$$V_{i+1}(s) \leftarrow \max_{a \in A} \sum_{s' \in S} P(s' \mid s, a) \Big[R(s, a, s') + \gamma V_i(s') \Big]$$

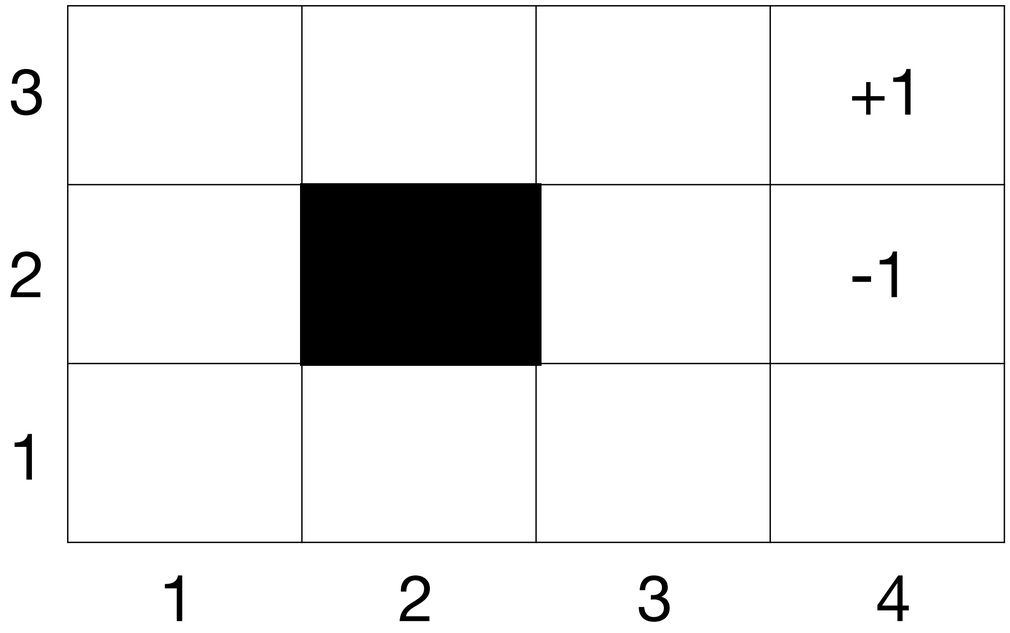
DP: refers to algorithms used to find optimal policies which have complete knowledge of the environment as an MDP.

Slide adapted from CIS 522

Example: Value iteration

Bellman update rule: $V_{i+1}(s) \leftarrow \max_{a \in A} \sum_{s' \in S} P(s' \mid s, a) \left[R(s, a, s') + \gamma V_i(s') \right]$

Example MDP



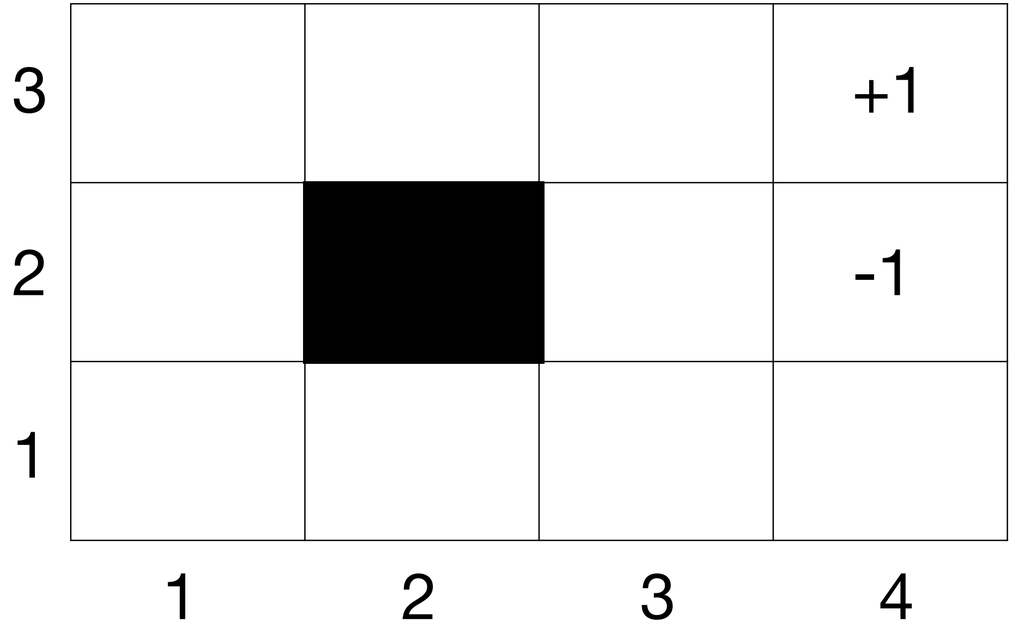
Rewards given in terminal states

$\gamma = 0.9$, living reward = 0, noise = 0.2

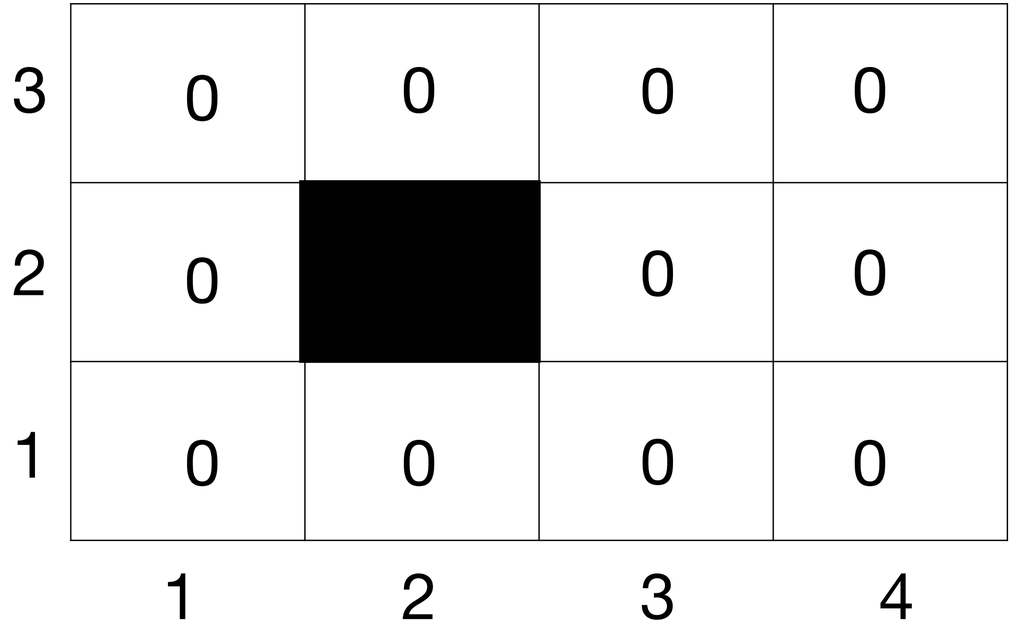
Example: Value iteration

Bellman update rule: $V_{i+1}(s) \leftarrow \max_{a \in A} \sum_{s' \in S} P(s' \mid s, a) \left[R(s, a, s') + \gamma V_i(s') \right]$

Example MDP



V_0



$\gamma = 0.9$, living reward = 0, noise = 0.2

$V_0(s) = 0$

Example: Value iteration

Bellman update rule: $V_{i+1}(s) \leftarrow \max_{a \in A} \sum_{s' \in S} P(s' \mid s, a) \left[R(s, a, s') + \gamma V_i(s') \right]$

V_0

3	0	0	0	0
2	0		0	0
1	0	0	0	0
	1	2	3	4

$V_0(s) = 0$

V_1

3	0	0	0	+1
2	0		0	-1
1	0	0	0	0
	1	2	3	4

$V_1(s) = 0$

$\gamma = 0.9$, living reward = 0, noise = 0.2

Example: Value iteration

Bellman update rule: $V_{i+1}(s) \leftarrow \max_{a \in A} \sum_{s' \in S} P(s' \mid s, a)\left[R(s, a, s') + \gamma V_i(s')\right]$

V_1

3	0	0	0	+1
2	0		0	-1
1	0	0	0	0
	1	2	3	4

$V_1(s) = 0$

V_2

3	0	0	? →	+1
2	0		0	-1
1	0	0	0	0
	1	2	3	4

Assume you move ->

0.8 [0+0.9x1] + 0.1 [0+0.9x0] + 0.1[0+0.9x0] =0.72

$\gamma = 0.9$, living reward = 0, noise = 0.2

Example: Value iteration

Bellman update rule: $V_{i+1}(s) \leftarrow \max_{a \in A} \sum_{s' \in S} P(s' \mid s, a)\left[R(s, a, s') + \gamma V_i(s')\right]$

V_1

3	0	0	0	+1
2	0		0	-1
1	0	0	0	0
	1	2	3	4

$V_1(s) = 0$

V_2

3	0	0	0.72	+1
2	0		0	-1
1	0	0	0	0
	1	2	3	4

Assume you move ->

0.8 [0+0.9x1] + 0.1 [0+0.9x0] + 0.1[0+0.9x0] =0.72

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Example: Value iteration

Bellman update rule: $V_{i+1}(s) \leftarrow \max_{a \in A} \sum_{s' \in S} P(s' \mid s, a)\left[R(s, a, s') + \gamma V_i(s')\right]$

V_1

3	0	0	0	+1
2	0		0	-1
1	0	0	0	0
	1	2	3	4

$V_1(s) = 0$

V_2

3	0	0	0.72	+1
2	0		0	-1
1	0	0	0	0
	1	2	3	4

Assume you move ->

0.8 [0+0.9x1] + 0.1 [0+0.9x0] + 0.1[0+0.9x0] =0.72

$\gamma = 0.9$, living reward = 0, noise = 0.2

Example: Value iteration

Bellman update rule: $V_{i+1}(s) \leftarrow \max_{a \in A} \sum_{s' \in S} P(s' \mid s, a)\left[R(s, a, s') + \gamma V_i(s')\right]$

V_2

3	0	0	0.72	+1
2	0		0	-1
1	0	0	0	0
	1	2	3	4

V_3

3	0	?	?	+1
2	0		?	-1
1	0	0	0	0
	1	2	3	4

$\gamma = 0.9$, living reward = 0, noise = 0.2

Example: Value iteration

Bellman update rule: $V_{i+1}(s) \leftarrow \max_{a \in A} \sum_{s' \in S} P(s' \mid s, a)\left[R(s, a, s') + \gamma V_i(s')\right]$

V_2

3	0	0	0.72	+1
2	0		0	-1
1	0	0	0	0
	1	2	3	4

V_3

3	0	0.52	0.78	+1
2	0		0.43	-1
1	0	0	0	0
	1	2	3	4

$\gamma = 0.9$, living reward = 0, noise = 0.2

Information propagates outward from terminal states

Eventually all states will have correct value estimates

Policy iteration: policy evaluation

How do we compute V's for a fixed policy?

Key idea: Bellman updates for arbitrary policy

$$V_0^\pi(s) = 0$$

$$V_{i+1}^\pi(s) \leftarrow \sum_{s'} P(s' \mid s, \pi(s)) \Big[R(s, \pi(s), s') + \gamma V_i^\pi(s') \Big]$$

Value iteration update rule

$$V_{i+1}(s) \leftarrow \max_{a \in A} \sum_{s' \in S} P(s' \mid s, a) \Big[R(s, a, s') + \gamma V_i(s') \Big]$$

Generalized Policy Iteration: Policy Iteration

Repeat two steps until convergence

1. Policy evaluation: keep current policy π fixed, find value function V^{π}

Iterate simplified Bellman update until values converge:

$$V_{i+1}^{\pi_k}(s) \leftarrow \sum_{s'} P(s' \mid s, \pi_k(s)) \Big[R(s, \pi_k(s), s') + \gamma V_i^{\pi_k}(s') \Big]$$

2. Policy improvement: find the best action for V^{π} via one-step lookahead

$$\pi_{k+1}(s) = \arg\max_a \sum_{s'} P(s' \mid s, a) \Big[R(s, a, s') + \gamma V^{\pi_k}(s') \Big]$$

Policy iteration is optimal too!

>Faster than value iteration in terms of number of (outer) loops, but remember that step 1 has an inner loop too.

Find the optimal Pi and V: $\pi_0 \xrightarrow{E} v_{\pi_0} \xrightarrow{I} \pi_1 \xrightarrow{E} v_{\pi_1} \xrightarrow{I} \pi_2 \xrightarrow{E} \cdots \xrightarrow{I} \pi_* \xrightarrow{E} v_*$

Temporal Differencing (TD)

Policy evaluation: Start V_0 (s) =0

Iterate until convergence: $V_{i+1}^{\pi_k}(s) \leftarrow \sum_{s'} P(s' \mid s, \pi_k(s)) \left[R(s, \pi_k(s), s') + \gamma V_i^{\pi_k}(s') \right]$

How can we extend this to when P and R is not known, and only revealed gradually through experience?

Every time you take action a from state s, you get a sample from the unknown P and corresponding reward R.

Temporal Differencing (TD)

Policy evaluation: Start V_0 (s) =0

Iterate until convergence: $V_{i+1}^{\pi_k}(s) \leftarrow \sum_{s'} P(s' \mid s, \pi_k(s)) \Big[R(s, \pi_k(s), s') + \gamma V_i^{\pi_k}(s') \Big]$

Key idea: Treat single sample you get as representative of the distribution, and apply an incremental update to reduce the “Bellman error”:

One sample of V(S): $\text{sample} = R(s, \pi(s), s') + \gamma V_i^{\pi}(s')$

TD update: $V^{\pi}(s) \leftarrow V^{\pi}(s) + \alpha\big(\text{sample} - V^{\pi}(s)\big)$

Doing this for each sample = computing the running average over samples.

Learning the optimal Q* function

Recall Bellman equation for optimal Q*

$$Q^*(s,a) = \sum_{s' \in \mathcal{S}} P(s' \mid s,a) \left[R(s,a,s') + \gamma \max_{a'} Q^*(s',a') \right]$$

The corresponding Q-value iteration equation (analogous to the state value iteration would be:

$$Q_{i+1}(s,a) \leftarrow \sum_{s' \in \mathcal{S}} P(s' \mid s,a) \left[R(s,a,s') + \gamma \max_{a'} Q_i(s',a') \right]$$

Again, this require access to P and R of which only have samples from experience.

Applying the TD to previous one

Q-value iteration $$Q_{i+1}(s,a) \leftarrow \sum_{s' \in \mathcal{S}} P(s' \mid s,a)\left[R(s,a,s') + \gamma \max_{a'} Q_i(s',a')\right]$$

TD: treat single sample as representative of the distribution and apply an incremental update to reduce the “Bellman error”:

1. Execute a single action a from state s and observe s’ and R:
   sample $= R(s,a,s') + \gamma \max_{a'} Q^*(s',a')$
2. Then incremental TD update is:

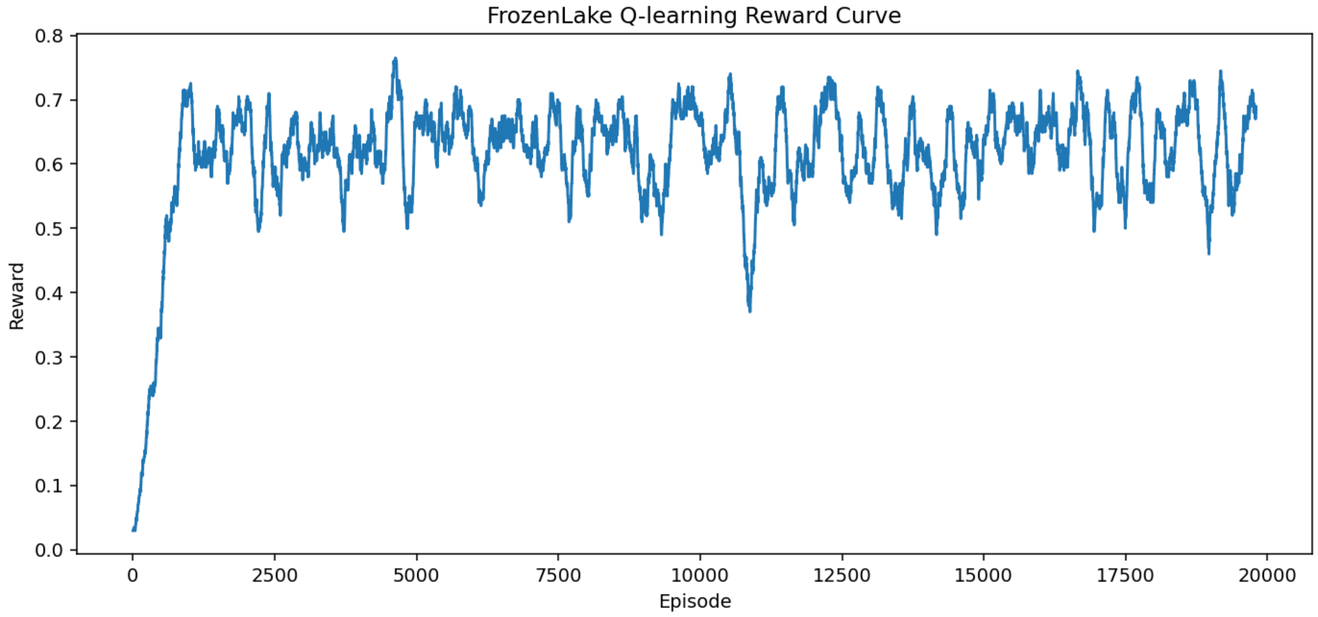
$$Q(s,a) \leftarrow Q(s,a) + \alpha \left(R(s,a,s') + \gamma \max_{a'} Q(s',a') - Q(s,a) \right)$$

Bellman error

This is Q-Learning.

Example of Q-Learning

Ep100 Ep1000 Ep5000 Ep10000 Ep20000



Solving unknown MDPs using function approximation

Q*(s,a) = expected utility starting in s, taking action a, and thereafter acting optimally.

Bellman Equation: $$Q^*(s,a) = \sum_{s'} P(s' \mid s,a)\left[R(s,a,s') + \gamma \max_{a'} Q^*(s',a')\right]$$

Q-value iteration: $$Q^*_{k+1}(s,a) \leftarrow \sum_{s'} P(s' \mid s,a)\left(R(s,a,s') + \gamma \max_{a'} Q^*_k(s',a')\right)$$

This is problematic when we do not known this transition function.

Slides from Fragkiadaki

Tabular Q-Learning

- Q-value iteration: $Q_{k+1}(s,a) \leftarrow \sum_{s'} P(s'|s,a)(R(s,a,s') + \gamma \max_{a'} Q_k(s',a'))$
- Rewrite as expectation: $Q_{k+1} \leftarrow \mathbb{E}_{s' \sim P(s'|s,a)} \left[R(s,a,s') + \gamma \max_{a'} Q_k(s',a') \right]$
- (Tabular) Q-Learning: replace expectation by samples
  - For an state-action pair (s,a), receive: $s' \sim P(s'|s,a)$ simulation and exploration
  - Consider your old estimate: $Q_k(s,a)$
  - Consider your new sample estimate:

$$\text{target}(s') = r(s,a,s') + \gamma \max_{a'} Q_k(s',a')$$

$$\text{error}(s') = \left(r(s,a,s') + \gamma \max_{a'} Q_k(s',a') - Q_k(s,a) \right)$$

Slides from Fragkiadaki

Tabular Q-Learning update

learning rate

$$Q_{k+1}(s,a) = Q_k(s,a) + \alpha \ \text{error}(s')$$
$$= Q_k(s,a) + \alpha \left(r(s,a,s') + \gamma \max_{a'} Q_k(s',a') - Q_k(s,a) \right)$$

Key idea: implicitly estimate the transitions via simulation

Slides from Fragkiadaki

Tabular Q-Learning overview

Bellman optimality

$$Q^*(s,a) = \mathbb{E}_{s'}\left[r(s,a,s') + \gamma \max_{a'} Q^*(s',a')\right]$$

Algorithm:

Start with $Q_0(s,a)$ for all s, a.

Get initial state s

For k = 1, 2, … till convergence

Sample action a, get next state s'

If s' is terminal:

$$\text{target} = r(s,a,s')$$

Sample new initial state s'

else:

$$\text{target} = r(s,a,s') + \gamma \max_{a'} Q_k(s',a')$$

$$Q_{k+1}(s,a) = Q_k(s,a) + \alpha\left(r(s,a,s') + \gamma \max_{a'} Q_k(s',a') - Q_k(s,a)\right)$$

$$s \leftarrow s'$$

Slides from Fragkiadaki

Tabular Q-Learning overview

Algorithm:

Start with $Q_0(s, a)$ for all s, a.

