Task and Motion Planning (TAMP)

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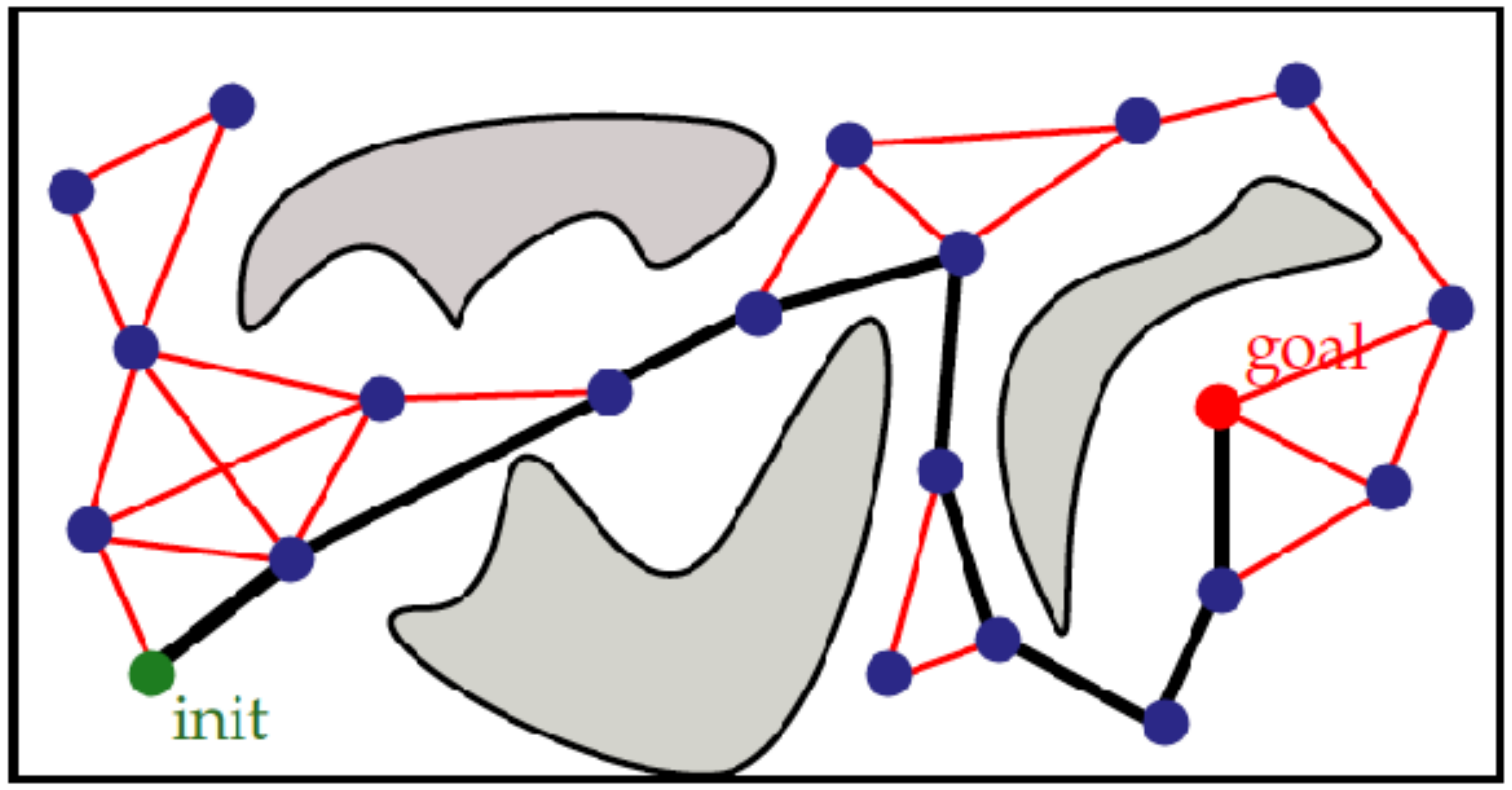
CSE 571: AI Robotics

05/20/2025

(Probable) Roadmap

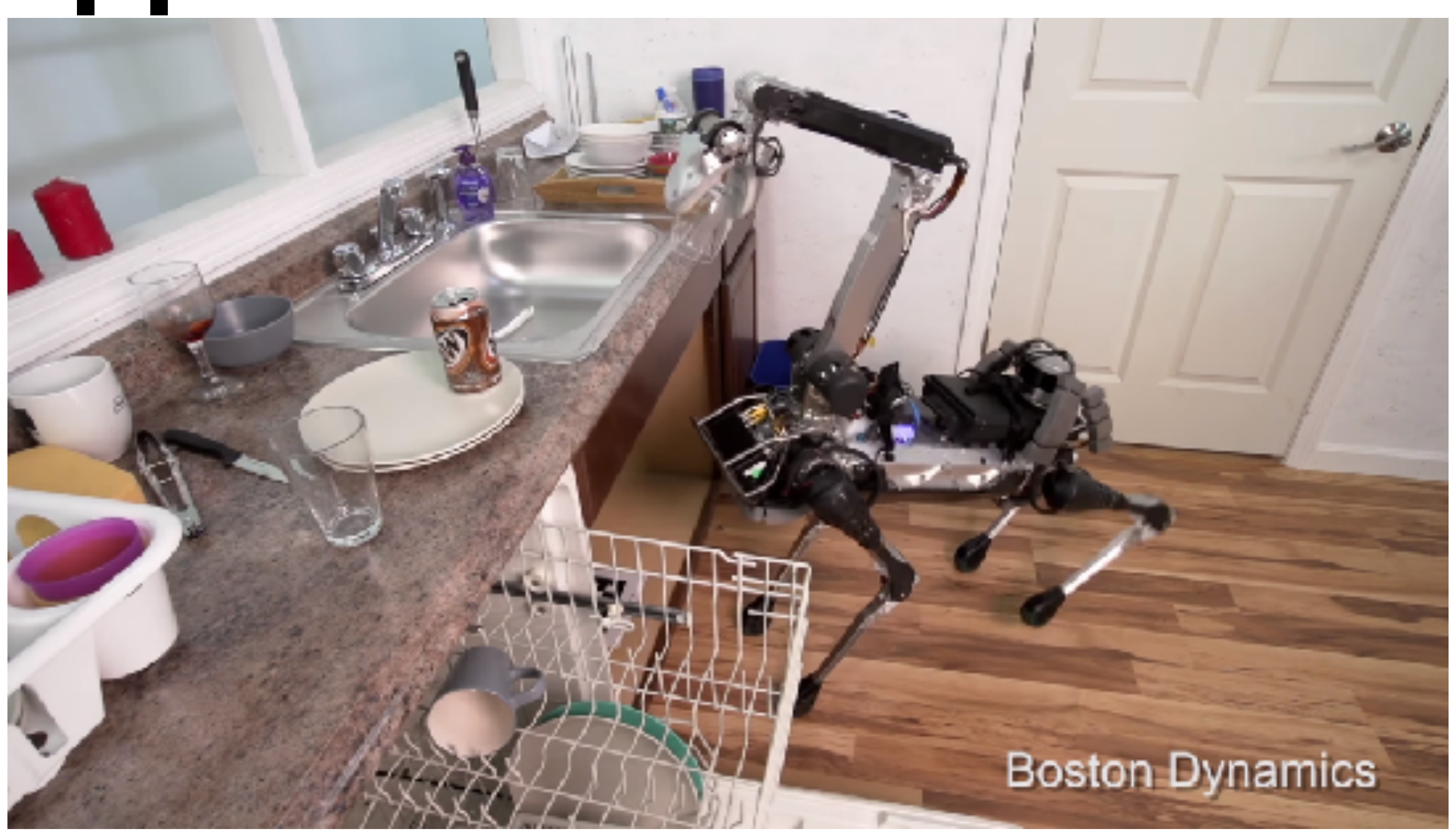
1. **Background**
   1. Task Planning
   2. Review: Motion Planning
2. **Hybrid Planning**
   1. Prediscretized Planning
   2. Multi-Modal Motion Planning
   3. Task and Motion Planning
3. **PDDLStream for TAMP**
4. **TAMP Extensions and Ongoing Work**
   1. GPU Acceleration, Stochasticity, Partially Observability, Imitation Learning



[Fig from Erion Plaku]

Planning for Autonomous Robots

- Robot must select both **high-level** actions & **low-level** controls
- **Application areas:** semi-structured and human environments



Household


Warehouse fulfilment

Food service


Construction

Task and Motion Planning (TAMP)

- Search in a **factored, hybrid** space
  - **Discrete** and **continuous** variables & actions

- **Variables**
  - **Continuous:** robot configuration, object poses, door joint positions,
  - **Discrete:** is-on, is-in-hand, is-holding-water, is-cooked, …

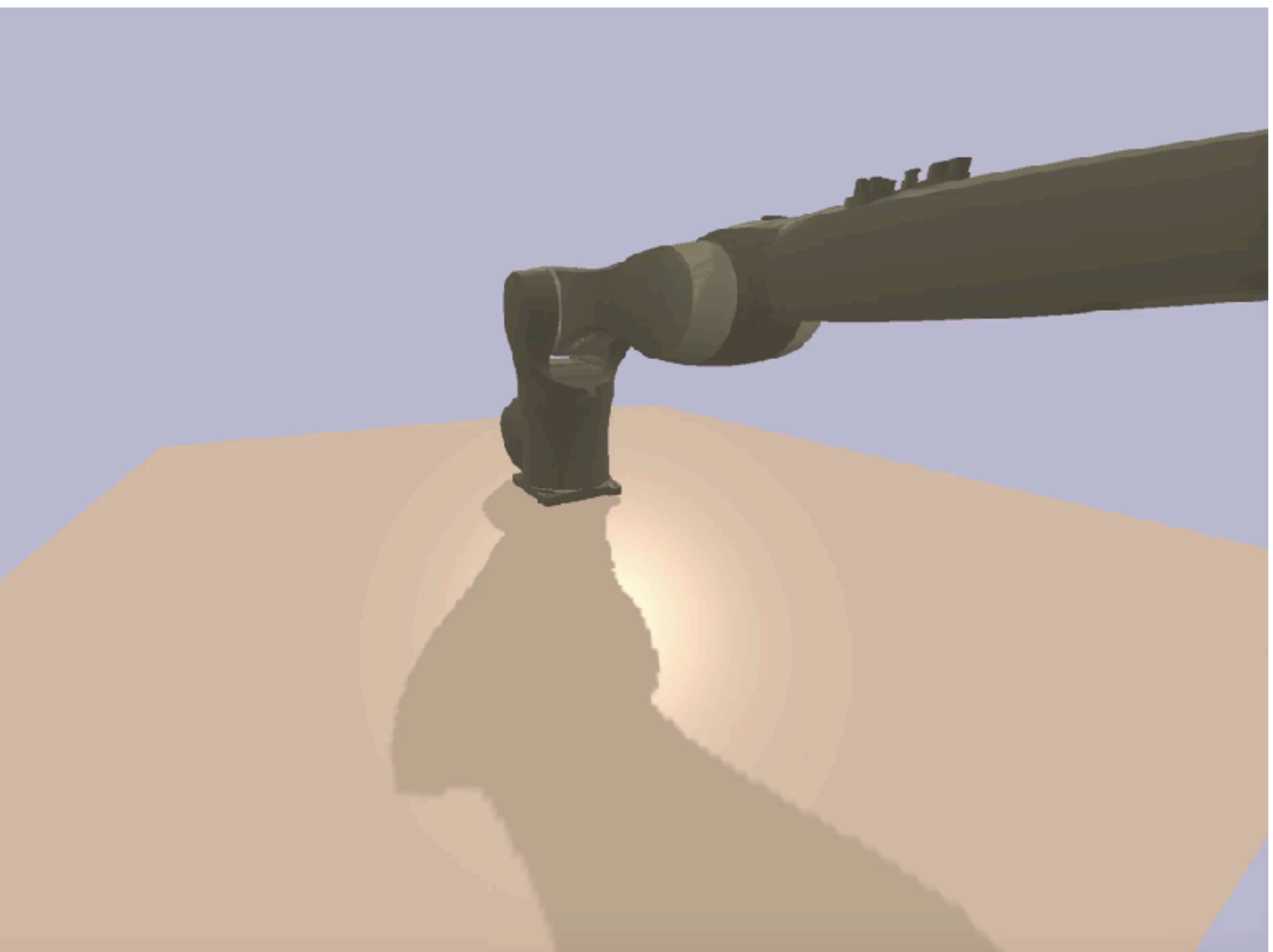
- **Actions:** move, pick, place, push, pull, pour, detect, cook, …

Cooking and Serving the “Blockoli”

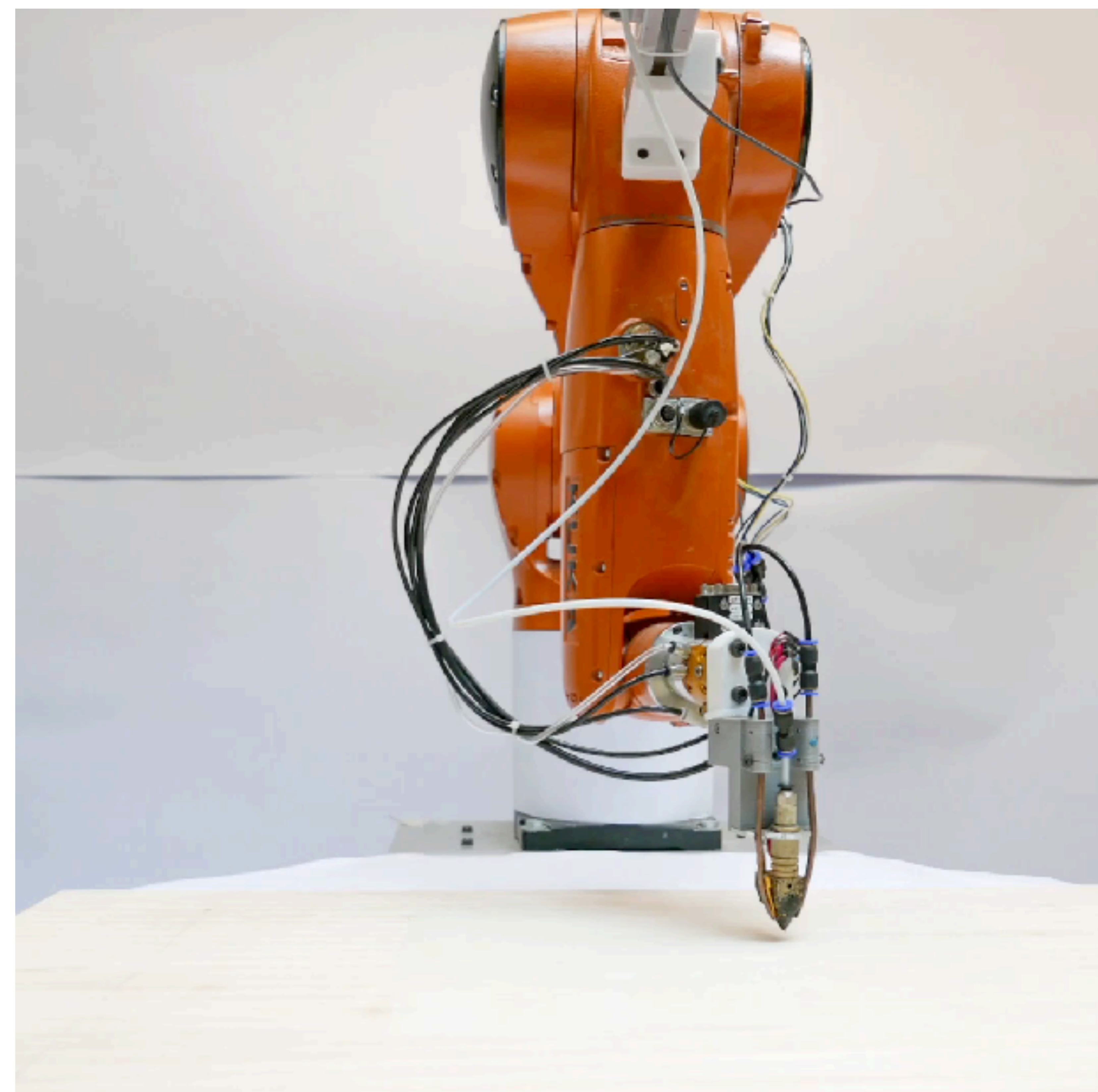
Preparing Coffee

Automated Fabrication

- Plan sequence of **306** 3D printing extrusions (actions)
- Collision, kinematic, **stability** and **stiffness** constraints



[Huang, Garrett, & Mueller 2018]



Problem Class

- **Discrete-time**
  - Plans are finite sequences of controls
- **Deterministic** (not an MDP for now)
  - Actions always produce the intended effect
  - Solutions are **plans** (instead of policies)
- **Observable** (not a POMDP for now)
  - Access to the full world state
- **Hybrid**
  - States & controls composed of mixed **discrete-continuous** variables

Task Planning

Task (Classical, Symbolic) Planning

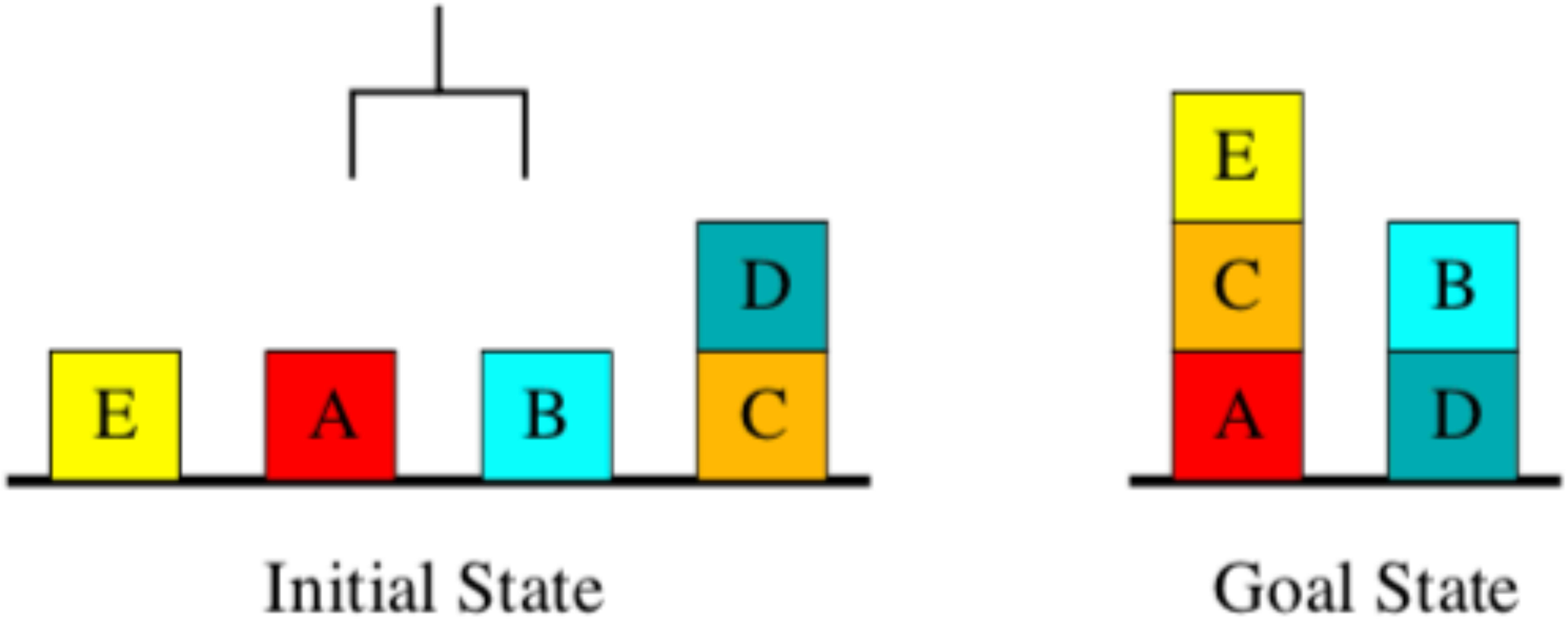
- **Discrete** problems with **many variables**
  - Often enormous (2^N) but **finite** state-spaces

- Problems typically described using an **action language**
  - **Propositional Logic** (STRIPS) [Fikes 1971][Aeronautiques 1998]
  - **Planning Domain Description Language** (PDDL)

- Develop **domain-independent** algorithms
  - Can apply to **any problem** expressible using PDDL
- Exploit **factoring** and **sparsity** to develop algorithms

Classical Planning Representations

Blocksworld domain

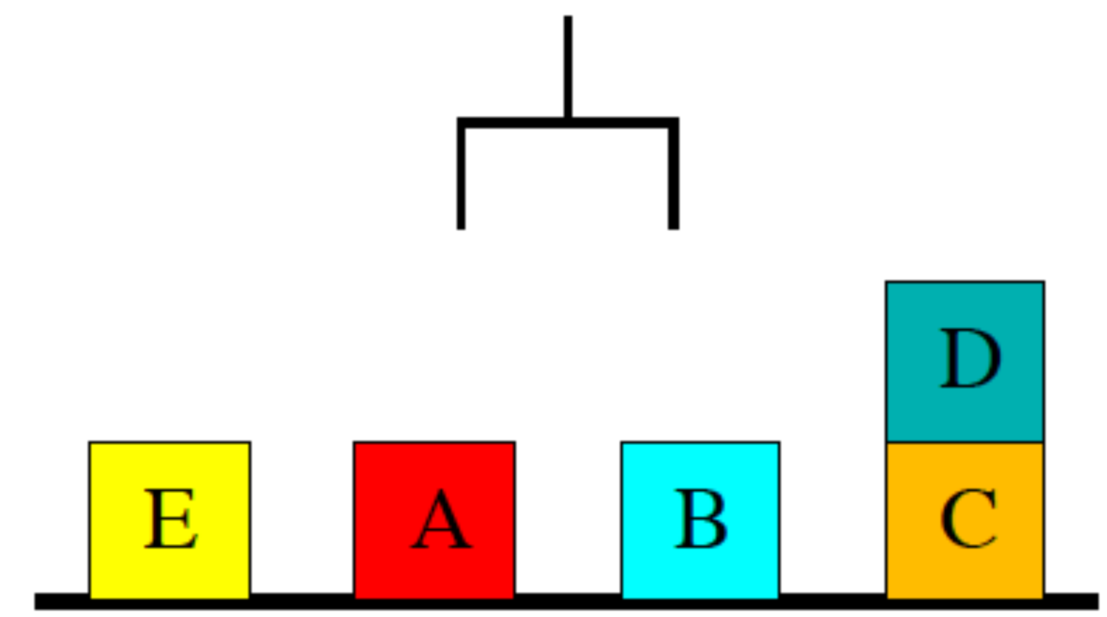


- Facts: $on(x, y)$, $onTable(x)$, $clear(x)$, $holding(x)$, $armEmpty()$.
- Initial state: $\{onTable(E), clear(E), \ldots, onTable(C), on(D, C), clear(D), armEmpty()\}$.
- Goal: $\{on(E, C), on(C, A), on(B, D)\}$.
- Actions: $stack(x, y)$, $unstack(x, y)$, $putdown(x)$, $pickup(x)$.
- $stack(x, y)$? $pre : \{holding(x), clear(y)\}$
  $add : \{on(x, y), armEmpty()\}$
  $del : \{holding(x), clear(y)\}$.

[Figs from Hector Geffner]

First-Order Action Languages

- **Predicate:** Boolean function `On(?b1, ?b2)=`**`True`**`/`**`False`**
- **Facts** (literals): instantiated predicates `On(D, C)=`**`True`**
- **State:** set of facts `{On(A, B)=`**`False`**`, On(D, C)=`**`True`**`, …}`
  - Equivalently, Boolean state variables
  - **Closed-world** assumption
    - Unspecified facts are **false**



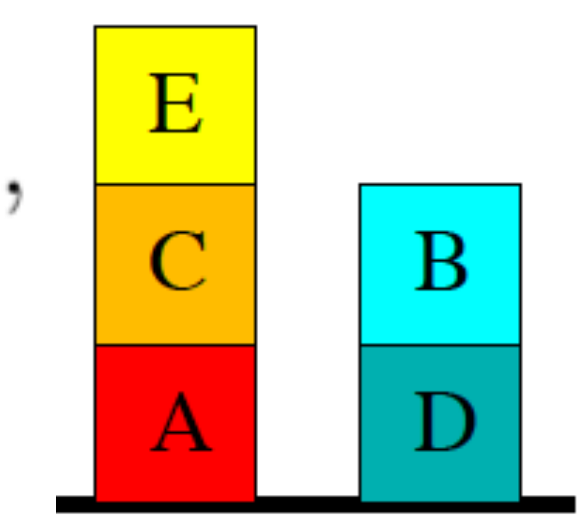
Initial State

Facts: $on(x, y)$, $onTable(x)$, $clear(x)$, $holding(x)$, $armEmpty()$.

Initial state: $\{onTable(E), clear(E), \ldots, onTable(C), on(D, C), clear(D), armEmpty()\}$.

Goal: $\{on(E, C), on(C, A), on(B, D)\}$.

Actions: $stack(x, y)$, $unstack(x, y)$, $putdown(x)$, $pickup(x)$.



Goal State

(Lifted) Action Schema

- A tuple of free **parameters**
- A **precondition** formula tests applicability
- An **effect** formula modifies the state (as a **delta**)
- Logical **conjunctions** encode factoring

```
(:action stack
 :parameters (?b1, ?b2)
 :precondition {
   Holding(?b1), Clear(?b2)}
 :effect {ArmEmpty(),
   On(?b1, ?b2),
   Clear(?b1)
   ¬Holding(?b1),
   ¬Clear(?b2)}
```

```
(:action unstack
 :parameters (?b1, ?b2)
 :precondition {ArmEmpty(),
   On(?b1, ?b2),
   Clear(?b1)}
 :effect {Holding(?b1),
  Clear(?b2),
   ¬Clear(?b1),
   ¬ArmEmpty(),
   ¬On(?b1, ?b2)}
```

Planning Approaches

- **State-space** search: [Bonet 2001] [Hoffman 2001] [Helmert 2006]
  - **Progression** (forward) or regression (backward)
  - Best-first **heuristic search** algorithms
- **Partial-order** planning [Penberthy 1992]
  - Search directly over plans (**plan-space**)
- Planning as **Satisfiability** [Kautz 1999]
  - Compile to **fixed-horizon** SAT instance
  - SAT is **NP-Complete,** Planning is **PSPACE-Complete**
  - **Increase horizon if formula unsatisfiable**
- Large Language Models (**LLMs**)

LLMs for Task Planning

- Large Language Models (LLMs) proficient at commonsense reasoning
- But they struggle at **easy** International Planning Competition (IPC) benchmark problems

Domain	Method	Instances correct				
		GPT-4	GPT-3.5	I-GPT3.5	I-GPT3	GPT-3
Blocksworld (BW)	One-shot	206/600 (34.3%)	37/600 (6.1%)	54/600 (9%)	41/600 (6.8%)	6/600 (1%)
	Zero-shot	210/600 (34.6%)	8/600 (1.3%)	-	-	-
	COT	214/600 (35.6%)	-	-	-	-
Logistics Domain	One-shot	28/200 (14%)	1/200 (0.5%)	6/200 (3%)	3/200 (1.5%)	-
	Zero-shot	15/200 (7.5%)	1/200 (0.5%)	-	-	-
Mystery BW (Deceptive)	One-shot	26/600 (4.3%)	0/600 (0%)	4/600 (0.6%)	14/600 (2.3%)	0/600 (0%)
	Zero-shot	1/600 (0.16%)	0/600 (0%)	-	-	-
	COT	54/600 (9%)	-	-	-	-

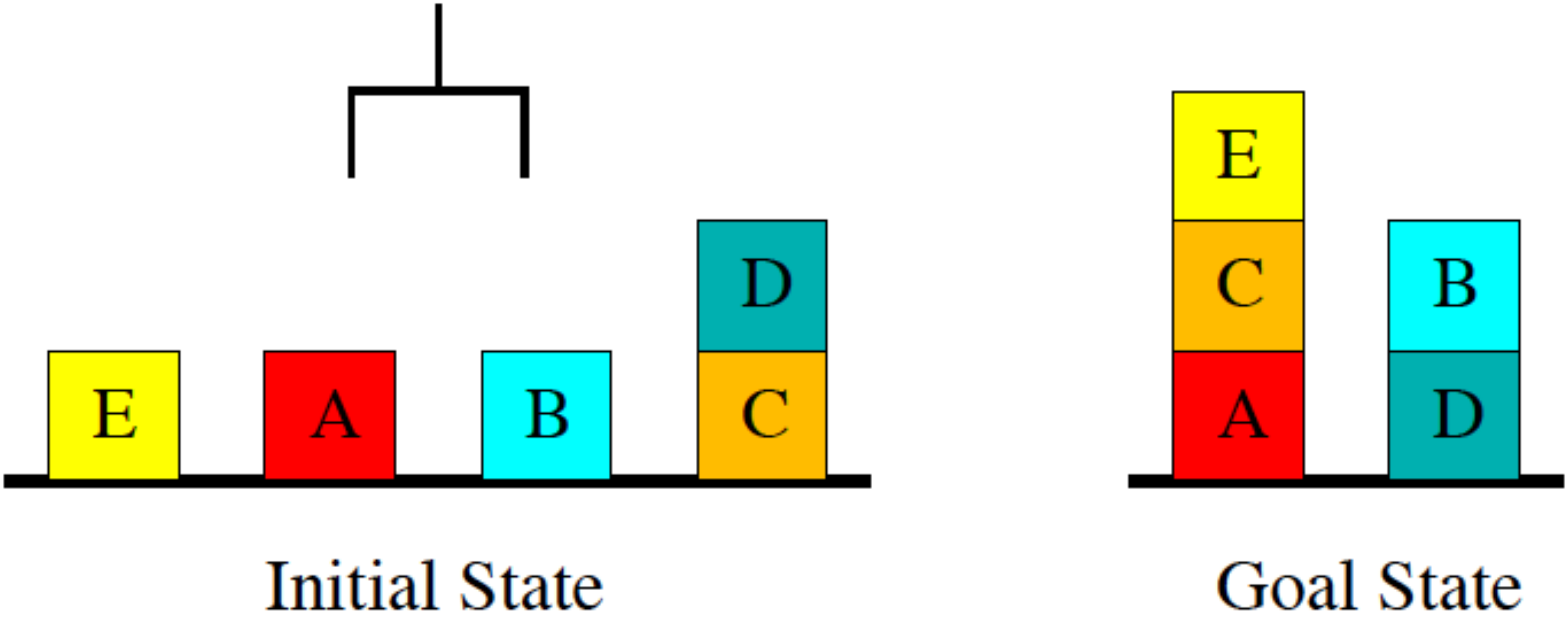
[Valmeekam 2023]

Forward Best-First Search

- For a state s
  - Path **cost:** $g(s)$
  - **Heuristic** estimate: $h(s)$
  - Open list **sorted** by priority $f(s)$
- **Weighted A*:** $f(s) = g(s) + wh(s)$
  - Uniform cost search: $w = 0 \implies f(s) = g(s)$
  - A* search: $w = 1 \implies f(s) = g(s) + h(s)$
  - **Greedy** best-first search: $w = \infty \implies f(s) = h(s)$
- How do we estimate $h(s)$?
  - No obvious metric (no metric-space embedding)

Predict the Minimum Plan Length

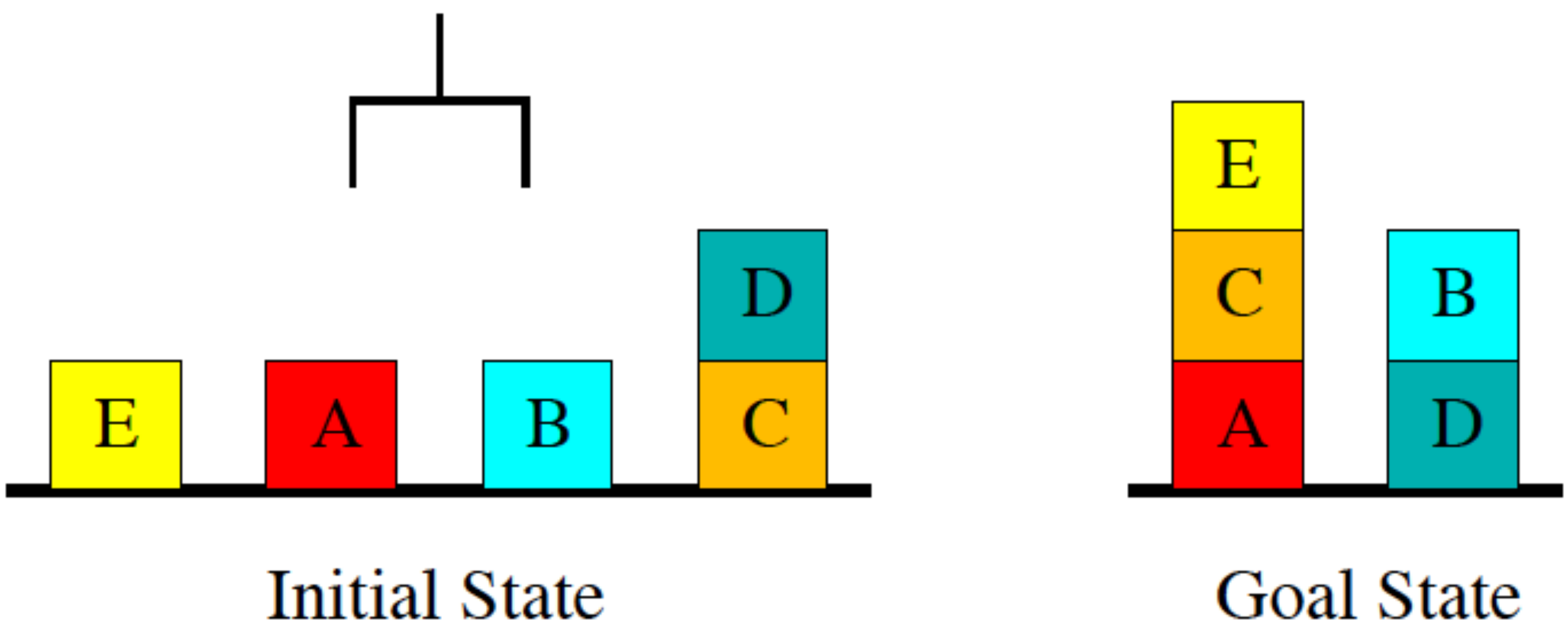
- Can stack / unstack anywhere on the ground
- Hint: is an **even** number