Get initial state s

For k = 1, 2, … ~~till convergence~~

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$$Q_{k+1}(s, a) = Q_k(s, a) + \alpha \left(r(s, a, s') + \gamma \max_{a'} Q_k(s', a') - Q_k(s, a) \right)$$

$$s \leftarrow s'$$

Bellman optimality

$$Q^*(s, a) = \mathbb{E}_{s'} \left[r(s, a, s') + \gamma \max_{a'} Q^*(s', a') \right]$$

How do we sample these actions?

>Choose random action all the time.

>Choose action that maximize Qk(s,a) greedily.

>Choose action sometimes randomly with prob of E, otherwise choose greedily..

Slides from Fragkiadaki

Epsilon-greedy

Initially, Q(s,a) is poor at the beginning, bad initial estimates in the first few cases can drive policy into sub-optimal region and never explore further.

$$\pi(s) = \begin{cases} \max_a \hat{Q}(s,a) & \text{with probability } 1-\epsilon \\ \text{random action} & \text{otherwise} \end{cases}$$

Gradually decrease epsilon as policy is learned.

Slides from Paul Liang

Tabular Q-Learning overview

Algorithm:

Start with $Q_0(s,a)$ for all s, a.

Get initial state s

For k = 1, 2, … till convergence

Sample action a, get next state s'

If s' is terminal:

$$\text{target} = r(s,a,s')$$

Sample new initial state s'

else:

$$\text{target} = r(s,a,s') + \gamma \max_{a'} Q_k(s',a')$$

$$Q_{k+1}(s,a) = Q_k(s,a) + \alpha \left(r(s,a,s') + \gamma \max_{a'} Q_k(s',a') - Q_k(s,a) \right)$$

$$s \leftarrow s'$$

Bellman optimality

$$Q^*(s,a) = \mathbb{E}_{s'} \left[r(s,a,s') + \gamma \max_{a'} Q^*(s',a') \right]$$

How do we sample these actions?

>Choose random action all the time.

>Choose action that maximize Qk(s,a) greedily.

>Choose action sometimes randomly with prob of E, otherwise choose greedily..

Slides from Fragkiadaki

Convergence

- Amazing result: Q-learning converges to optimal policy -- even if you're acting suboptimally!
- This is called off-policy learning
- Caveats:
  - You have to explore enough
  - You have to eventually make the learning rate small enough
  - … but not decrease it too quickly

Limitation: Still requires small and discrete state and action space, as it is tabular learning, it keeps a |S| X |A| table of Q(s,a).

Slides from Abbeel

Things to cover today:

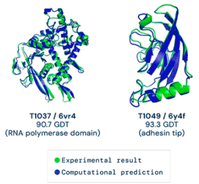
- Introduction and motivation to RL
- Markov Decision Processes (MDPs)
- Solving known MDPs using value and policy iteration
- Temporal Difference & Q-Learning
- Tabular Q-Learning & Deep Q-Learning
- Policy Gradient Methods
- PPO
- Imitation Learning

Quick recap:

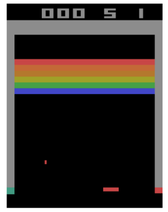
Motivation for RL

Self-driving

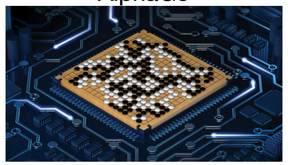
AlphaFold



Atari

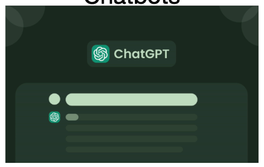


AlphaGo

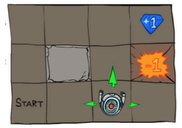


Plasma control

Chatbots

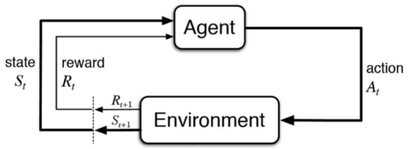


Markov Decision Process (MDPs)



An MDP consist of 7 terms:

- Set of states, S
- Set of actions, A
- Transition function P(s' | s, a)
- Reward function R(s, a, s')
- Start state s_0
- Discount factor γ
- Horizon H



$$s_0, a_0, r_0, s_1, a_1, r_1, s_2, a_2, r_2 \dots$$

Andrey Andreyevich Markov (1856-1922)

Russian mathematician known for his work on stochastic processes.

State-value function

$$V^{\pi}(s) = \mathbb{E}_{\pi}\left[\sum_{k=0}^{\infty} \gamma^k R_{t+k+1} \,\middle|\, S_t = s\right]$$

Action-value function

$$Q^{\pi}(s,a) = \mathbb{E}_{\pi}\left[\sum_{k=0}^{\infty} \gamma^k R_{t+k+1} \,\middle|\, S_t = s, A_t = a\right]$$

Bellman Equation

$$V^{\pi}(s) = \sum_{a} \pi(a|s) \sum_{s',r} p(s', r|s, a)\left[r + \gamma V^{\pi}(s')\right]$$

$$V^{*}(s) = \max_{a} \sum_{s',r} p(s', r|s, a)\left[r + \gamma V^{*}(s')\right]$$

Value & Policy iteration

$$V_{i+1}(s) \leftarrow \max_{a \in A} \sum_{s' \in S} P(s' \mid s, a)\left[R(s, a, s') + \gamma V_i(s')\right]$$

1. Policy evaluation: keep current policy π fixed, find value function V^{π}

Iterate simplified Bellman update until values converge:

$$V_{i+1}^{\pi_k}(s) \leftarrow \sum_{s'} P(s' \mid s, \pi_k(s))\left[R(s, \pi_k(s), s') + \gamma V_i^{\pi_k}(s')\right]$$

2. Policy improvement: find the best action for V^{π} via one-step lookahead

$$\pi_{k+1}(s) = \arg\max_{a} \sum_{s'} P(s' \mid s, a)\left[R(s, a, s') + \gamma V^{\pi_k}(s')\right]$$

TD & Q-Learning

$$V(s) \leftarrow V(s) + \alpha\left[R_{t+1} + \gamma V(S_{t+1}) - V(s)\right]$$

$$Q(s,a) \leftarrow Q(s,a) + \alpha\left[R_{t+1} + \gamma \max_{a'} Q(S_{t+1}, a') - Q(s,a)\right]$$

Things to cover today:

- Introduction and motivation to RL
- Markov Decision Processes (MDPs)
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- Temporal Difference & Q-Learning
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- Policy Gradient Methods
- PPO
- Imitation Learning

Problems with Q-Learning

In many real situations, we cannot possibly learn about every single state+action!

>Too many state-action pairs to visit them all in training

>Too many state-action pairs to hold the q-tables in memory

Ideally what we want:
-Generalize by learning about some small number of training q-states from experience

-Generalize that experience to new, similar q-states

-Core idea in ML, and we will see it over and over again.

Example with Pacman

>We discover through experience that this is a bad state:

Slides from CIS522

Example with Pacman

>We discover through experience that this is a bad state:

>In naive Q-learning, we know nothing about this state or its Q states:

Slides from CIS522

Example with Pacman

>We discover through experience that this is a bad state:

>In naive Q-learning, we know nothing about this state or its Q states:

>Or even this:

Slides from CIS522

Solution

- Solution: describe a state using a vector of features
  - Features are functions from states to real numbers (often 0/1) that capture important properties of the state
  - Example features:
    - Distance to closest ghost
    - Distance to closest dot
    - Number of ghosts
    - $1 / (\text{dist to dot})^2$
    - Is Pacman in a tunnel? (0/1)
    - …… etc.
  - Can also describe a q-state (s, a) with features
    - e.g. action moves closer to food

Deep Q-Learning

Solution: Describe a state using a vector of features

Predict Q-values with a deep neural network

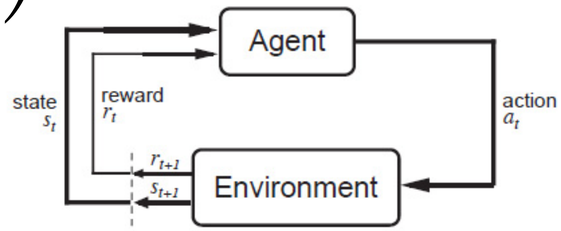
Input: the state

Output: Q-value of various actions

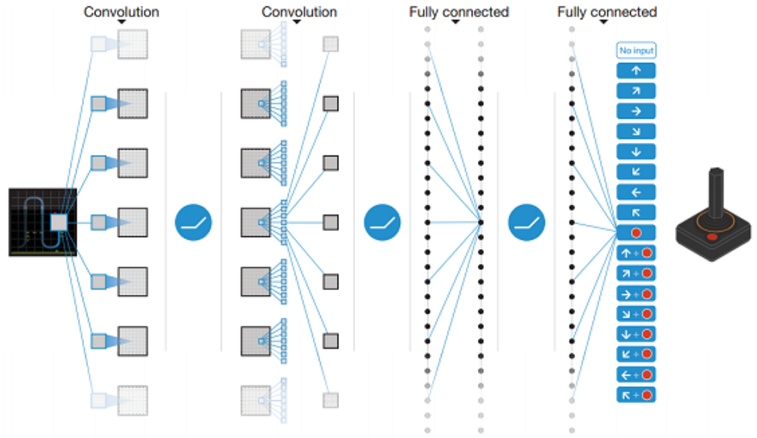
Learning: gradient descent with the squared Bellman error loss:

$$\left(\left(R(s,a,s') + \gamma \max_{a'} Q(s',a') \right) - Q(s,a) \right)^2$$

The policy action is the one with the highest predicted Q-value.



AlphaGo



Deep Q-Learning

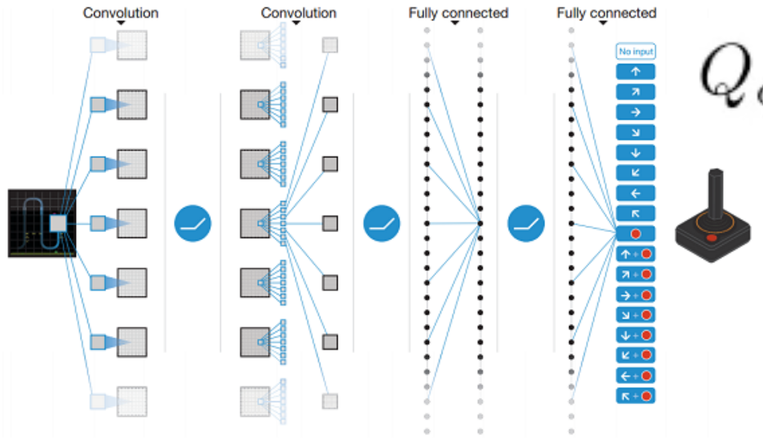
1. take some action $\mathbf{a}_i$ and observe $(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)$
2. $\mathbf{y}_i = r_i + \gamma \max_{\mathbf{a}'} Q_\phi(\mathbf{s}'_i, \mathbf{a}'_i)$
3. $\phi \leftarrow \phi - \alpha \frac{dQ_\phi(\mathbf{s}_i, \mathbf{a}_i)}{d\phi} \left(Q_\phi(\mathbf{s}_i, \mathbf{a}_i) - \mathbf{y}_i\right)$

$$= \frac{d}{d\phi}(Q_\phi - y_i)^2$$

Incremental update step → gradient descent* on the squared Bellman error loss!

S

A



$Q_\phi(\mathbf{s}, \mathbf{a})$

Value-based VS Policy-based RL

>Value based : Learned value function, Implicit policy (e.g E-greedy)

>Policy based : No value function, just directly learn a policy.

Why do we want to learn directly the policy?

- Often π can be simpler than Q or V
  - E.g., robotic grasp

Q(s,a) and V(s) very high-dimensional
But policy could be just 'open/close hand'

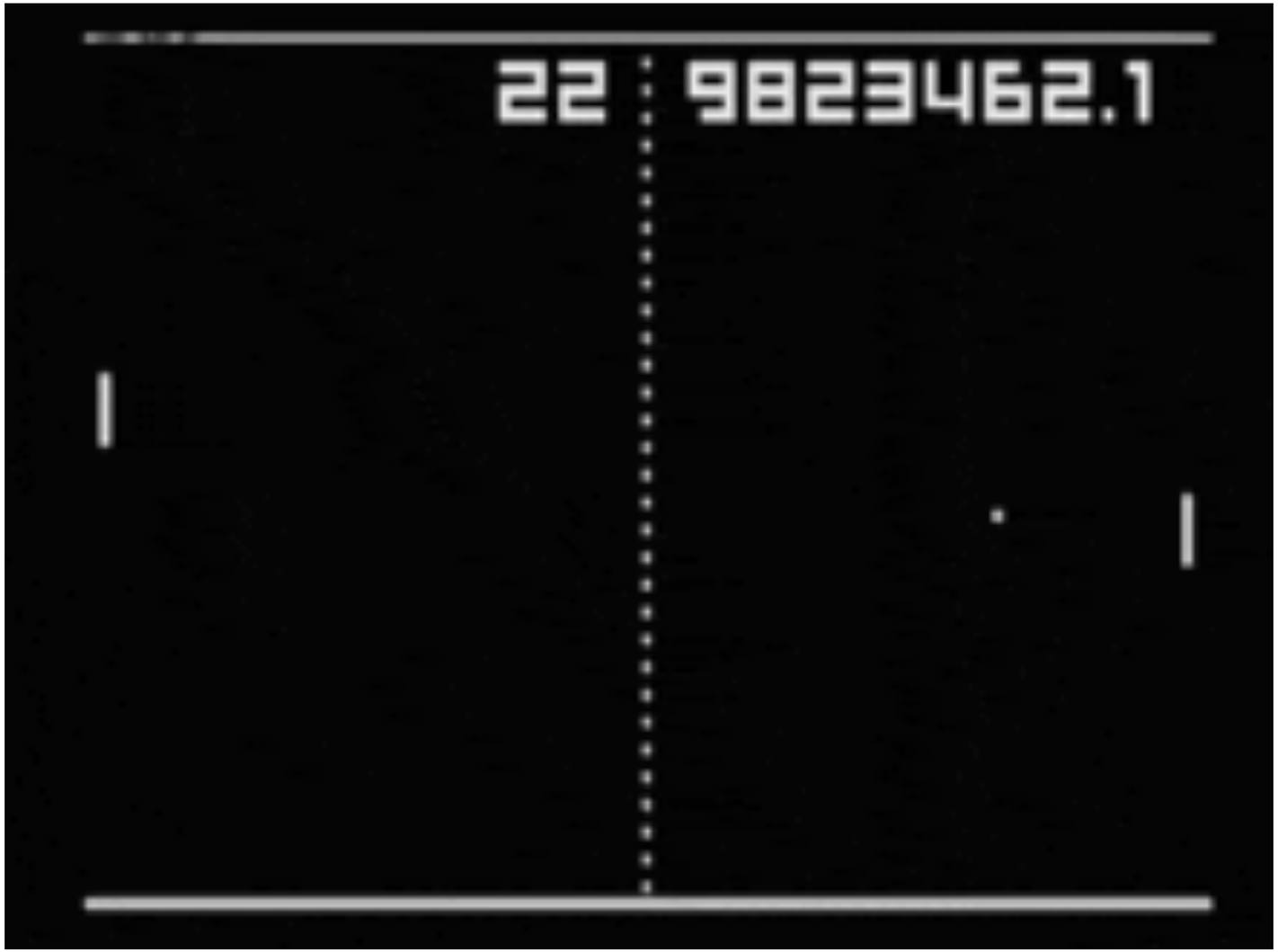
- V: doesn't prescribe actions
  - Would need dynamics model (+ compute 1 Bellman back-up)
- Q: need to be able to efficiently solve $\arg\max_u Q_\theta(s,u)$
  - Challenge for continuous / high-dimensional action spaces*

$$\pi^*(a|s) = \begin{cases} 1-\epsilon, & \text{if } a = \arg\max_a \mathbb{E}_{s'}\left[r(s,a,s') + \gamma V^*(s')\right] \\ \epsilon, & \text{else} \end{cases} \qquad \pi^*(a|s) = \begin{cases} 1-\epsilon, & \text{if } a = \arg\max_a Q^*(s,a) \\ \epsilon, & \text{else} \end{cases}$$

Value-based VS Policy-based RL

	Policy-based	**Value-based**
Conceptually:	Optimize what you care about	Indirect, exploit the problem structure, self-consistency
Empirically:	More compatible with rich architectures	More compatible with exploration and off-policy learning
	More versatile	More sample-efficient when they work.
	More compatible with auxiliary objectives	

Policy Gradient Methods

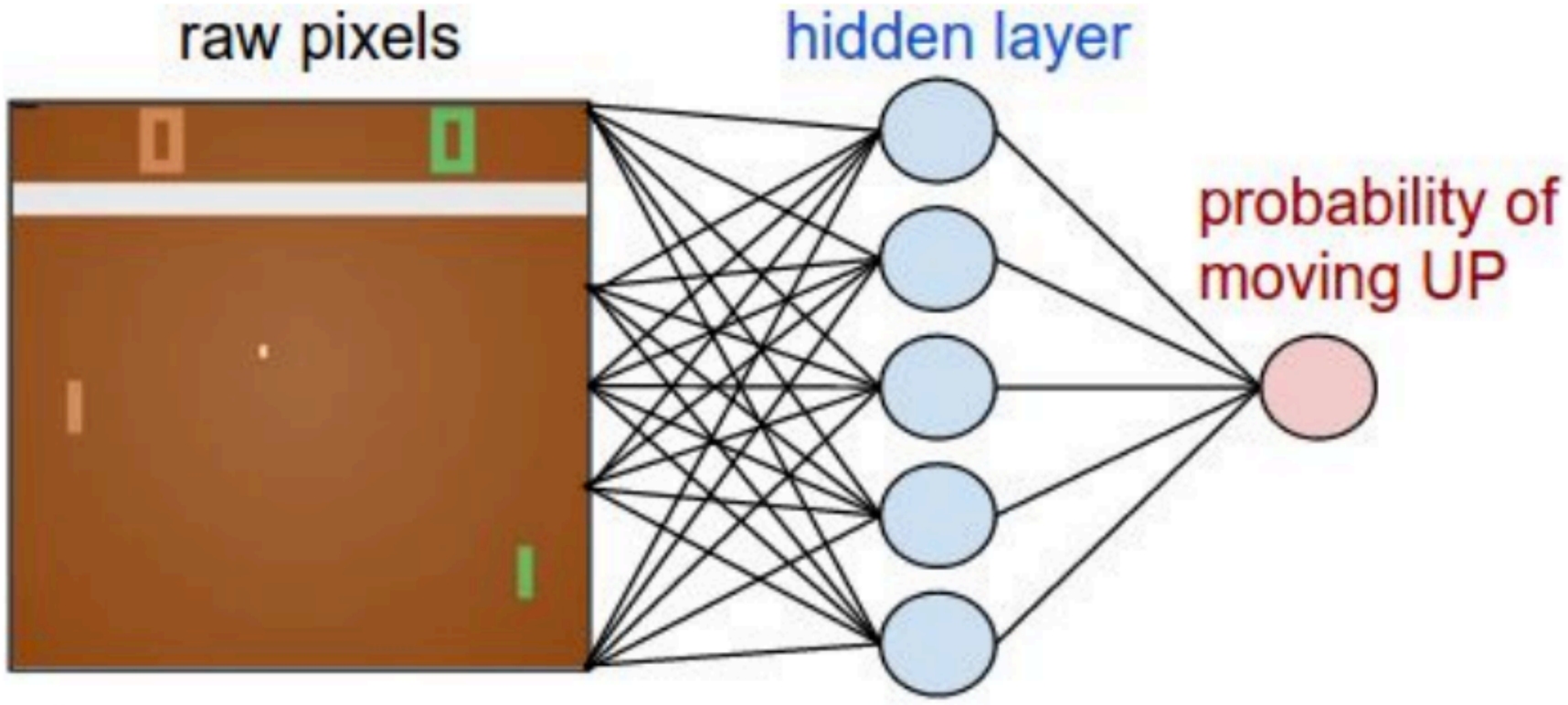


Pong from pixels

e.g.,

height width

[80 x 80]
array of



NN see +1 if it scored a point, -1 if it was scored against. How do we learn these parameters?

Slides from Karpathy

Pong from pixels

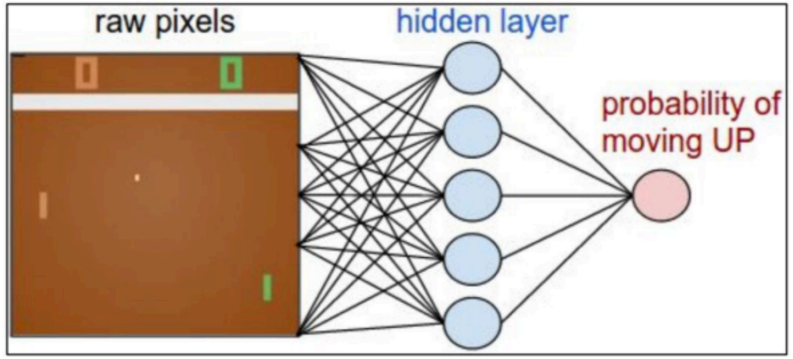
Given training labels from a human expert.

(x1,UP)
(x2,DOWN)
(x3,UP)
...

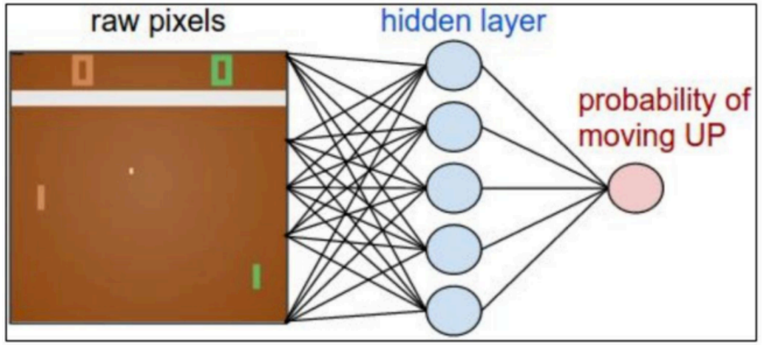
Maximum Likelihood Estimation (MLE).

maximize:

$$\sum_i \log p(y_i | x_i)$$



Except, we don't have labels...

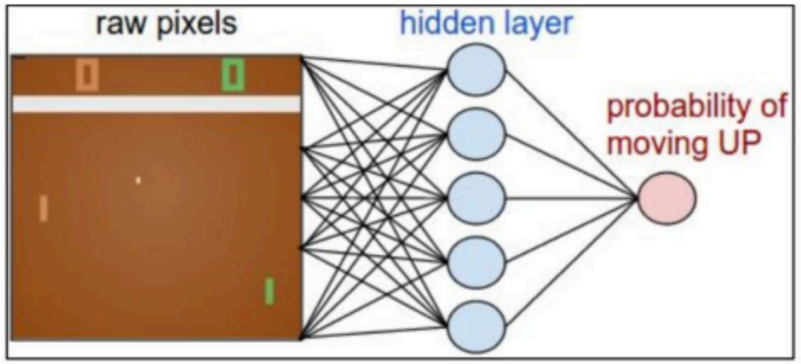


Should we go UP or DOWN?

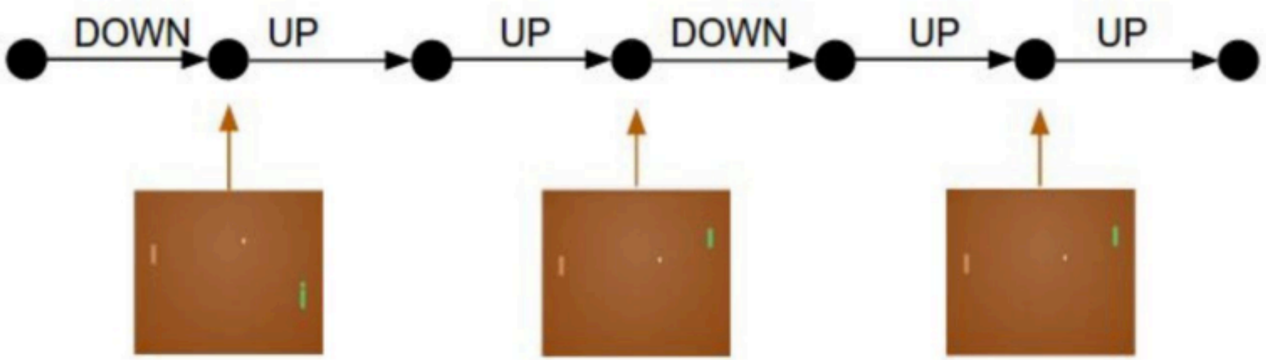
Slides from Karpathy

Pong from pixels

No data, lets just act according to our current policy.



Rollout the policy and collect an episode



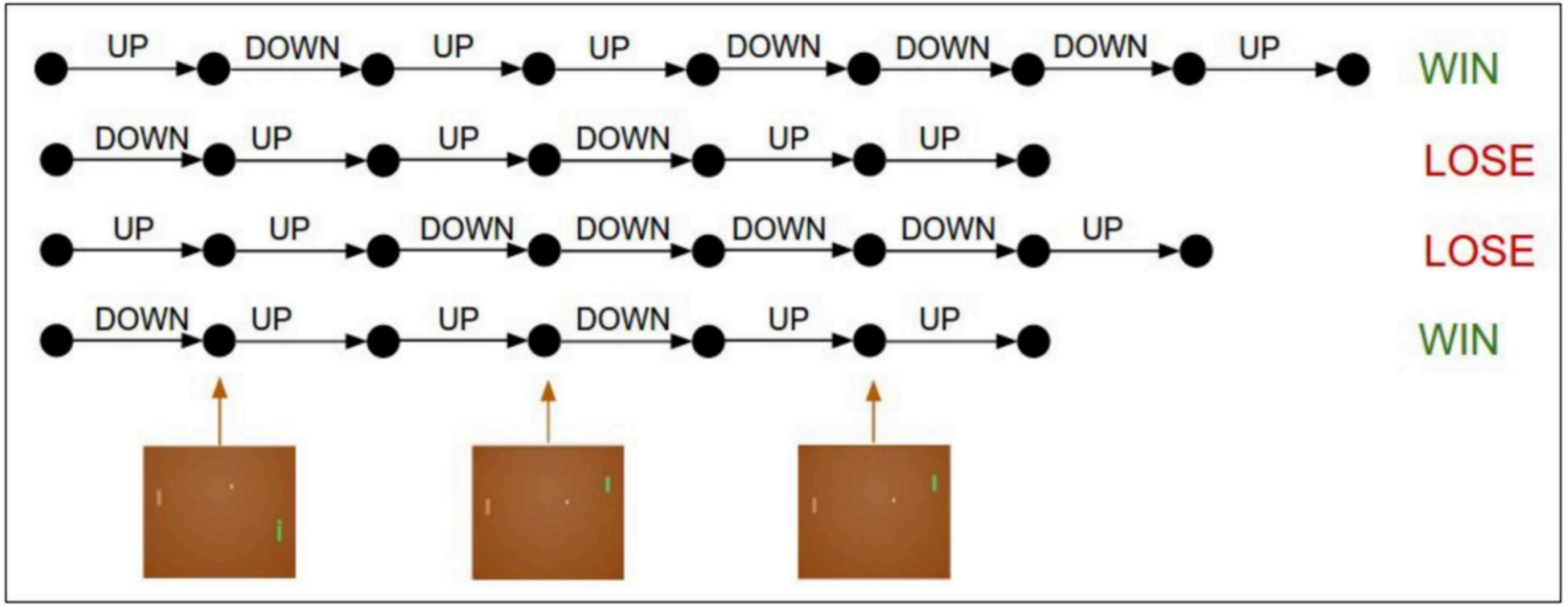
WIN

Slides from Karpathy

Pong from pixels

Collect many rollouts...

4 rollouts:

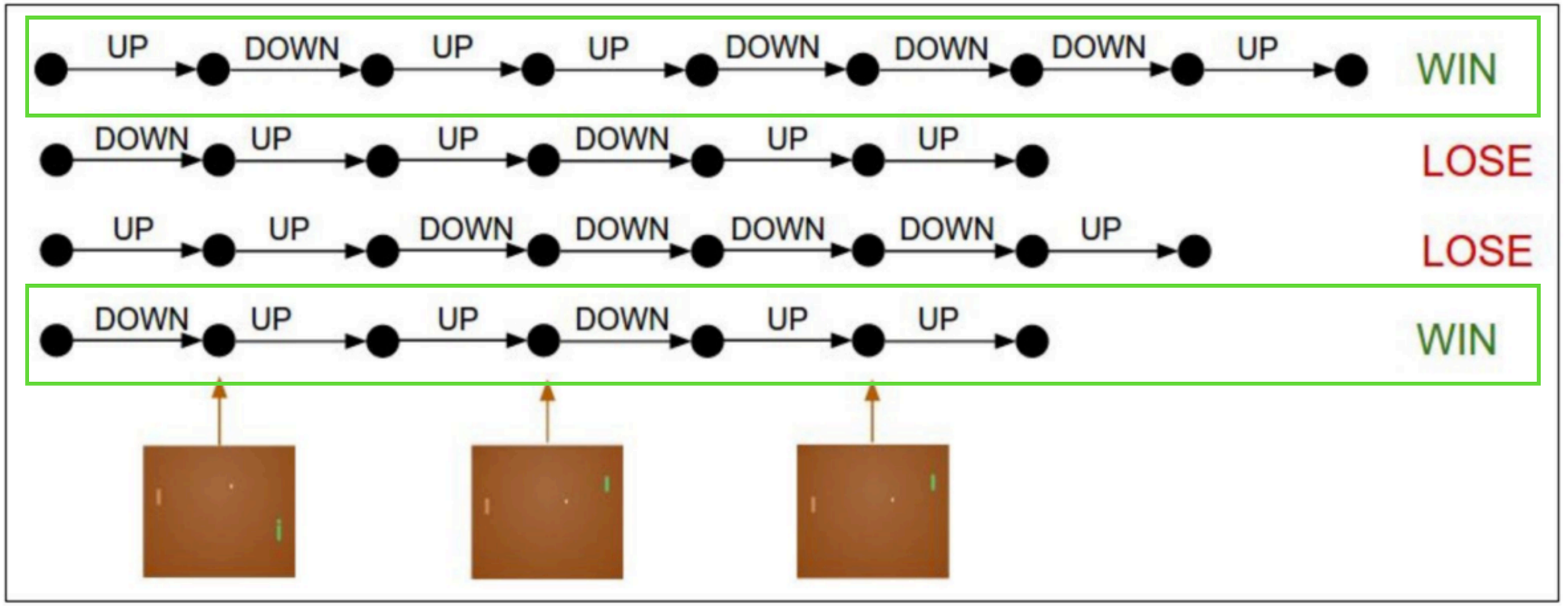


Slides from Karpathy

Pong from pixels

I am not sure what we did here, but these are good actions cause we won.

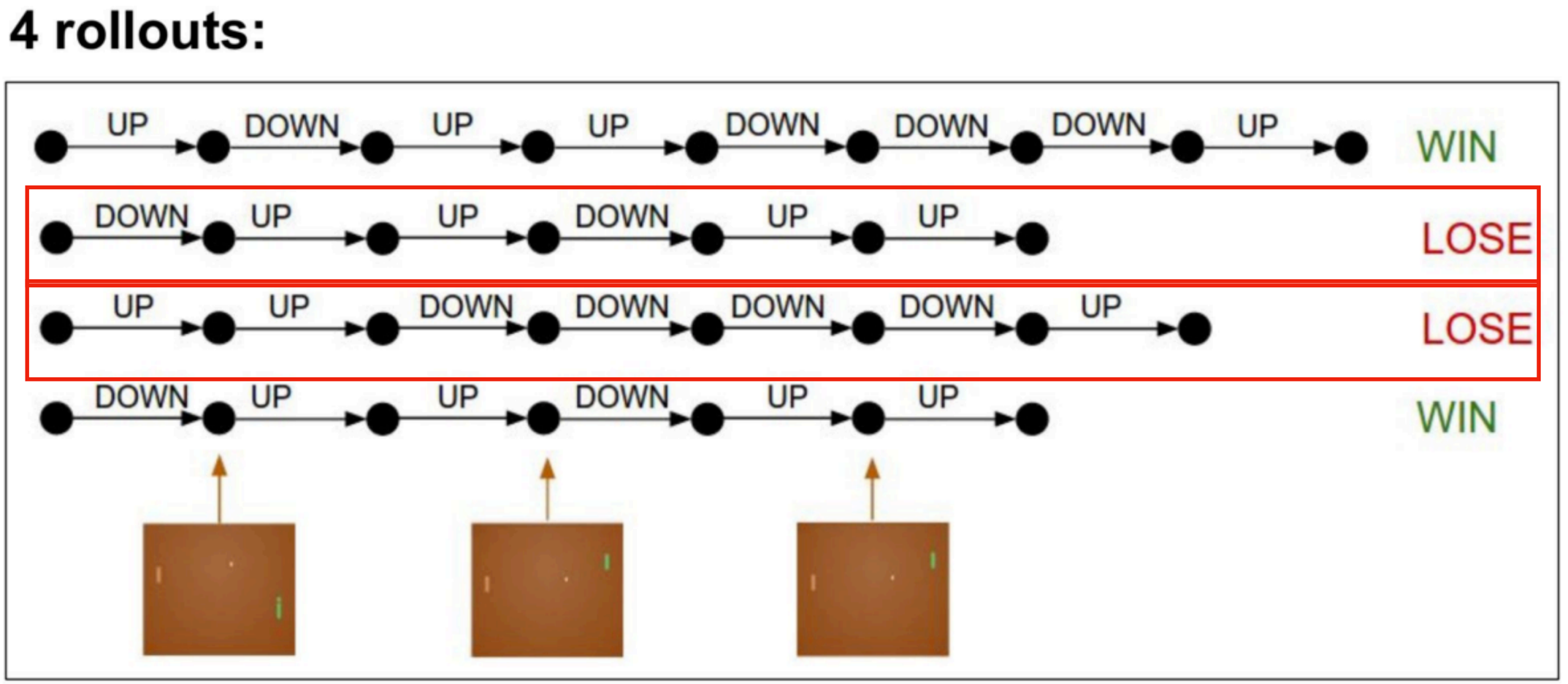
4 rollouts:



Slides from Karpathy

Pong from pixels

Not sure whatever we did here, but it was bad.

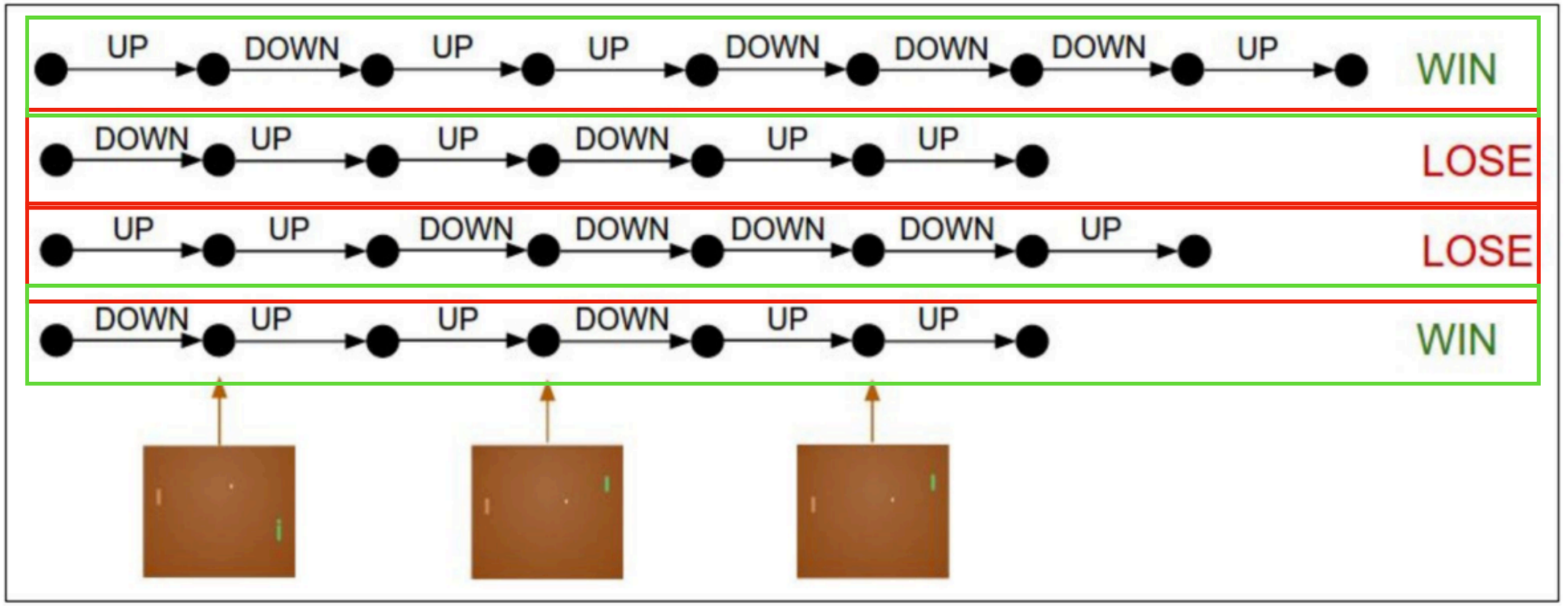


Slides from Karpathy

Pong from pixels

Pretend every action we took here was the correct label or wrong label based on the outcome.

4 rollouts:



Slides from Karpathy

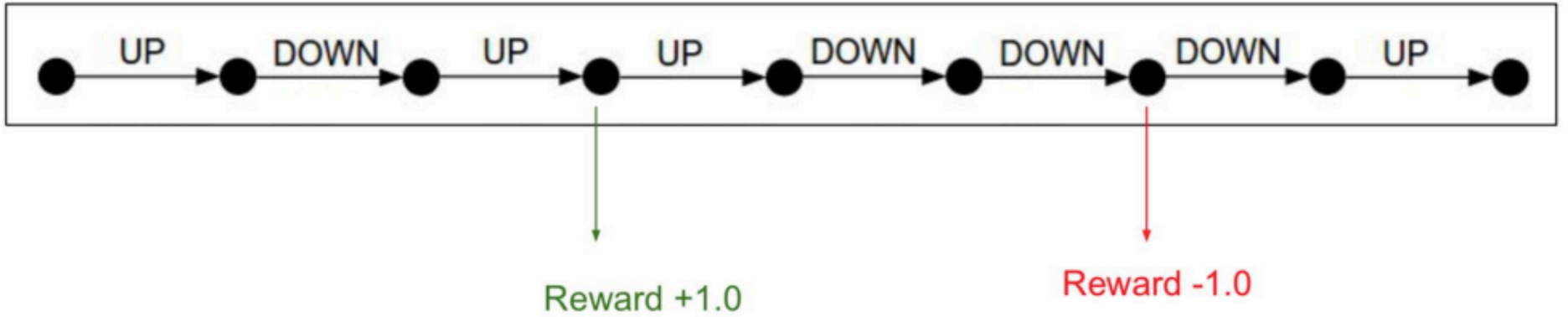
Discounting

Blame each action assuming that its effects have exponentially decaying impact into the future.

$$\sum_i A_i * \log p(y_i|x_i)$$

Discounted rewards

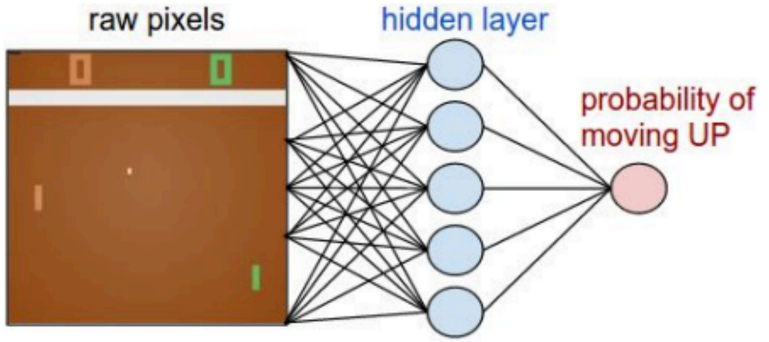
0.21 0.24 0.27 -0.81 -0.9 -1 0 0



\gamma = 0.9

Slides from Karpathy

$\pi(a \mid s)$



1. Initialize a policy network at random
2. **Repeat Forever:**
3. Collect a bunch of rollouts with the policy
4. Increase the probability of actions that worked well

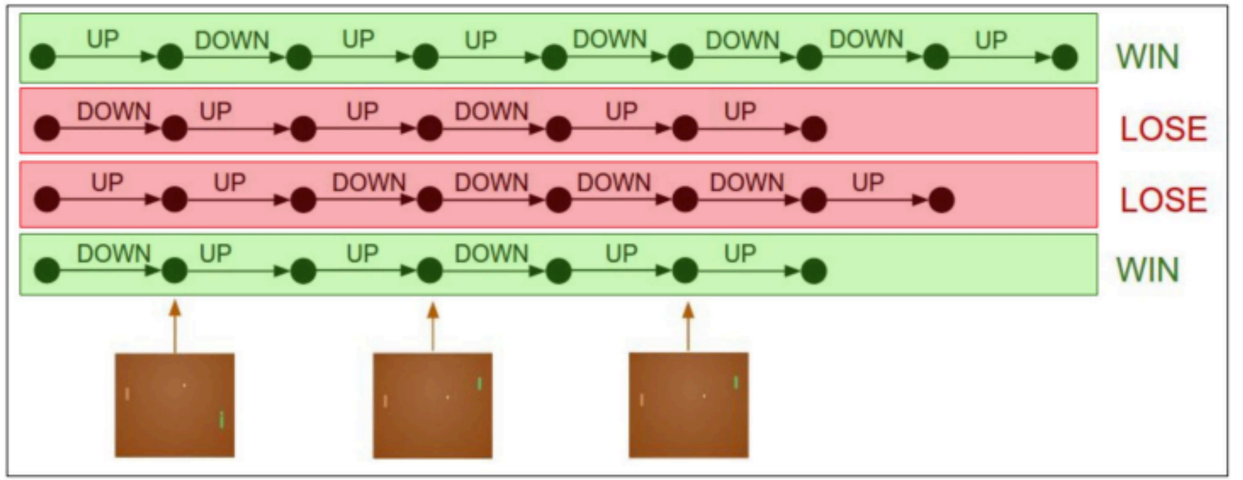
epsilon greedy!