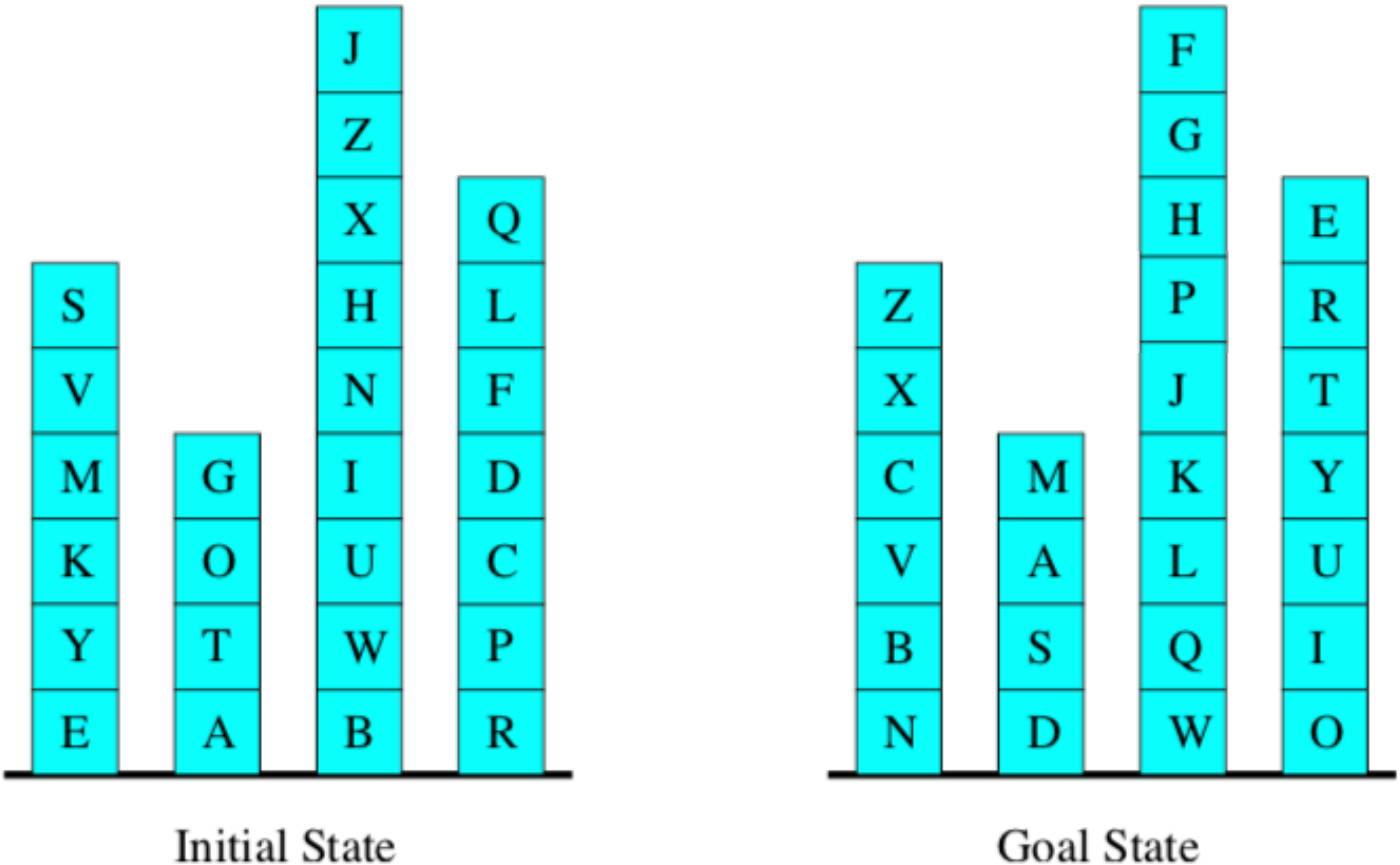
Predict the Minimum Plan Length

- **Solution** (length=8):
  - **unstack**(D, C)
  - **stack**(D, ground)
  - **unstack**(B, ground)
  - **stack**(B, D)
  - **unstack**(C, ground)
  - **stack**(C, A)
  - **unstack**(E, ground)
  - **stack**(E, C)

4/5 Blocks
Move Once



Predict the Minimum Plan Length



Domain-Independent Heuristics

- Estimating $h(s)$ is **nontrivial**
- Can we do it in an a **domain-independent** manner?
- Solve a relaxed, **approximate** planning problem

- Suggestions for how to do this?
  - **Independently** plan for each goal [Lipovetzky 2012]
  - **Remove** some action preconditions [Helmert 2006]
  - <u>Remove negative (**delete**) **effects**</u> [Bonet 2001] [Hoffman 2001]
  - Learn a **value function** on PDDL [Shen 2020]
  - …

Delete-Relaxation Heuristics

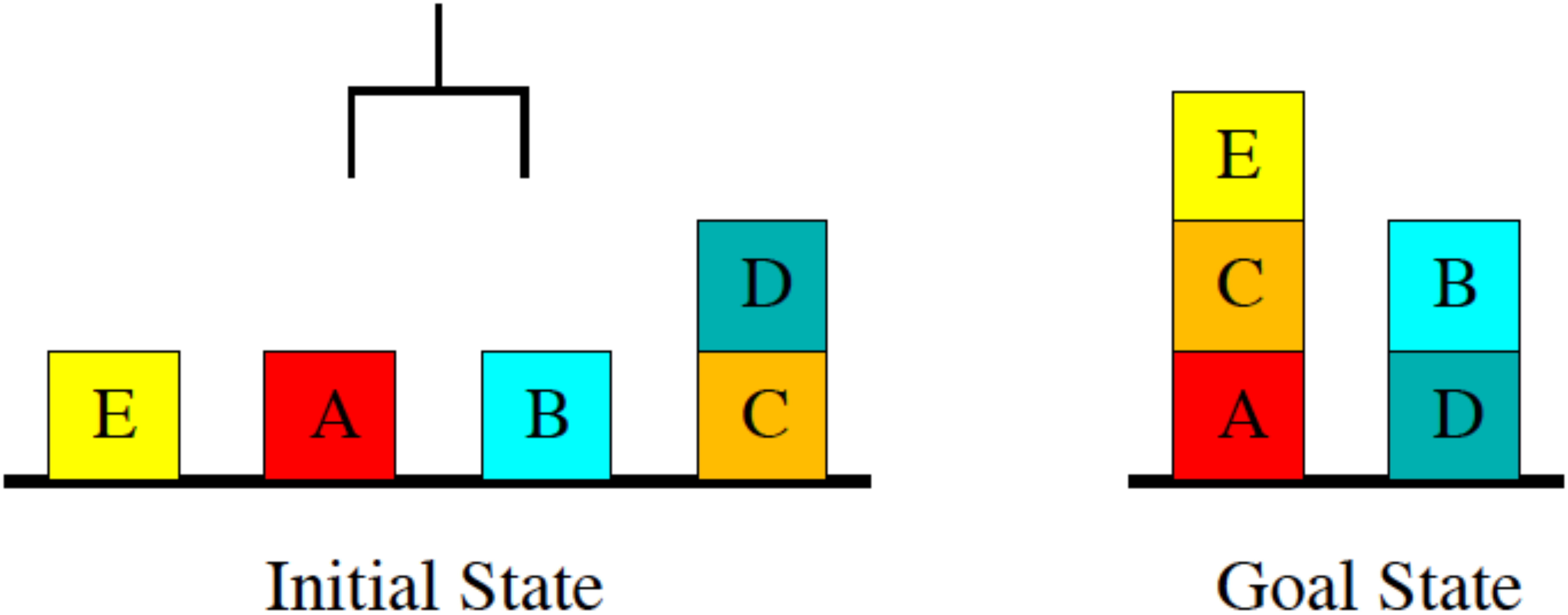
- Remove all negative (¬) effects
  - Solving optimally is **NP-Complete**
  - Can greedily find a short plan in polynomial time
- Basis for both **admissible** and **greedier**, non-admissible heuristics

(:**action stack**
 :**parameters** (?b1, ?b2)
 :**precondition** {
  Holding(?b1), Clear(?b2)}
 :**effect** {ArmEmpty(),
  On(?b1, ?b2),
  Clear(?b1)
  ~~¬Holding(?b1),~~
  ~~¬Clear(?b2)}~~

(:**action unstack**
 :**parameters** (?b1, ?b2)
 :**precondition** {ArmEmpty(),
  On(?b1, ?b2),
  Clear(?b1)}
 :**effect** {Holding(?b1),
  Clear(?b2),
  ~~¬Clear(?b1),~~
  ~~¬ArmEmpty(),~~
  ~~¬On(?b1, ?b2)}~~

Predict the Minimum Delete-Relaxed Plan Length

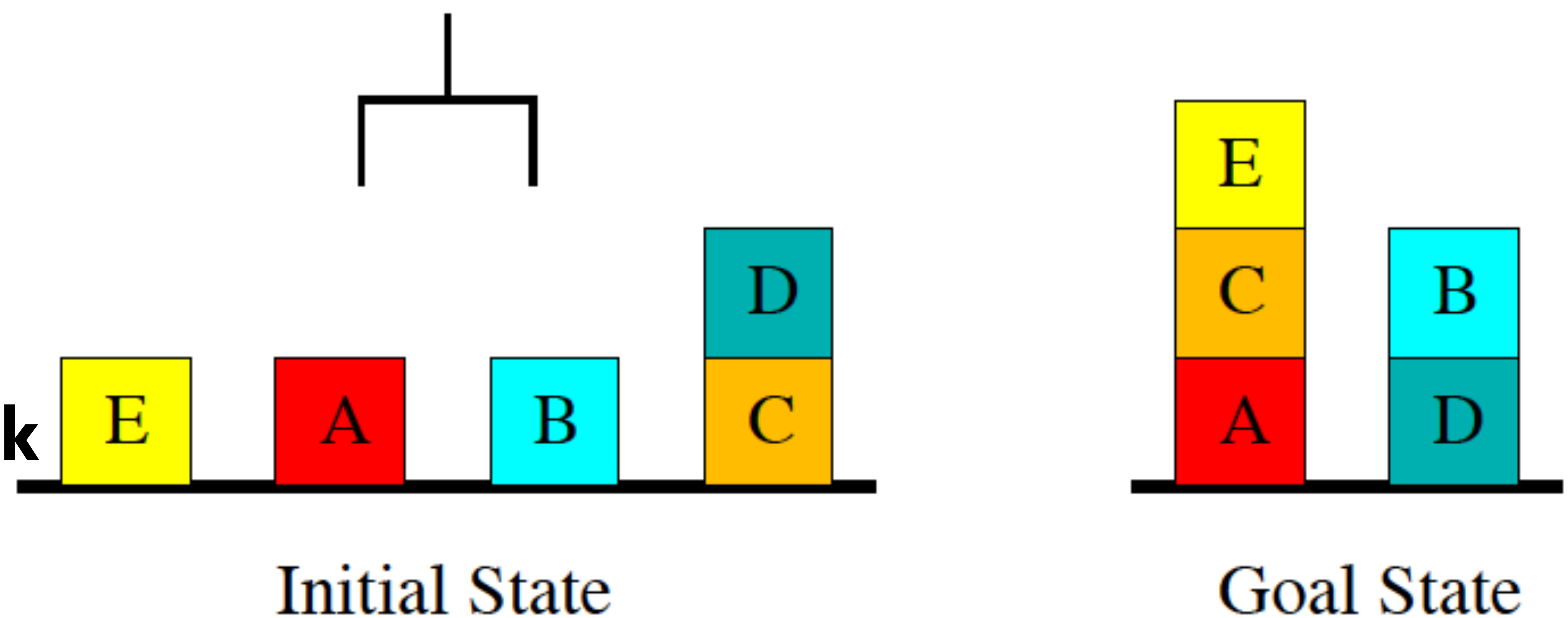
- Can stack / unstack anywhere on the ground
- Hint: is **no greater** than 8



Predict the Minimum Delete-Relaxed Plan Length

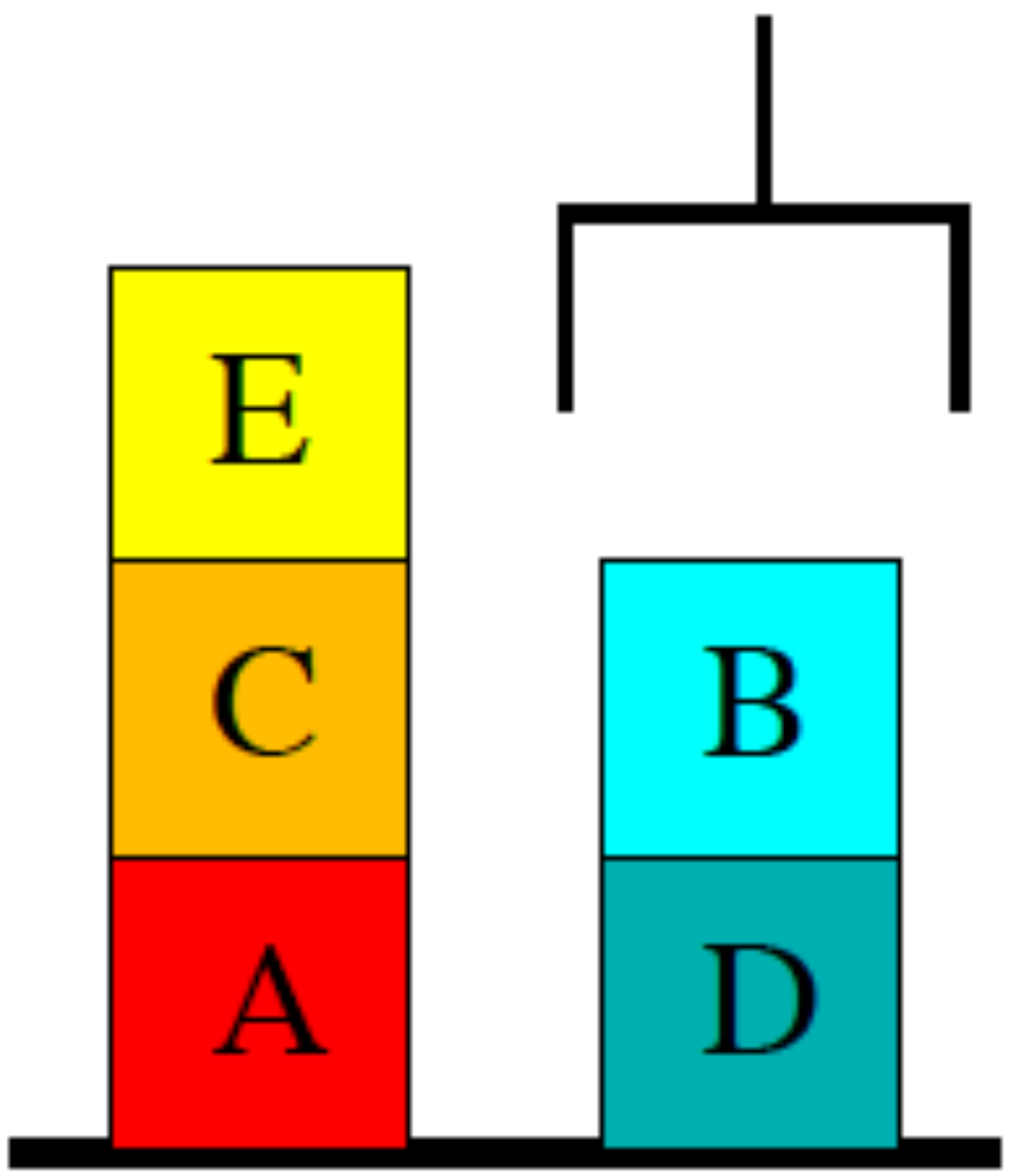
- **Solution** (length=8):
  - **`unstack`**`(D, C)`
  - **`stack`**`(D, ground)`
  - **`unstack`**`(B, ground)`
  - **`stack`**`(B, D)`
  - **`unstack`**`(C, ground)`
  - **`stack`**`(C, A)`
  - **`unstack`**`(E, ground)`
  - **`stack`**`(E, C)`

Still Need to
Unstack + **Stack**
4/5 Blocks

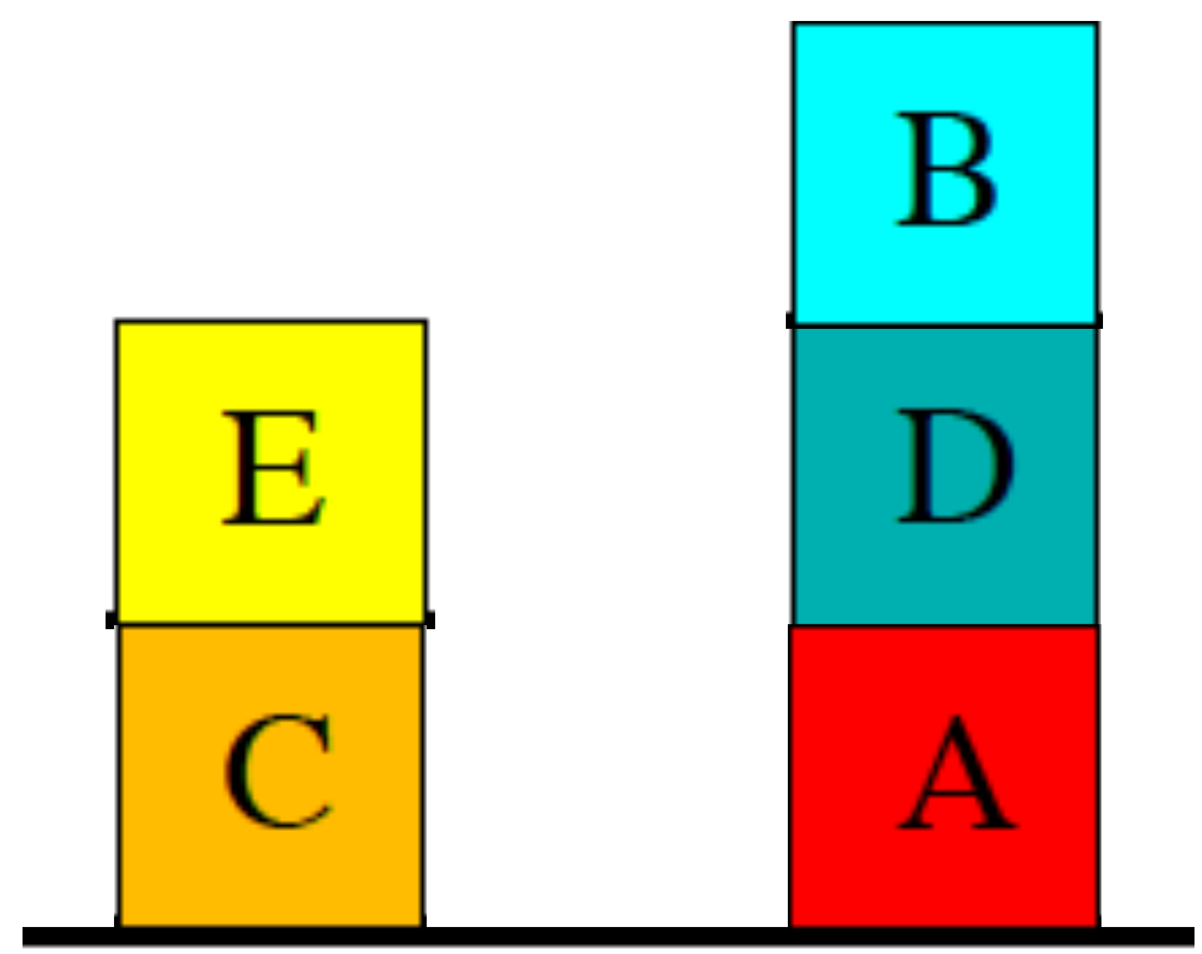


Predict the Minimum Plan Length

- Can **stack** / **unstack** anywhere on the ground
- Hint: is an **even** number



Initial State



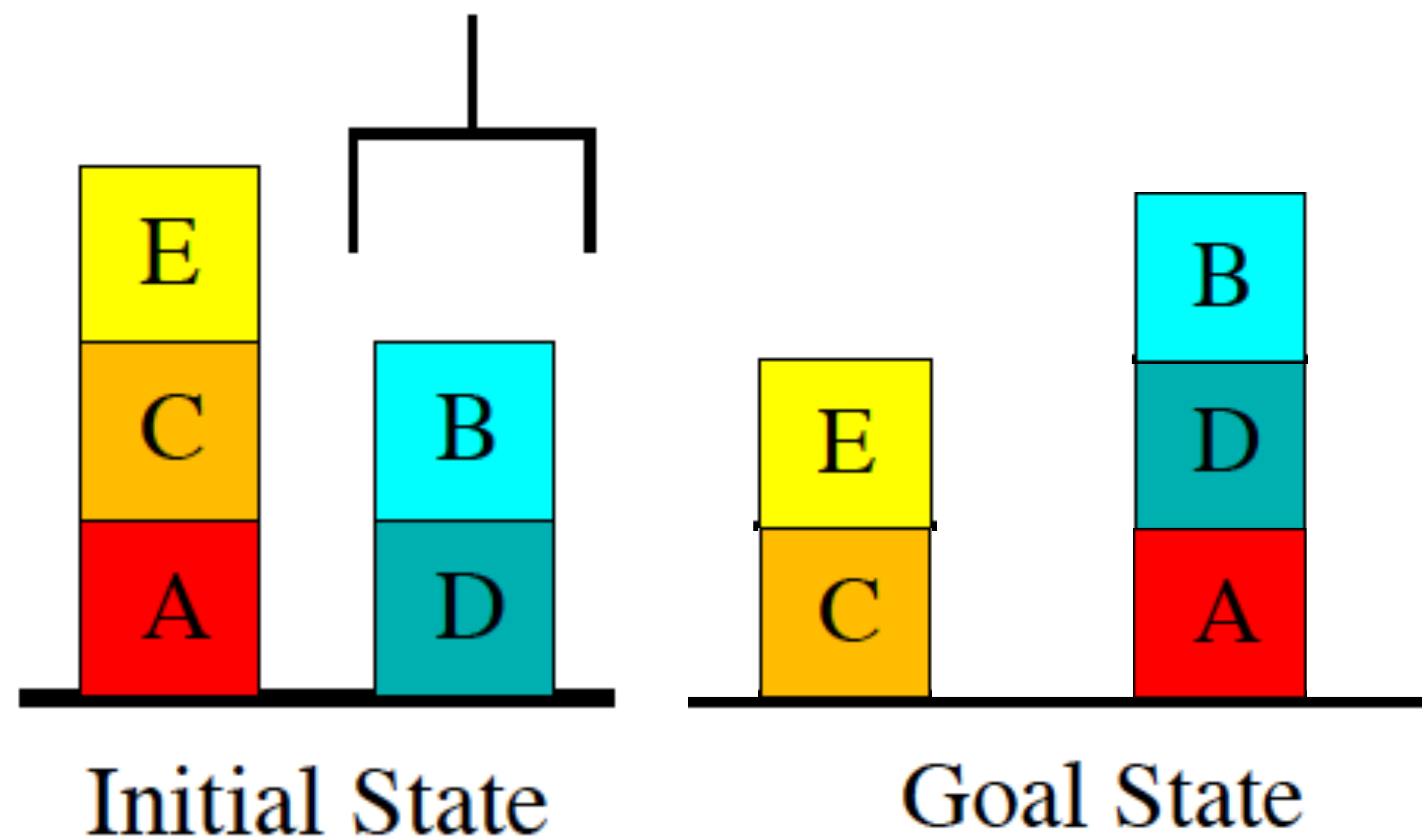
Goal State

Predict the Minimum Plan Length

- **Solution** (length=12):
  - **unstack**(E, C)
  - **stack**(E, ground)
  - **unstack**(C, A)
  - **stack**(C, ground)
  - **unstack**(E, ground)
  - **stack**(E, C)
  - **unstack**(B, D)
  - **stack**(B, ground)
  - **unstack**(D, ground)
  - **stack**(D, A)
  - **unstack**(B, ground)
  - **stack**(B, D)

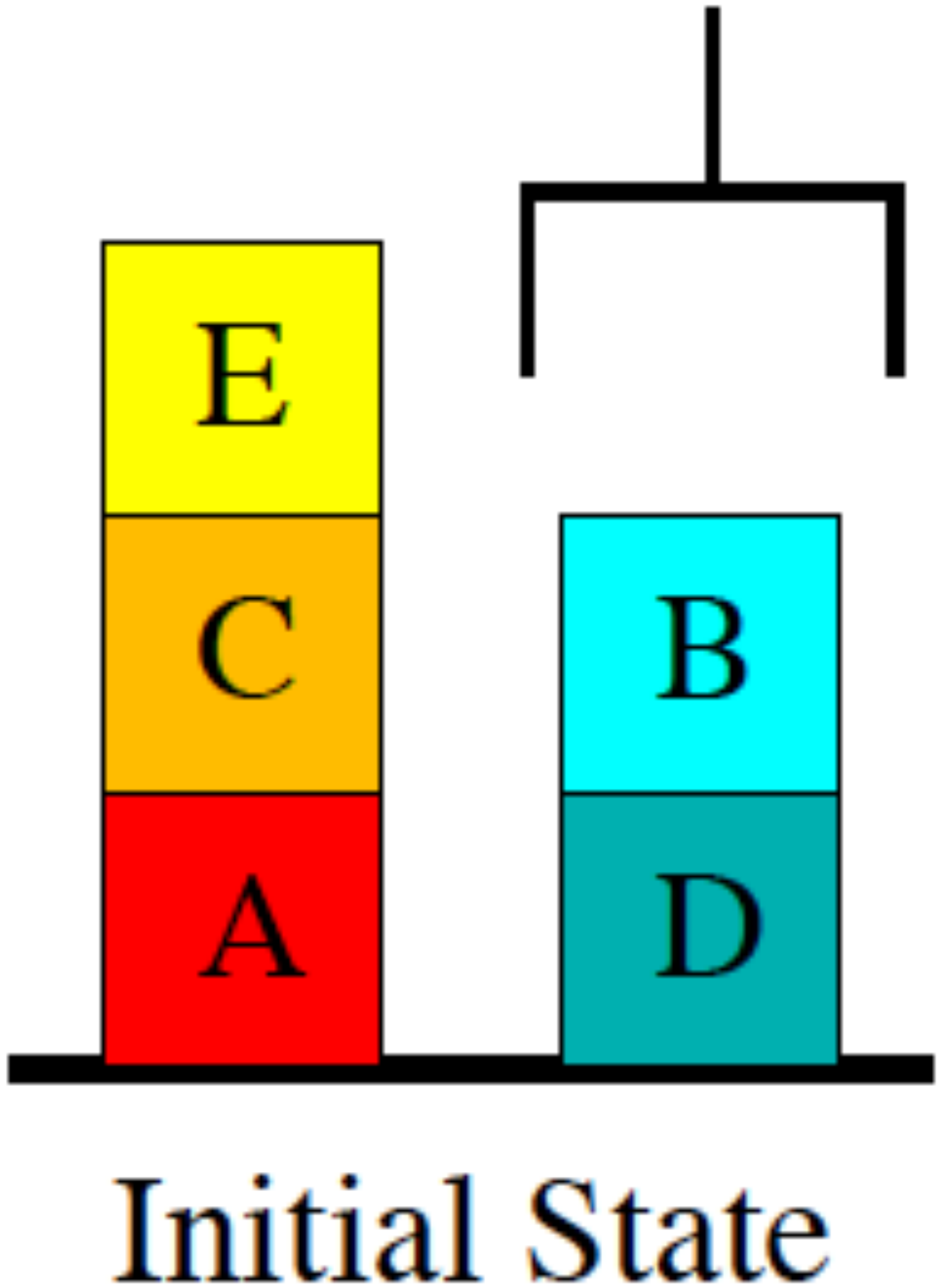
Need to **Restack**

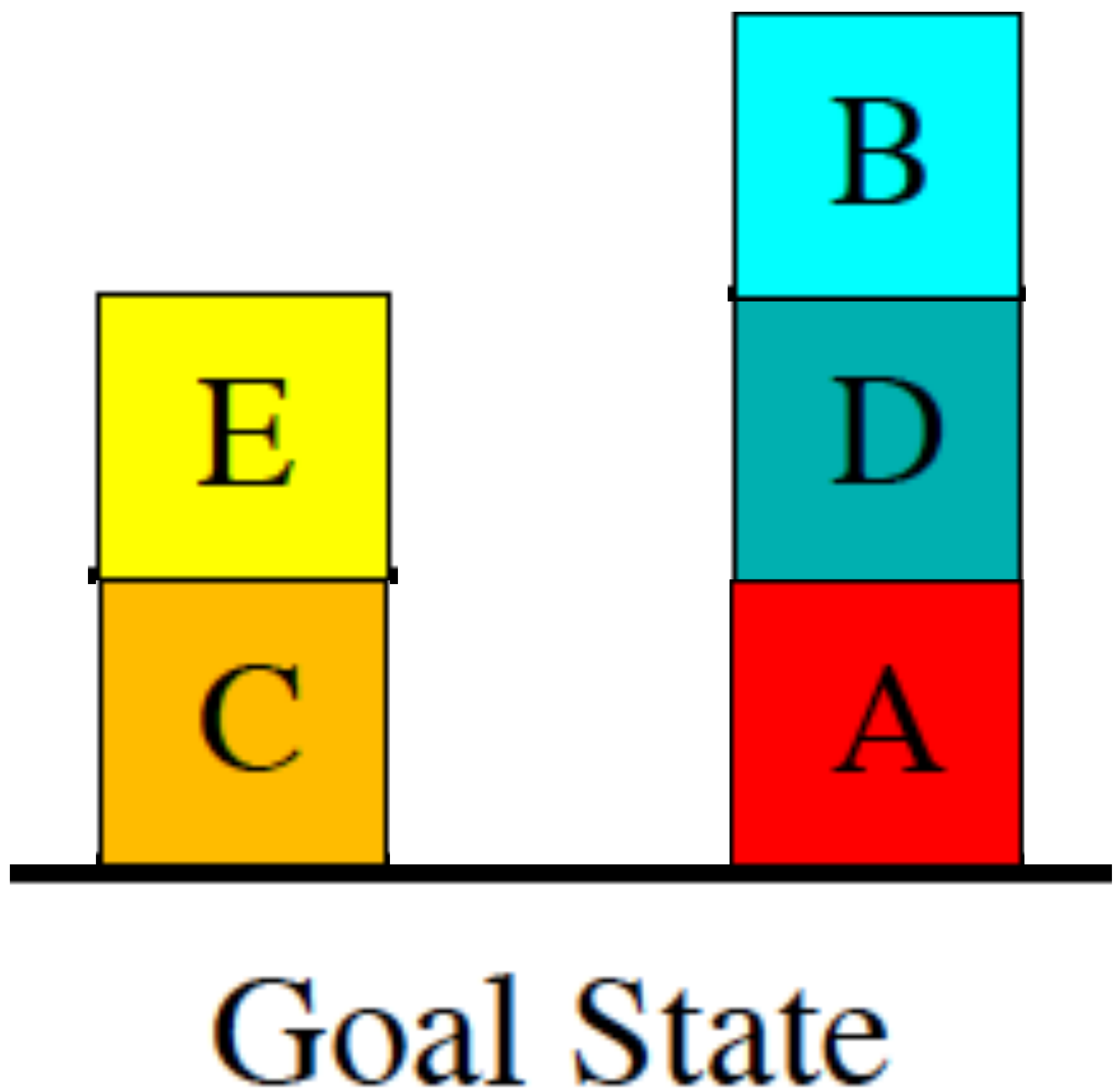
B and **E**



Predict the Minimum Delete-Relaxed Plan Length

- Can stack / unstack anywhere on the ground
- Hint: is **no greater** than 12

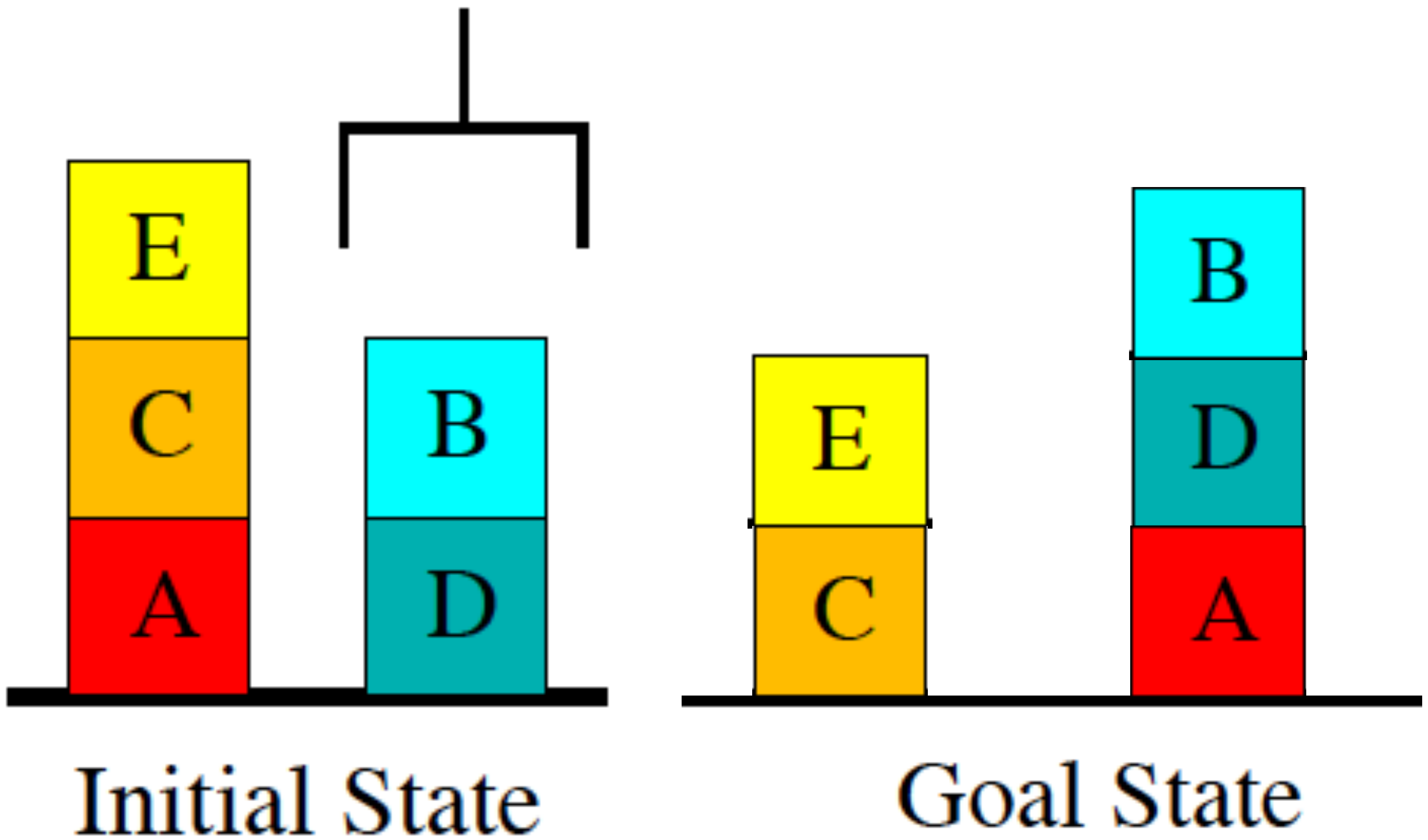




Predict the Minimum Delete-Relaxed Plan Length

- **Solution** (length=5):
  - **unstack**(E, C)
  - **unstack**(C, A)
  - **unstack**(B, D)
  - **unstack**(D, ground)
  - **stack**(D, A)