Pretend every action we took here was the correct label.

maximize: $\log p(y_i \mid x_i)$

Pretend every action we took here was the wrong label.

maximize: $(-1) * \log p(y_i \mid x_i)$



$$\sum_i A_i * \log p(y_i \mid x_i)$$

Does not require transition probabilities
Does not estimate Q(), V()
Predicts policy directly

Policy gradient

Slides from Karpathy

Does this algorithm actually work well?

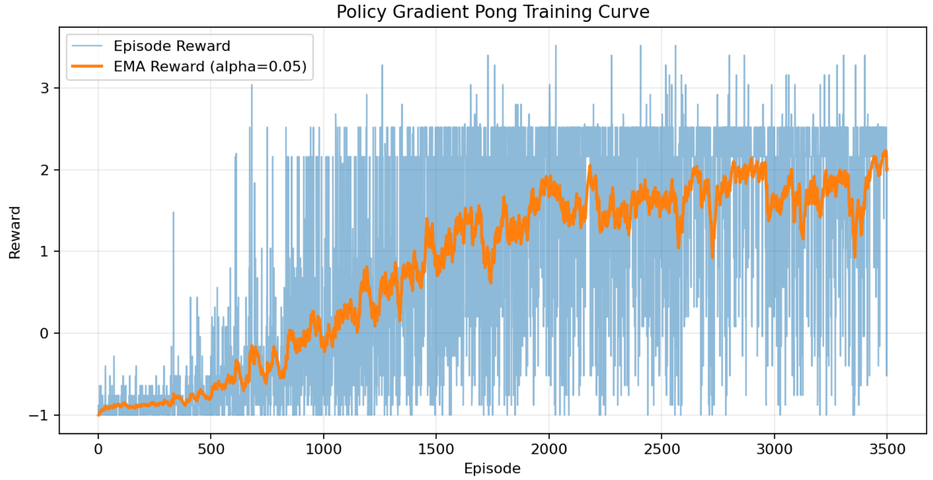
Ep 100

Ep 500

Ep 1500

Ep 2500

Ep 3500



But why does it work?

Policy gradients

Let's define a class of parameterized policies: $\Pi = \{\pi_\theta \mid \theta \in \mathbb{R}^m\}$.

For each policy, define its value:

$$J(\theta) = \mathbb{E}\left[\sum_{t\geq 0} \gamma^t r_t \,\middle|\, \pi_\theta\right].$$

Writing the term of trajectories: $\tau = (s_0, a_0, r_0, s_1, a_1, r_1, \ldots)$

Probability of a trajectory $p(\tau;\theta) = \pi_\theta(a_0 \mid s_0)\, p(s_1 \mid s_0, a_0) \times \pi_\theta(a_1 \mid s_1)\, p(s_2 \mid s_1, a_1) \times \pi_\theta(a_2 \mid s_2)\, p(s_3 \mid s_2, a_2) \times \ldots$

Reward of a trajectory $r(\tau) = \sum_{t\geq 0} \gamma^t r_t$

$$J(\theta) = \mathbb{E}\left[\sum_{t\geq 0} \gamma^t r_t \,\middle|\, \pi_\theta\right] = \mathbb{E}_{\tau\sim p(\tau;\theta)}\left[r(\tau)\right].$$

Adapted from Paul liang

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Adapted from Paul liang

But why does it work?

Formally, let's define a class of parameterized policies $\Pi = \{\pi_\theta, \theta \in \mathbb{R}^m\}$

For each policy, define its value:

$$J(\theta) = \mathbb{E}\left[\sum_{t \geq 0} \gamma^t r_t | \pi_\theta\right]$$

We want to find the optimal policy $\theta^* = \arg\max_\theta J(\theta)$

How can we do this?

Gradient ascent on policy parameters

Adapted from Paul liang

REINFORCE algorithm

Expected reward: $J(\theta) = \mathbb{E}_{\tau \sim p(\tau;\theta)} [r(\tau)]$

$$= \int_\tau r(\tau) p(\tau;\theta) \, d\tau$$

$$p(\tau;\theta) = \prod_{t \geq 0} p(s_{t+1}|s_t, a_t) \pi_\theta(a_t|s_t)$$

Now let's differentiate this: $\nabla_\theta J(\theta) = \int_\tau r(\tau) \nabla_\theta p(\tau;\theta) \, d\tau$ Intractable! Gradient of an expectation is problematic when p depends on θ

However, we can use a nice trick: $\nabla_\theta p(\tau;\theta) = p(\tau;\theta) \frac{\nabla_\theta p(\tau;\theta)}{p(\tau;\theta)} = p(\tau;\theta) \nabla_\theta \log p(\tau;\theta)$

If we inject this back:

$$\nabla_\theta J(\theta) = \int_\tau (r(\tau) \nabla_\theta \log p(\tau;\theta)) \, p(\tau;\theta) \, d\tau$$

$$= \mathbb{E}_{\tau \sim p(\tau;\theta)} [r(\tau) \nabla_\theta \log p(\tau;\theta)]$$

Tractable :-)

Adapted from Paul liang

REINFORCE algorithm

Can we compute these without knowing the transition probabilities?

We have: $p(\tau;\theta) = \prod_{t\geq 0} p(s_{t+1}|s_t, a_t)\pi_\theta(a_t|s_t)$

Thus: $\log p(\tau;\theta) = \sum_{t\geq 0} (\log p(s_{t+1}|s_t, a_t) + \log \pi_\theta(a_t|s_t))$

And when differentiating: $\nabla_\theta \log p(\tau;\theta) = \sum_{t\geq 0} \nabla_\theta \log \pi_\theta(a_t|s_t)$

Doesn't depend on transition probabilities

Therefore when sampling a trajectory, we can estimate gradients:

$$\nabla_\theta J(\theta) = \mathbb{E}_{\tau\sim p(\tau;\theta)} [r(\tau)\nabla_\theta \log p(\tau;\theta)] \approx \sum_{t\geq 0} r(\tau)\nabla_\theta \log \pi_\theta(a_t|s_t)$$

Adapted from Paul liang

Intuition for Policy Gradient

Gradient estimator:

$$\nabla_\theta J(\theta) \approx \sum_{t \geq 0} r(\tau) \nabla_\theta \log \pi_\theta(a_t | s_t)$$

Interpretation:

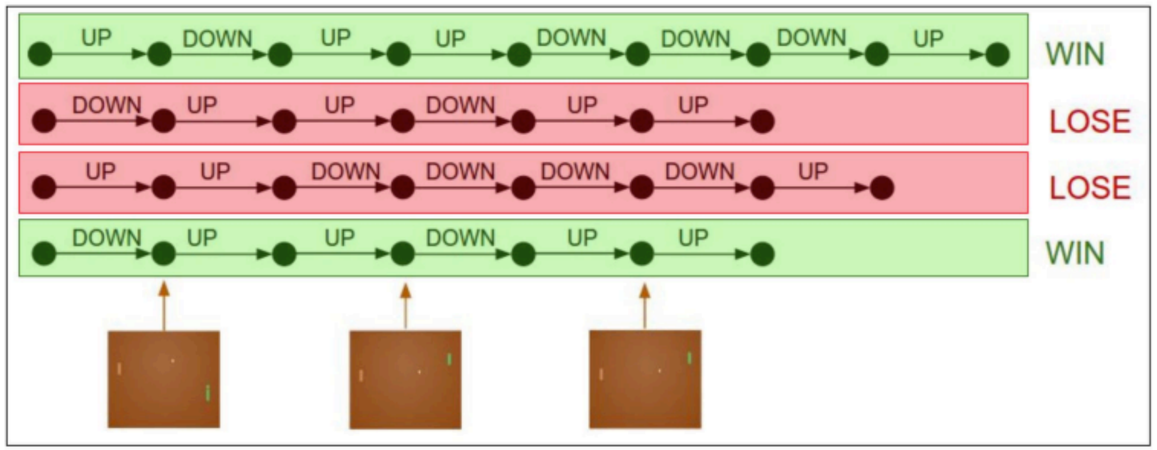
- If **r(trajectory)** is high, push up the probabilities of the actions seen
- If **r(trajectory)** is low, push down the probabilities of the actions seen

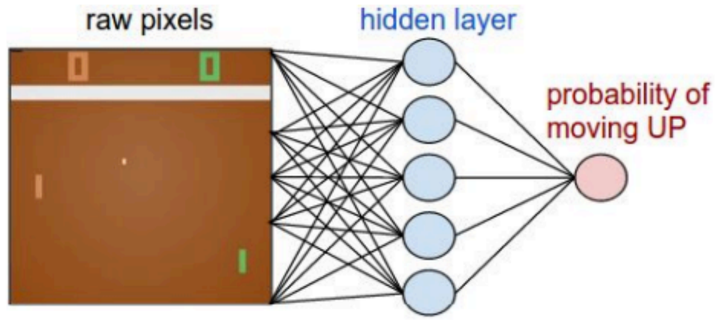
Pretend every action we took here was the correct label.

maximize: $\log p(y_i \mid x_i)$

Pretend every action we took here was the wrong label.

maximize: $(-1) * \log p(y_i \mid x_i)$





$$\sum_i A_i * \log p(y_i | x_i)$$

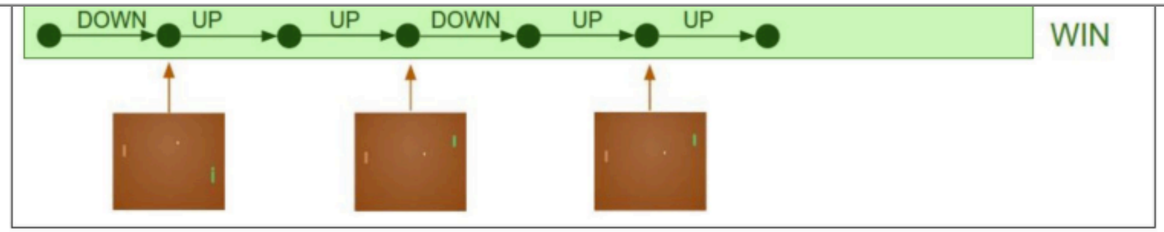
Adapted from Paul liang

Intuition for Policy Gradient

Gradient estimator: $\nabla_\theta J(\theta) \approx \sum r(\tau) \nabla_\theta \log \pi_\theta(a_t|s_t)$

Algorithm 1 REINFORCE Algorithm

1: Initialize a policy π_θ
2: **while** NOT stopping criterion **do**
3: Collect N trajectories $\{\tau^i\}$ from the environment using π_θ
4: Calculate $\nabla_\theta \mathcal{J}(\pi_\theta)$ using Equation (16)
5: Update $\theta = \theta + \alpha \nabla_\theta \mathcal{J}(\pi_\theta)$
6: **end while**



$$\sum_i A_i * \log p(y_i|x_i)$$

Adapted from Paul liang

Policy Gradient Method- Proximal Policy Optimization

Proximal Policy Optimization Algorithms

John Schulman, Filip Wolski, Prafulla Dhariwal, Alec Radford, Oleg Klimov
OpenAI
{joschu, filip, prafulla, alec, oleg}@openai.com

Abstract

We propose a new family of policy gradient methods for reinforcement learning, which alternate between sampling data through interaction with the environment, and optimizing a "surrogate" objective function using stochastic gradient ascent. Whereas standard policy gradient methods perform one gradient update per data sample, we propose a novel objective function that enables multiple epochs of minibatch updates. The new methods, which we call proximal policy optimization (PPO), have some of the benefits of trust region policy optimization (TRPO), but they are much simpler to implement, more general, and have better sample complexity (empirically). Our experiments test PPO on a collection of benchmark tasks, including simulated robotic locomotion and Atari game playing, and we show that PPO outperforms other online policy gradient methods, and overall strikes a favorable balance between sample complexity, simplicity, and wall-time.

Policy Gradient updates:

$$\theta \leftarrow \theta + \alpha \nabla_\theta \log \pi_\theta(a|s) A(s,a)$$

Issue:

- Large update → policy changes too much
- Training becomes unstable
- Performance can collapse

Key idea of PPO:
From "Increase probability of good actions." To "increase probability of good action but don't move too far from the old policy."

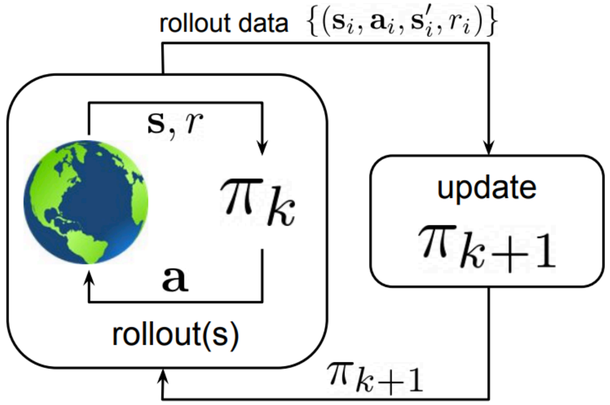
$$r_t(\theta) = \frac{\pi_\theta(a_t \mid s_t)}{\pi_{\theta_{\text{old}}}(a_t \mid s_t)}$$

Clipping prevents large destructive updates

$$L^{\text{CLIP}}(\theta) = \mathbb{E}_t \left[\min \left(r_t(\theta)\hat{A}_t, \text{clip}\left(r_t(\theta), 1-\epsilon, 1+\epsilon\right)\hat{A}_t \right) \right]$$

The difference kinds of RL algorithms

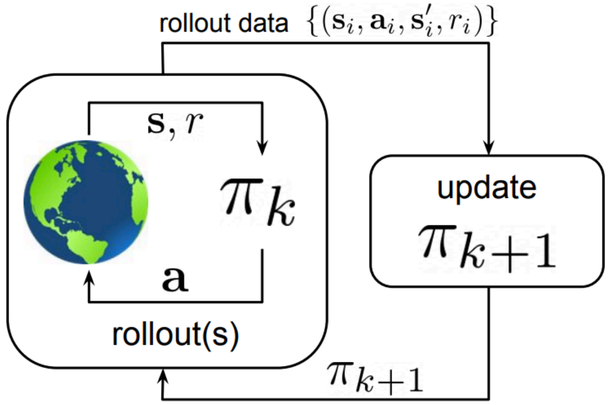
On-policy RL



[SARSA, REINFORCE, PG]

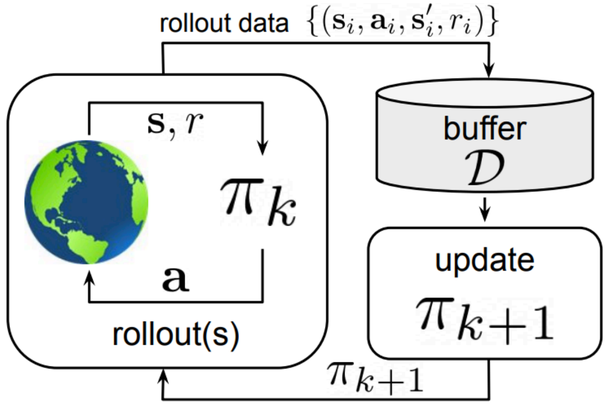
The difference kinds of RL algorithms

On-policy RL



[SARSA, REINFORCE, PG]

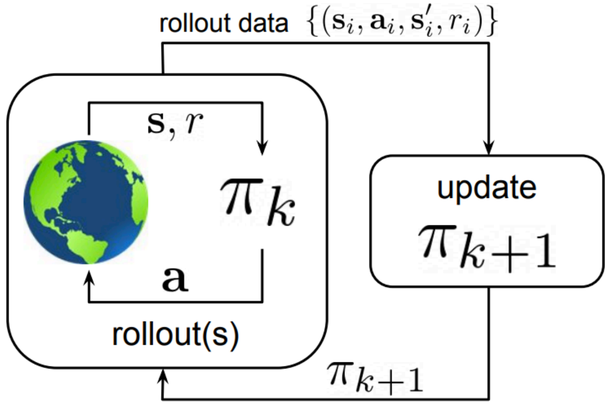
Off-policy RL



[Q-learning, DQN, and etc..]

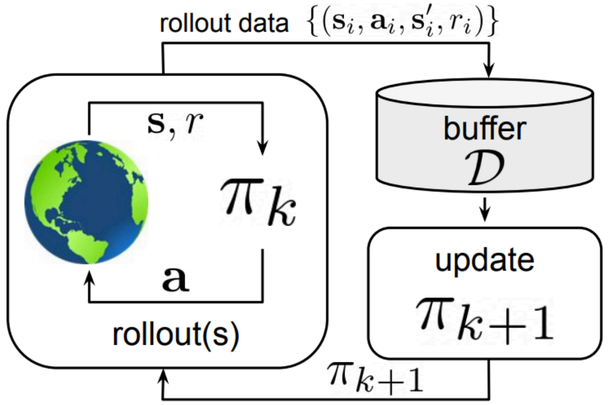
The difference kinds of RL algorithms

On-policy RL



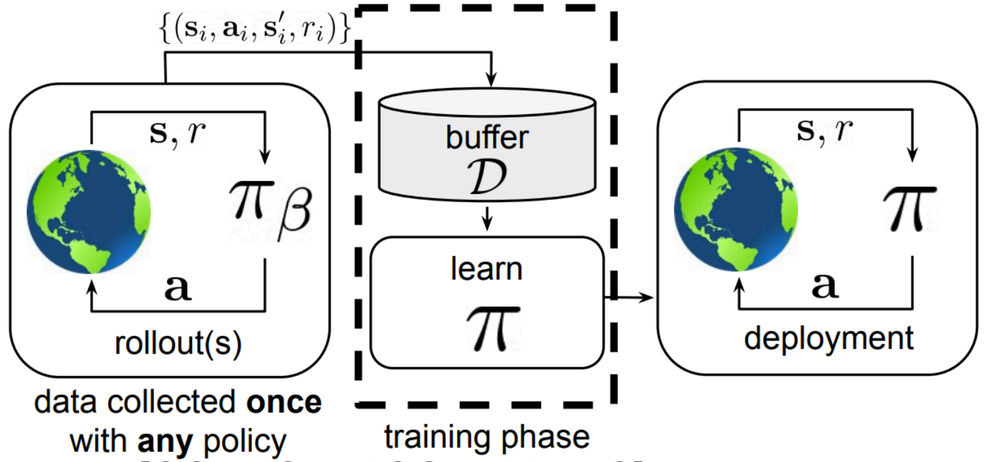
[SARSA, REINFORCE, PG]

Off-policy RL



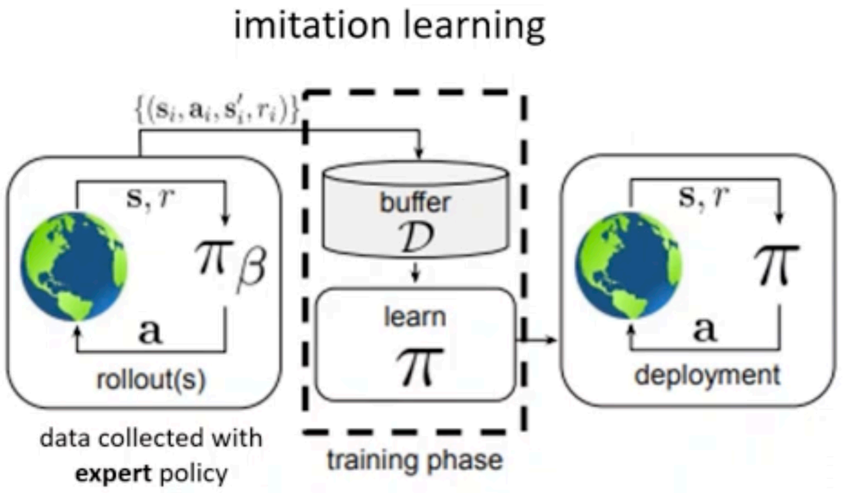
[Q-learning, DQN, and etc..]