Only Need to
Unstack + Stack
1/5 Blocks



Motion Planning

Review: Motion Planning

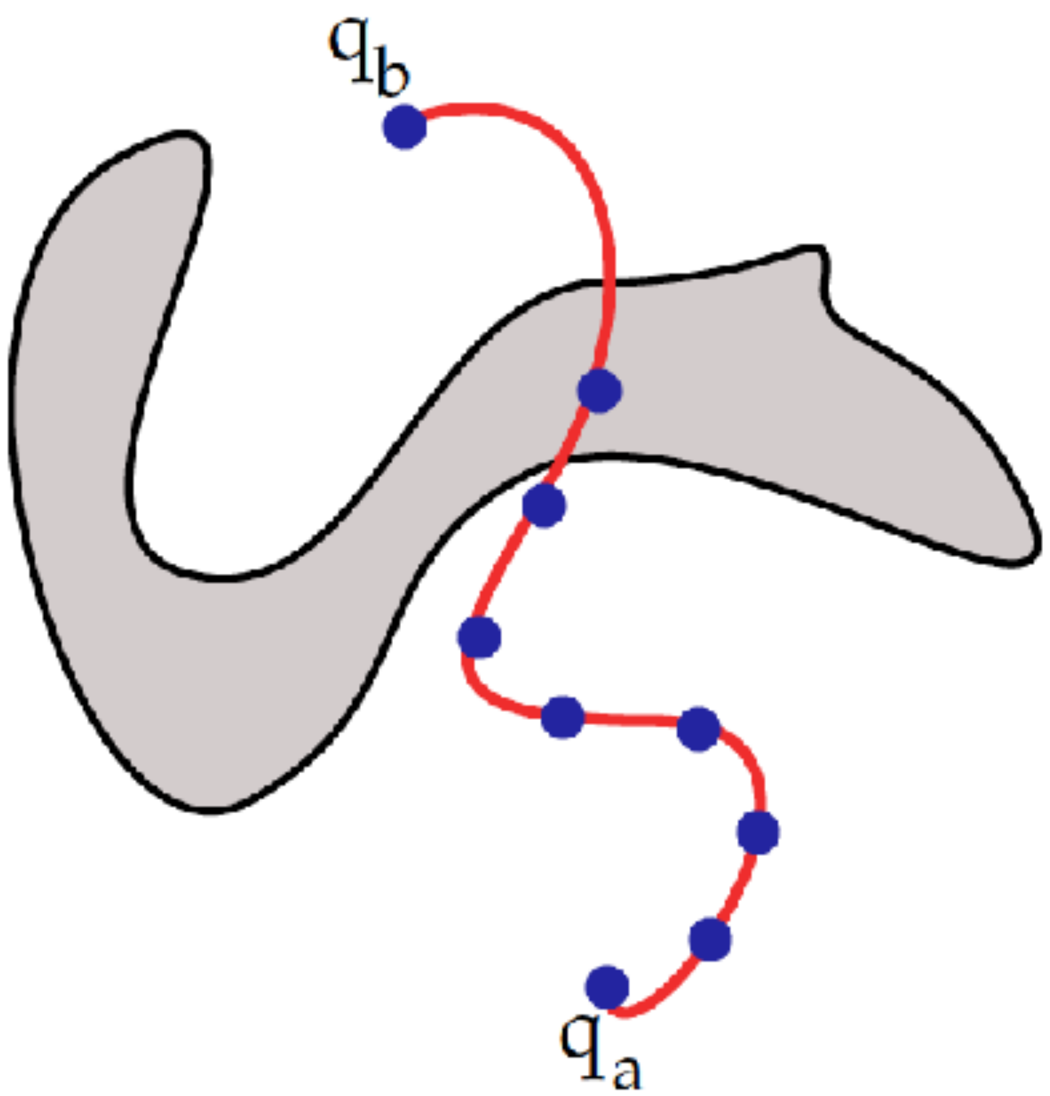
- Plan a **path** for a robot from an initial configuration to a goal configuration that **avoids obstacles**
  - Sequence of **continuous** configurations
  - Configurations often are **high-dimensional**
    - Example: 7 DOFs

- **High-level approaches:**
  - Geometric decomposition
  - <u>Sampling-based</u>
  - Grid-based
  - Optimization-based

Sampling-Based Motion Planning

- **Discretize** configuration space by **sampling**
  - Sampling be deterministic or **random**
- **Implicitly** represent the collision-free configuration space using an blackbox **collision checker**

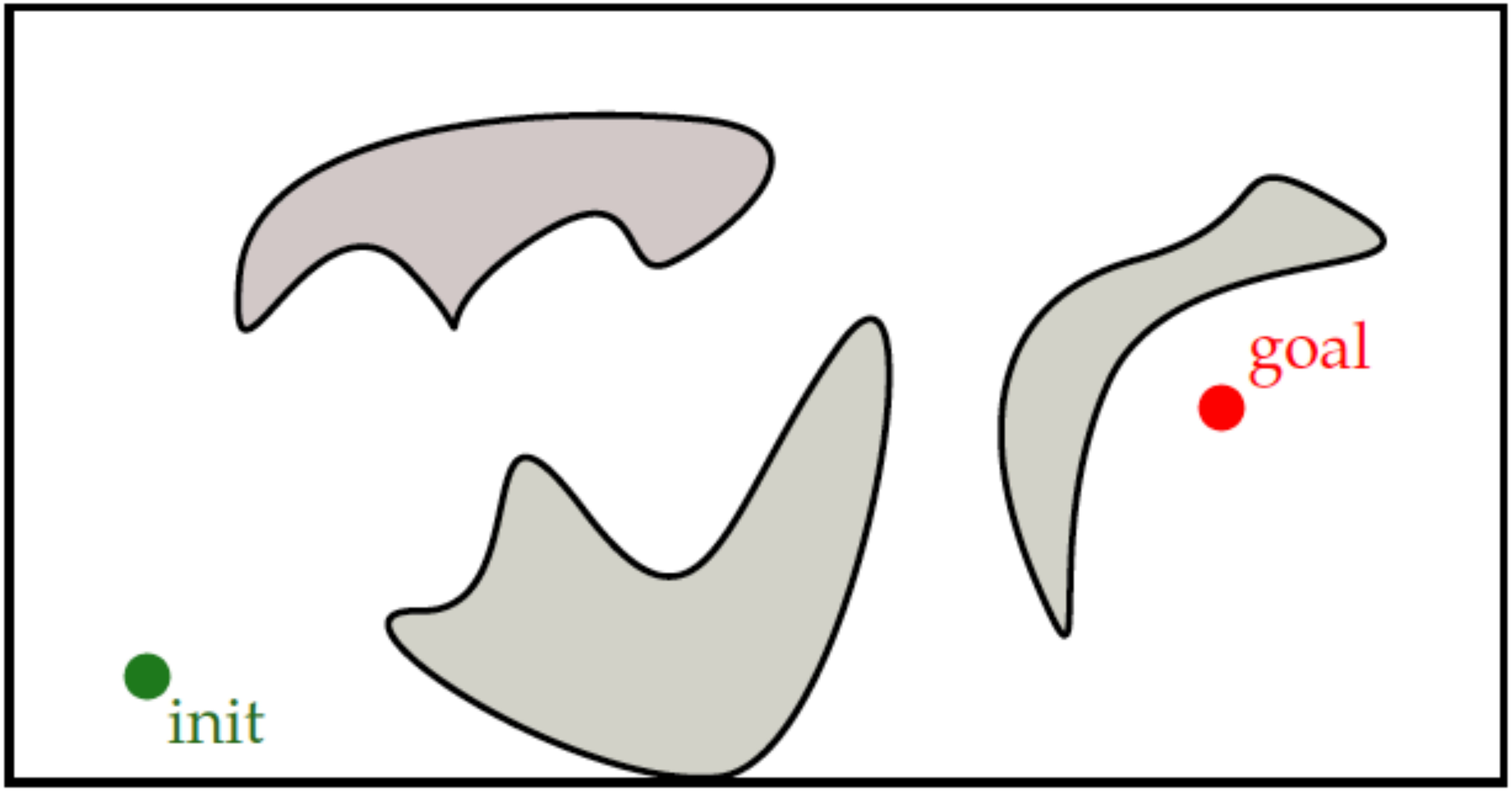
- **Algorithms**
  - Probabilistic Roadmap (PRM)
  - Rapidly-Exploring Random Tree (RRT)
  - Bidirectional RRT (BiRRT)
  - **cuRobo** PRM Global Planning



[Kavraki 1994][Kuffner 2000][LaValle 2006][Sundaralingam 2022]

[Fig from Erion Plaku]

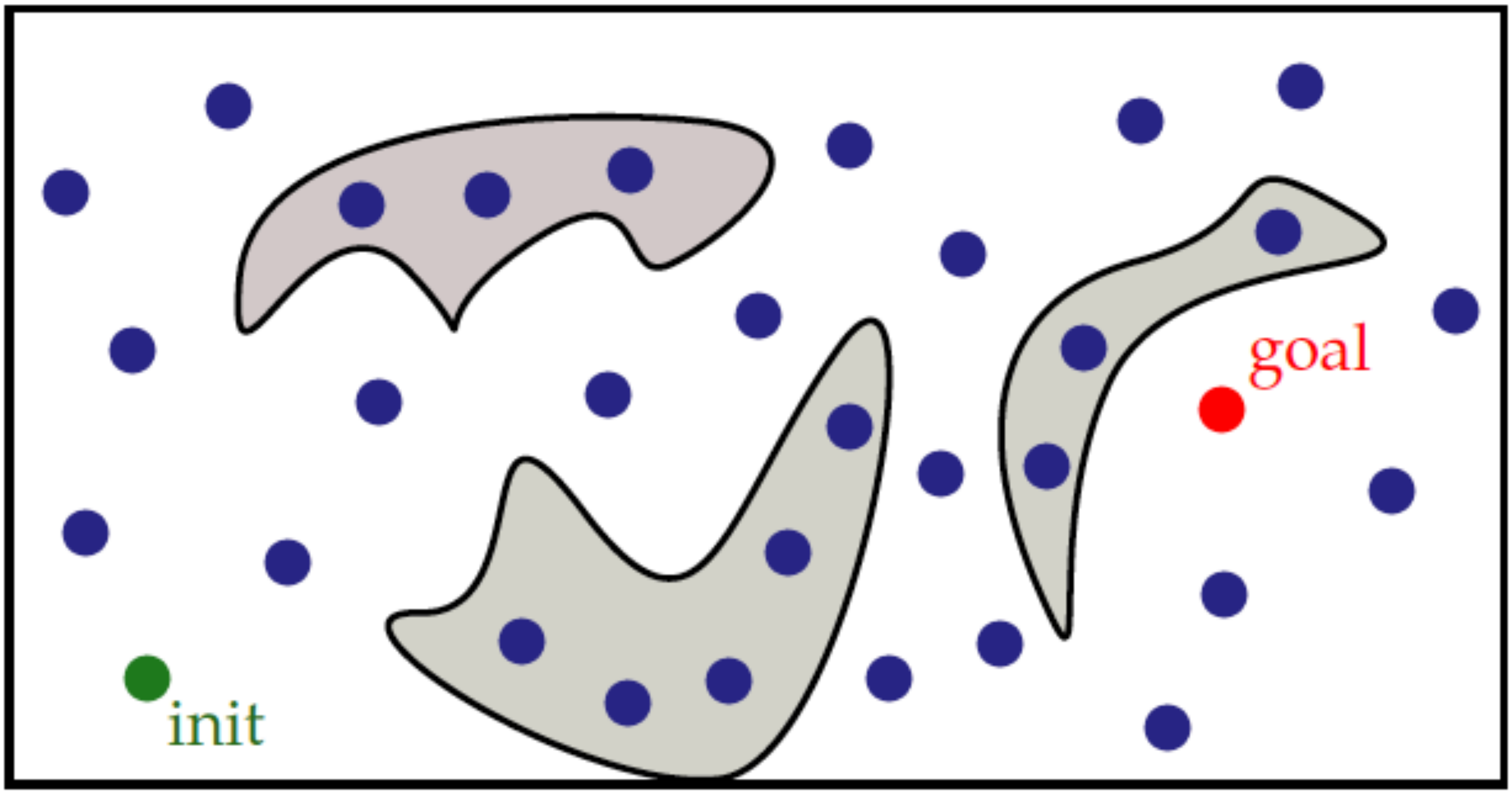
Probabilistic Roadmap (1/7)



[Fig from Erion Plaku]

Find a path from init to goal that avoids the obstacles

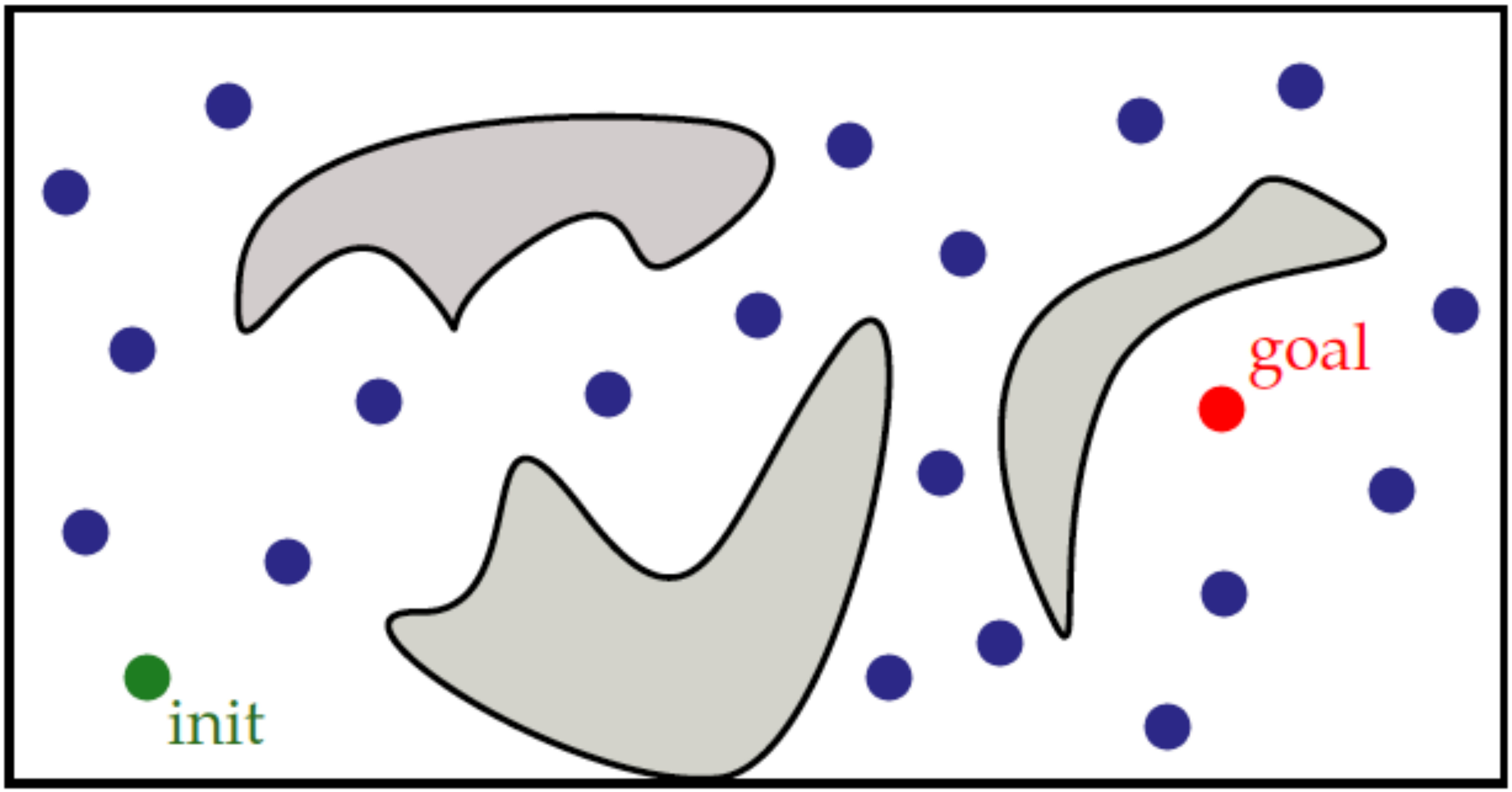
Probabilistic Roadmap (2/7)



[Fig from Erion Plaku]

Sample a set of configurations

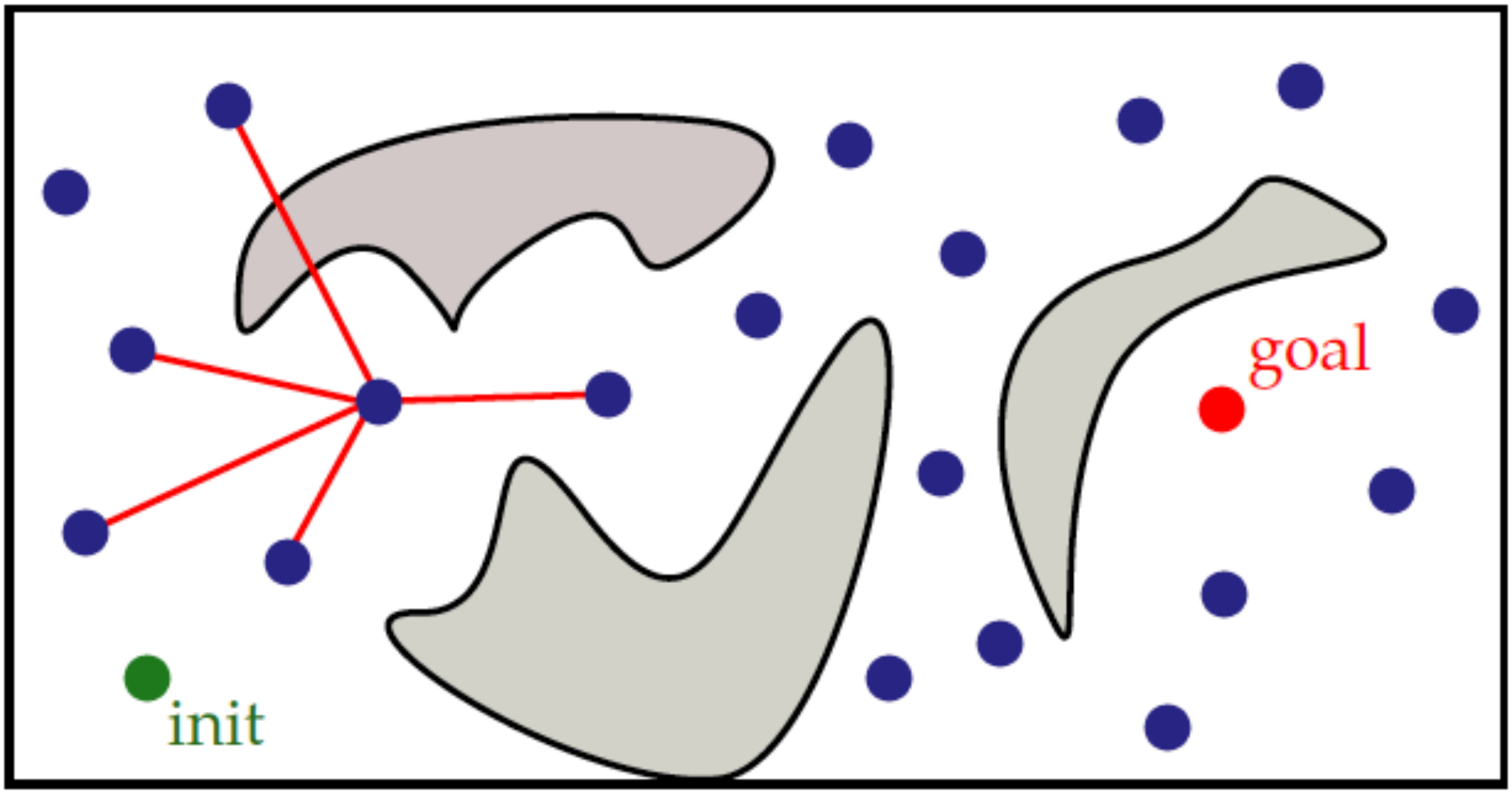
Probabilistic Roadmap (3/7)



[Fig from Erion Plaku]

Remove configurations that collide with the obstacles

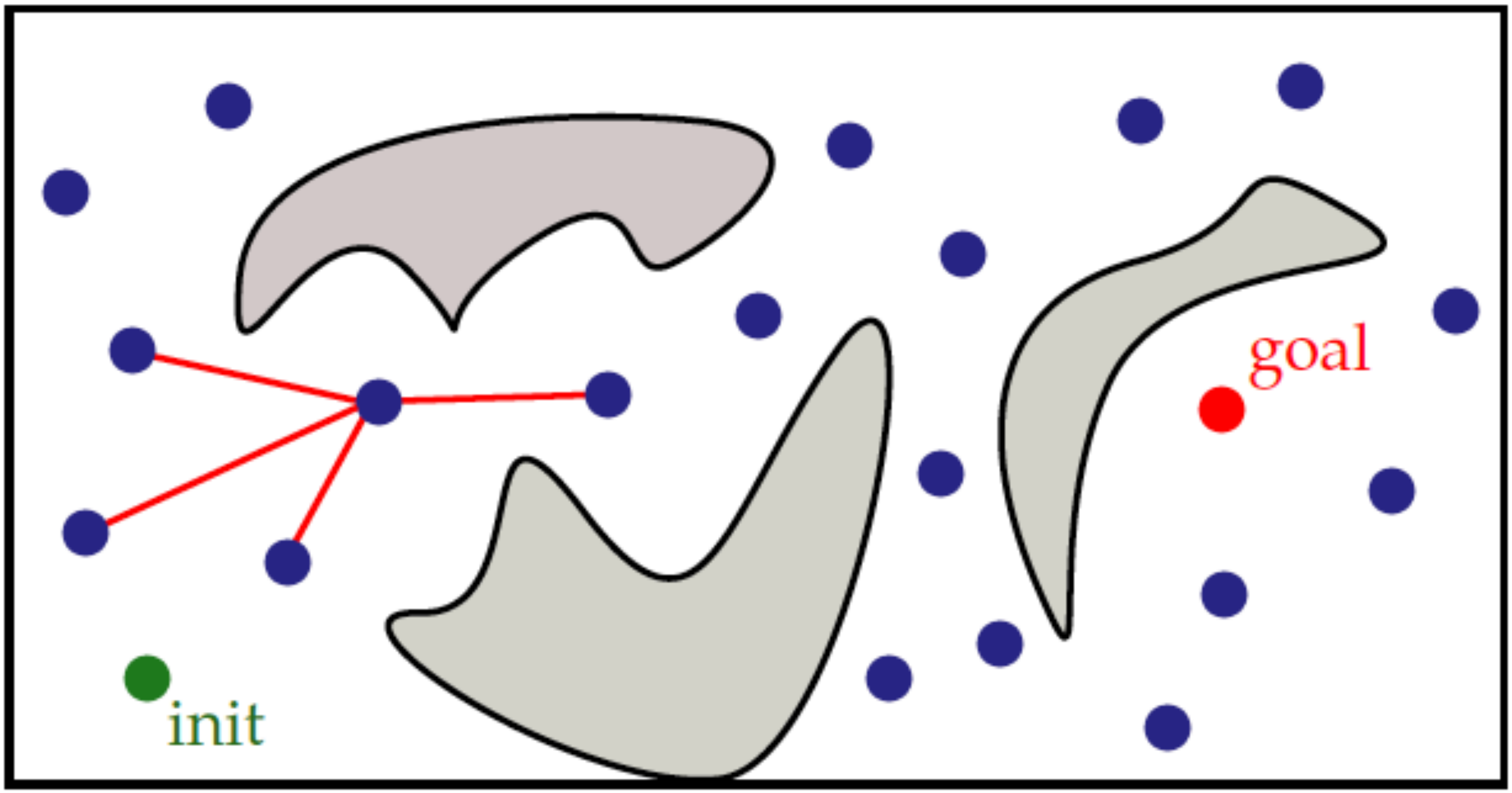
Probabilistic Roadmap (4/7)



[Fig from Erion Plaku]

Connect nearby configurations

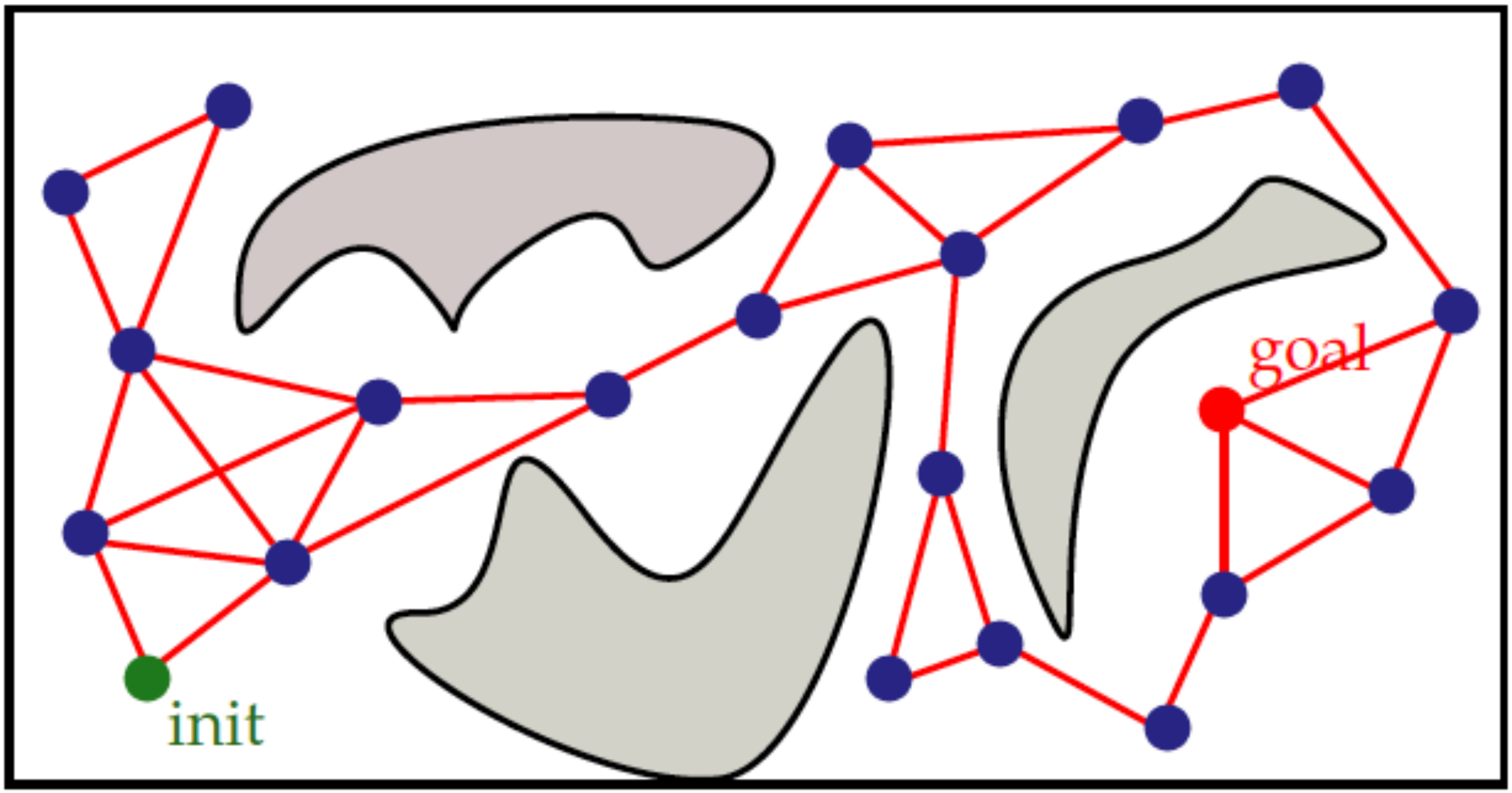
Probabilistic Roadmap (5/7)



[Fig from Erion Plaku]

Prune connections that collide with the obstacles

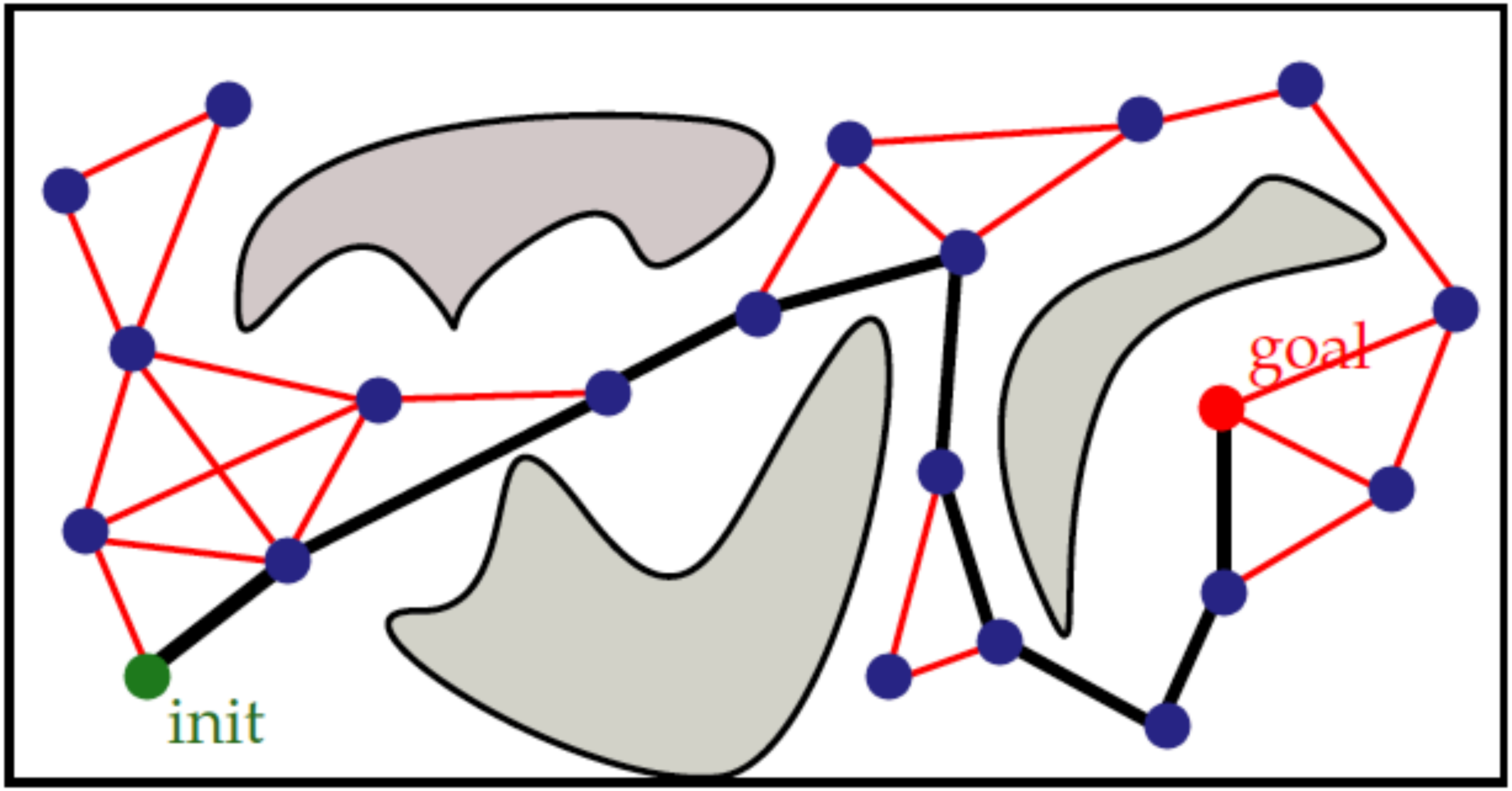
Probabilistic Roadmap (6/7)



[Fig from Erion Plaku]

The resulting structure is a finite roadmap (graph)

Probabilistic Roadmap (7/7)