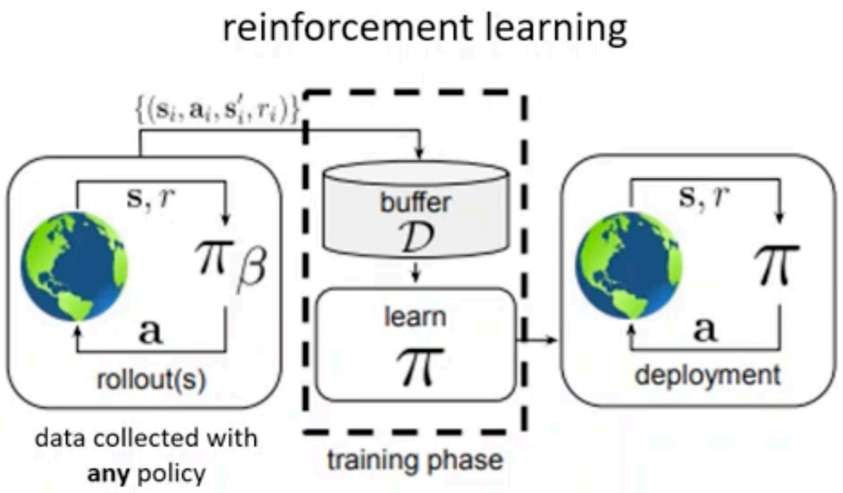
Offline RL

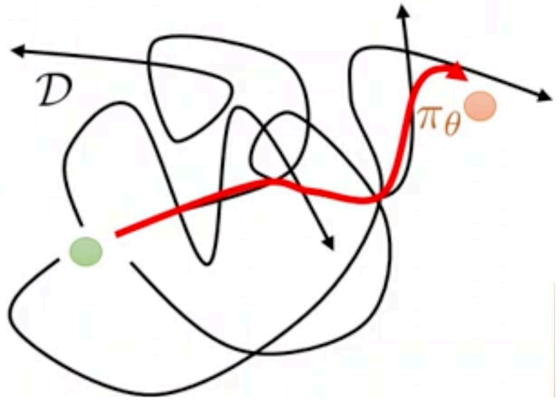


[CQL, IQL, BCQ, TD3+BC]

What's the difference between imitation learning and reinforcement learning?





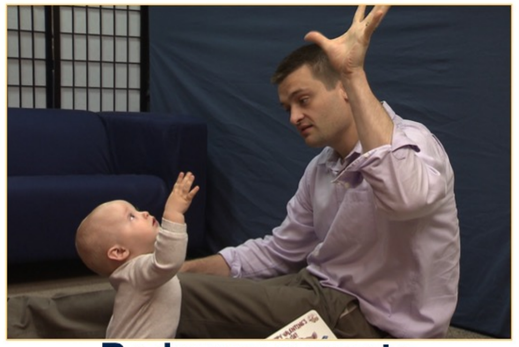


Imitation Learning

Do we do imitation learning?

Facial movements

Vocal imitation



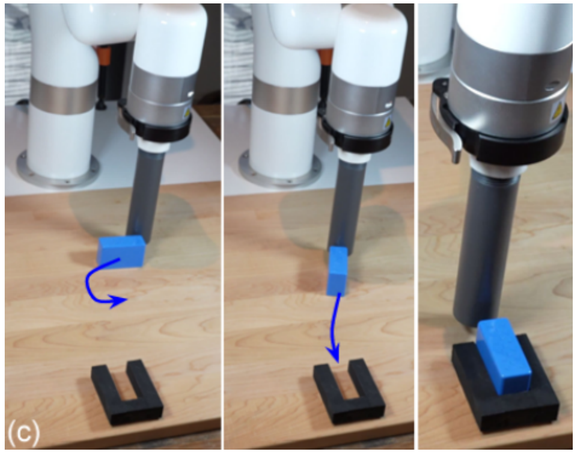
Body movements

Actions on objects

Photo Credit: Meltzoff & Moore, 1977, Science

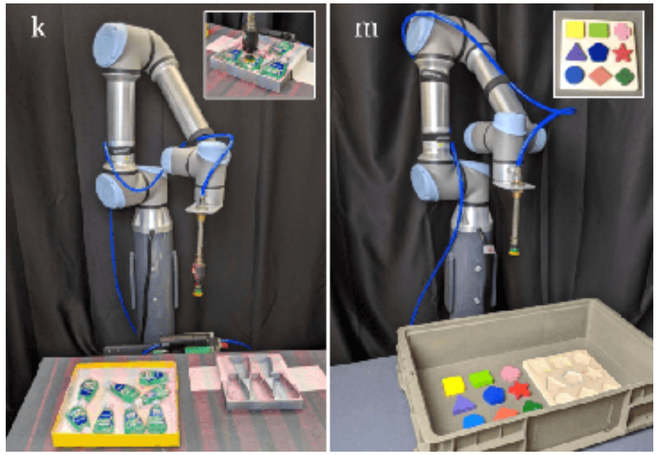
Imitation Learning

Robot in learning in the real world



IBC, Florence et al 2021.

kPAM, Florence et al 2019



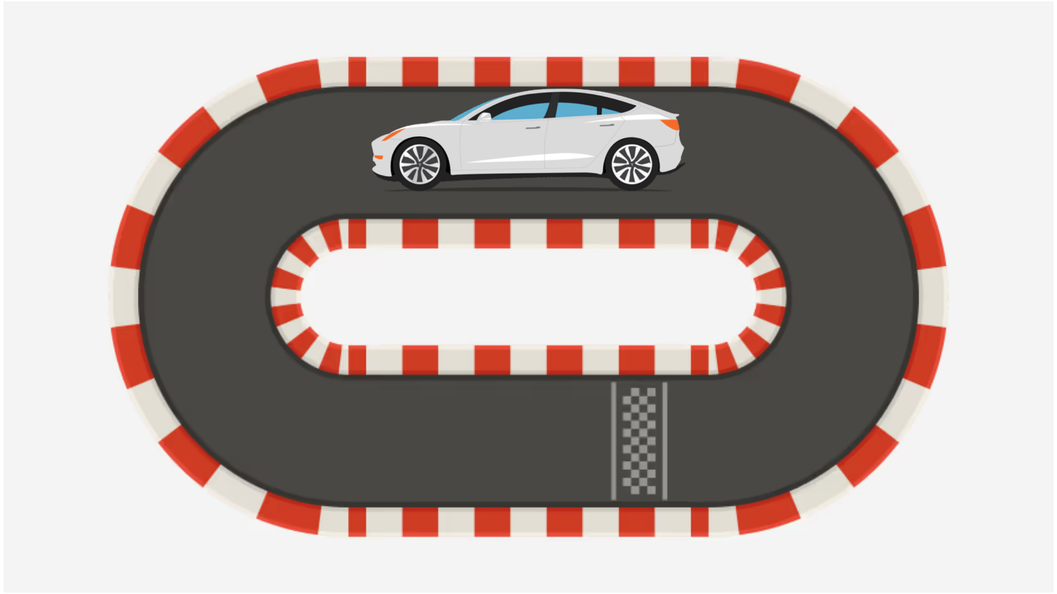
Transporter Network, Zeng et al 2020

RT-2, GDM et al 2023

Imitation Learning

Can we learn a direct mapping?

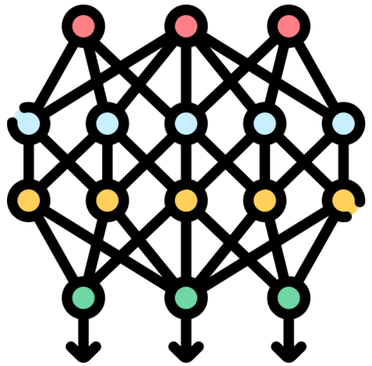
1. Get demonstration from Me driving my Tesla.



Database of “good” trajectories

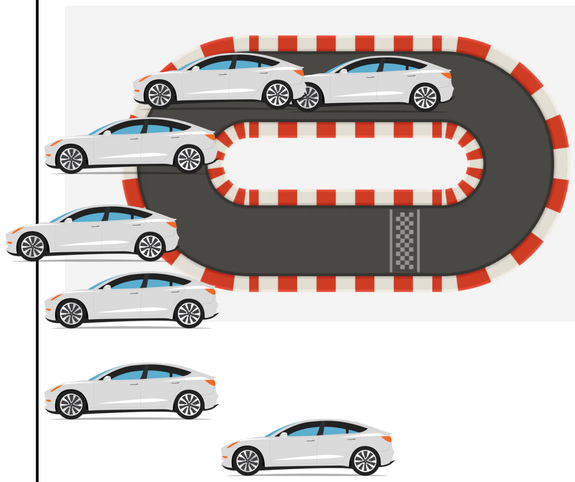
$$s_1, a_1, s_2, a_2 \ldots$$

2. Use favorite supervised learner



Actions

3. Test time



Imitation Learning

Can we learn a direct mapping?

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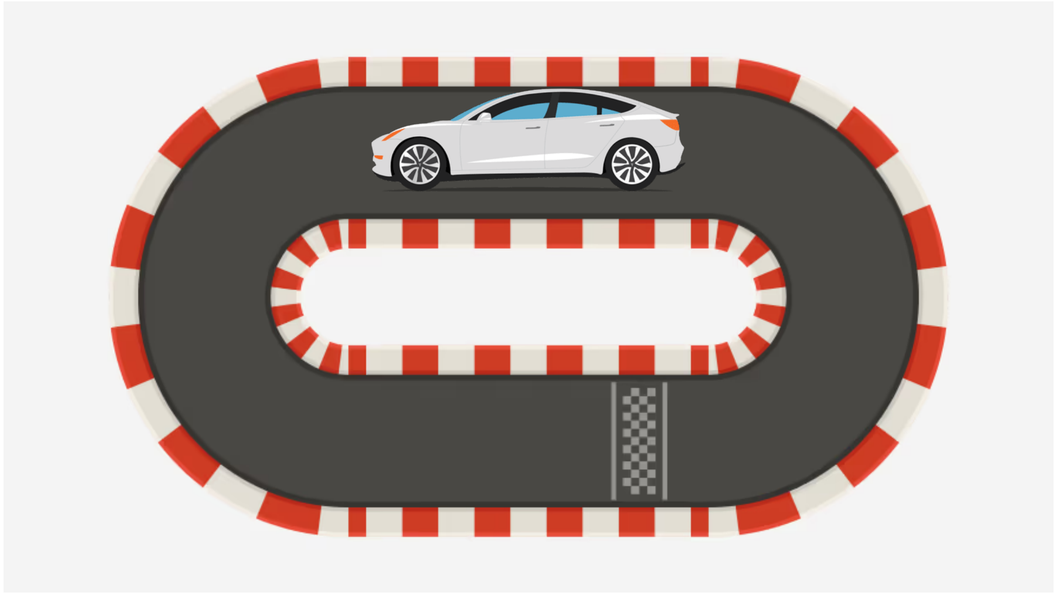

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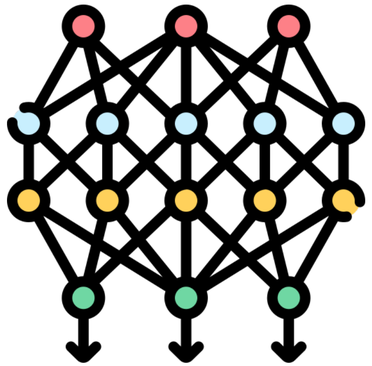

Actions

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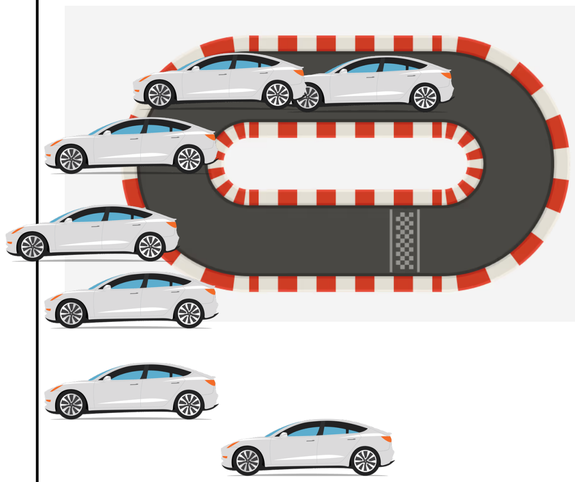
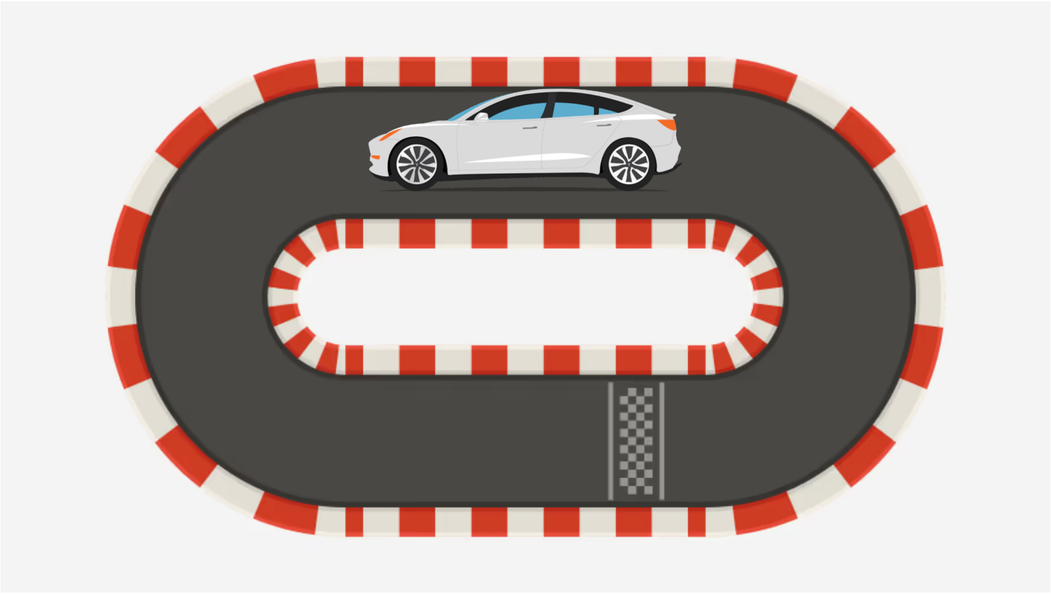


Imitation Learning

What went wrong?



1. Learner follows demonstrations
2. Learner makes a small mistakes
3. Learner enters a state it has NOT SEEN in training data
4. Makes another mistake
5. Repeat, and mistake compound
6. Fails beyond recovery

Imitation Learning

An old problem since 1989

ALVINN:
AN AUTONOMOUS LAND VEHICLE IN A NEURAL NETWORK

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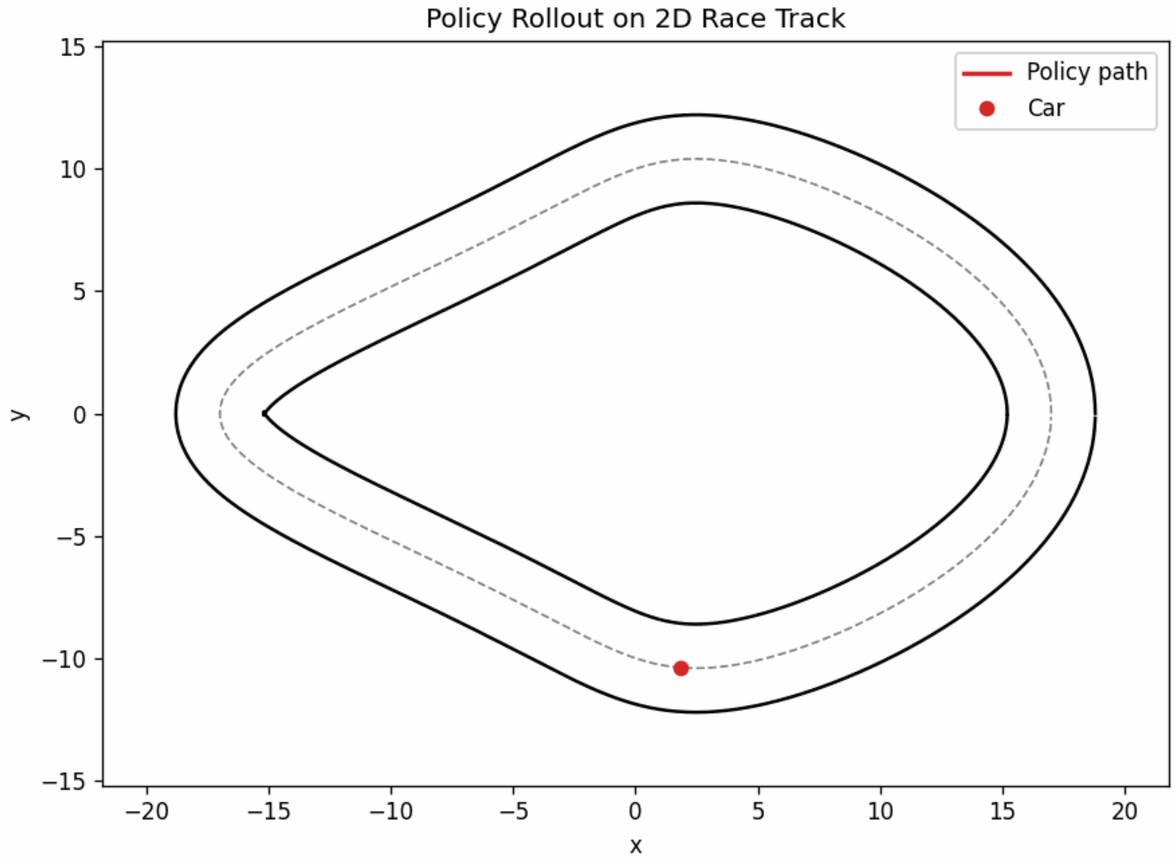
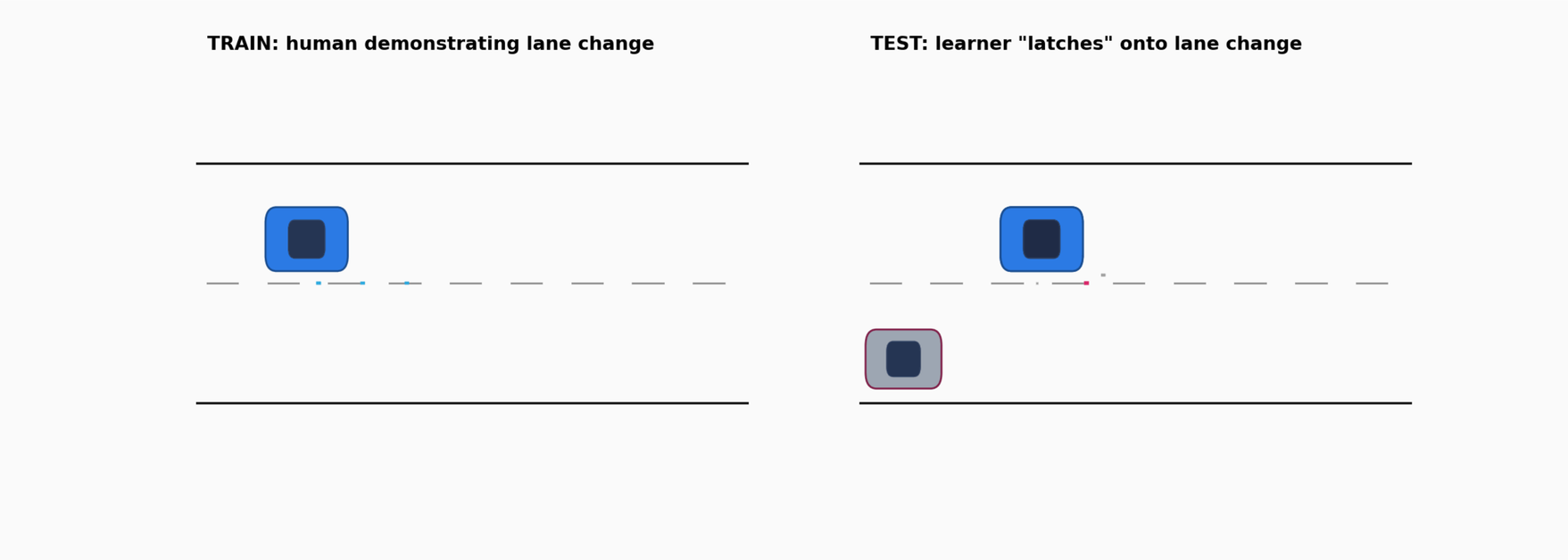
ABSTRACT

ALVINN (Autonomous Land Vehicle In a Neural Network) is a 3-layer back-propagation network designed for the task of road following. Currently ALVINN takes images from a camera and a laser range finder as input and produces as output the direction the vehicle should travel in order to follow the road. Training has been conducted using simulated road images. Successful tests on the Carnegie Mellon autonomous navigation test vehicle indicate that the network can effectively follow real roads under certain field conditions. The representation developed to perform the task differs dramatically when the network is trained under various conditions, suggesting the possibility of a novel adaptive autonomous navigation system capable of tailoring its processing to the conditions at hand.