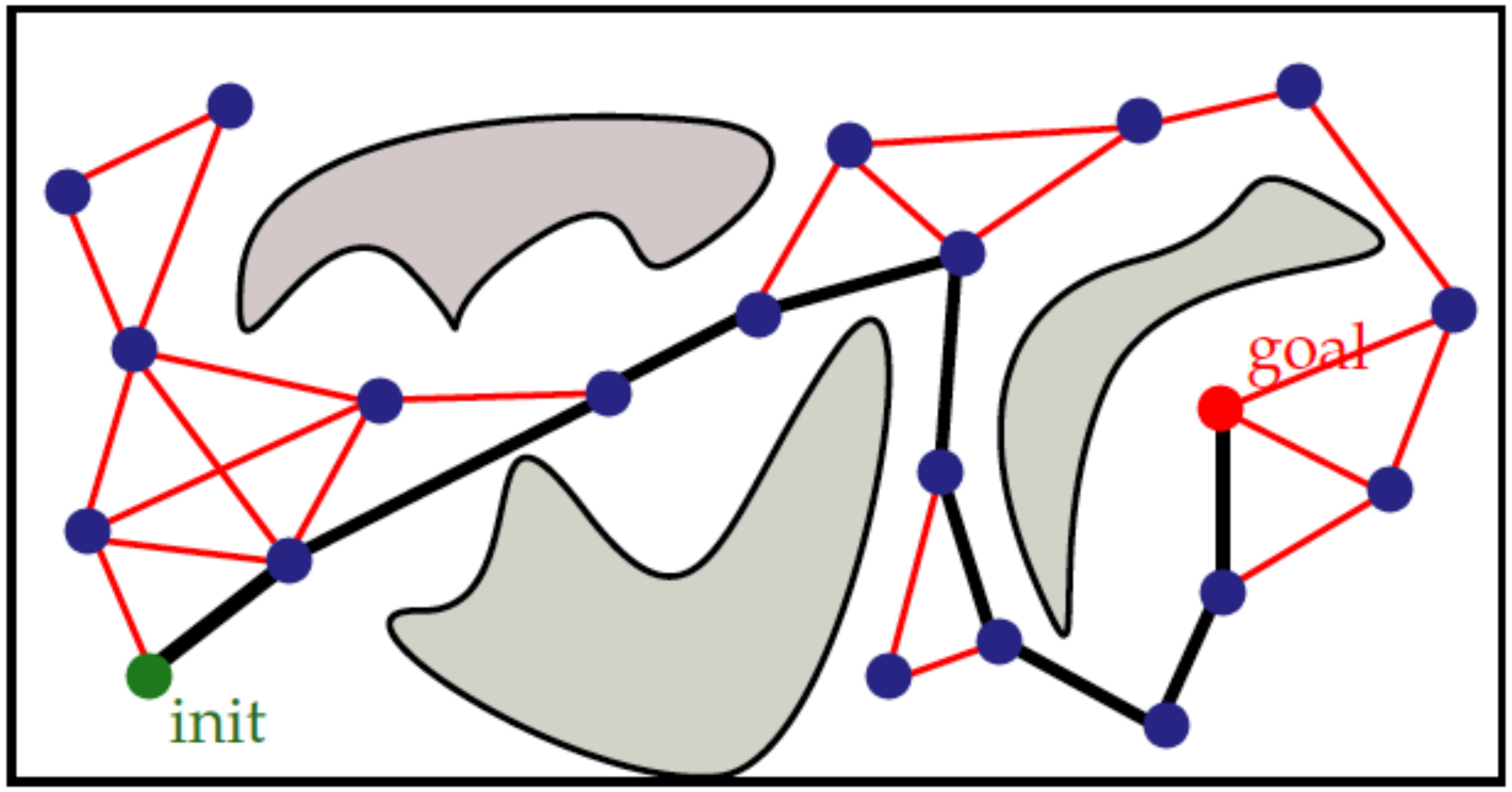


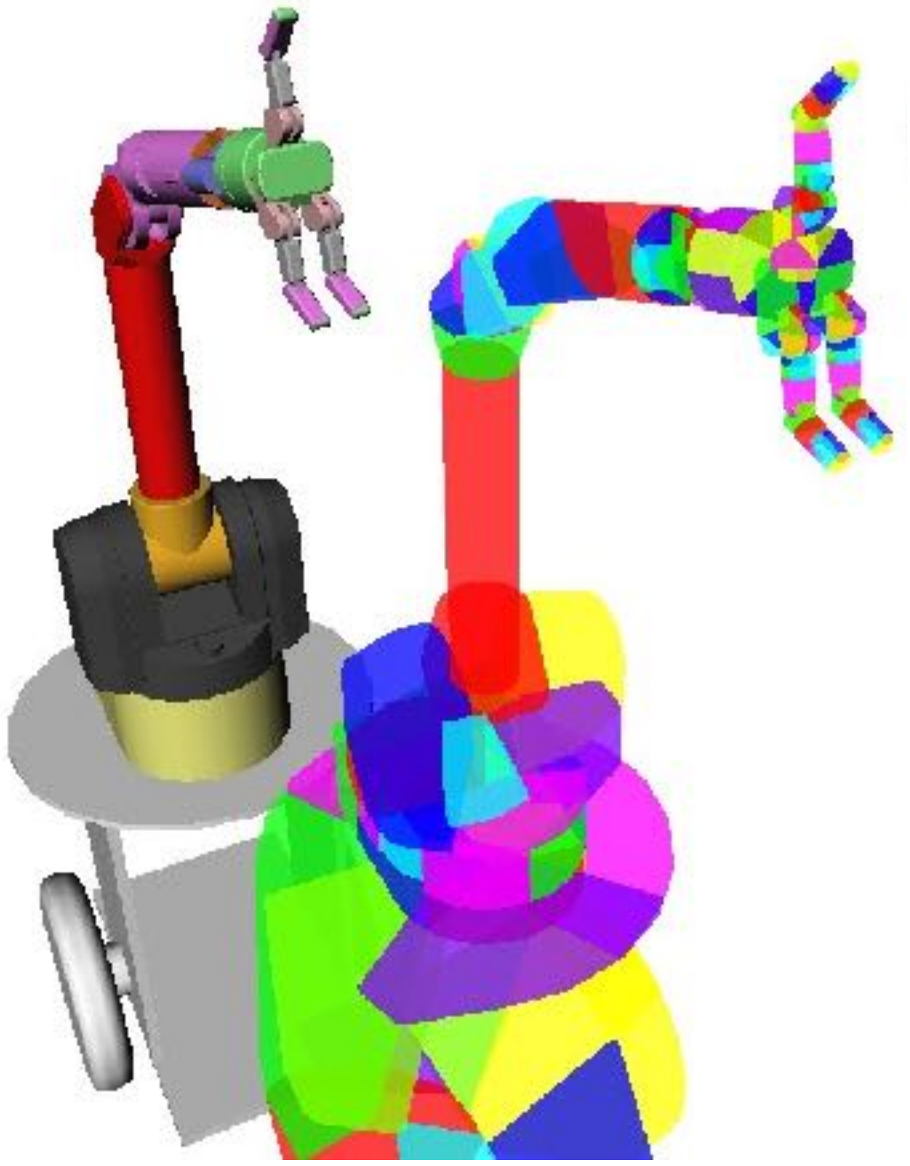
[Fig from Erion Plaku]

Search for the shortest-path on the roadmap

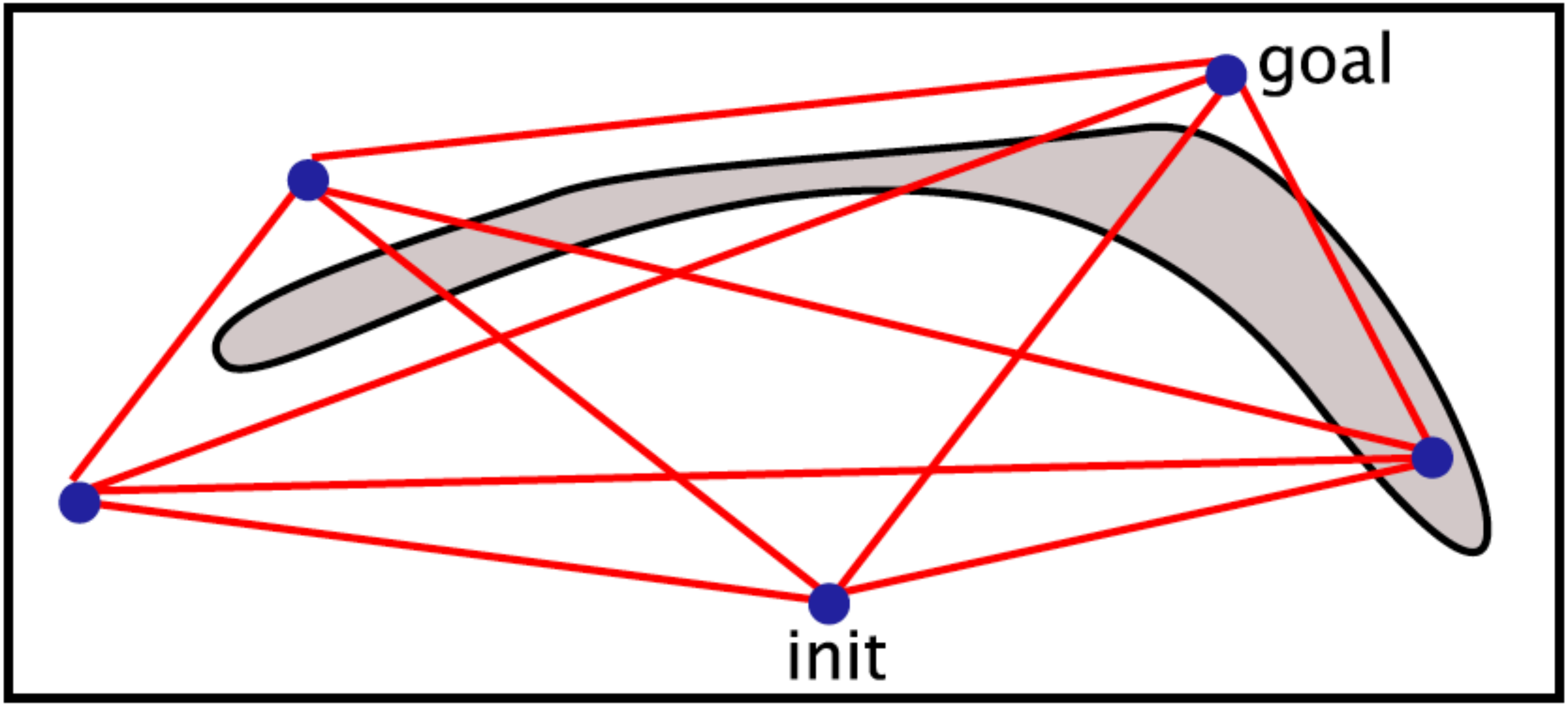
Collision Checking is Expensive

- Collision checking **dominates** runtime
  - **Complex geometries** & **fine resolutions** (for safety)
- Many edges clearly do not lie on a low-cost path
- **Optimistically** plan **without collisions**
- Check collisions **lazily** by evaluating only on **candidate plans**





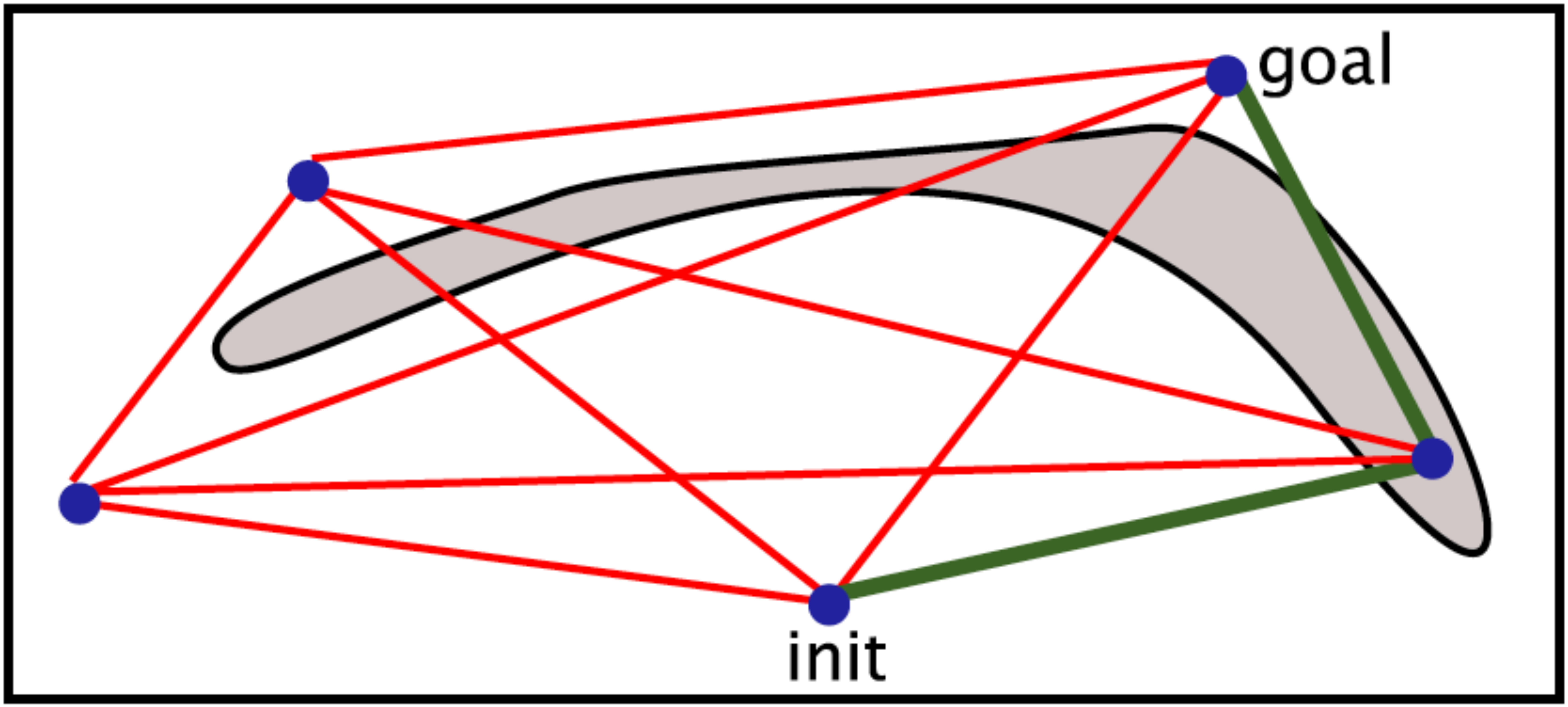
Lazy PRM (1/10)



[Fig from Erion Plaku]

Construct a PRM ignoring collisions

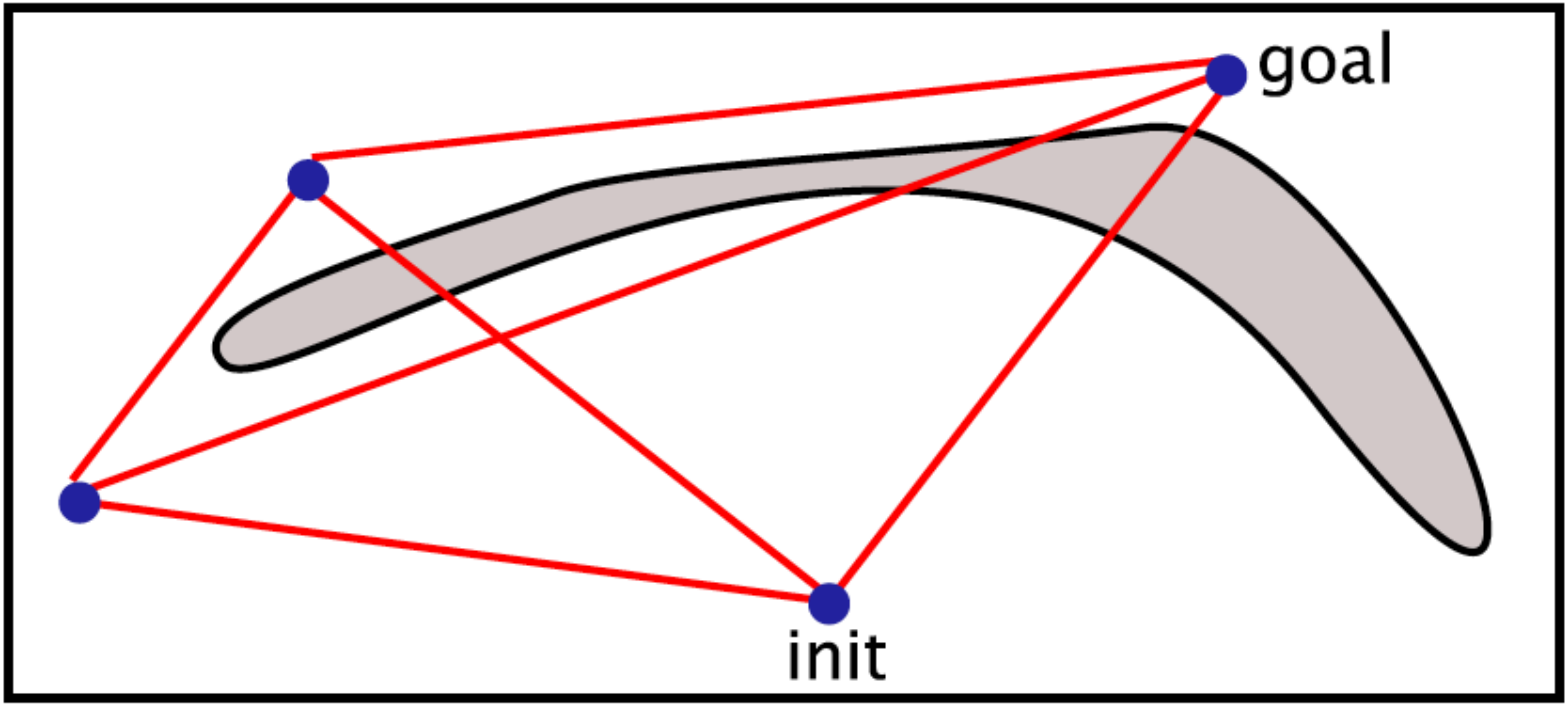
Lazy PRM (2/10)



[Fig from Erion Plaku]

Search for the shortest-path on the roadmap

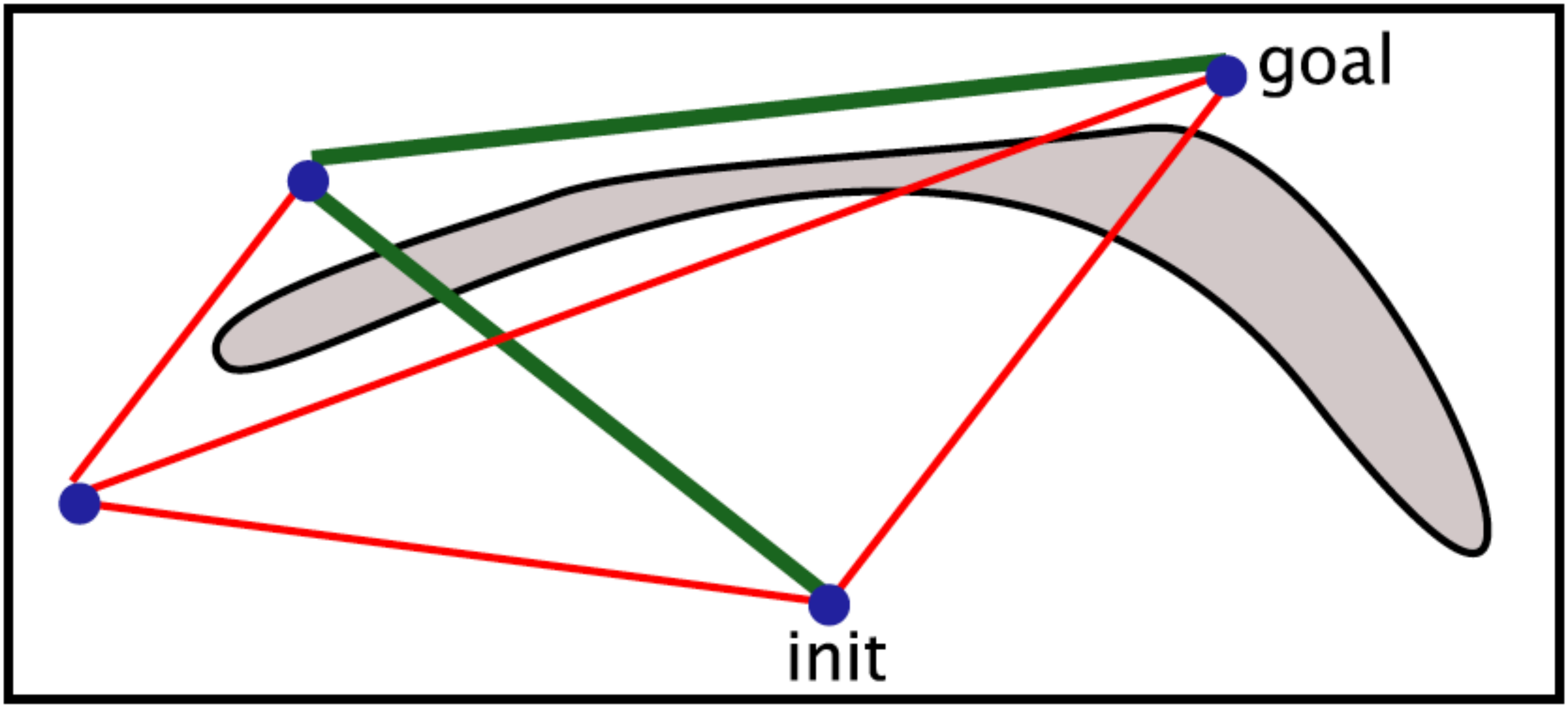
Lazy PRM (3/10)



[Fig from Erion Plaku]

Remove plan edges that collide with obstacles

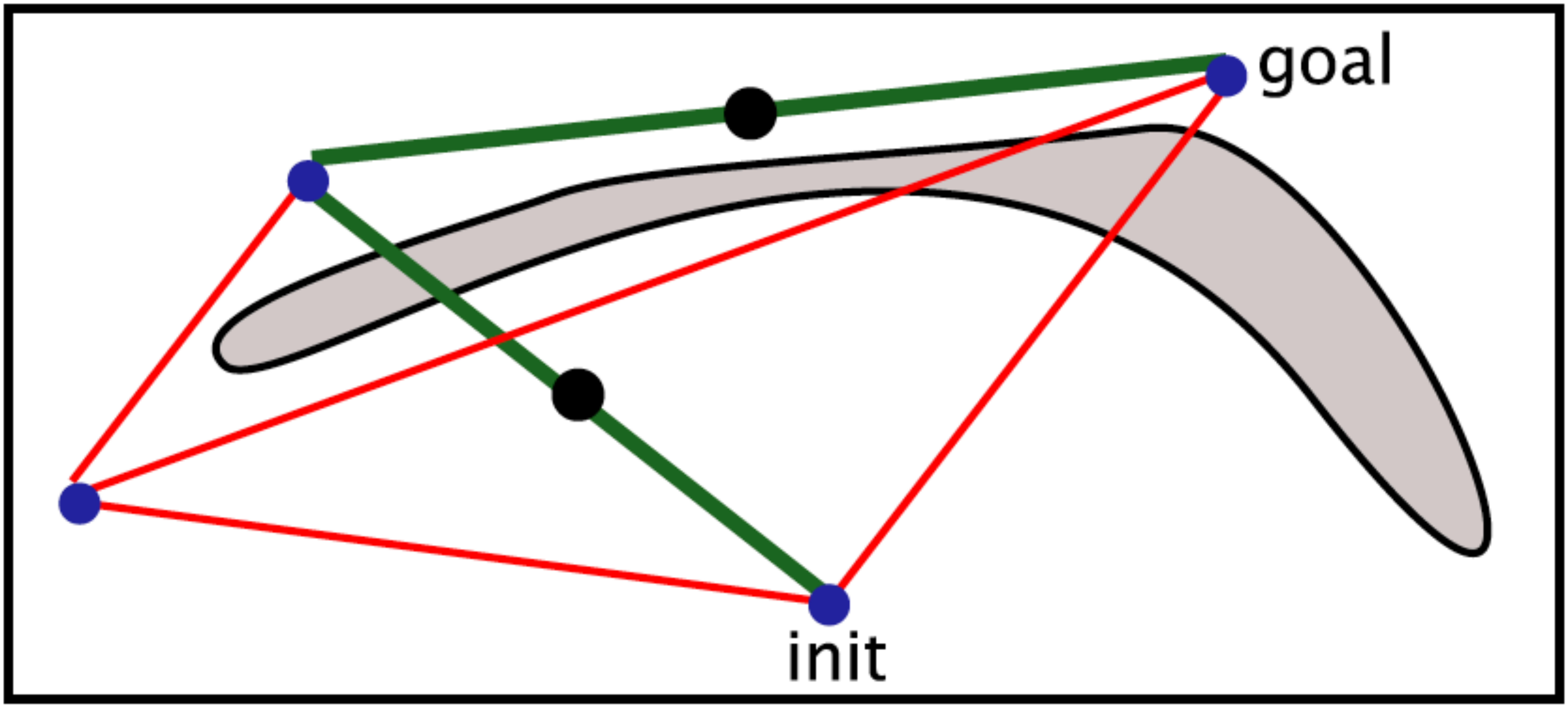
Lazy PRM (4/10)



[Fig from Erion Plaku]

Search for the new shortest-path on the roadmap

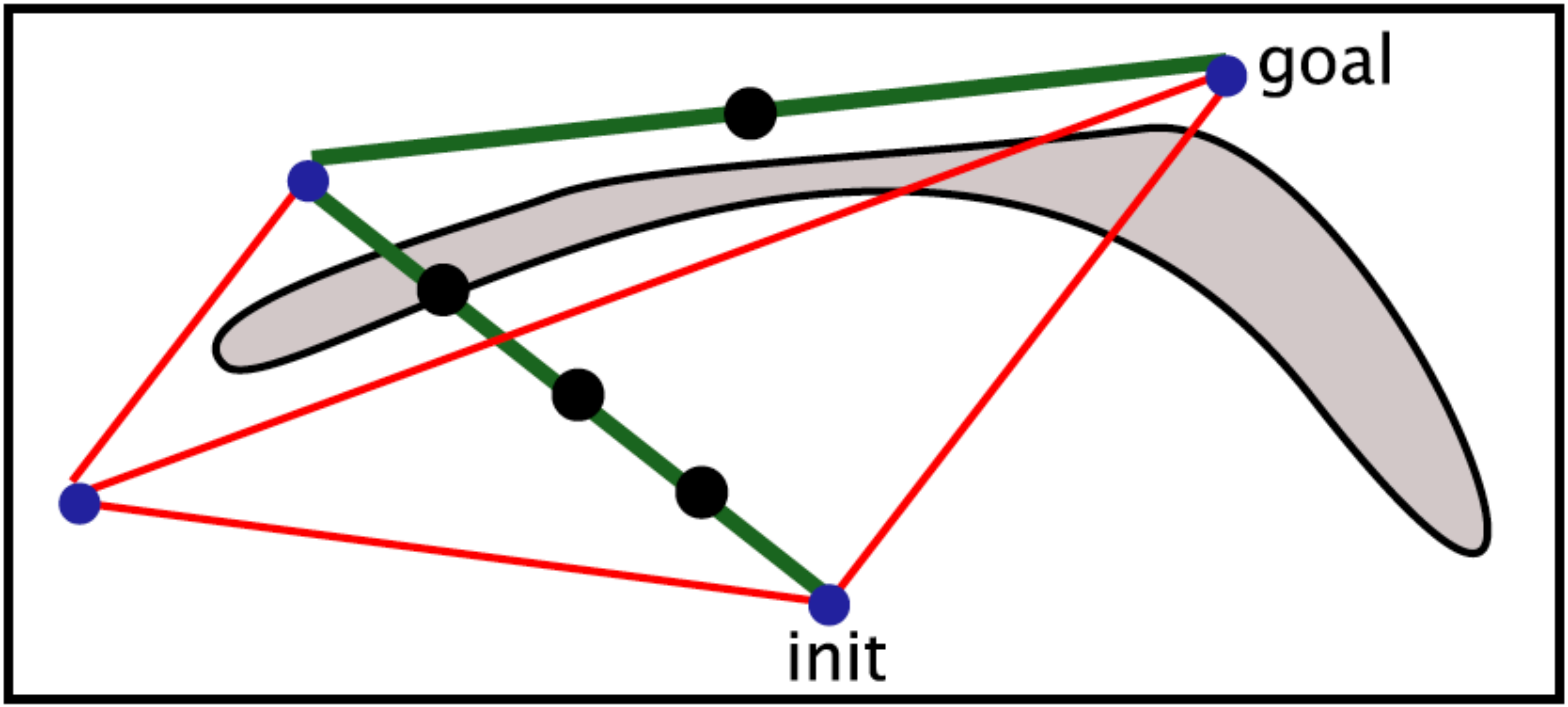
Lazy PRM (5/10)



[Fig from Erion Plaku]

Check the edges on the plan for collisions

Lazy PRM (6/10)

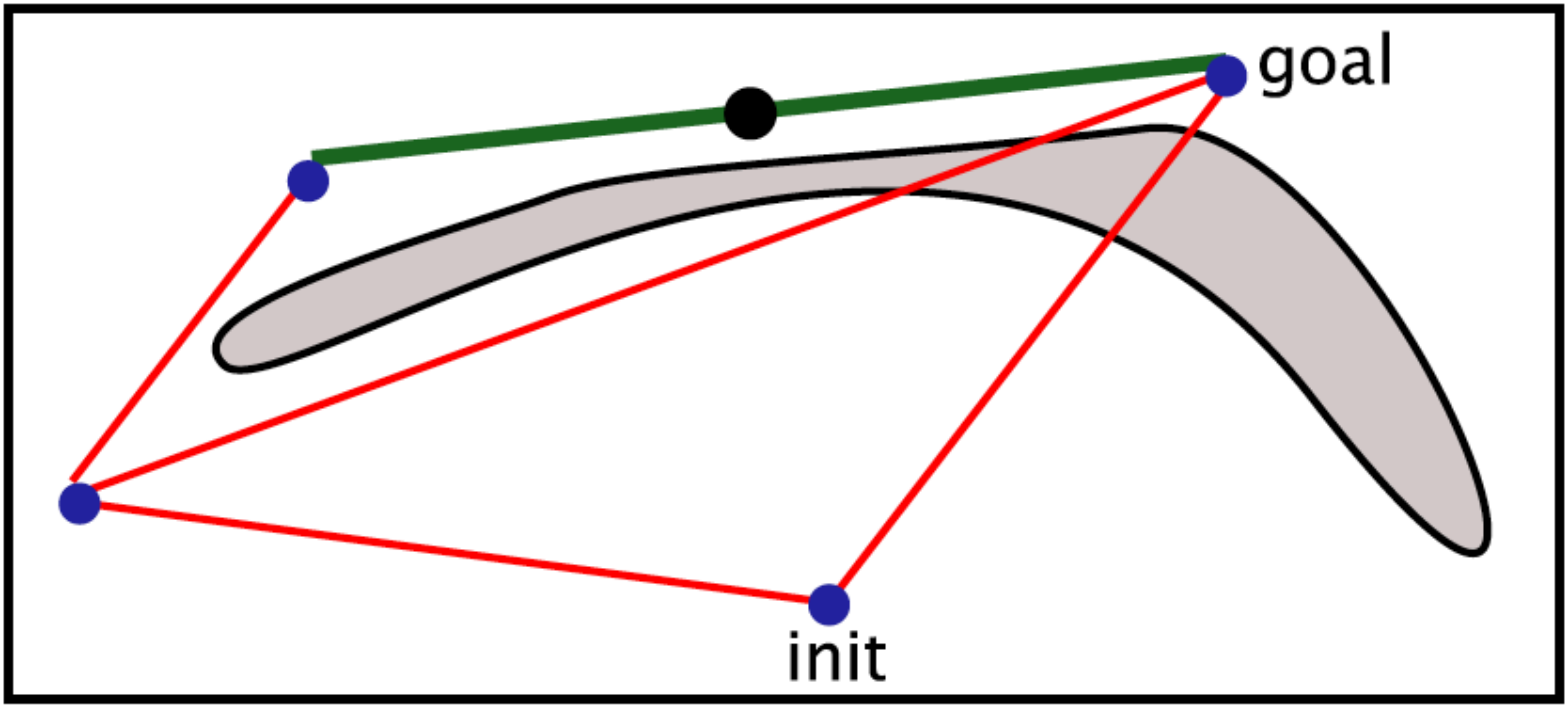


[Fig from Erion Plaku]

Check the edges on the plan for collisions

(with increased resolution)

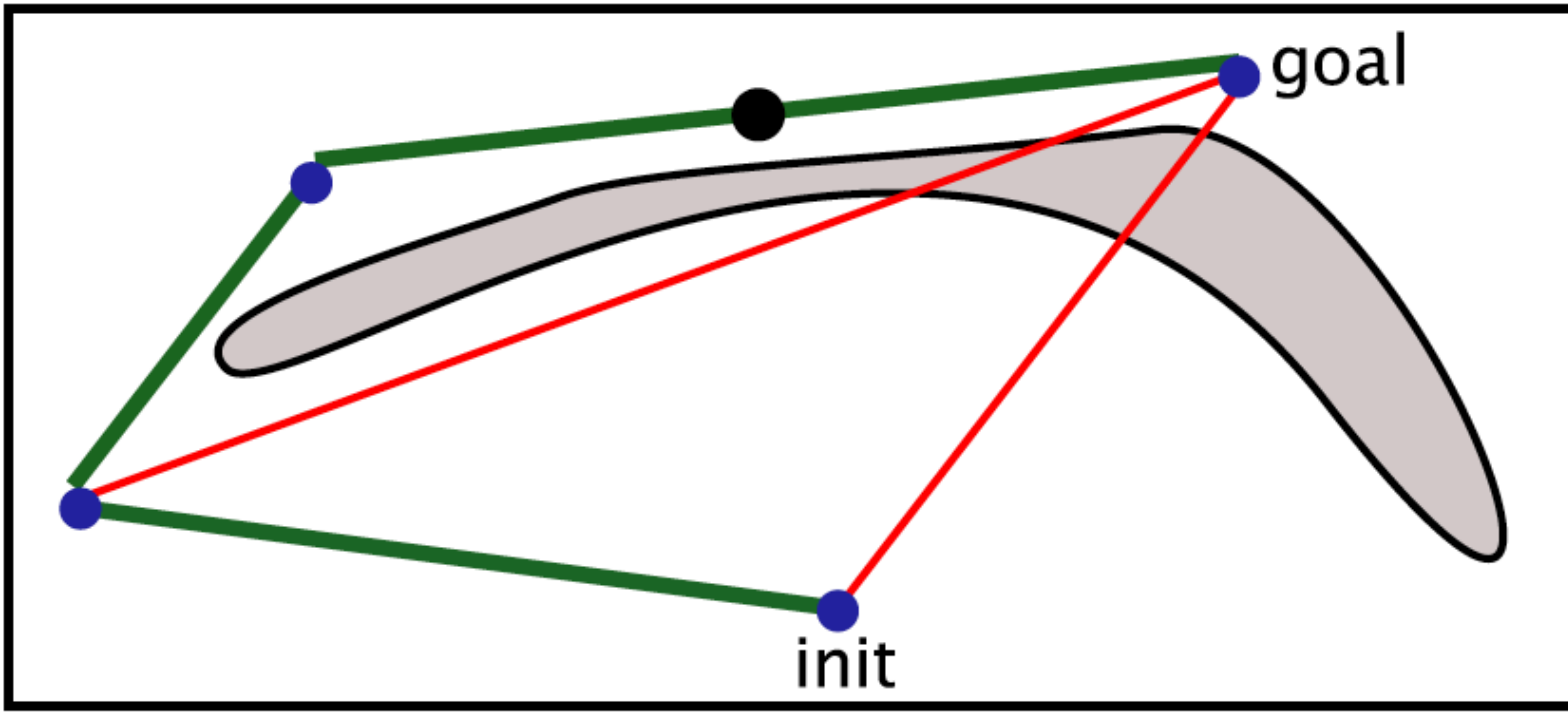
Lazy PRM (7/10)



[Fig from Erion Plaku]