There are difficulties involved with training "on-the-fly" with real images. If the network is not presented with sufficient variability in its training exemplars to cover the conditions it is likely to encounter when it takes over driving from the human operator, it will not develop a sufficiently robust representation and will perform poorly. In addition, the network must not solely be shown examples of accurate driving, but also how to recover (i.e. return to the road center) once a mistake has been made. Partial initial training on a variety of simulated road images should help eliminate these difficulties and facilitate better performance.

Imitation Learning



STATE

Velocity

Clearance

Lane Change

Neural Network

ACTION

Execute Lane Change

In all training data:
Lane departure > 0, execute LC = 1

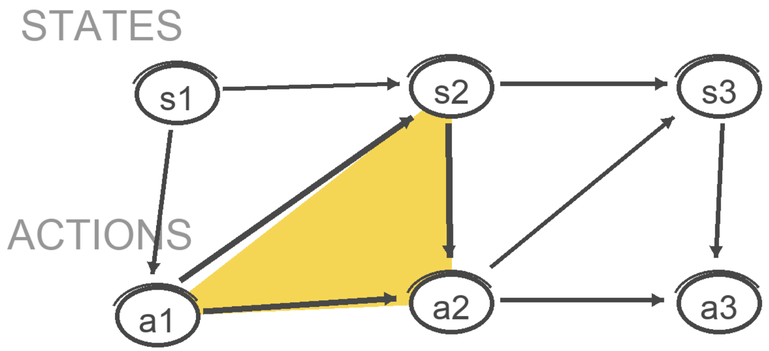
Execute Lane Change

Positive Feedback

Lane Departure

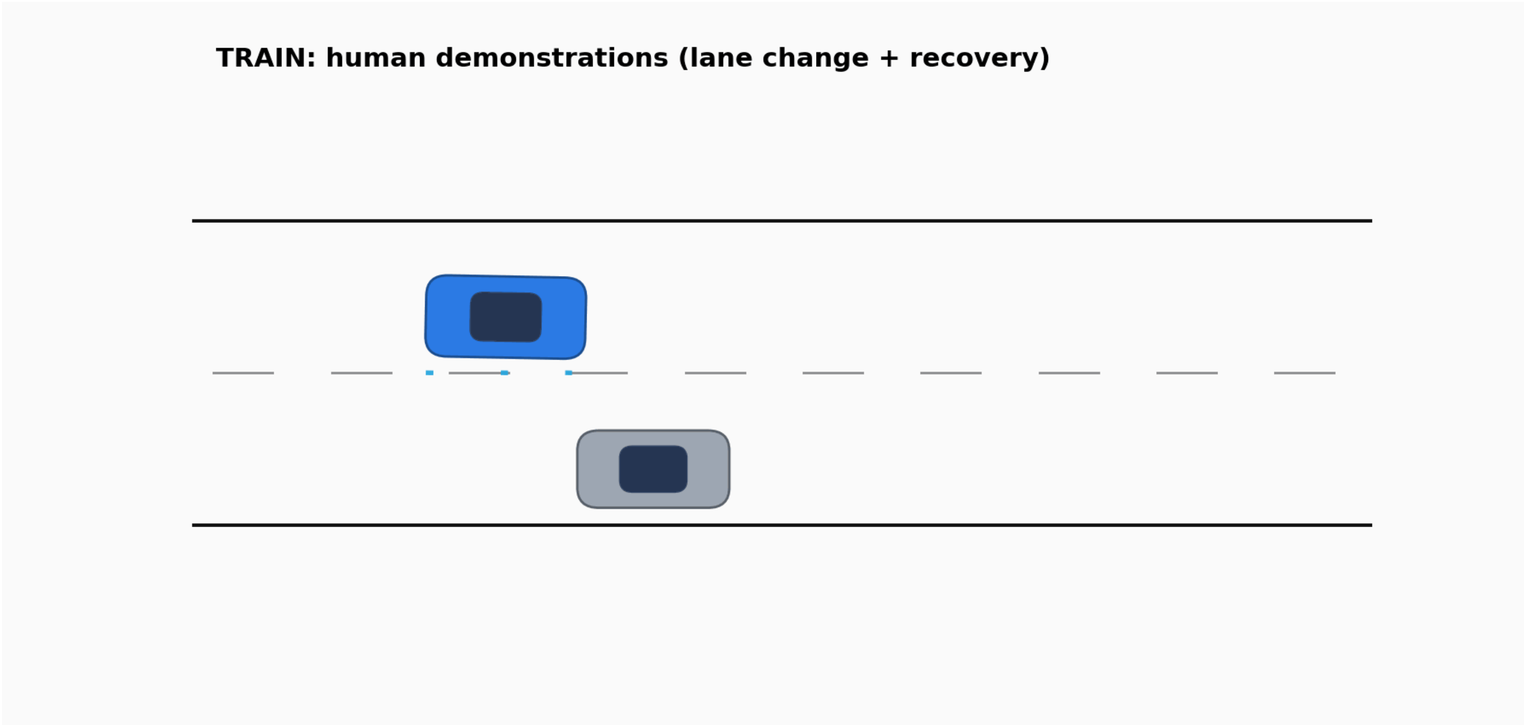
Imitation Learning

Feedback is Ubiquitous



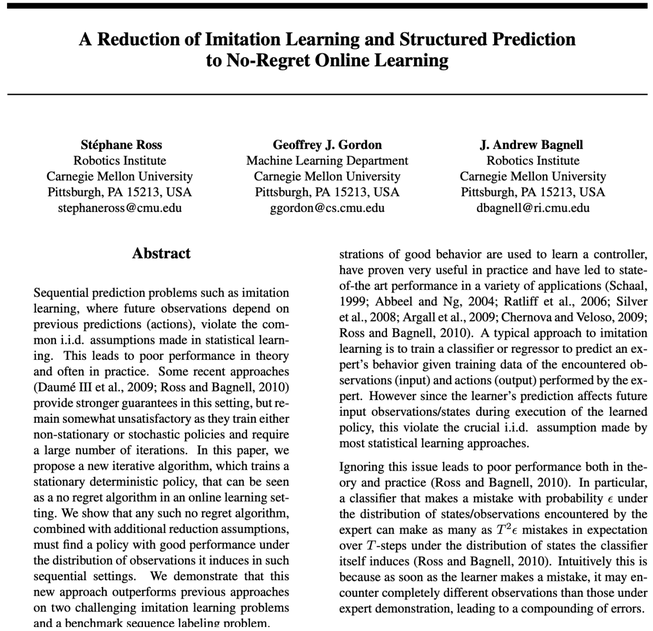
Past action errors feedback into current action through multiple paths.

Is feedback always a problem?



Imitation Learning

What is the problem? Feedback drives Covariate Shift.



A Reduction of Imitation Learning and Structured Prediction to No-Regret Online Learning

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Abstract

Sequential prediction problems such as imitation learning, where future observations depend on previous predictions (actions), violate the common i.i.d. assumptions made in statistical learning. This leads to poor performance in theory and often in practice. Some recent approaches (Daumé III et al., 2009; Ross and Bagnell, 2010) provide stronger guarantees in this setting, but remain somewhat unsatisfactory as they train either non-stationary or stochastic policies and require a large number of iterations. In this paper, we propose a new iterative algorithm, which trains a stationary deterministic policy, that can be seen as a no regret algorithm in an online learning setting. We show that any such no regret algorithm, combined with additional reduction assumptions, must find a policy with good performance under the distribution of observations it induces in such sequential settings. We demonstrate that this new approach outperforms previous approaches on two challenging imitation learning problems and a benchmark sequence labeling problem.

strations of good behavior are used to learn a controller, have proven very useful in practice and have led to state-of-the art performance in a variety of applications (Schaal, 1999; Abbeel and Ng, 2004; Ratliff et al., 2006; Silver et al., 2008; Argall et al., 2009; Chernova and Veloso, 2009; Ross and Bagnell, 2010). A typical approach to imitation learning is to train a classifier or regressor to predict an expert's behavior given training data of the encountered observations (input) and actions (output) performed by the expert. However since the learner's prediction affects future input observations/states during execution of the learned policy, this violates the crucial i.i.d. assumption made by most statistical learning approaches.

Ignoring this issue leads to poor performance both in theory and practice (Ross and Bagnell, 2010). In particular, a classifier that makes a mistake with probability ϵ under the distribution of states/observations encountered by the expert can make as many as $T^2\epsilon$ mistakes in expectation over T-steps under the distribution of states the classifier itself induces (Ross and Bagnell, 2010). Intuitively this is because as soon as the learner makes a mistake, it may encounter completely different observations than those under expert demonstration, leading to a compounding of errors.

(Ross et al)

Low training error, compounding test error

Lets define things:

- $s_t \in \mathcal{S}$: state at time t
- $a_t \in \mathcal{A}$: action at time t
- $P(s_{t+1} \mid s_t, a_t)$: dynamics (often **unknown**)
- $c(s_t, a_t)$: cost function

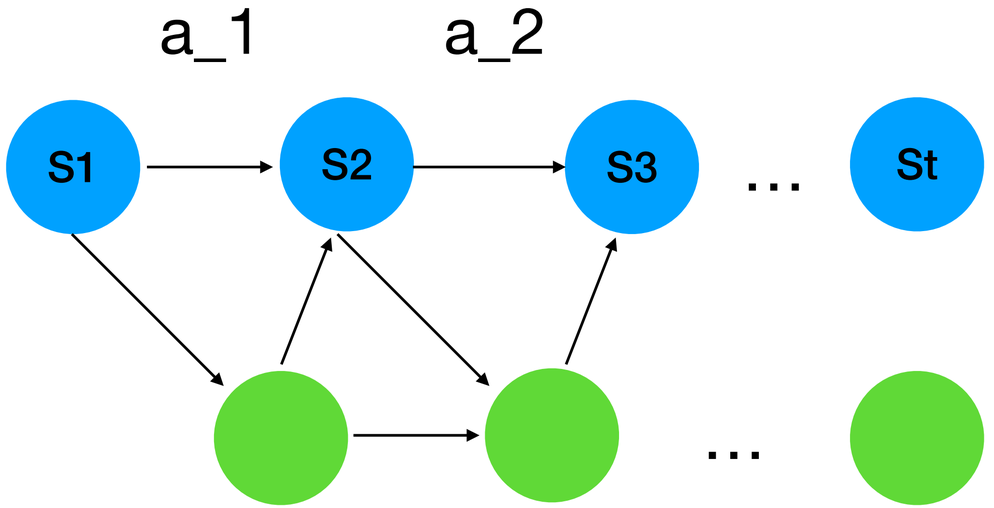
Define a policy and the induced state distribution:

- $\pi : \mathcal{S} \to \mathcal{P}(\mathcal{A})$: a (possibly stochastic) policy mapping states to a distribution over actions.
- d_π^t: the distribution of states you see at time t **when you follow** π.

$$J(\pi) = \sum_{t=1}^{T} \mathbb{E}_{s_t \sim d_\pi^t} \left[c(s_t, a_t) \right] \text{ (with } a_t \sim \pi(\cdot \mid s_t))$$

Imitation Learning

How do we train? Behavior Cloning



- You're given a dataset

$$D = \{(s_t, a_t)\}$$

collected by the **expert/human** policy π^h:

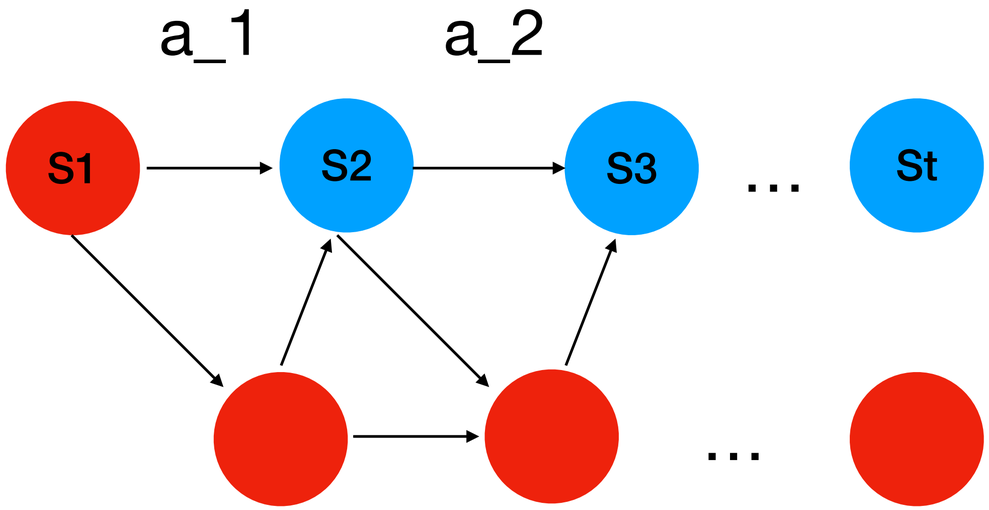
- $a_t \sim \pi^h(\cdot \mid s_t)$
- $s_t \sim d^t_{\pi^h}$

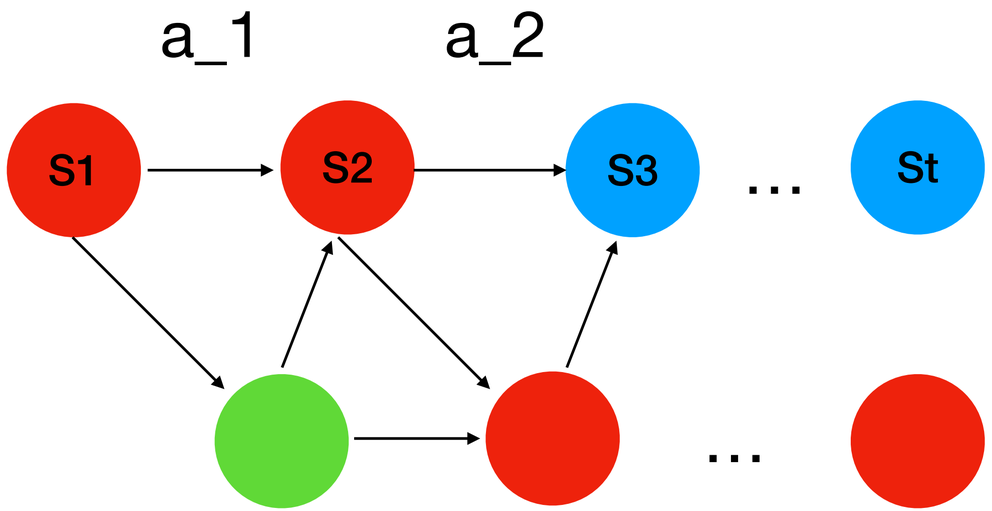
- BC learns $\hat{\pi}$ by minimizing a supervised loss:

$$\hat{\pi} = \arg\min_{\pi} \mathbb{E}_{(s,a)\sim D}\left[\ell\big(a, \pi(s)\big)\right]$$

Imitation Learning

What happen at test time?





Imitation Learning

How to fix? Interactively query human (aka DAGGER)

A Reduction of Imitation Learning and Structured Prediction to No-Regret Online Learning

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DAGGER: Dataset Aggregation

Initialize $\mathcal{D} \leftarrow \emptyset$.
Initialize $\hat{\pi}_1$ to any policy in Π.
for $i = 1$ **to** N **do**
  Let $\pi_i = \beta_i \pi^* + (1 - \beta_i)\hat{\pi}_i$.
  Sample T-step trajectories using π_i.
  Get dataset $\mathcal{D}_i = \{(s, \pi^*(s))\}$ of visited states by π_i and actions given by expert.
  Aggregate datasets: $\mathcal{D} \leftarrow \mathcal{D} \bigcup \mathcal{D}_i$.
  Train classifier $\hat{\pi}_{i+1}$ on $\mathcal{D}$.
end for
Return best $\hat{\pi}_i$ on validation.

DAGGER algo

Theory: Low training error -> guarantee a low test time error.

Imitation Learning

State Space

distribution induced by expert policy

DAgger (Dataset Aggregation)

Step 0: seed with expert dataset D0

Collect trajectories from expert policy pi*.

D <- {(s, pi*(s)) sampled from expert}

Dataset size |D| = 0

Mixing beta = 1.00 (expert -> learner)

Key idea:
Collect states visited by current policy
and label them with expert actions.

Imitation Learning

What is the problem with DAGGER?

Require to query the human every time.

Can we counter Covariate Shift using Offline Cache human demo?

Feedback in Imitation Learning:
The Three Regimes of Covariate Shift

Jonathan Spencer [1] **Sanjiban Choudhury** [2] **Arun Venkatraman** [2] **Brian Ziebart** [2] **J. Andrew Bagnell** [2 3]

Abstract

Imitation learning practitioners have often noted that conditioning policies on previous actions leads to a dramatic divergence between "held out" error and performance of the learner *in situ*. Interactive approaches (Ross et al., 2011) can provably address this divergence but require repeated querying of a demonstrator. Recent work identifies this divergence as stemming from a "causal confound" (de Haan et al., 2019; Wen et al., 2020) in predicting the current action, and seek to ablate causal aspects of current state using tools from causal inference. In this work, we argue instead that this divergence is simply another manifestation of *covariate shift*, exacerbated particularly by settings of *feedback* between decisions and input

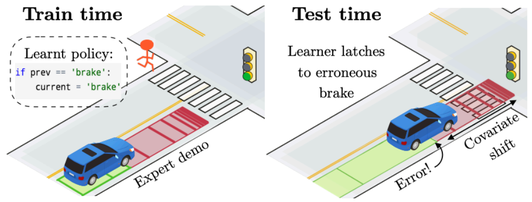


Figure 1. A common example of feedback-driven covariate shift in self-driving. At train time, the robot learns that the previous action (BRAKE) accurately predicts the current action almost all the time. At test time, when the learner mistakenly chooses to BRAKE, it continues to choose BRAKE, creating a bad feedback cycle that causes it to diverge from the expert.

EASY
(Realizable)

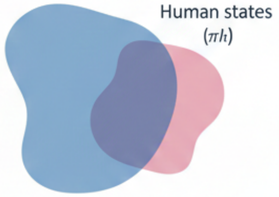
$\pi^h h \in \Pi$

Demonstrator in policy class.

Solution

Behavioral Cloning with large datasets.