Remove plan edges that collide with obstacles

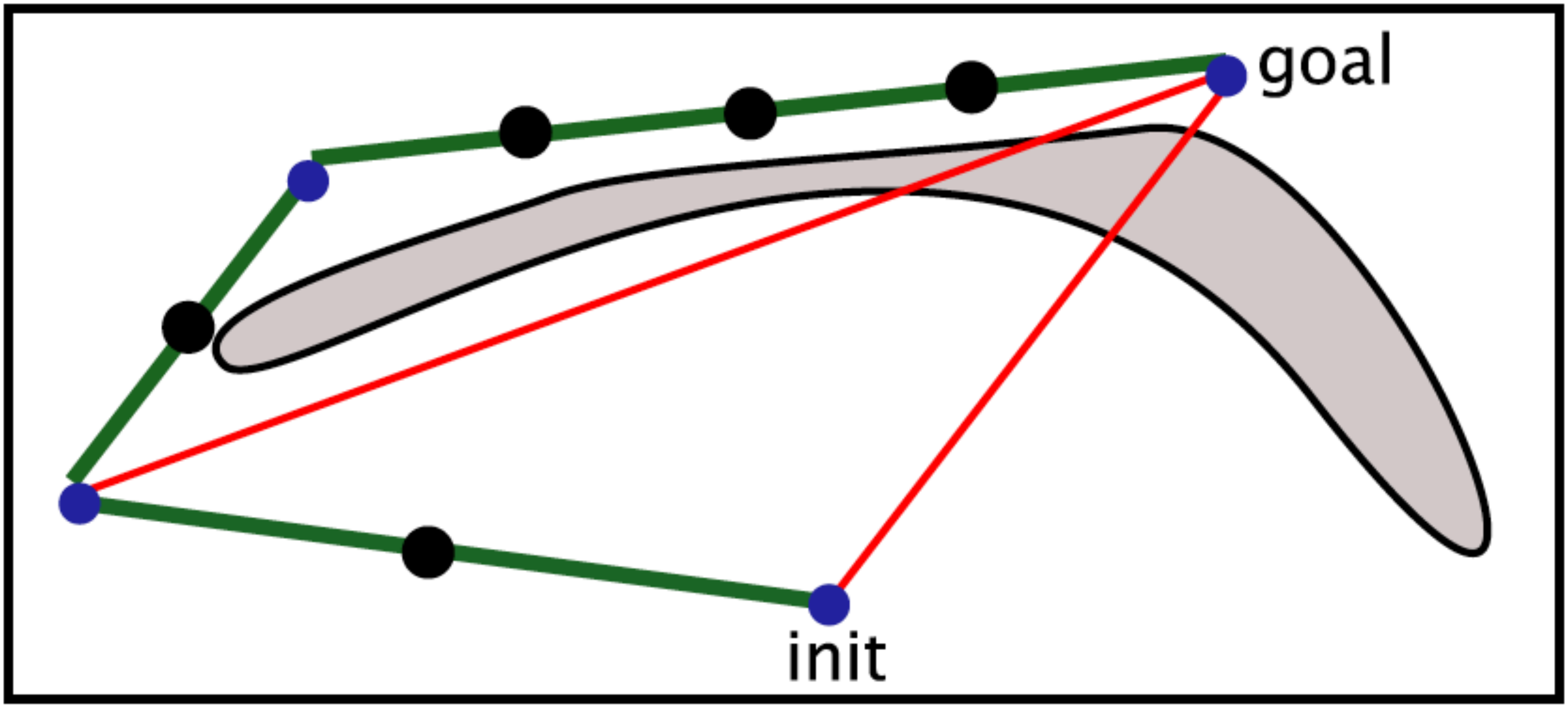
Lazy PRM (8/10)



[Fig from Erion Plaku]

Search for the new shortest-path on the roadmap

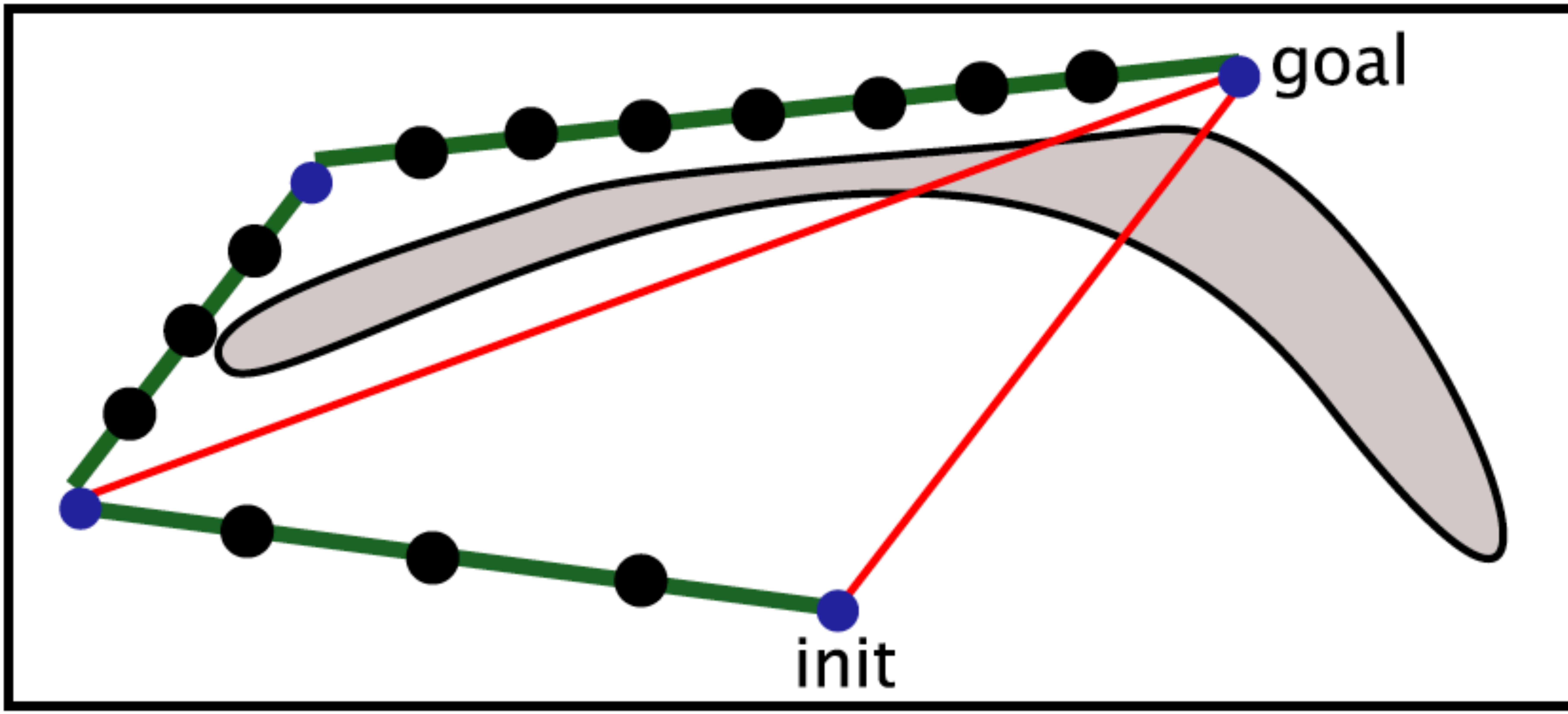
Lazy PRM (9/10)



[Fig from Erion Plaku]

Check the edges on the plan for collisions

Lazy PRM (10/10)



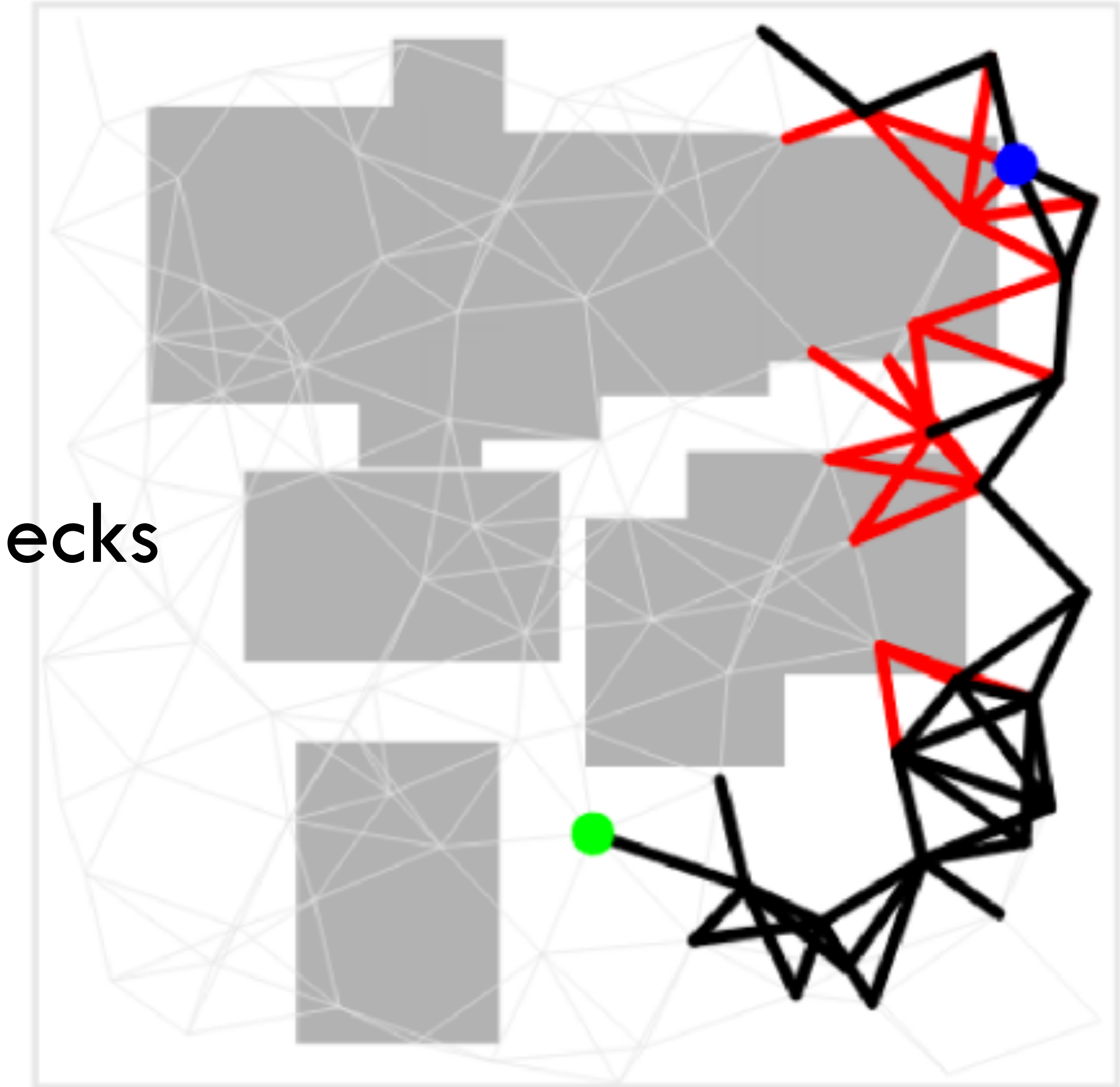
[Fig from Erion Plaku]

Return the current path as a solution

Lazy Motion Planning

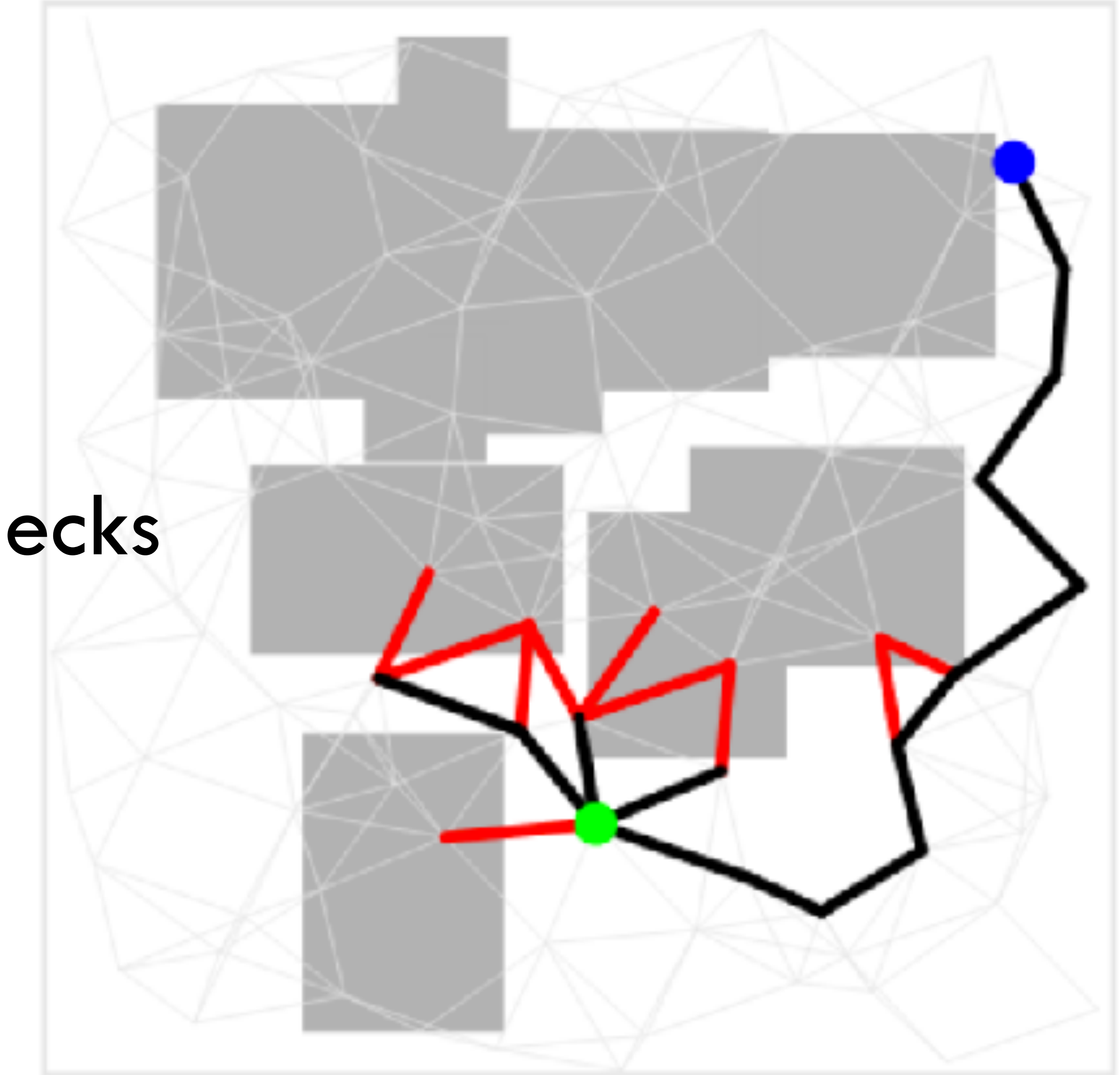
- **Defer** collision checking until a path is found
- **Remove** colliding edges path from the roadmap
- **Repeat** this process with a new path
- **Terminate** when a collision-fee path is found

77 checks

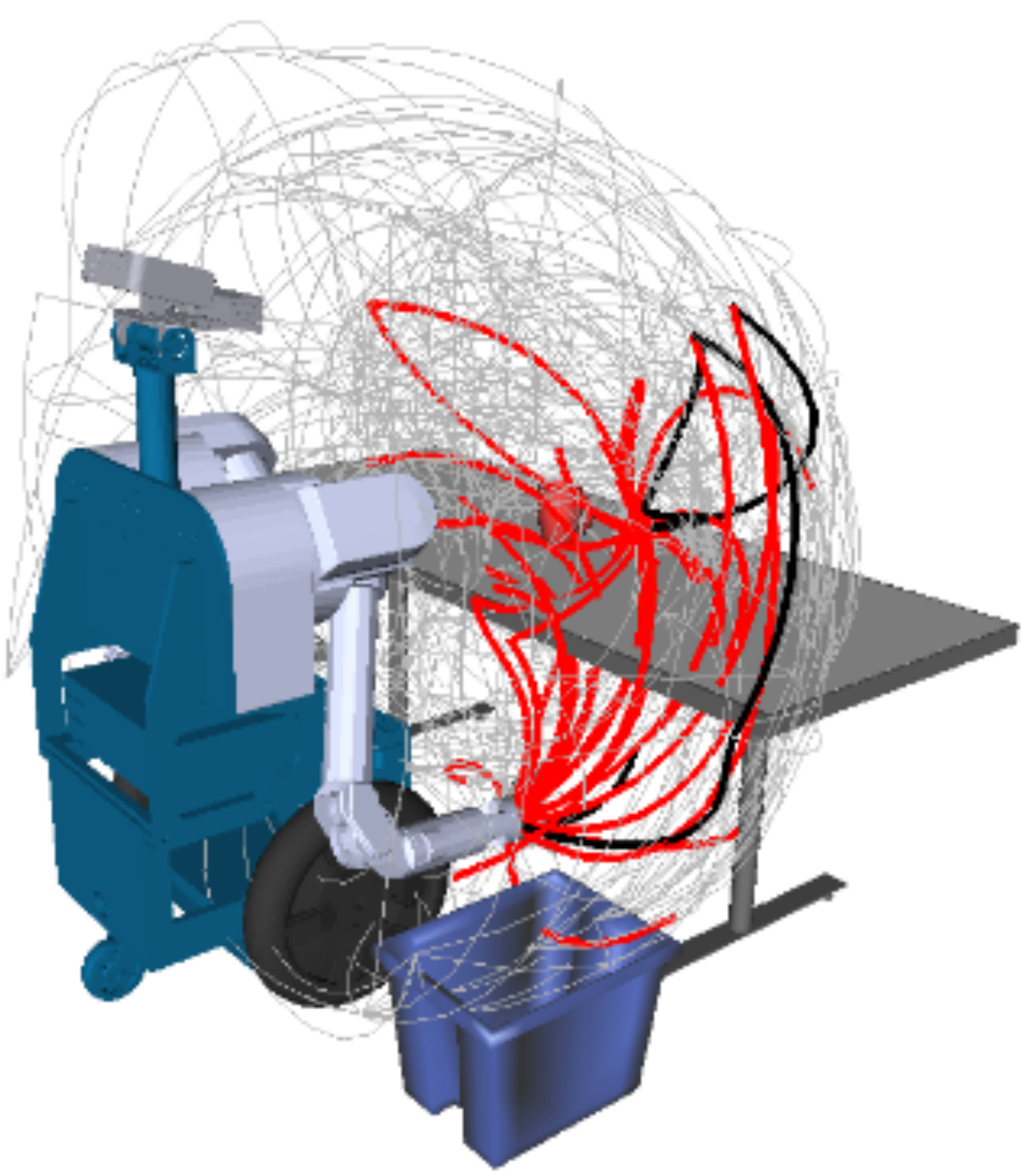


Eager (during search)

23 checks



Lazy



[Bohlin 2000][Dellin 2016]

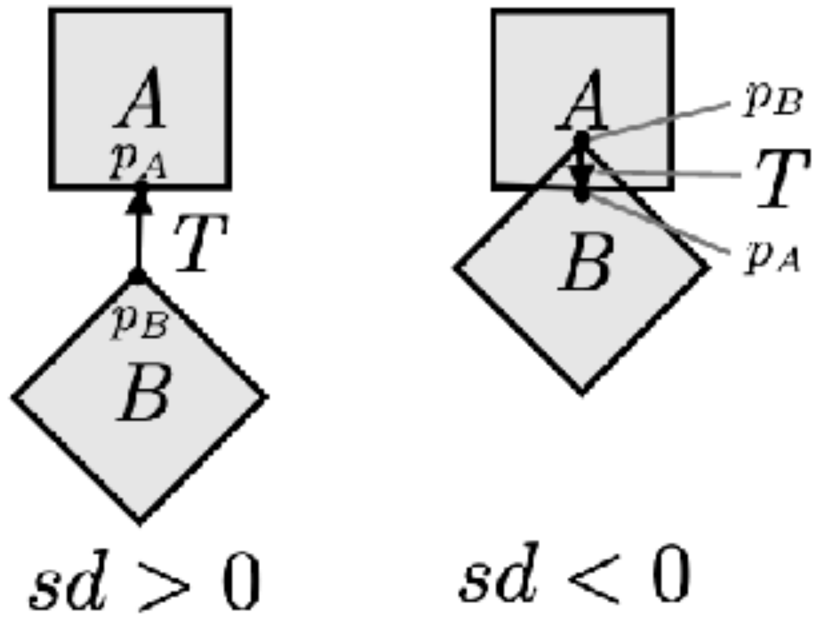
Trajectory Optimization

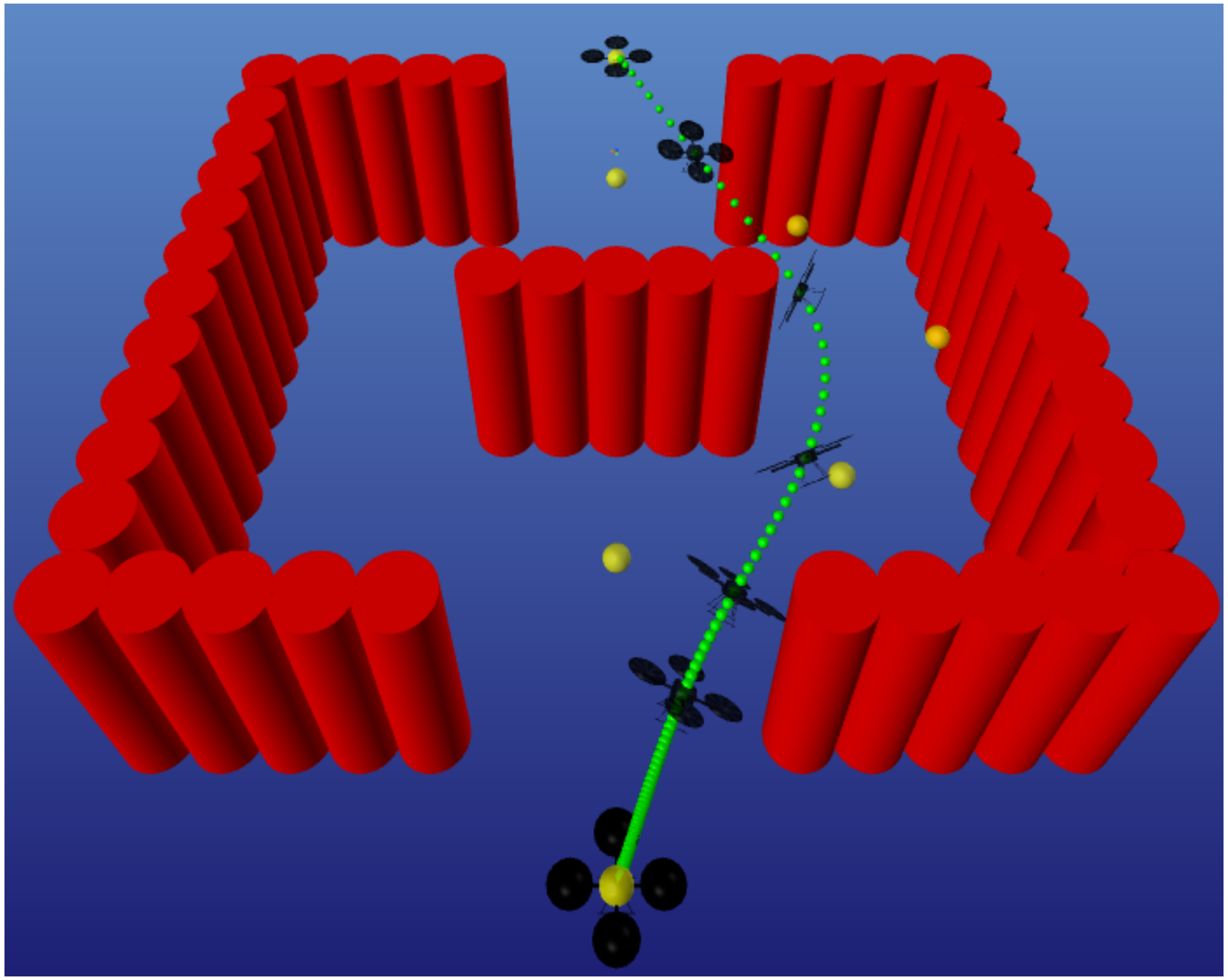
- Frame motion planning as a **non-convex constrained optimization** problem & converge to **local minima**

$$\begin{aligned}&\text{minimize } f(\mathbf{x})\\&\text{subject to}\\&\quad g_i(\mathbf{x}) \le 0, \quad i = 1, 2, \ldots, n_{ineq}\\&\quad h_i(\mathbf{x}) = 0, \quad i = 1, 2, \ldots, n_{eq}\end{aligned}$$

- Collision constraints enforced via **signed distance** (sd)

cuRobo
Local
TrajOpt





[Ratliff 2009][Schulman 2013][Sundaralingam 2022]

Task and Motion Planning (TAMP)

Shakey the Robot (1969)

- **First autonomous mobile manipulator** (via pushing)
  - Visibility graph, A* search, and STRIPS!
- **Decoupled** task and motion planning
  - Task planning **then** motion planning

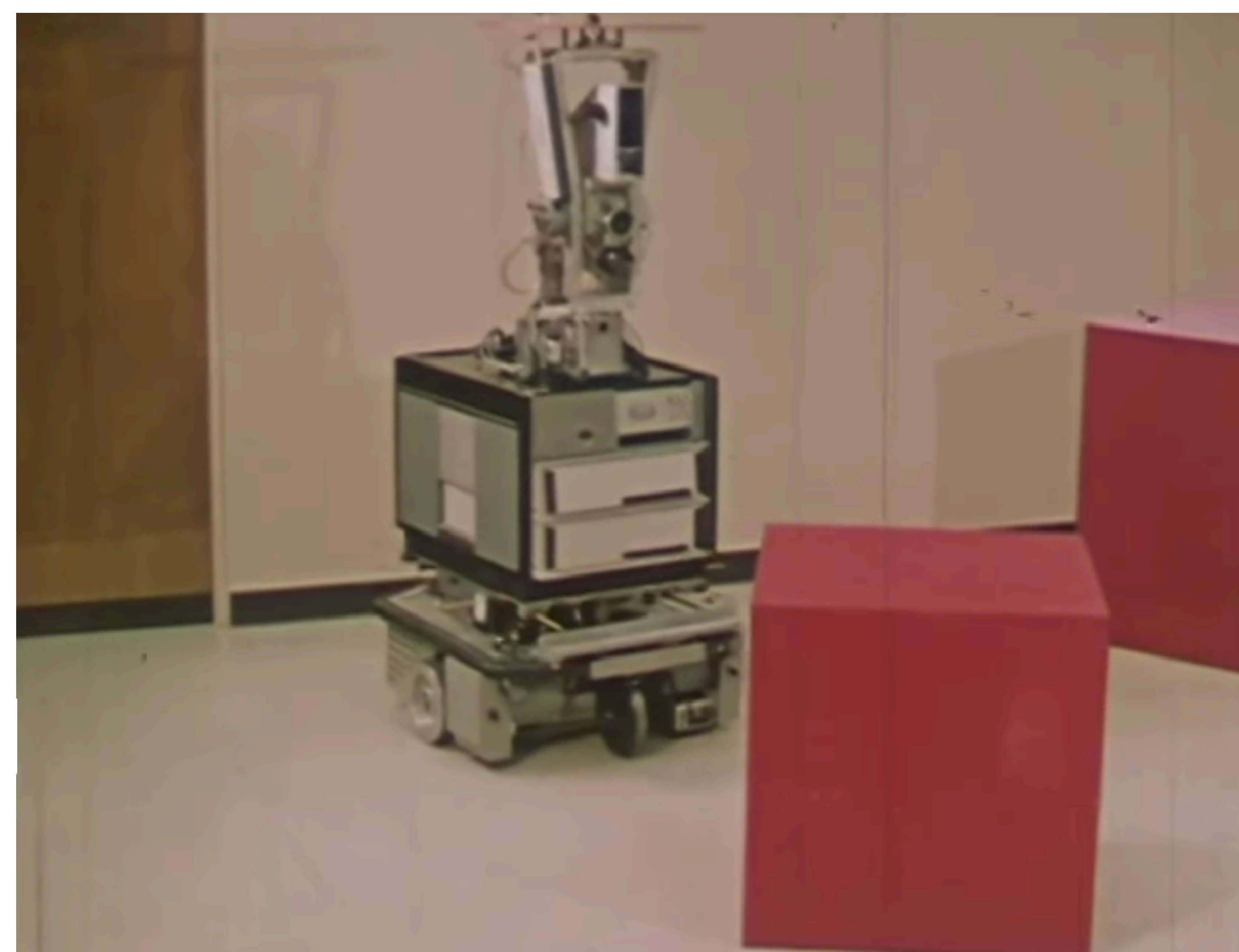
[Fikes 1971]

[Nilsson 1984]

```
type(robot robot)        type(o1 object)
name(robot shakey)       name(o1 box1)
at(robot 4.1 7.2)        at(o1 3.1 5.2)
theta(robot 90.1)        inroom(o1 r1)
                         shape(o1 wedge)
                         radius(o1 3.1)
```

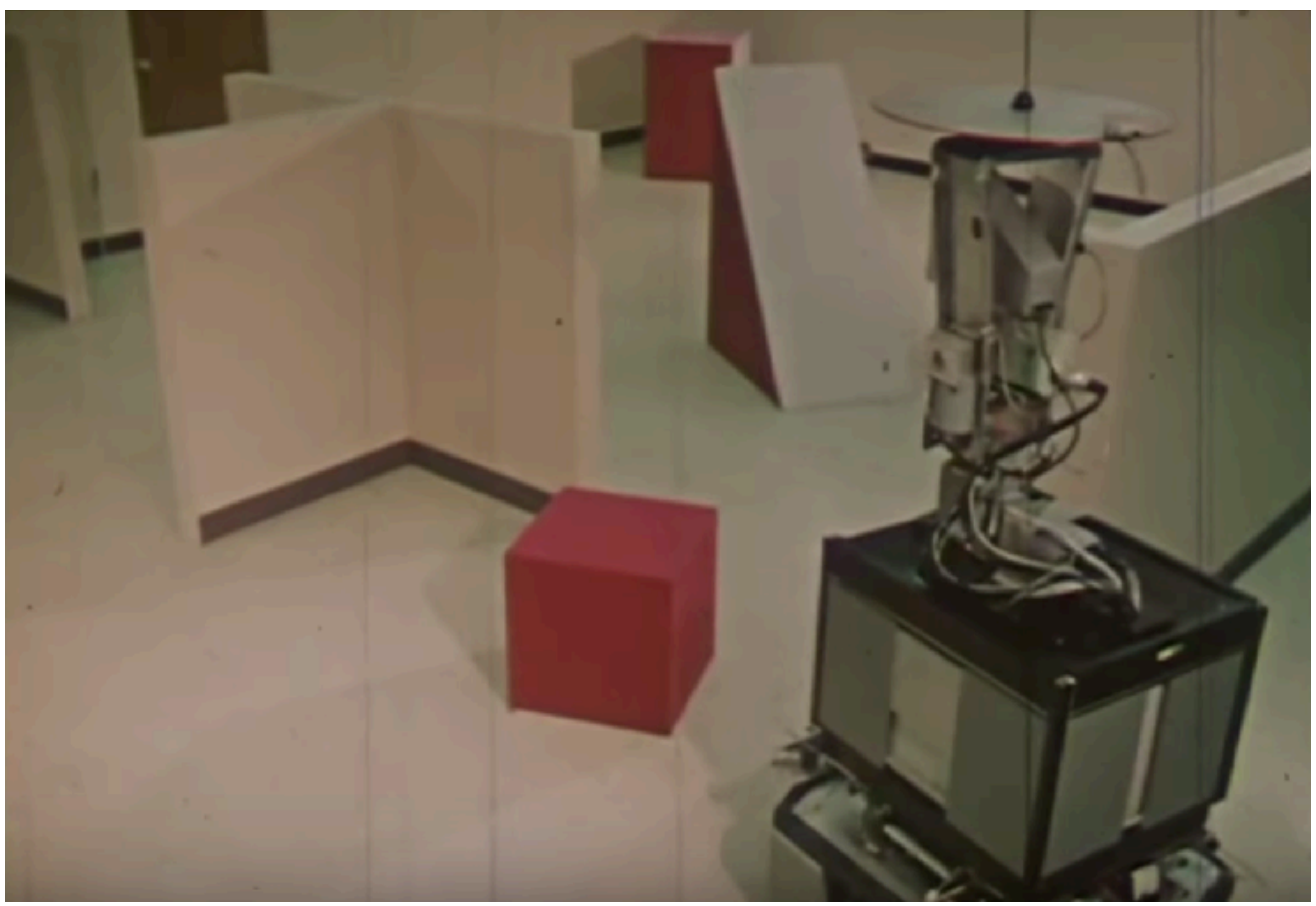
GOTHRU(d,r1,r2)

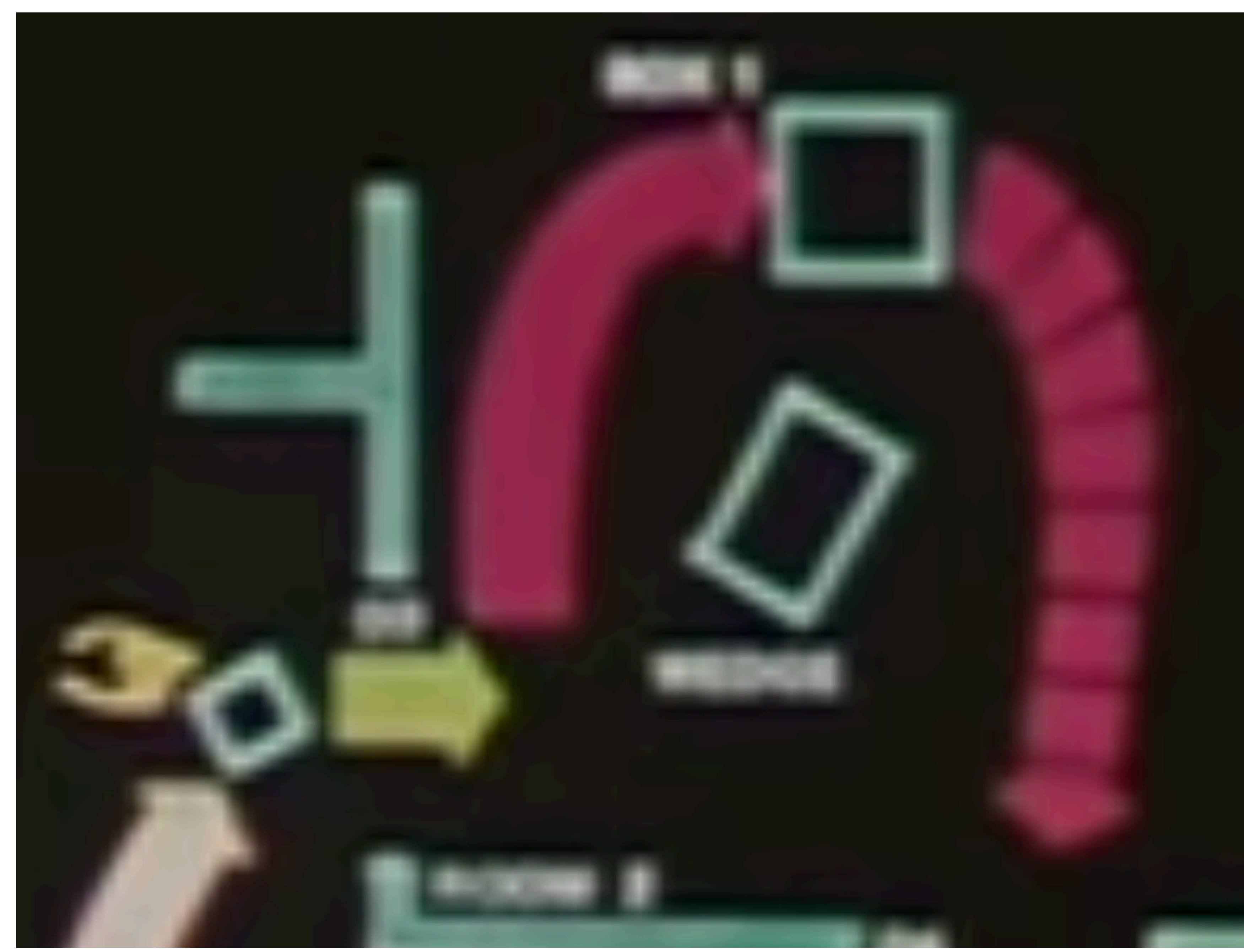
Precondition	INROOM(ROBOT,r1) ∧ CONNECTS(d,r1,r2)
Delete List	INROOM(ROBOT,$)
Add List	INROOM(ROBOT,r2)



Obstacle Blocks Shakey's Path

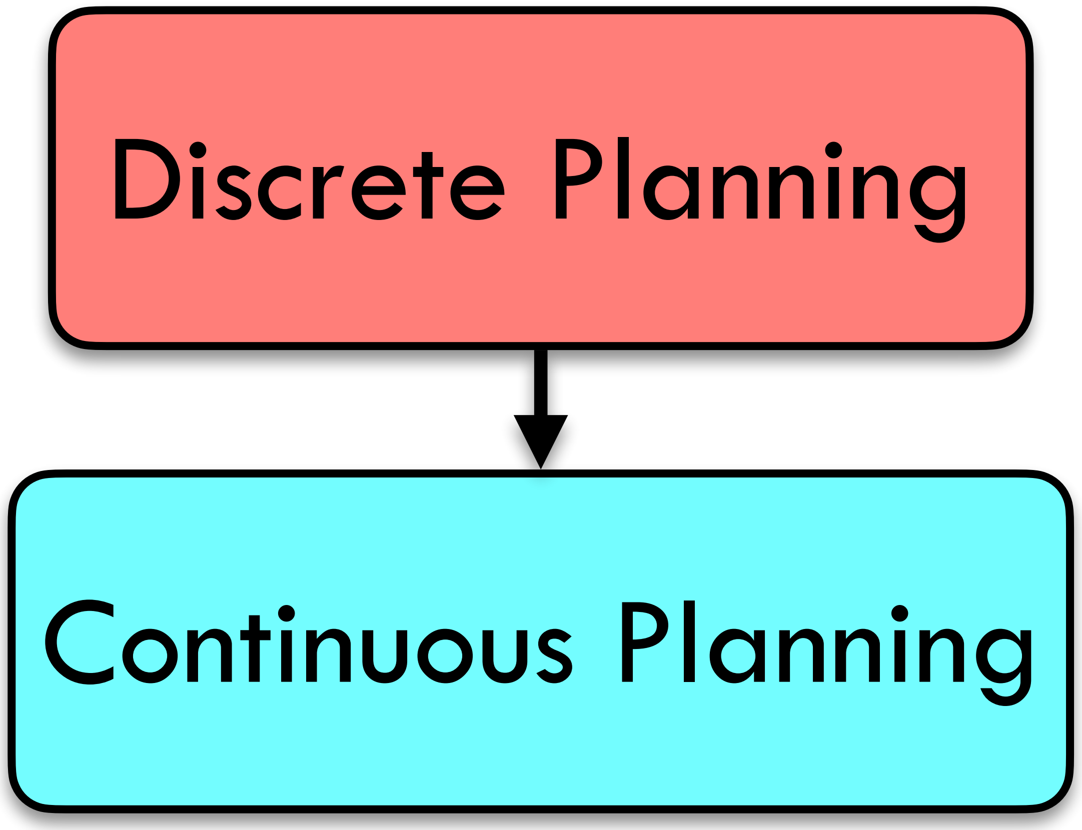
- What if a movable block **prevented** Shakey from safely moving into the adjacent room?
- Shakey could **push** it out of the way or **go around** it
  - What's more efficient? How to push it? …



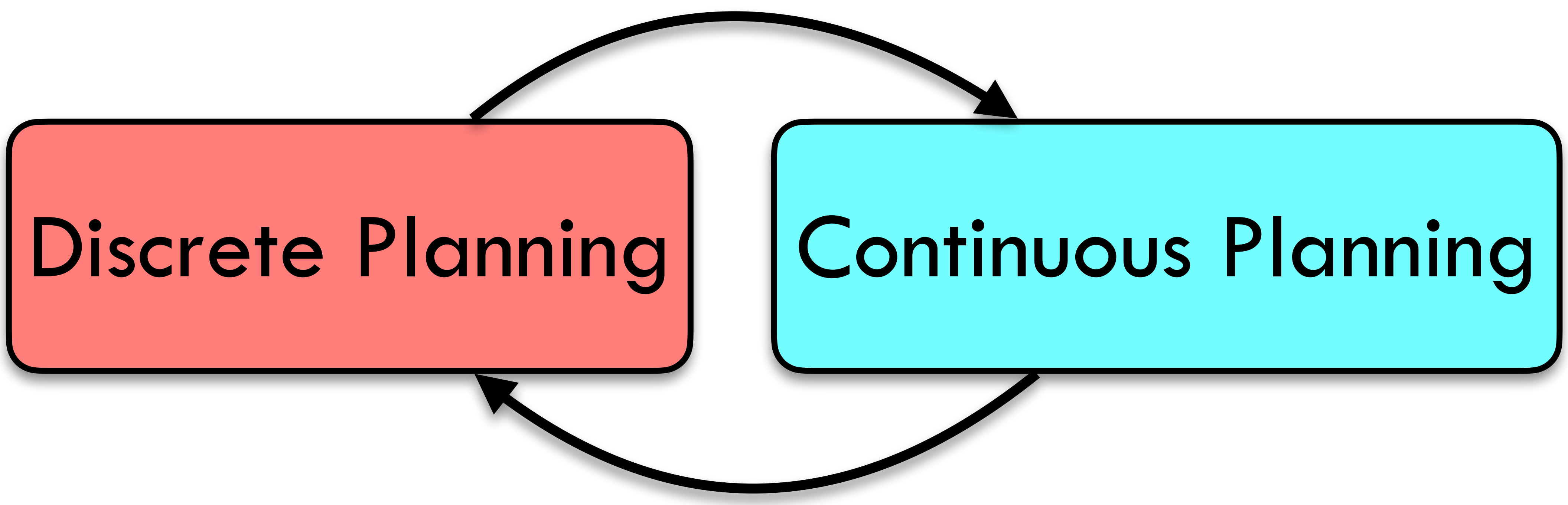


Decoupled vs Integrated TAMP

- **Decoupled:** discrete (task) planning **then** continuous (motion) planning
- Requires a strong **downward refinement** assumption
  - **<u>Every</u>** correct discrete plan can be **refined** into a correct continuous plan (from hierarchal planning)
- **Integrated:** **<u>simultaneous</u>** discrete & continuous planning



Decoupled



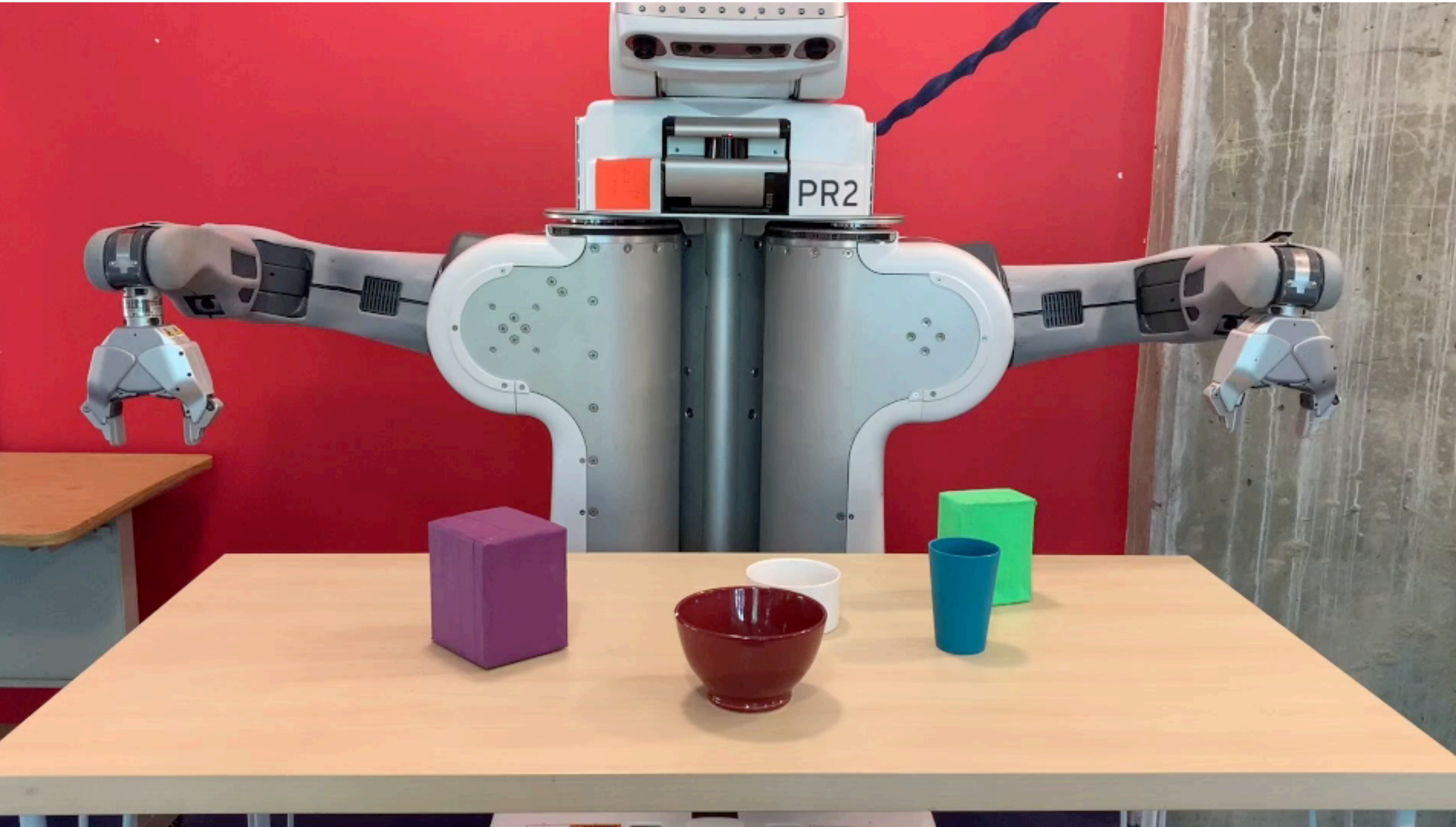
Integrated

Geometric Constraints Affect Plan

- **Inherits challenges** of both motion & classical planning
  - **High-dimensional, continuous** state-spaces
  - State-space **exponential** in number of variables
  - **Long horizons**

- Continuous constraints **limit high-level strategies**
  - Kinematics, reachability, joint limits, collisions grasp, visibility, stability, stiffness, torque limits, …

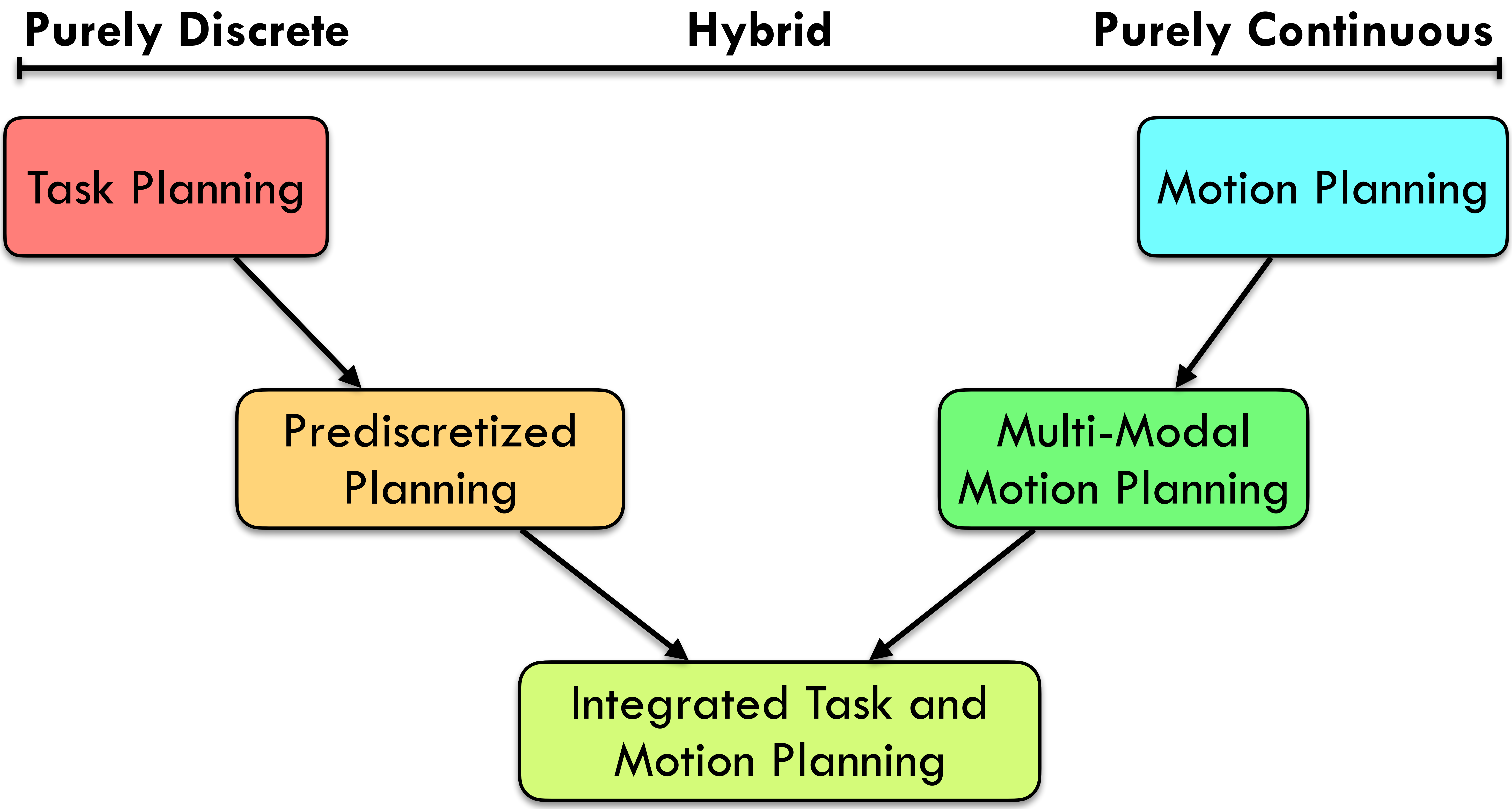
Pouring Among Obstacles



Block in Left Cabinet & Doors Closed

- Robot forced to **regrasp** the object
  - Change from a **top** grasp to a **side** grasp
- **Non-monotonic** problem
  - Plan must **undo goals** to solve
  - **Open** then **close** the cabinet door
- Physical constraints can be subtle!

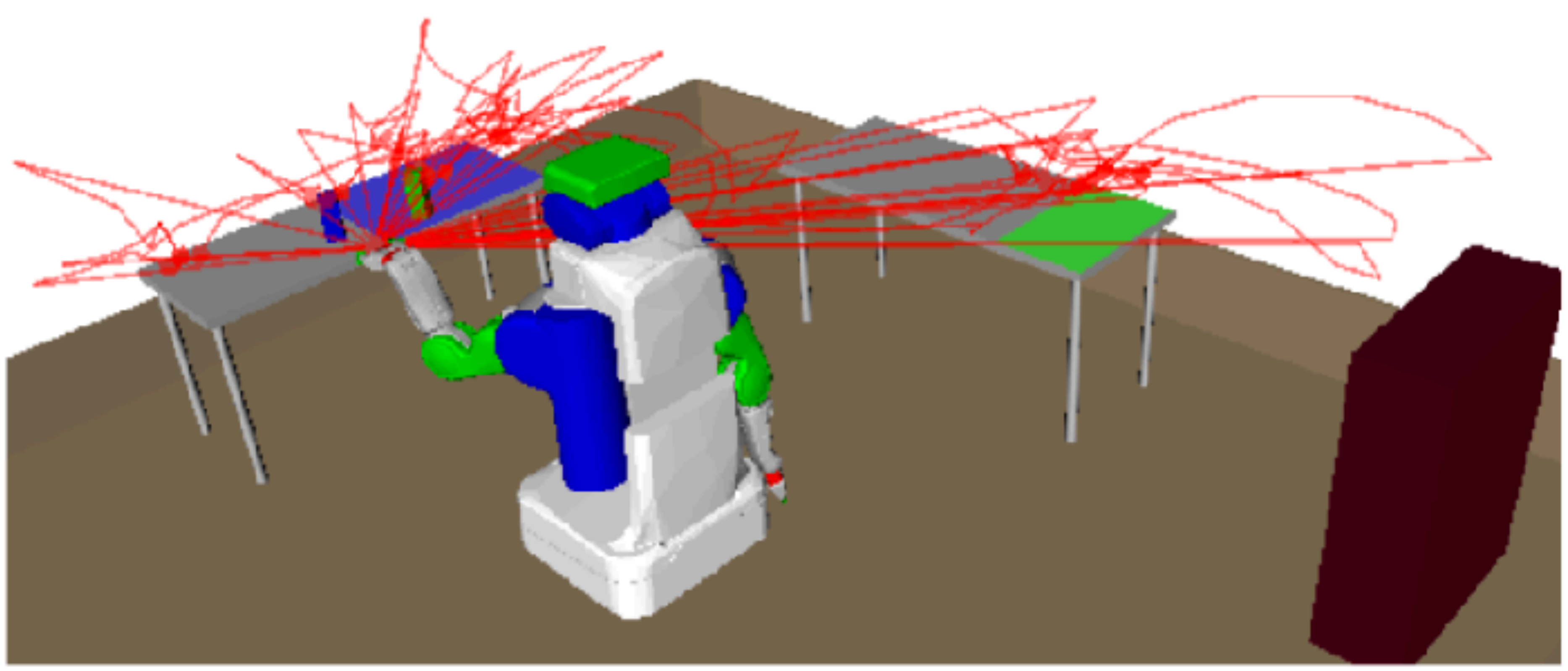
Hybrid Planning Spectrum

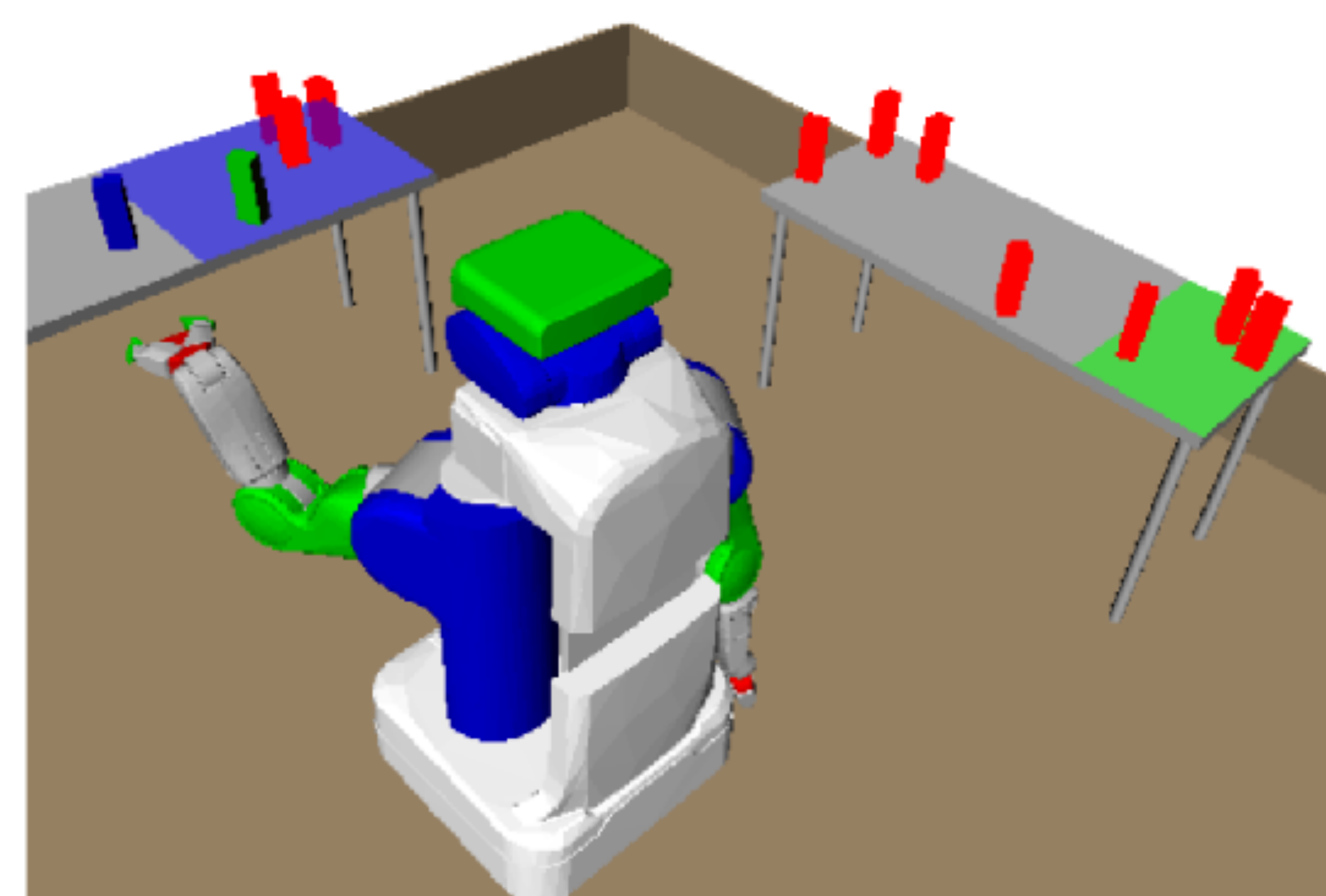


Prediscretized Planning

Prediscretized Continuous Variables

- Assume a **finite set** of object placements, object grasps, and (sometimes) robot configurations are **given**
- **Directly** perform discrete task planning
- Still need to evaluate **reachability**
  - Eagerly in **batch** [Lozano-Pérez 2014][Garrett 2017][Ferrer-Mestres 2017]
  - Eagerly during **search** [Dornhege 2009]
  - **Lazily** [Erdem 2011][Dantam 2018][Lo 2018]

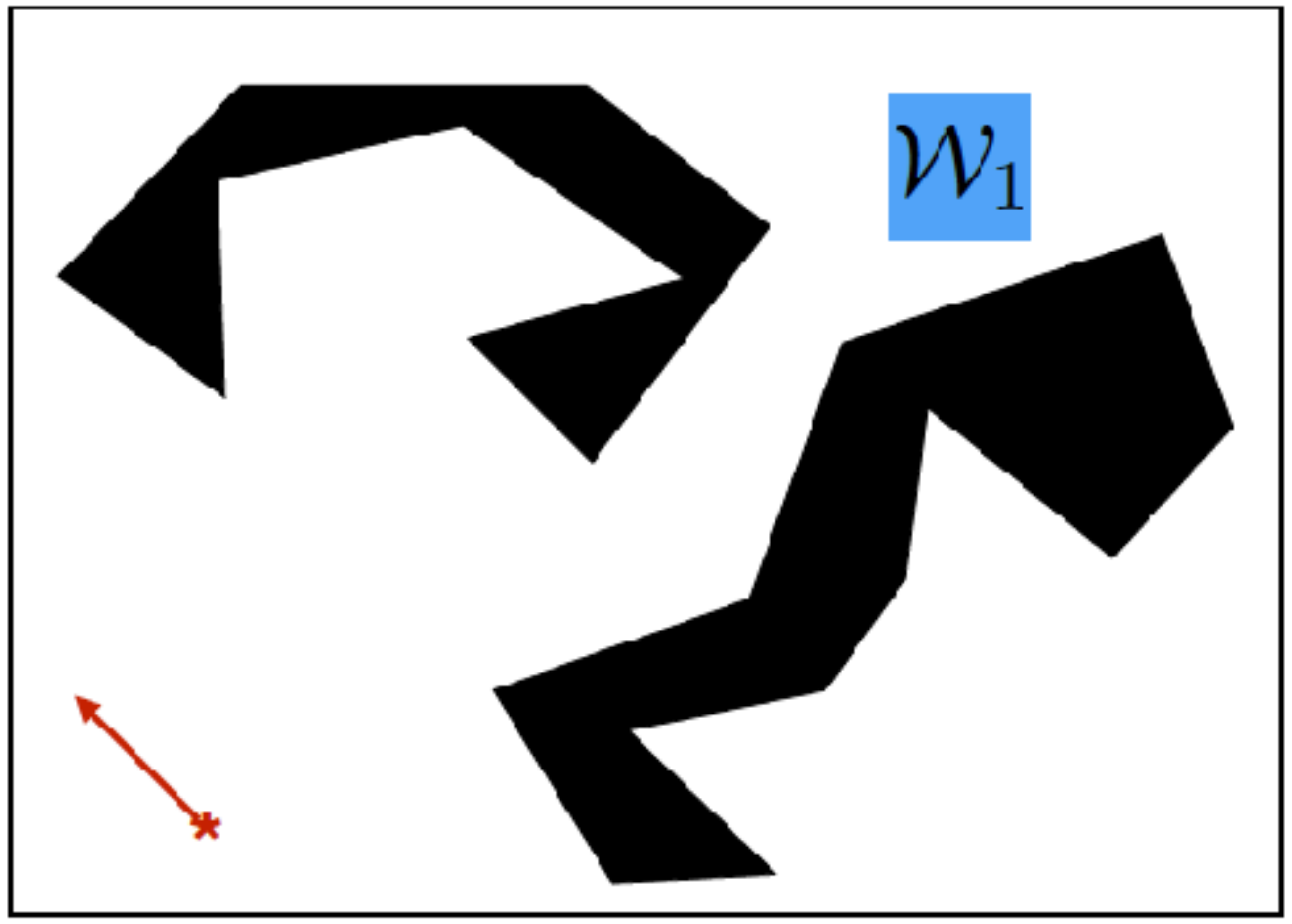


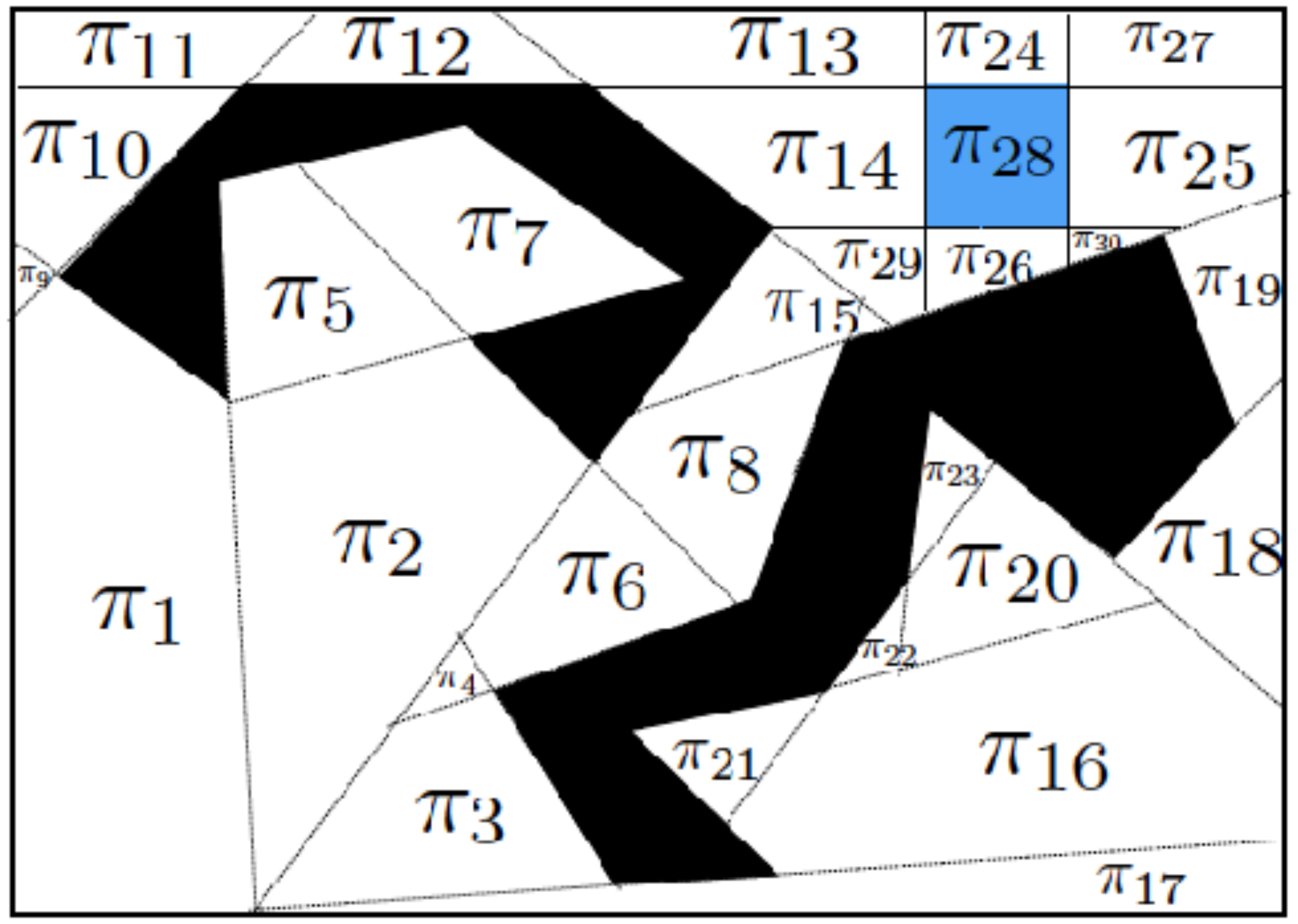


Prepartitioned Workspace

- Non-convexity handled by **partitioning** the workspace
- **Continuous control** parameters
- Tackle **convex dynamics** using **cone programming**

- **In contrast,** TAMP is often:
  - **High-dimensional**
  - **Non-convex**
    - 3D collision constraints
  - Less dynamically sophisticated



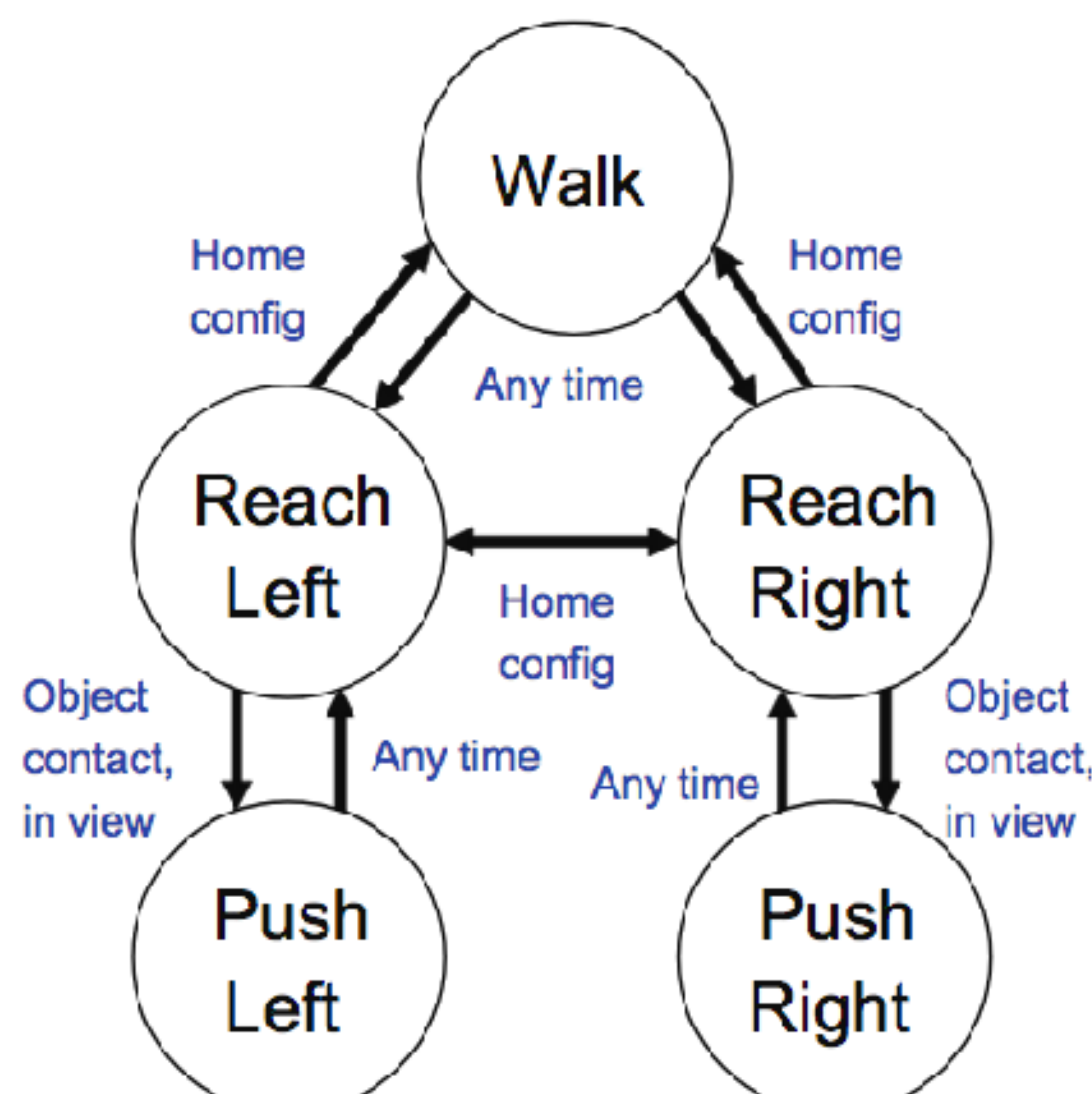


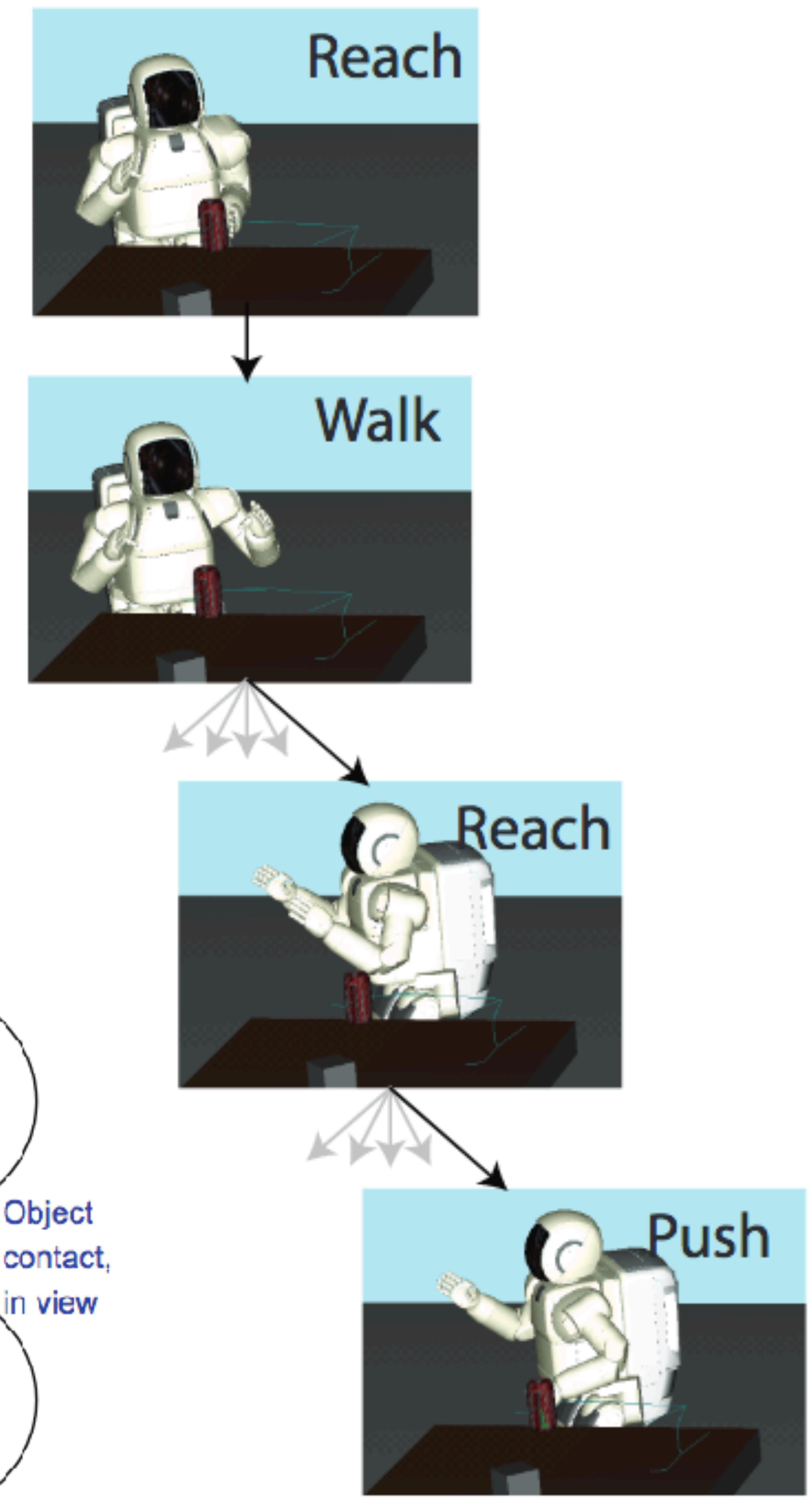
[Deits 2015][Shoukry 2016]
[Fernandez-Gonzalez 2018]