GOLDILOCKS
(Finite Density Ratio)

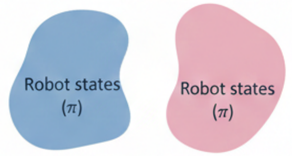


$\Pi\, d_\pi(s)\, \frac{1}{s}\, d_{\pi h}(s)\, \Pi < \theta$

FINITE ⟶ density ratio

Interactive Simulator

HARD
(Infinite Densiity Ratio)



$\Pi\, d_\pi(s)\, \frac{1}{\pi}\, d_{\pi h}(s) \rightarrow 0$

INFINITE ⟶ density ratio

Solution

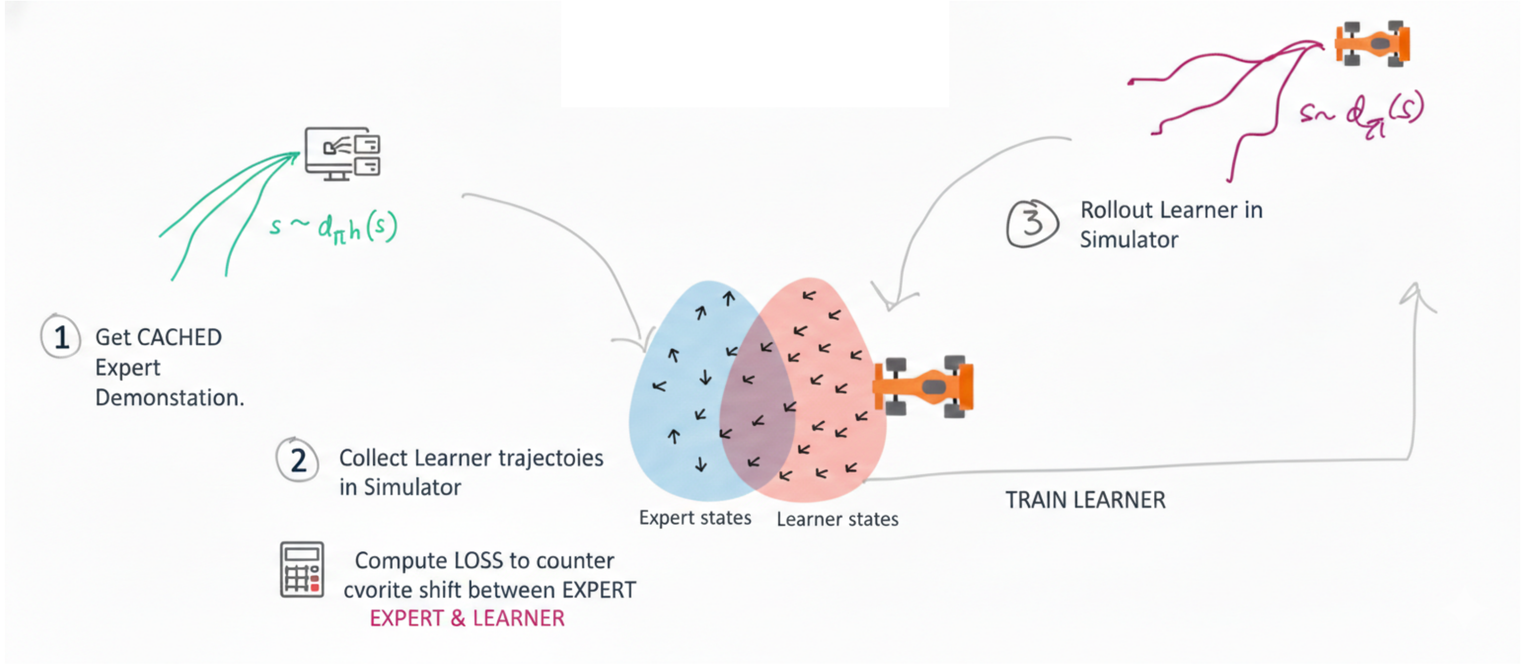
Interactive Expert (e.g., DAgger)

Imitation Learning

ALICE: Aggregate Losses to Imitate Cached Experts

Algorithm 1 ALICE

Input: Cached expert demonstrations $\mathcal{D}_{\text{exp}} = \{(s_t^*, a_t^*)\}$, Simulator $\Sigma : \pi \to \rho_\pi$
Initialize dataset $\mathcal{D} \leftarrow \emptyset$
Initialize learner $\hat{\pi}^1$ to any policy in Π
for $i = 1$ **to** N **do**
  Sample states $s_t \sim \rho^t_{\hat{\pi}^i}$ by running $\hat{\pi}^i$ in simulator Σ
  Compute a dataset of losses $\mathcal{D}_i = \{\ell_t(\pi, \mathcal{D}_{\text{exp}}, \rho^t_{\hat{\pi}^i})\}$.
  Aggregate datasets: $\mathcal{D} \leftarrow \mathcal{D} \bigcup \mathcal{D}_i$
  Train learner $\hat{\pi}^{i+1}$ on aggregated $\mathcal{D}$
end for
Return best $\hat{\pi}^i$ on validation



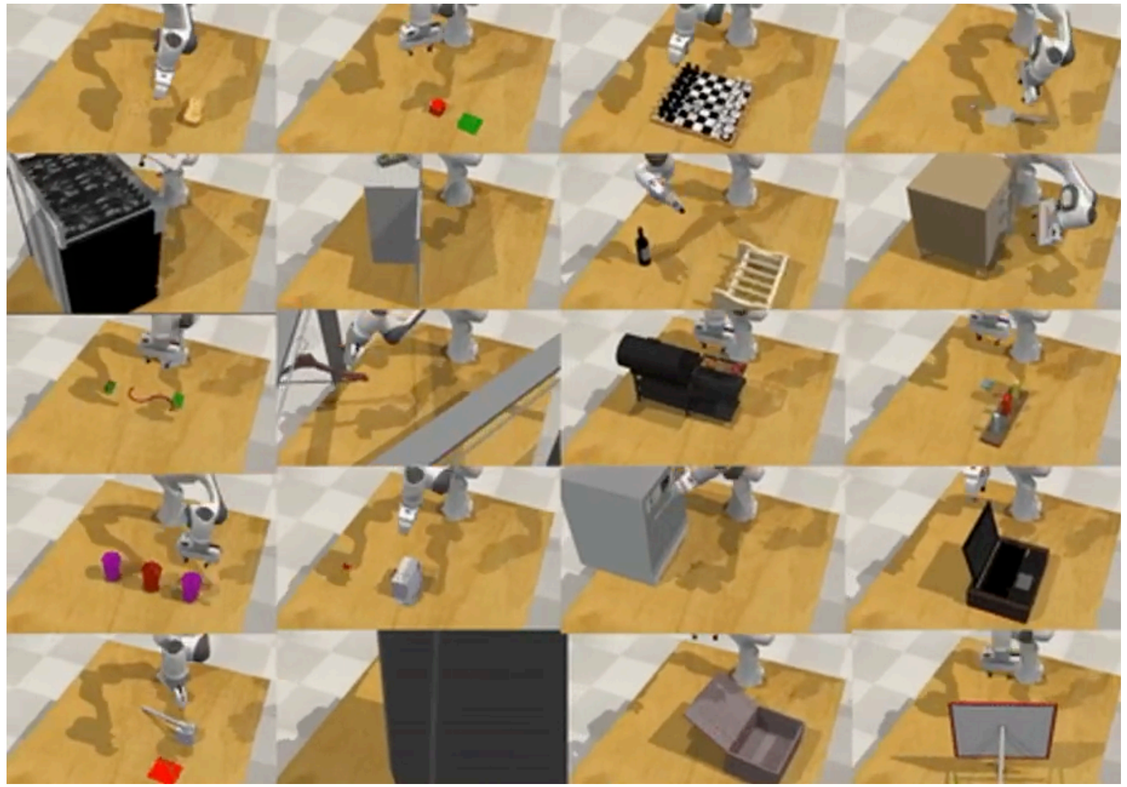
Imitation Learning

Evaluating covariate shift at scale with the Colosseum.

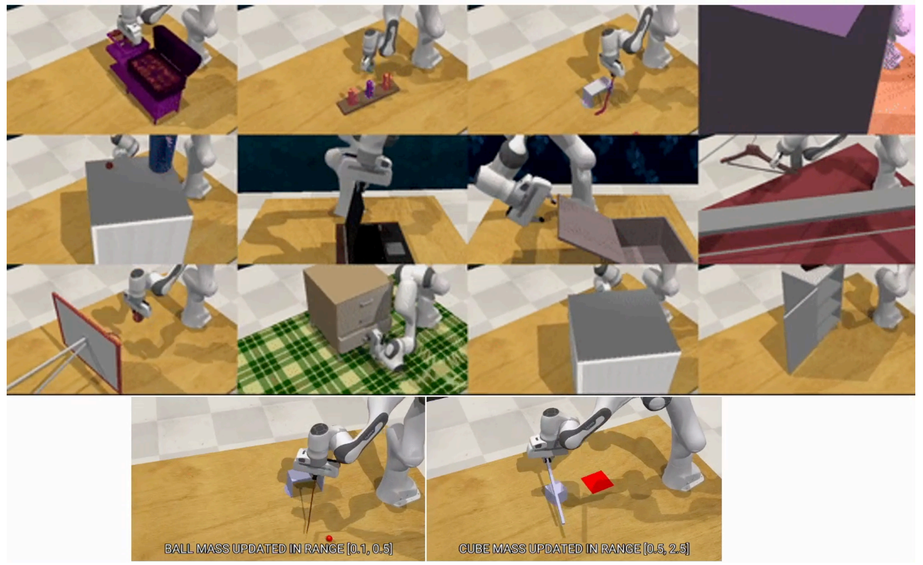
The COLOSSEUM:

A Benchmark for Evaluating Generalization for Robotic Manipulation

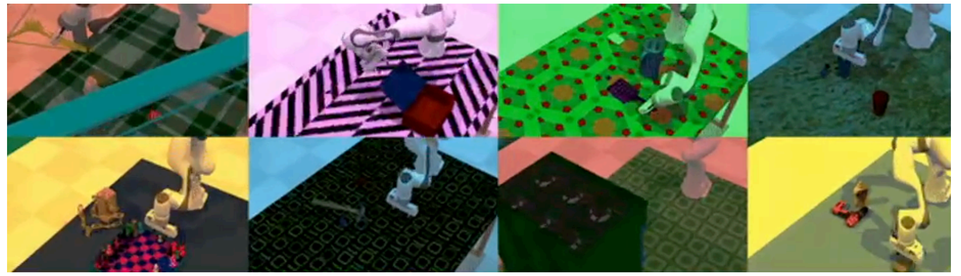
20 RLBench Tasks



Perturbations

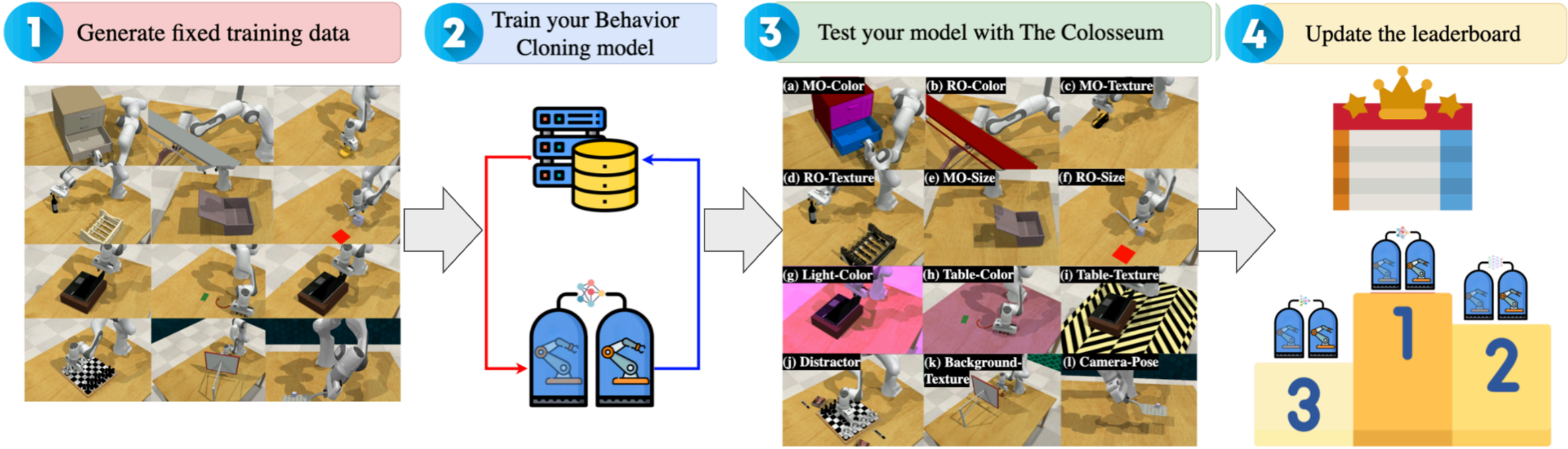


All variations



Imitation Learning

Evaluating covariate shift at scale with the Colosseum.

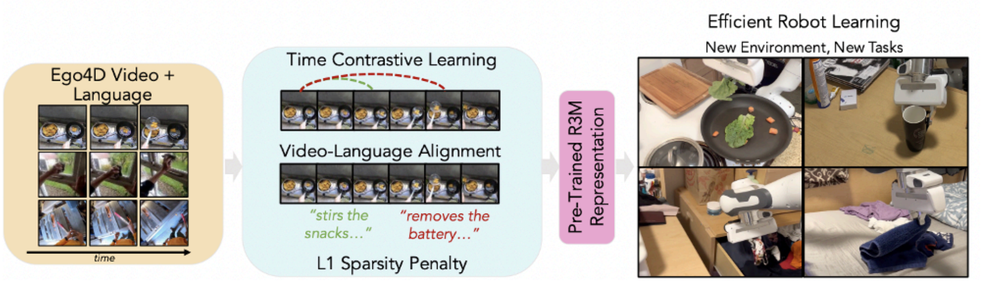


$$p(x_{test}) \neq p(x_{train}) \qquad p(y_{test}|x_{test}) = p(y_{train}|x_{train})$$

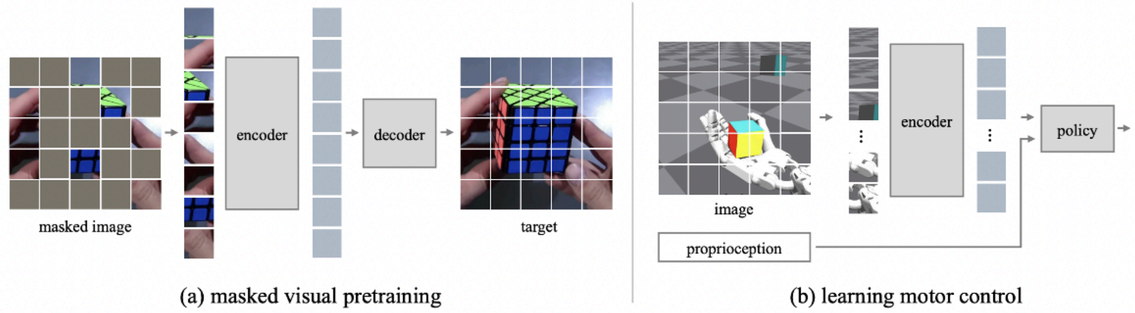
Out-of-Distribution (OoD) formulation for covariate shift

Imitation Learning

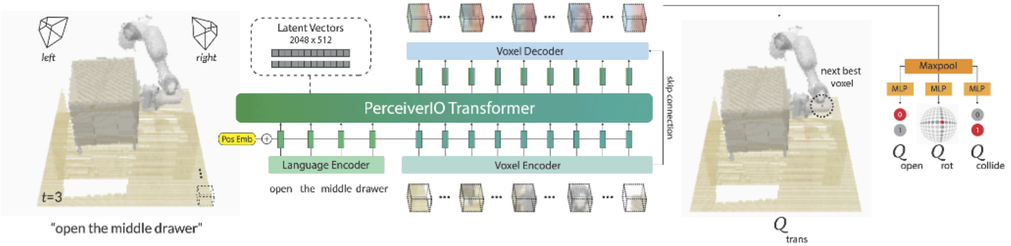
Benchmarking with The Colosseum



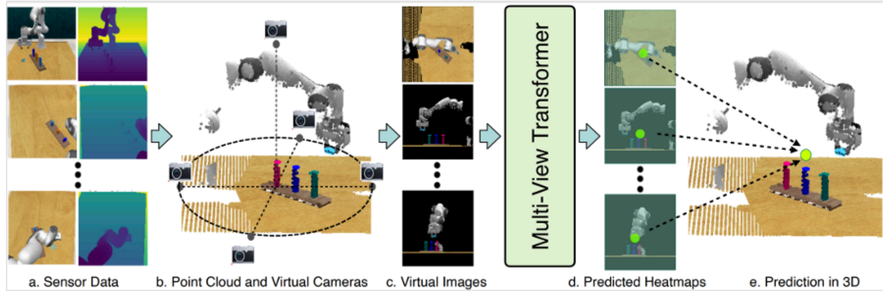
R3M-MLP (Nair et al.)



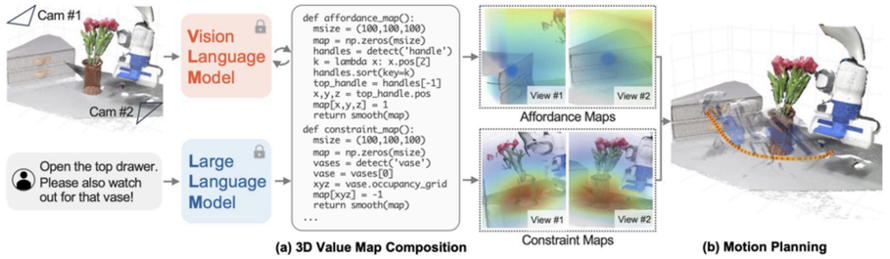
MVP-MLP (Xiao et al.)



PerAct (Shridhar et al.)

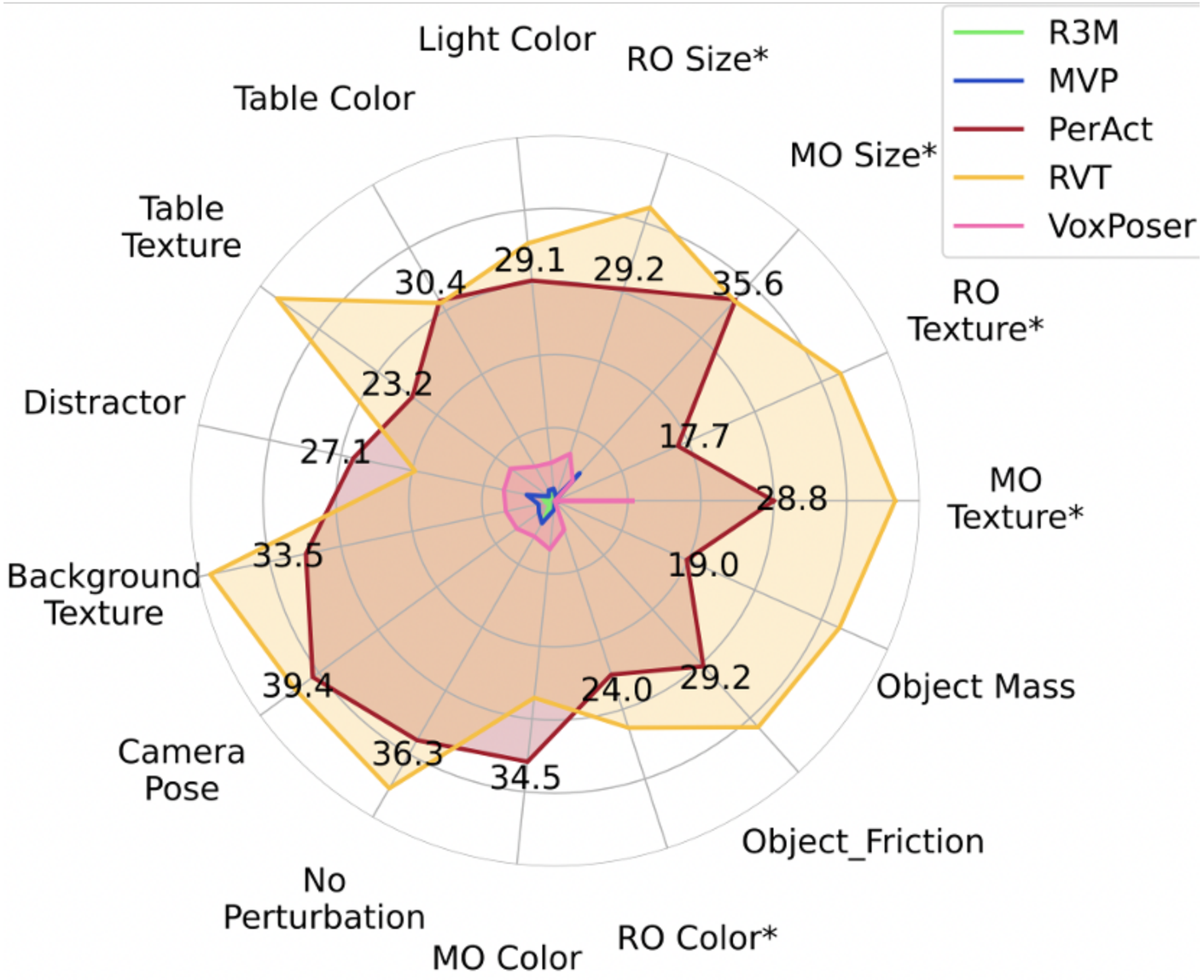


RVT (Goyal et al.)



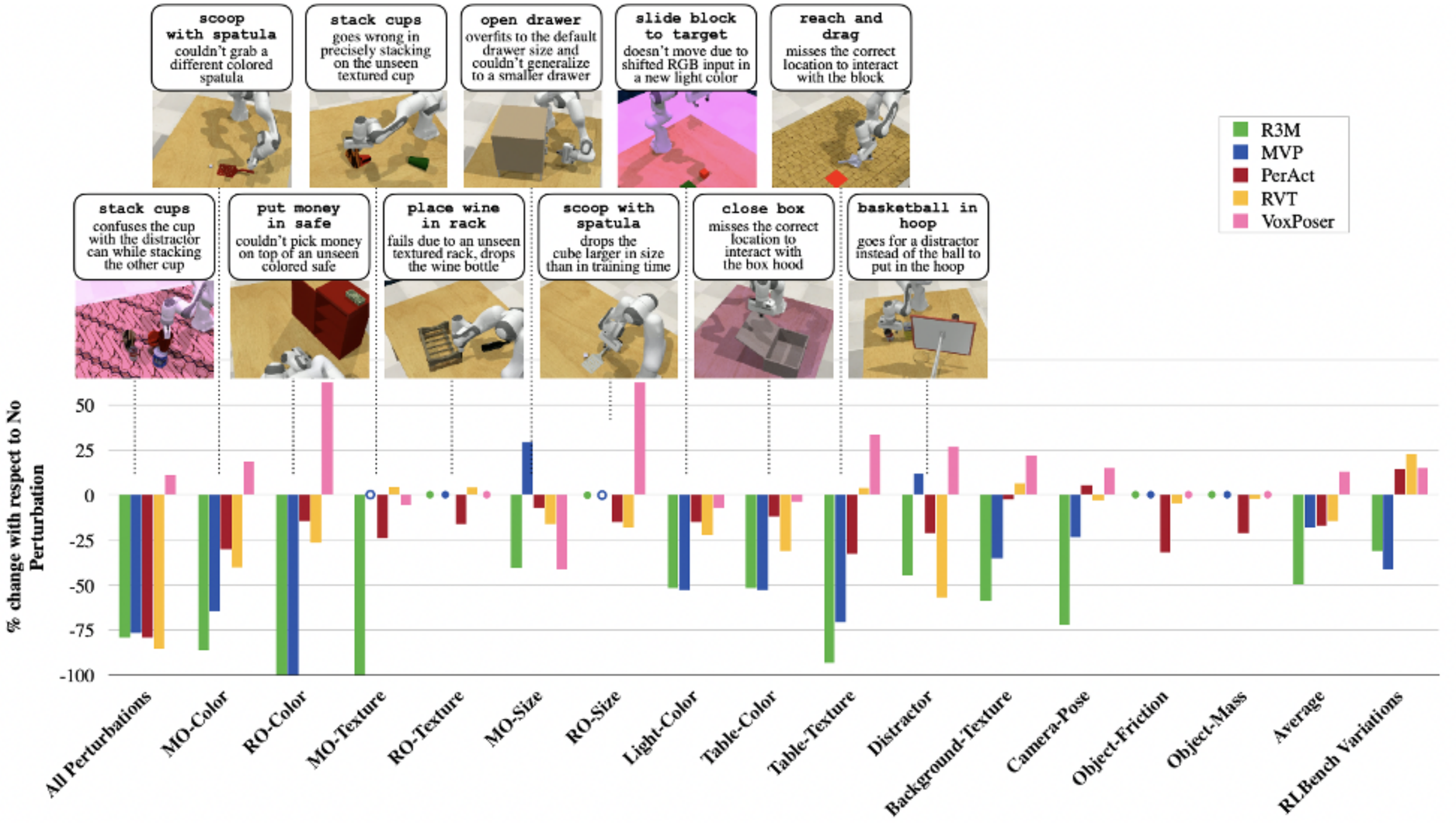
VoxPoser (Huang et al.)

Imitation Learning

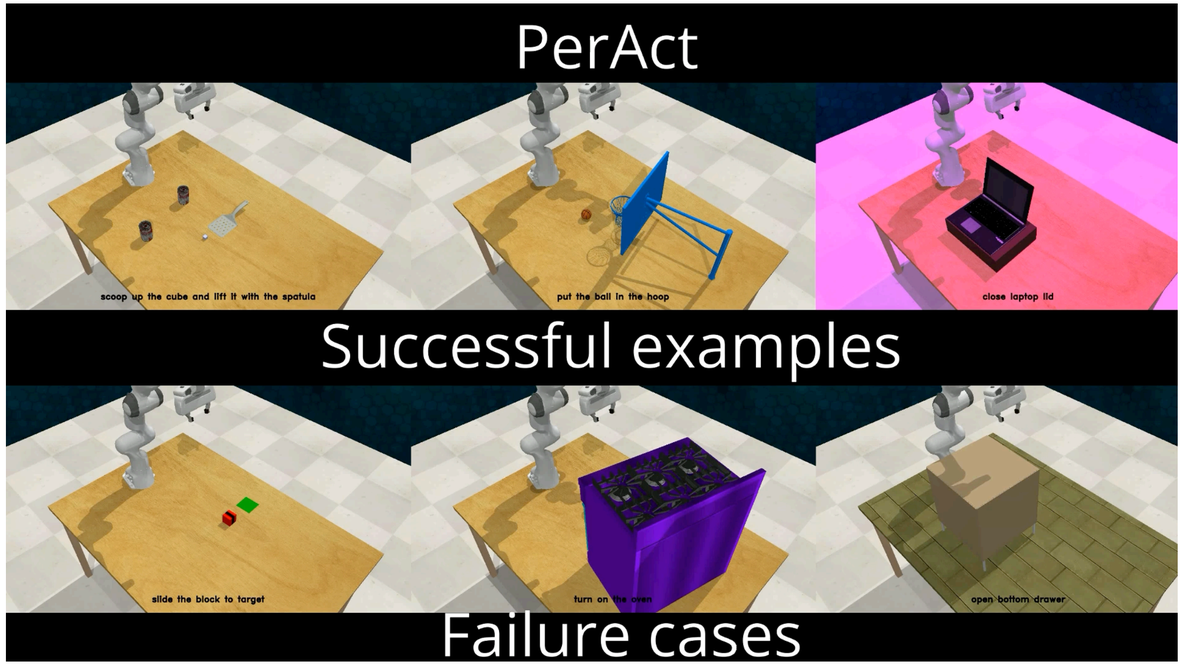


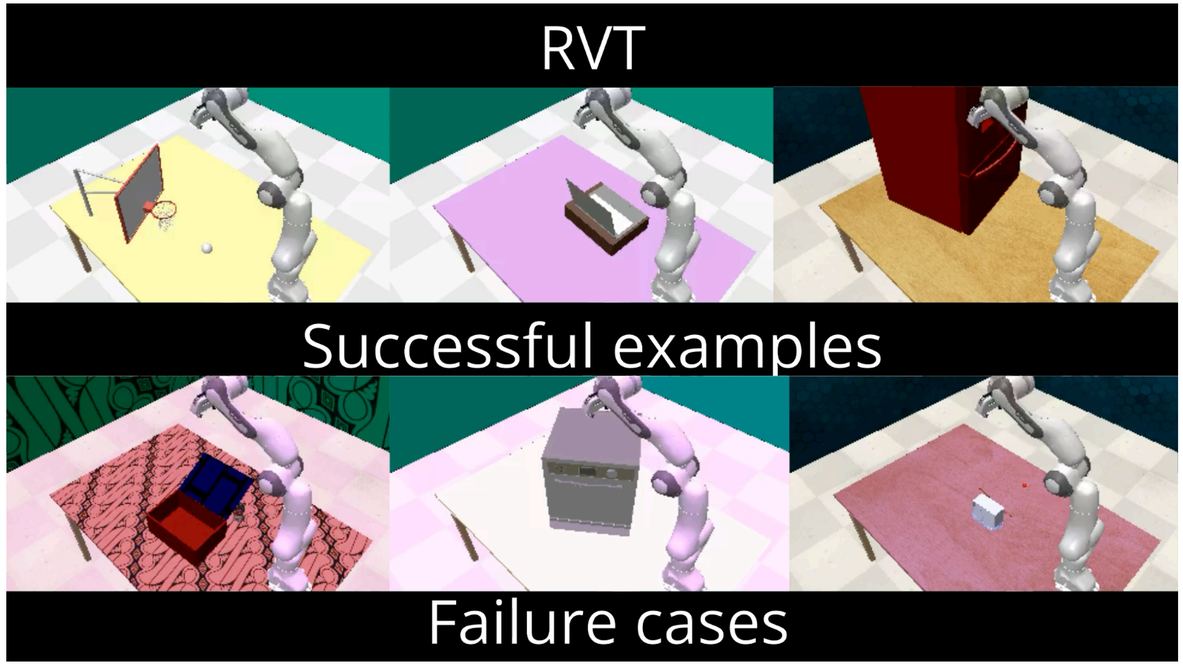
Insights to our evaluation results

- For 2D models, light colors, object textures and camera poses are the most affecting factor.
- For 3D models, they are most affected by color-related perturbation as well as distractors.
- For models that are robust to camera poses, they do not directly learn on capture view.
- 3D models outperform 2D models in general, hence making them more robust to generalization.



Examples of failure modes





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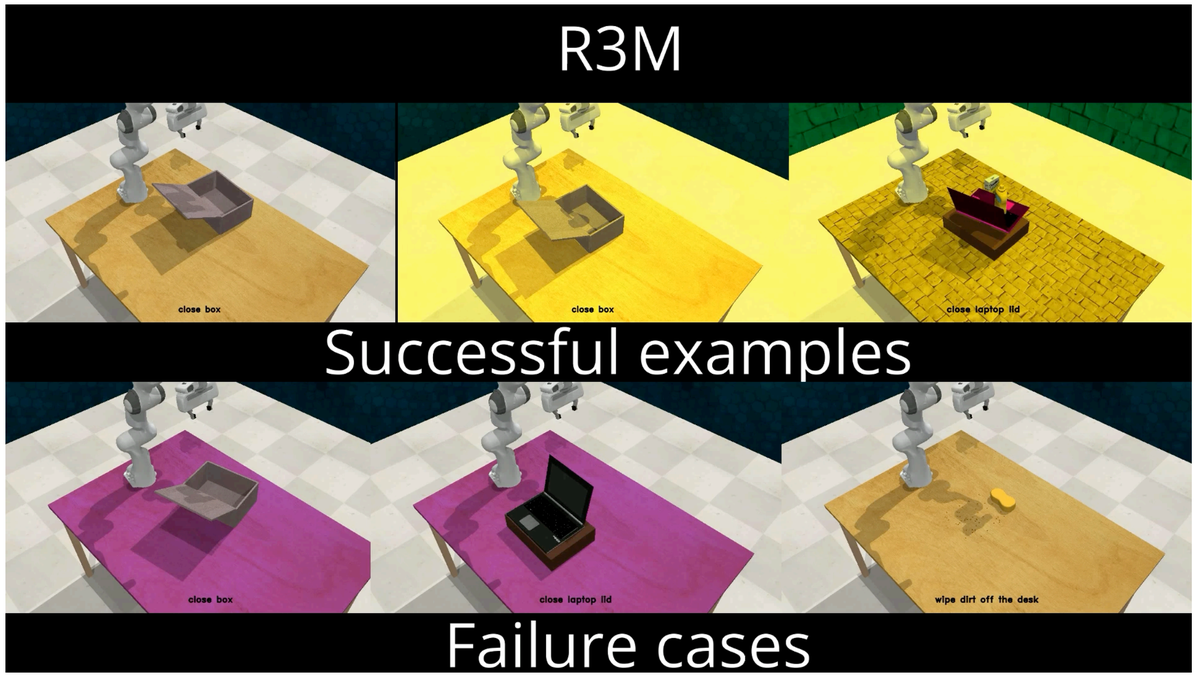

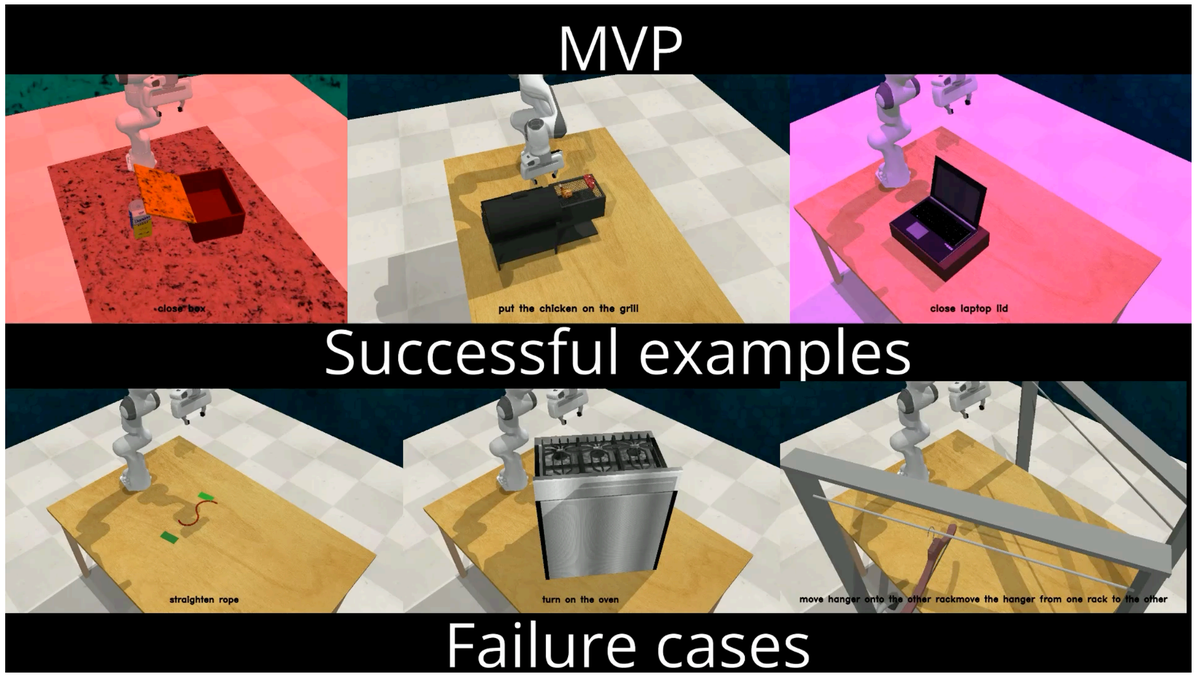

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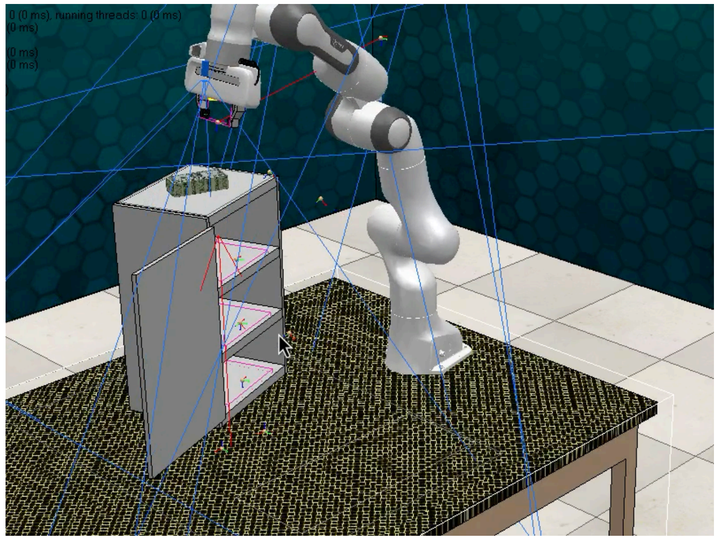
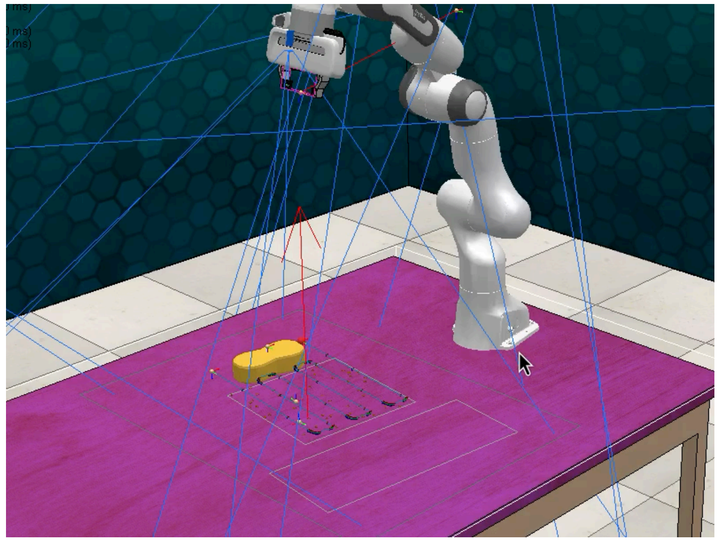
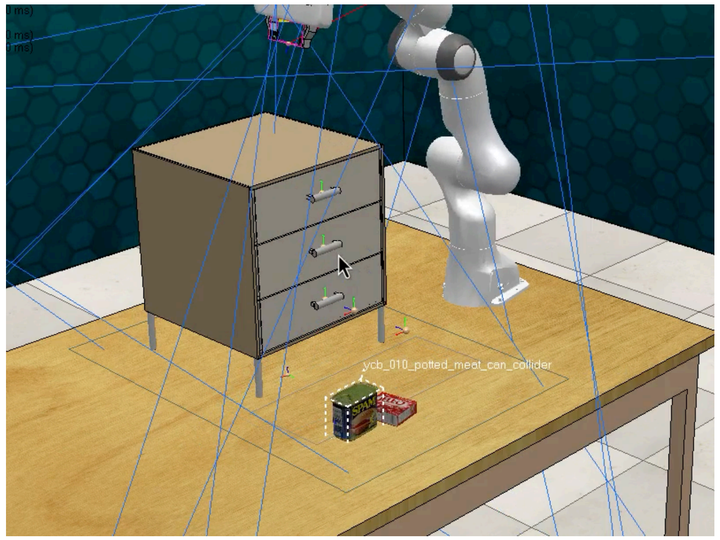
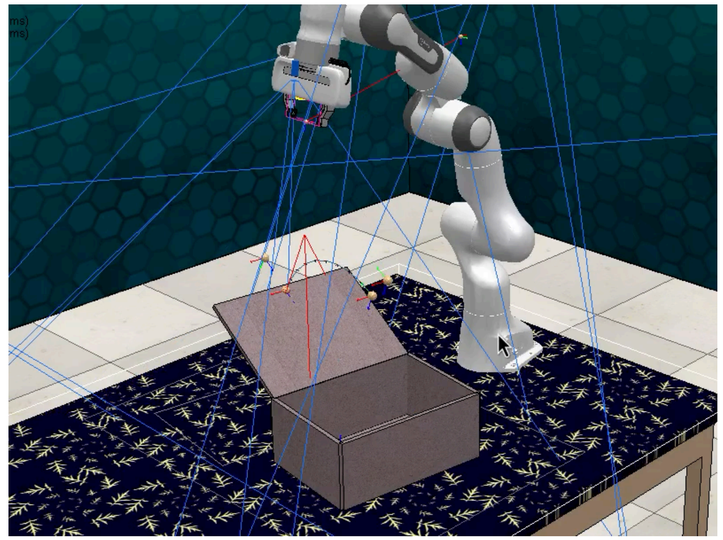




Voxposer





Simulation and Real-world correlation