Multi-Modal Motion Planning

Multi-Modal Motion Planning

- Collision-free configuration space **changes** when objects are manipulated
- Use a **sequence** of motion planning problems each defined by a **mode**
- **Mode:** a set of motion constraints
  - Gripper is empty
  - Object pose remains **constant**

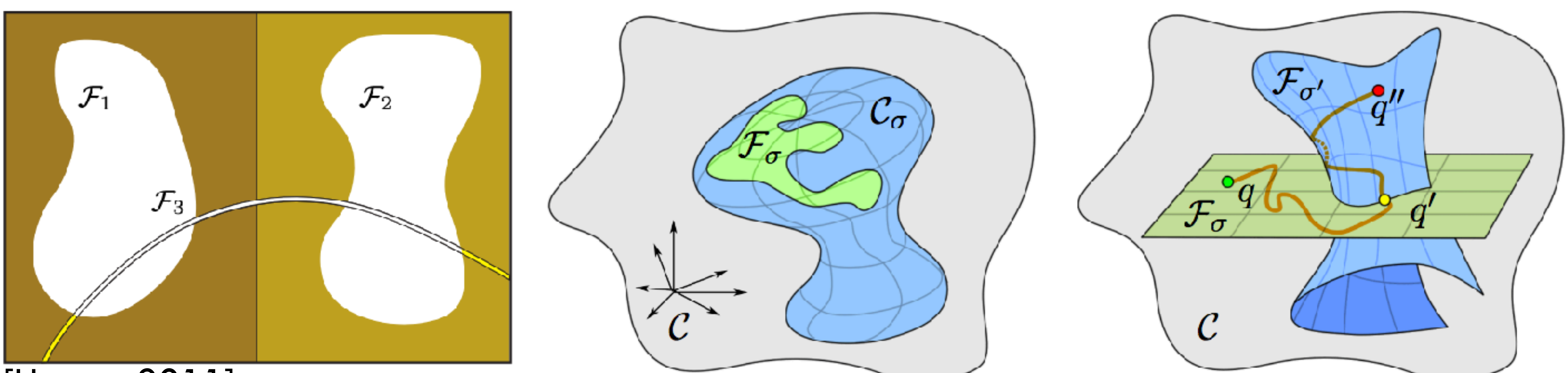




[Alami 1994][Siméon 2004][Hauser 2011]
[Barry 2013][Vega-Brown 2016]

Low-dimensional Intersections

- Need samples that **connect** adjacent modes
- Intersection of two modes is often **low-dimensional**
  - **Special-purpose** samplers are needed
- **Example:** transition from gripper **empty** to **holding**
- Configurations at the **intersection** obtained using **inverse kinematics** (IK)

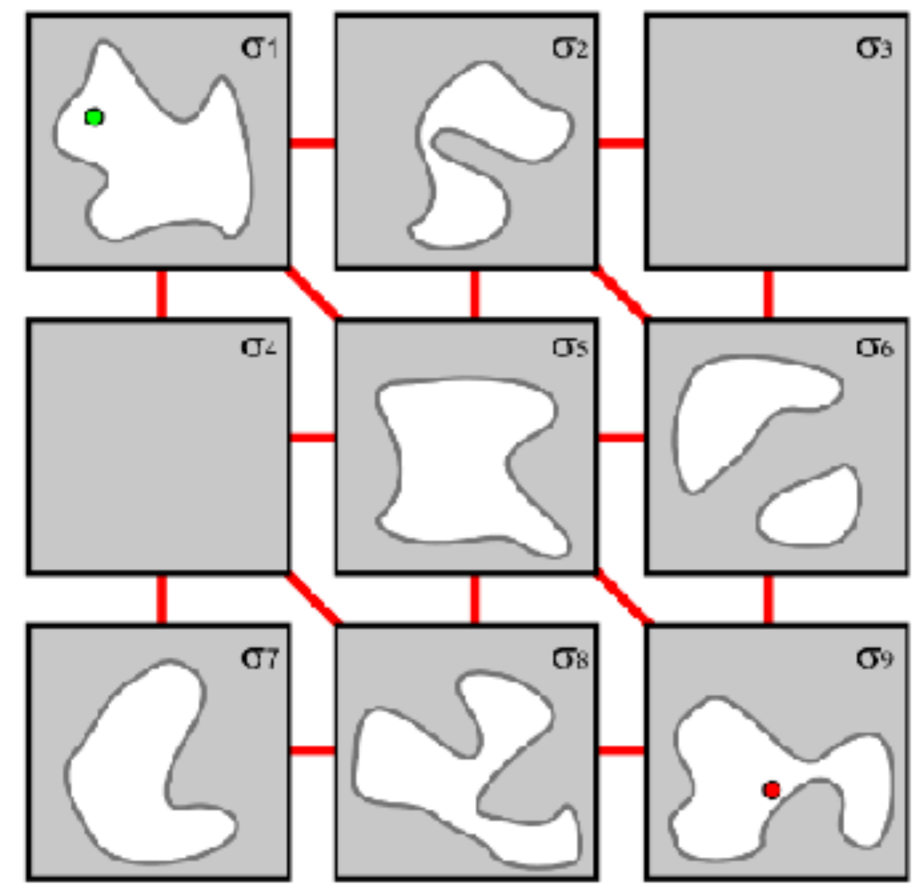


[Hauser 2011]

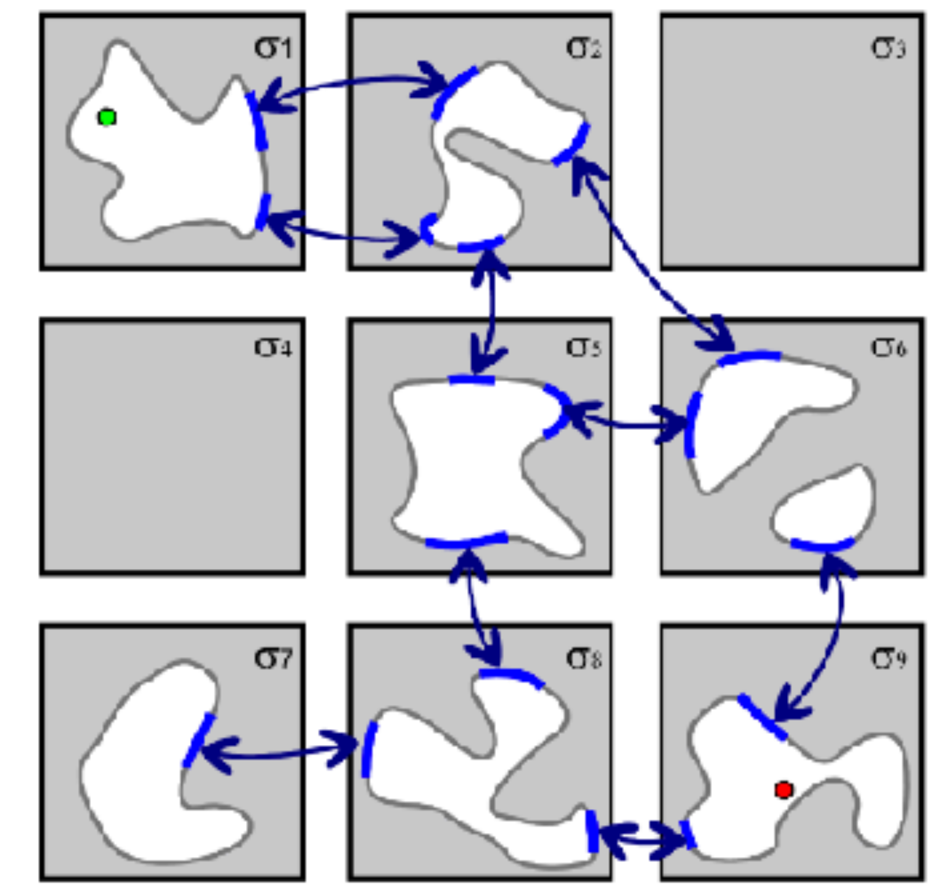
Sampling-Based Multi-Modal Planning

1. Sample from the set of **modes**
2. Sample at the **low-dimensional intersection** of adjacent modes
3. Sample a roadmap **within** each mode
4. Discrete search on the multi-modal **roadmap**

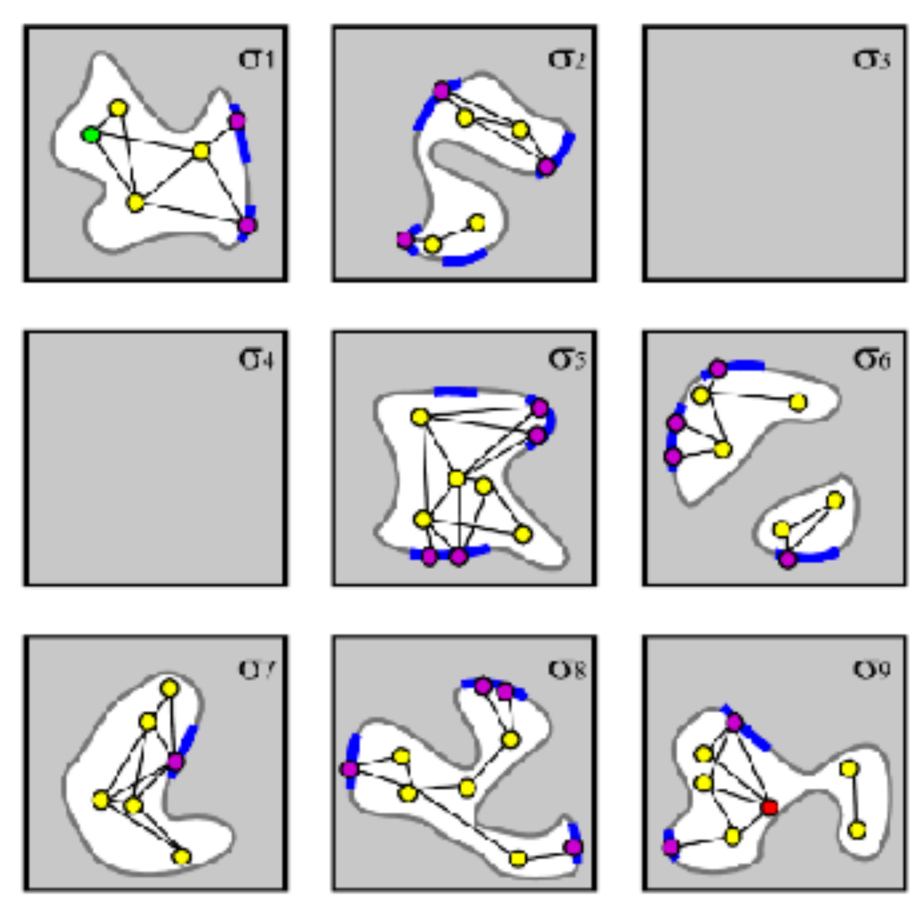
[Hauser 2011]



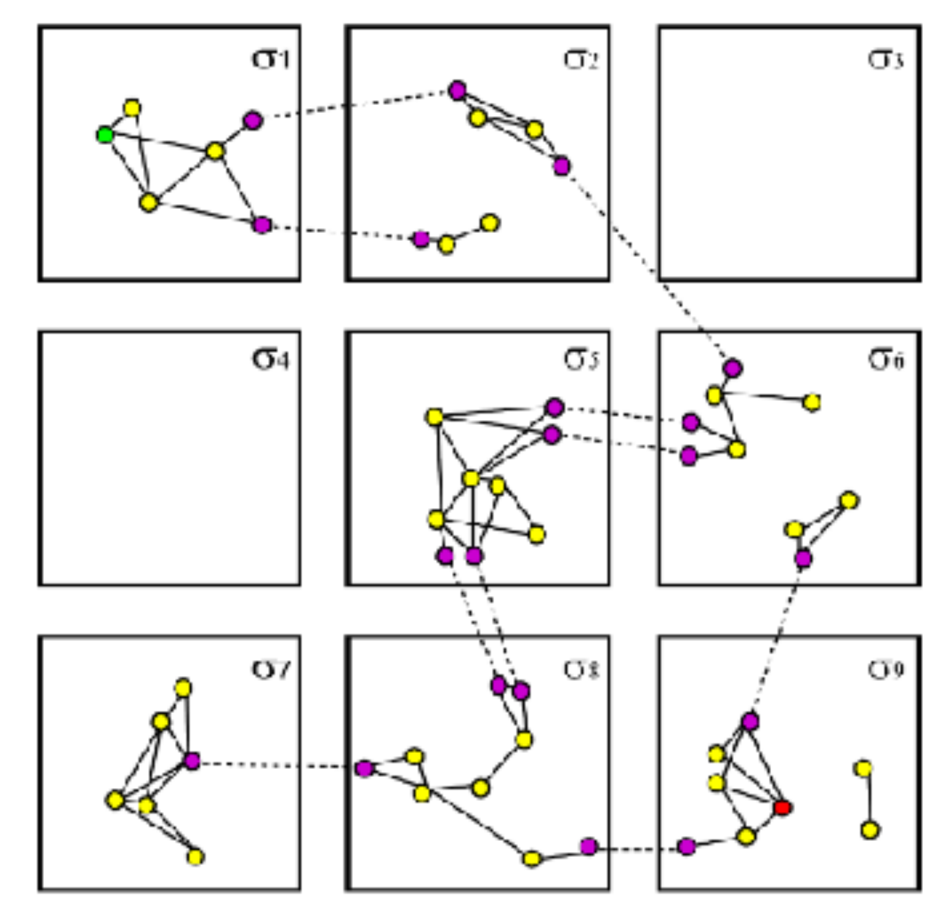
Adjacent modes



Intersections



Individual mode roadmaps

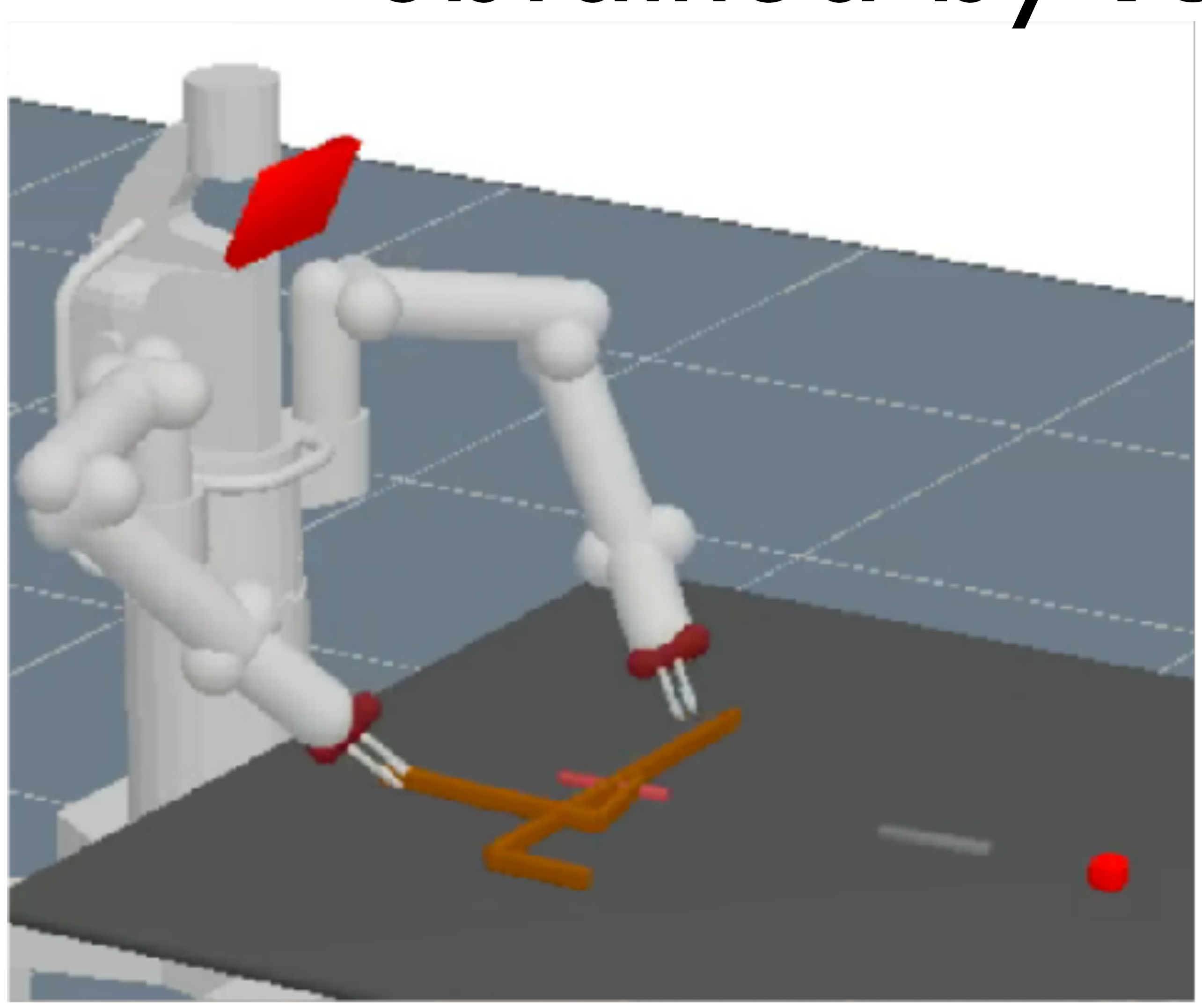


Combined Roadmap

Optimization-Based Multi-Modal Motion Planning

- Discrete search over sequences of **mode switches**
  - Sequences have **varying length**
  - Each sequence induces a **non-convex constrained optimization problem**
- Sequences can be pruned using **lower bounds** obtained by **relaxing** some constraints

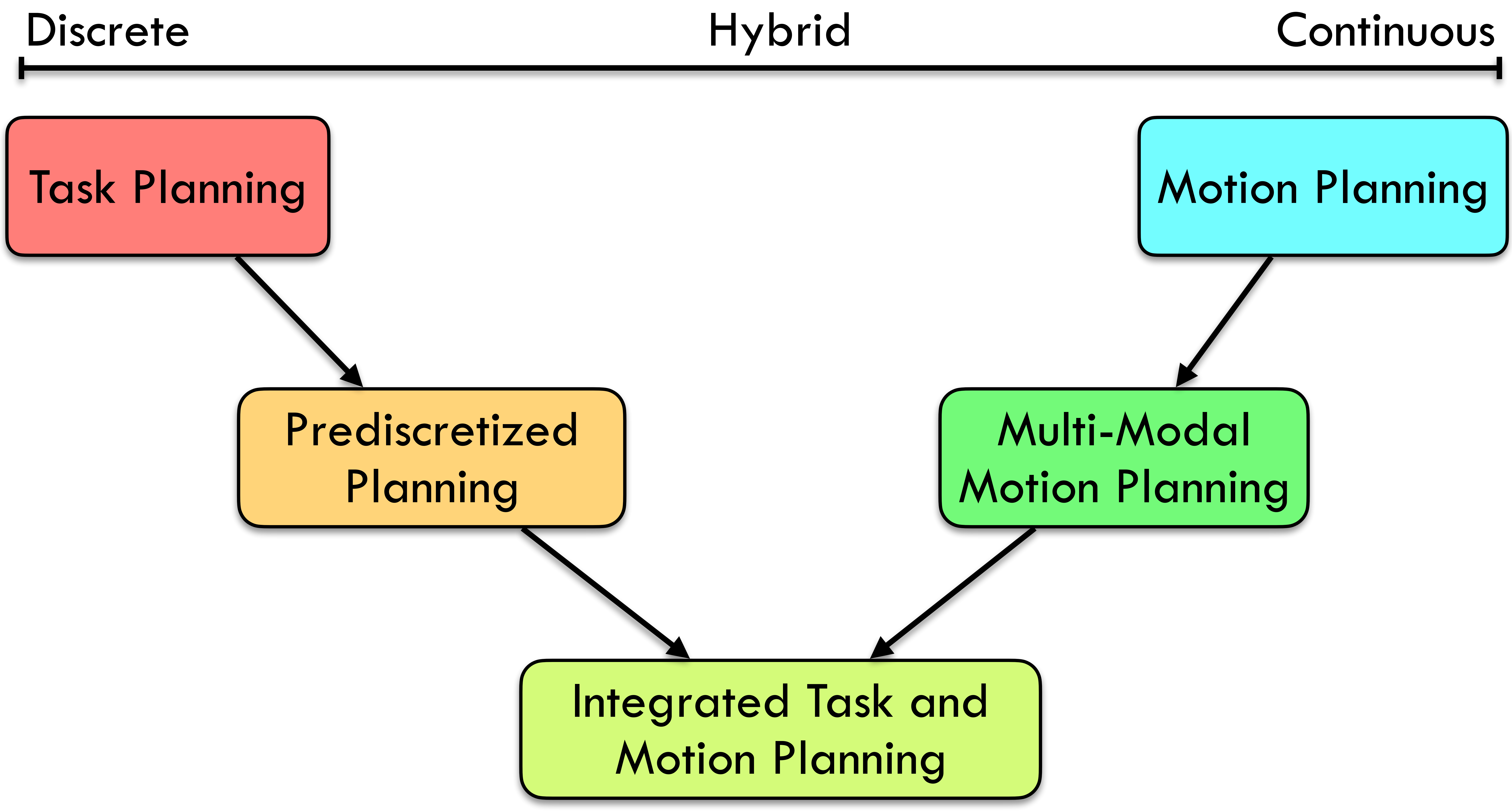
[Lagriffoul 2014]



[Toussaint 2015]
[Toussaint 2018]

$$\begin{aligned}
\min_{x, a_{1:K}, s_{1:K}} \quad & \int_0^T f_{\text{path}}(\bar{x}(t))\, dt + f_{\text{goal}}(x(T)) \\
\text{s.t.} \quad & x(0) = x_0,\ h_{\text{goal}}(x(T)) = 0,\ g_{\text{goal}}(x(T)) \le 0, \\
& \forall t \in [0, T]:\ h_{\text{path}}(\bar{x}(t), s_{k(t)}) = 0, \\
& \qquad\qquad\quad g_{\text{path}}(\bar{x}(t), s_{k(t)}) \le 0 \\
& \forall k \in \{1, .., K\}:\ h_{\text{switch}}(\hat{x}(t_k), a_k) = 0, \\
& \qquad\qquad\qquad g_{\text{switch}}(\hat{x}(t_k), a_k) \le 0, \\
& \qquad\qquad\qquad s_k \in \text{succ}(s_{k\text{-}1}, a_k)\ .
\end{aligned}$$

Hybrid Planning Spectrum Revisited



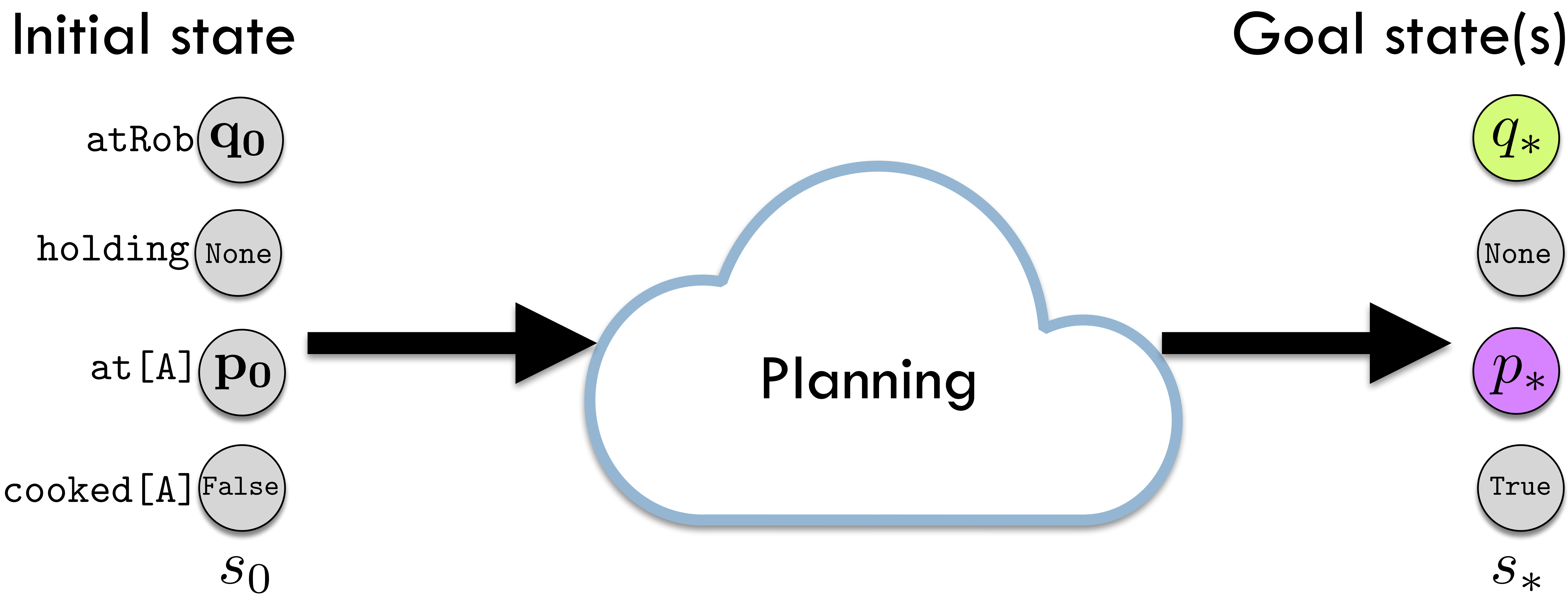
TAMP Example

Integrated Task and Motion Planning. Caelan Reed Garrett, Rohan Chitnis, Rachel Holladay, Beomjoon Kim, Tom Silver, Leslie Pack Kaelbling, Tomás Lozano-Pérez. *Annual Review of Control, Robotics, and Autonomous Systems*, 2021.

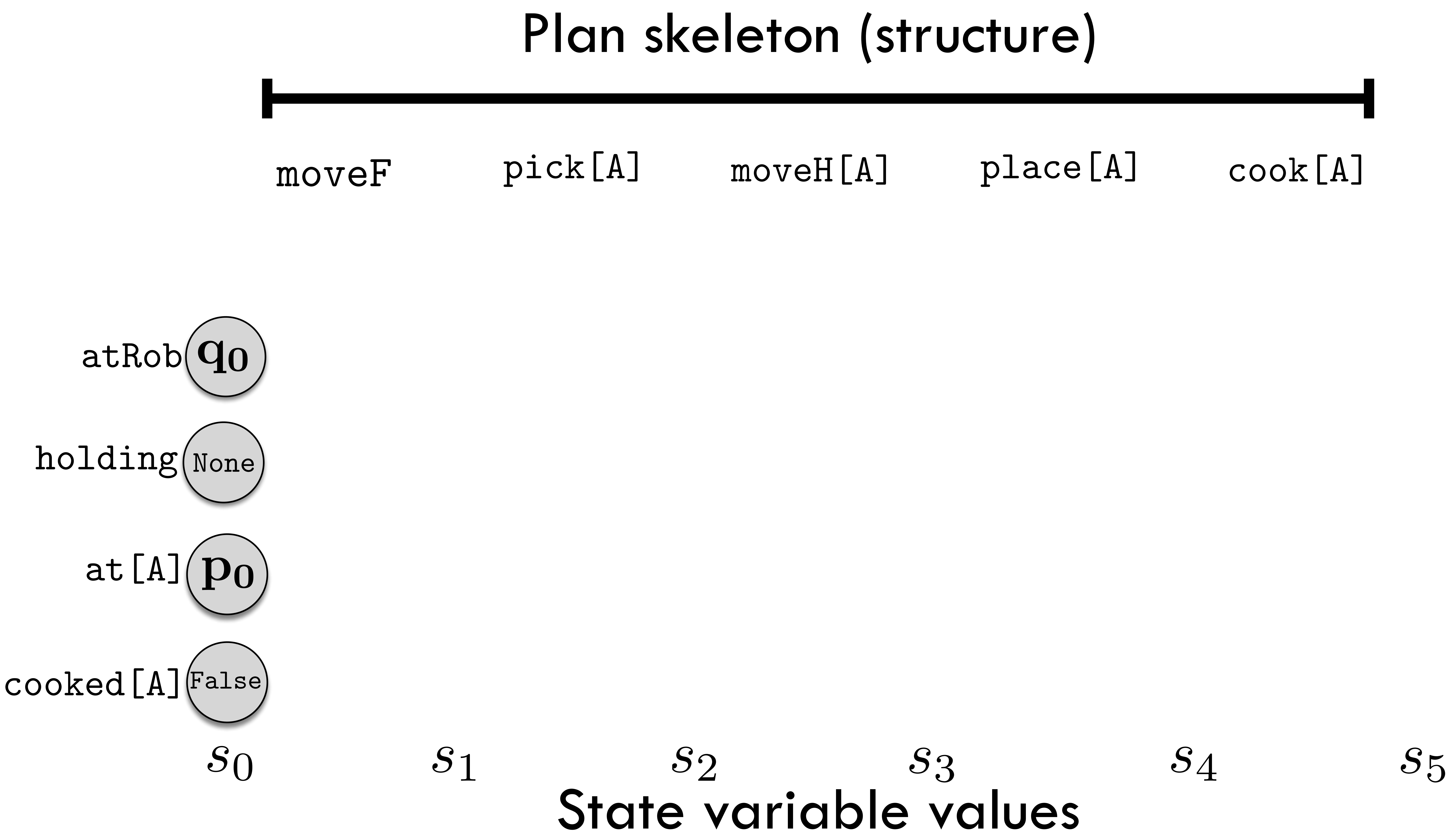
TAMP Example: Cook Object A

s_0 = {atRob=$\mathbf{q}_0$, at[A]=$\mathbf{p}_0$, holding=None, cooked[A]=False}

Goal conditions: cooked[A]=True

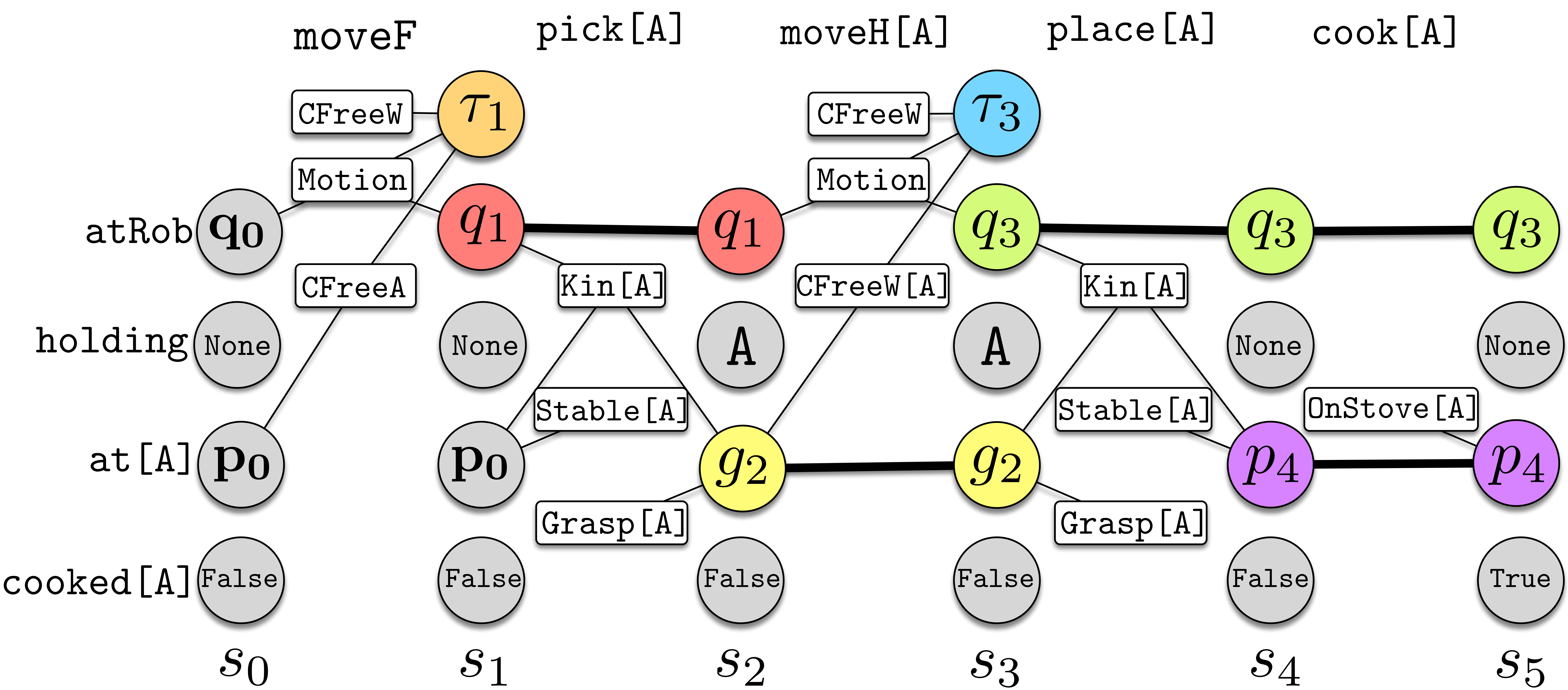


Plan Skeleton & Action Parameters



Plan Constraints & Parameter Values

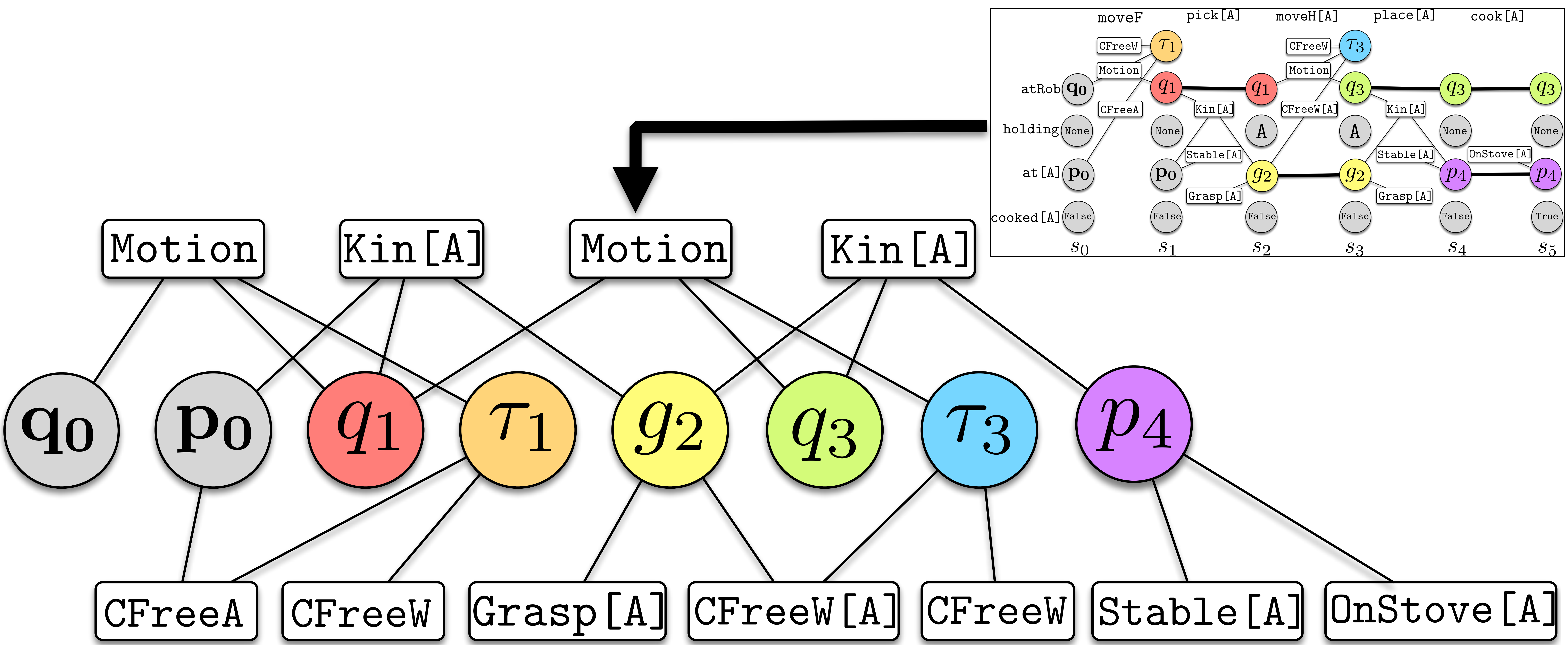
Example plan: $\pi = [\texttt{moveF}(\mathbf{q_0}, \tau_1, q_1), \texttt{pick[A]}(q_1, \mathbf{p_0}, g_2), \texttt{moveH[A]}(q_1, \tau_3, q_3), \texttt{place[A]}(q_3, p_4, g_2), \texttt{cook[A]}(p_4)]$



Constraints

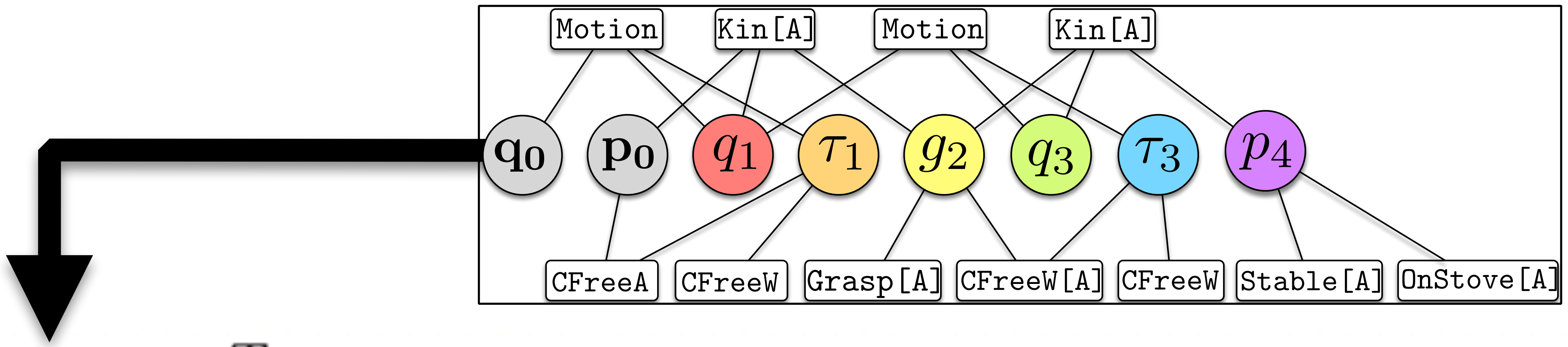
Constraint Network (Factor Graph)

- Compress plan skeleton into a constraint network
- Undirected bipartite graph of **variables & constraints**
- Can address with **optimization** and/or **sampling**



Joint Optimization

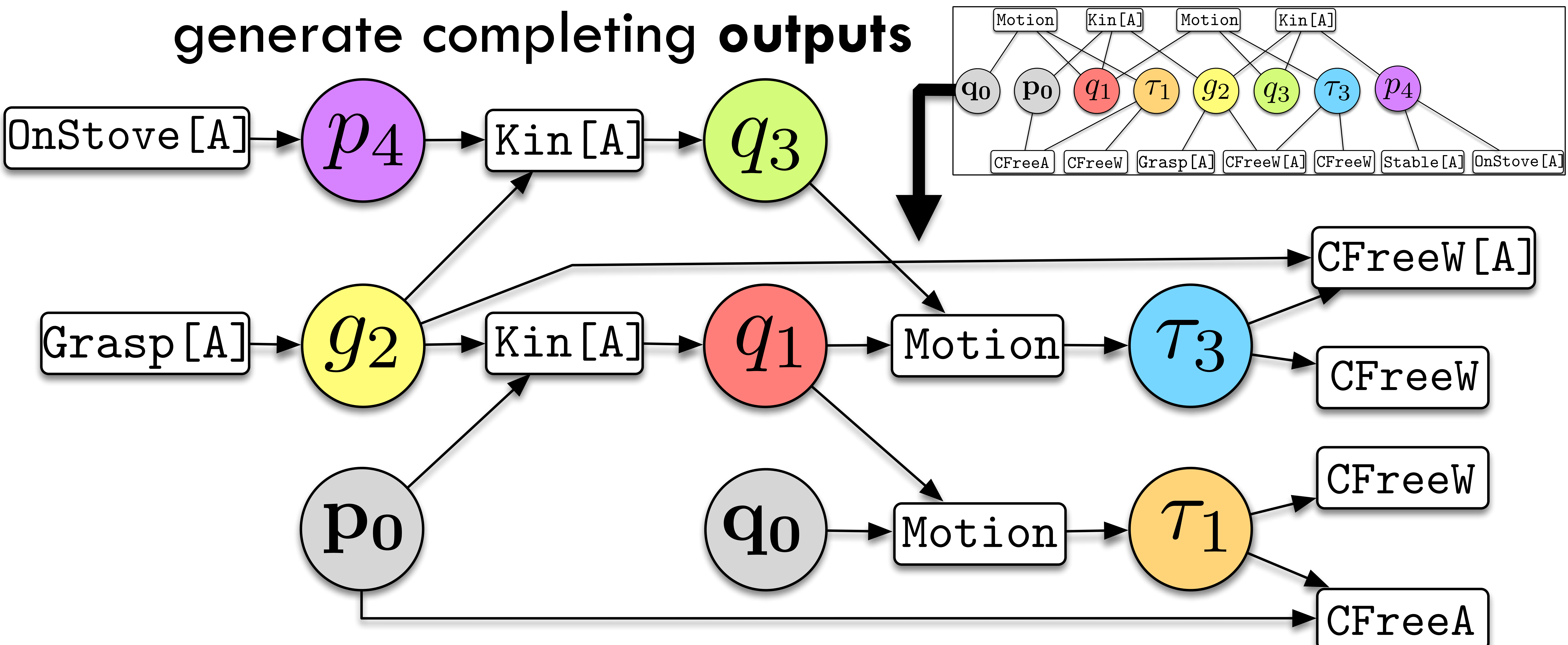
- Constraint network is a **mathematical program**
- **Hard to solve:** non-convex, constrained, mixed integer
- Often reduce to a sequence of simpler (unconstrained, quadratic) programs
- First- (**gradient**) vs second-order (**Hessian**) methods



$$\begin{aligned}
\underset{q_1,\tau_1,g_2,\tau_3,q_3,p_4}{\text{minimize}} \quad & \textstyle\sum_{t=1}^{T} f_{\texttt{moveF}}(\tau_1[t], \tau_1[t-1]) + \sum_{t=1}^{T} f_{\texttt{moveH[A]}}(g_1, \tau_3[t], \tau_3[t-1]) \\
\text{subject to} \quad & g_{\texttt{Grasp[A]}}(g_1) = 0,\ g_{\texttt{Stable[A]}}(p_4) = 0,\ h_{\texttt{OnStove[A]}}(p_4) \le 0 \\
& g_{\texttt{Kin[A]}}(q_1, \mathbf{p_0}, g_2) = 0,\ g_{\texttt{Kin[A]}}(q_3, p_4, g_1) = 0 \\
& h_{\texttt{Motion}}(\tau_1[t], \tau_1[t-1]) \le 0,\ h_{\texttt{Motion}}(\tau_3[t], \tau_3[t-1]) \le 0 \qquad \text{for } t \in [T] \\
& h_{\texttt{CFreeW}}(\tau_1[t]) \le 0,\ h_{\texttt{CFreeA}}(\mathbf{p_0}, \tau_1[t]) \le 0 \qquad \text{for } t \in [T] \\
& h_{\texttt{CFreeW}}(\tau_3[t]) \le 0,\ h_{\texttt{CFreeW[A]}}(g_1, \tau_3[t]) \le 0 \qquad \text{for } t \in [T]
\end{aligned}$$

Sampling Network

- Satisfy constraint network **compositionally**
- Directed Acyclic Graph (DAG) **computation graph**
  - **Conditional** samplers consume **inputs** and generate completing **outputs**



Taxonomy of TAMP Approaches

	Pre-discretized	Sampling	Optimization
Satisfaction first	Ferrer-Mestres et al. (84, 85)[b]	Siméon et al. (22)[a] Hauser et al. (13, 14, 29)[a] Garrett et al. (21, 86)[b] Krontiris & Bekris (87, 88)[a] Akbari & Rosell (89)[b] Vega-Brown & Roy (90)[a]	
Interleaved	Dornhege et al. (62, 63, 91)[b] Gaschler et al. (92–94)[b] Colledanchise et al. (95)[b]	Gravot et al. (96, 97)[b] Stilman et al. (23, 98, 99)[a] Plaku & Hager (100)[a] Kaelbling & Lozano-Pérez (101, 102)[b] Barry et al. (30, 103, 104)[a] Garrett et al. (70, 71)[b] Thomason & Knepper (105)[b] Kim et al. (106, 107)[b] Kingston et al. (108)[a]	Fernández-González et al. (109)[b]
Sequencing first	Nilsson (3)[b] Erdem et al. (74, 75)[b] Lagriffoul et al. (65–67)[b] Pandey et al. (110, 111)[b] Lozano-Pérez & Kaelbling (112)[b] Dantam et al. (77–79)[b] Lo et al. (113)[b]	Wolfe et al. (114)[b] Srivastava et al. (60, 76)[b] Garrett et al. (55, 73)[b]	Toussaint et al. (61, 68, 69)[b] Shoukry et al. (81–83)[b] Hadfield-Menell et al. (115)[b]

[a]Approaches for MMMP.
[b]Approaches for TAMP.

Garrett et al., 2021. *"Integrated Task and Motion Planning", Annual Review of Control, Robotics, and Autonomous Systems.*

My Approach: PDDLStream

- **No general-purpose, flexible framework** for planning in a variety of TAMP domains
- Extends **PDDL** to incorporate **sampling procedures**
  - Can model domains with **infinitely-many** actions

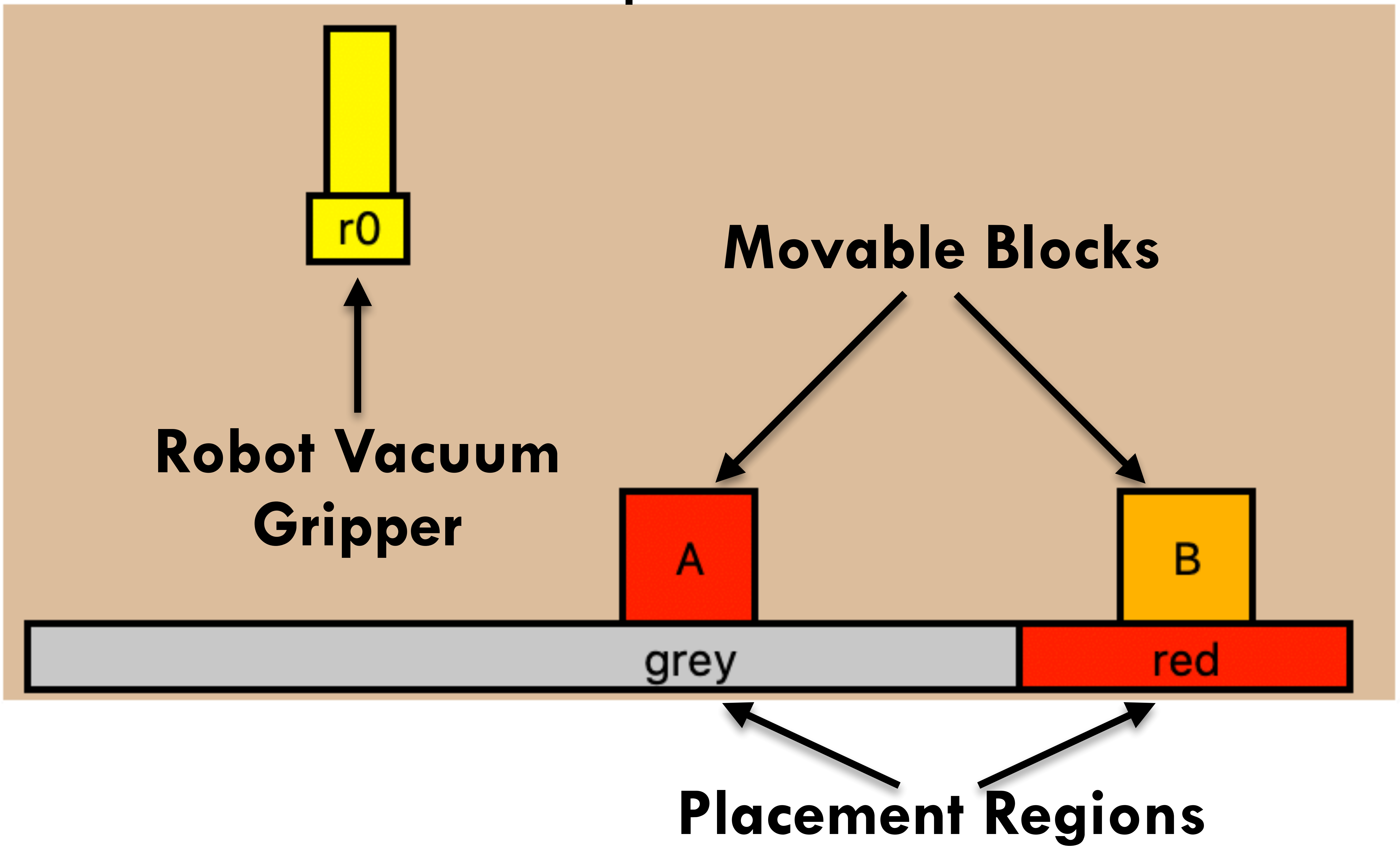
- Develop **domain-independent** algorithms that treat the samplers as **blackbox inputs**
- Algorithms solve a **sequence of finite** PDDL problems
  - Leverage existing **classical planners** as subroutines
- Algorithms are particularly **fast when downward refinement holds** while remaining **complete**

PDDLStream Language

PDDLStream: Integrating Symbolic Planners and Blackbox Samplers via Optimistic Adaptive Planning. Caelan Reed Garrett, Tomás Lozano-Pérez, Leslie Pack Kaelbling. *International Conference on Automated Planning and Scheduling (ICAPS)*, 2020.

2D Pick-and-Place Domain

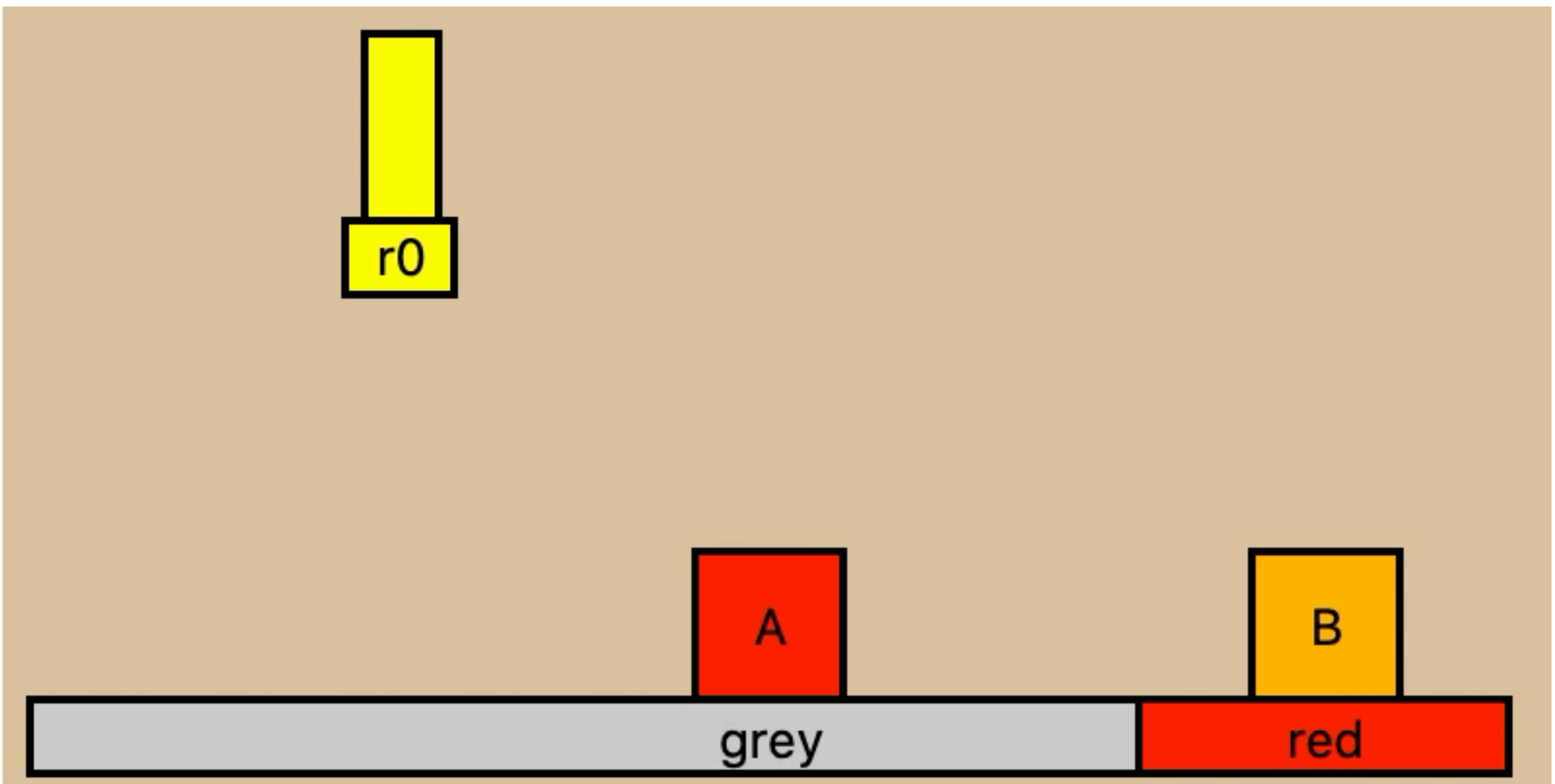
- Robot and block poses are continuous `[x y]` pairs
- **Goal:** block **`A`** within the **`red`** region
  - Block **`B`** obstructs the placement of **`A`**



2D Pick-and-Place Solution

- One (of **infinitely many**) possible solutions

```
[move(…), pick(B,…), move(…), place(B,…),
 move(…), pick(A,…), move(…), place(A,…)]
```



2D Pick-and-Place Initial & Goal

- Not all values are discrete, some are **continuous**
- **Static** (constant) initial **facts - satisfied constraints**

$F =$ `{Block(`**`A`**`), Block(`**`B`**`), Region(`**`red`**`), Region(`**`grey`**`), Conf([-7.5, 5]), Pose(`**`A`**`, [0. 0.]), Pose(`**`B`**`, [7.5 0.])}`

- **Fluent** (changing) initial facts - **state variables**

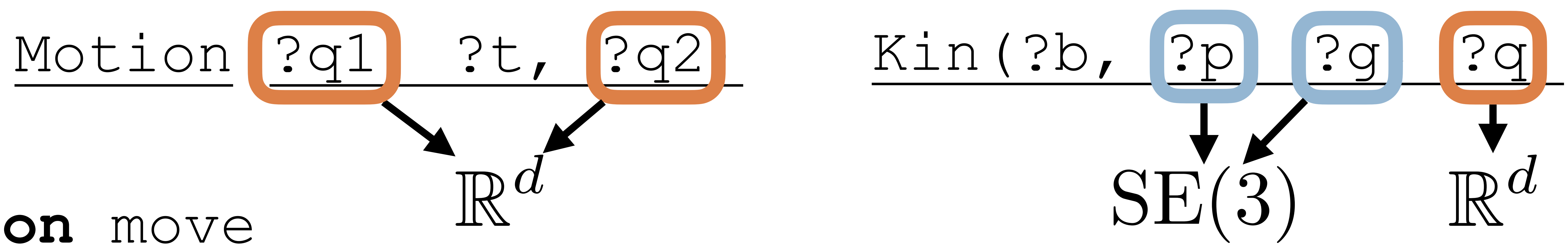
$s_0 =$ `{AtConf([-7.5 5.]), HandEmpty(), AtPose(`**`A`**`, [0. 0.]), AtPose(B, [7.5 0.])}`

- **Goal** logical formula - set of **goal states**

$S_* =$ **`exists`**`(?p){Contained(`**`A`**`,?p,`**`red`**`),AtPose(`**`A`**`,?p)}`

Pick-and-Place Actions

- Typical PDDL action description except that arguments are **high-dimensional & continuous**!
- To use, must **satisfy** static facts **(constraints)**




```
(:action move
 :parameters (?q1, ?t, ?q2)
 :precondition {Motion(?q1, ?t, ?q2), AtConf(?q1)}
 :effect {AtConf(?q2), ¬AtConf(?q1)})
```

$A_{\texttt{move}}$

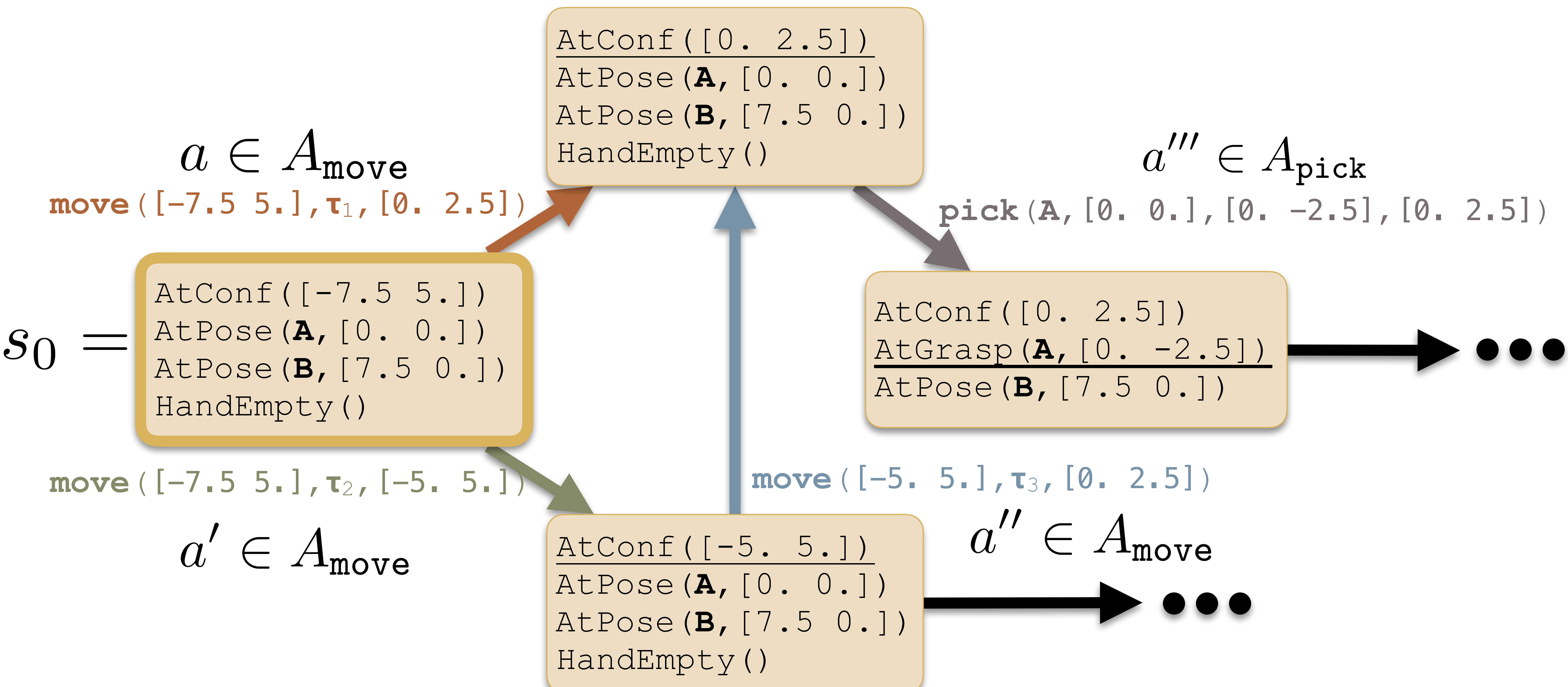
```
(:action pick
 :parameters (?b, ?p, ?g, ?q)
 :precondition {Kin(?b, ?p, ?g, ?q), AtConf(?q),
                AtPose(?b, ?p), HandEmpty()}
 :effect {AtGrasp(?b,?g), ¬AtPose(?b,?p), ¬HandEmpty()})
```

$A_{\texttt{pick}}$

Search in Discretized State-Space

Suppose an **oracle** gave use the following values and facts:

$F =$ {**Motion**([-7.5 5.],τ1,[0. 2.5]), **Motion**([-7.5 5.],τ2,[-5. 5.]), **Motion**([-5. 5.],τ3,[0. 2.5]), **Kin**(**A**,[0. 0.],[0. -2.5],[0. 2.5]), ...}

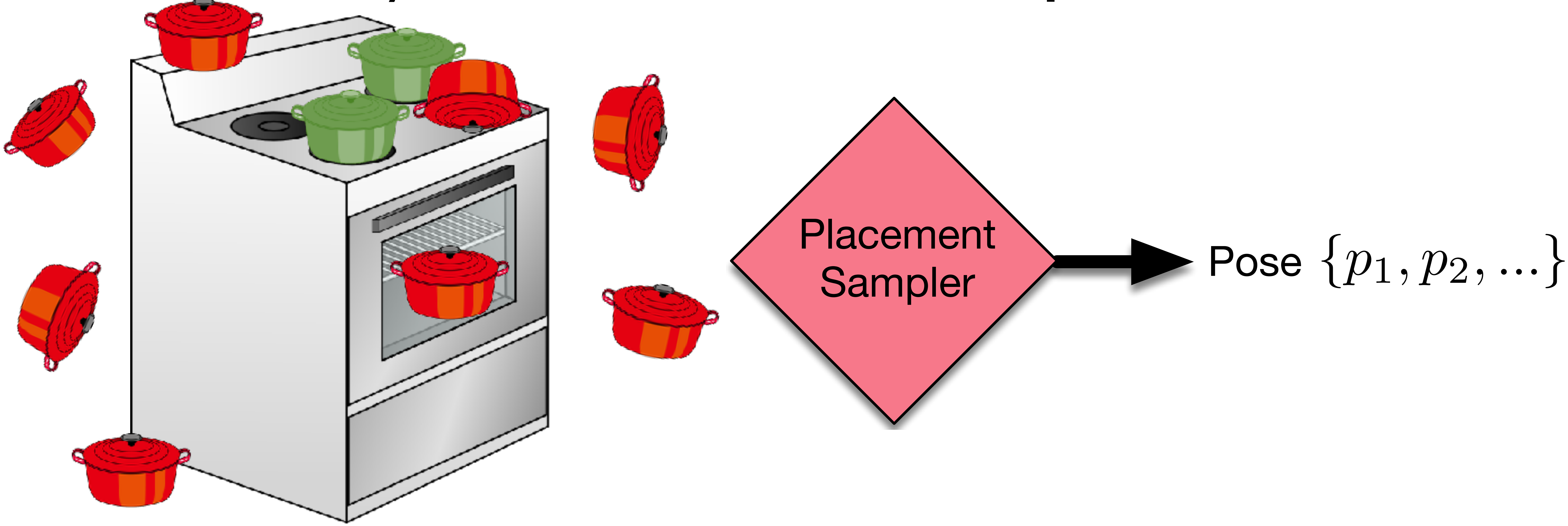


No a Priori Discretization

- Values **given** at start:
  - 1 initial configuration: `Conf([-7.5 5.])`
  - 2 initial poses: `Pose(A, [0. 0.])` `Pose(B, [7.5 0.])`
- **Planner** needs to **find:**
  - 1 pose for **A** within **red**: `Contain(A, ?p, red)`
  - 1 collision-free pose for **B**: `CFree(A, ?p, B, ?p2)`
  - 1 grasp for **A** and **B**: `Grasp(A,?g),Grasp(B,?g)`
  - 4 grasping configurations: `Kin(?b, ?p, ?g, ?q)`
  - 4 robot trajectories: `Motion(?q1, ?t, ?q2)`

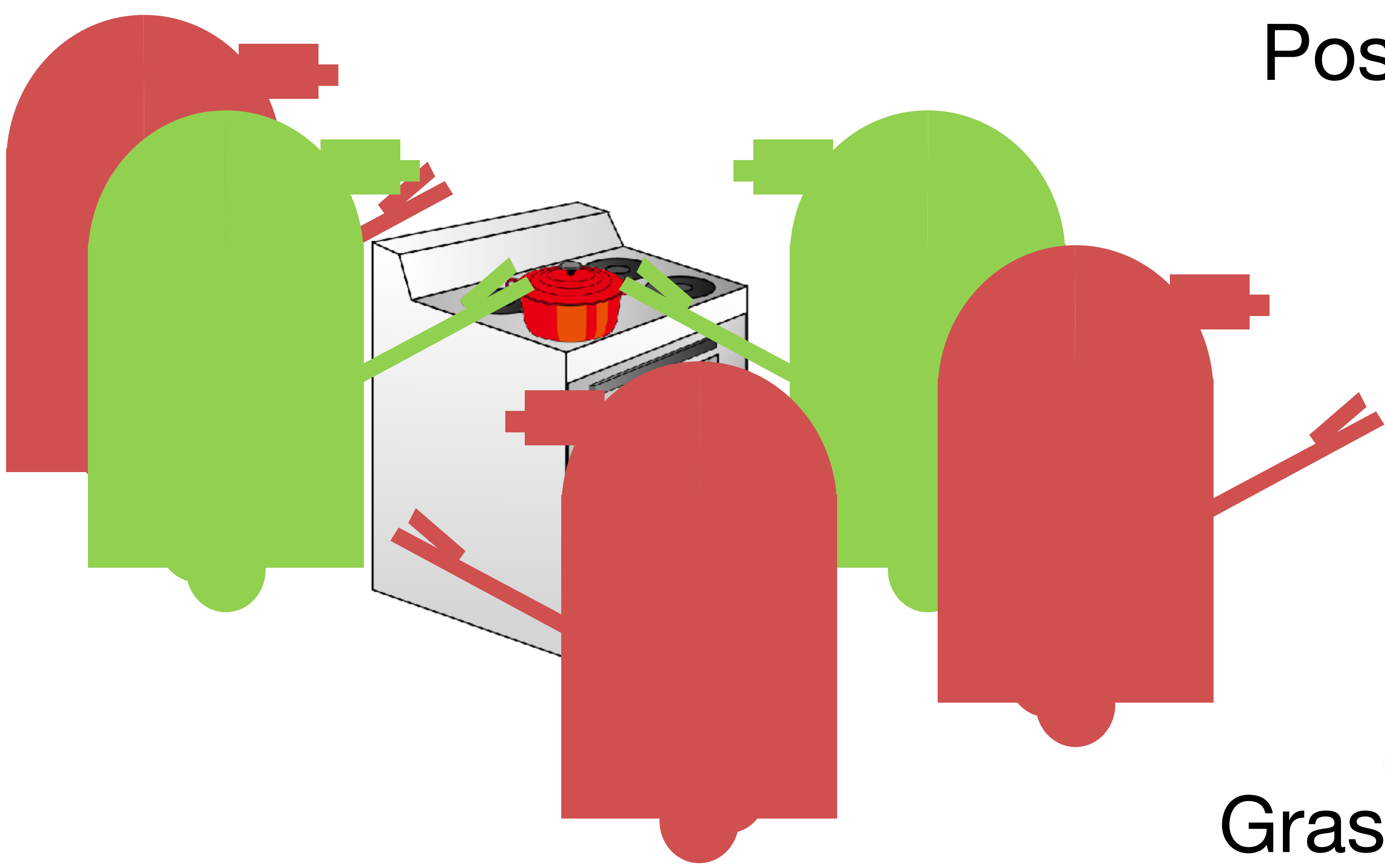
What Samplers Do We Need?

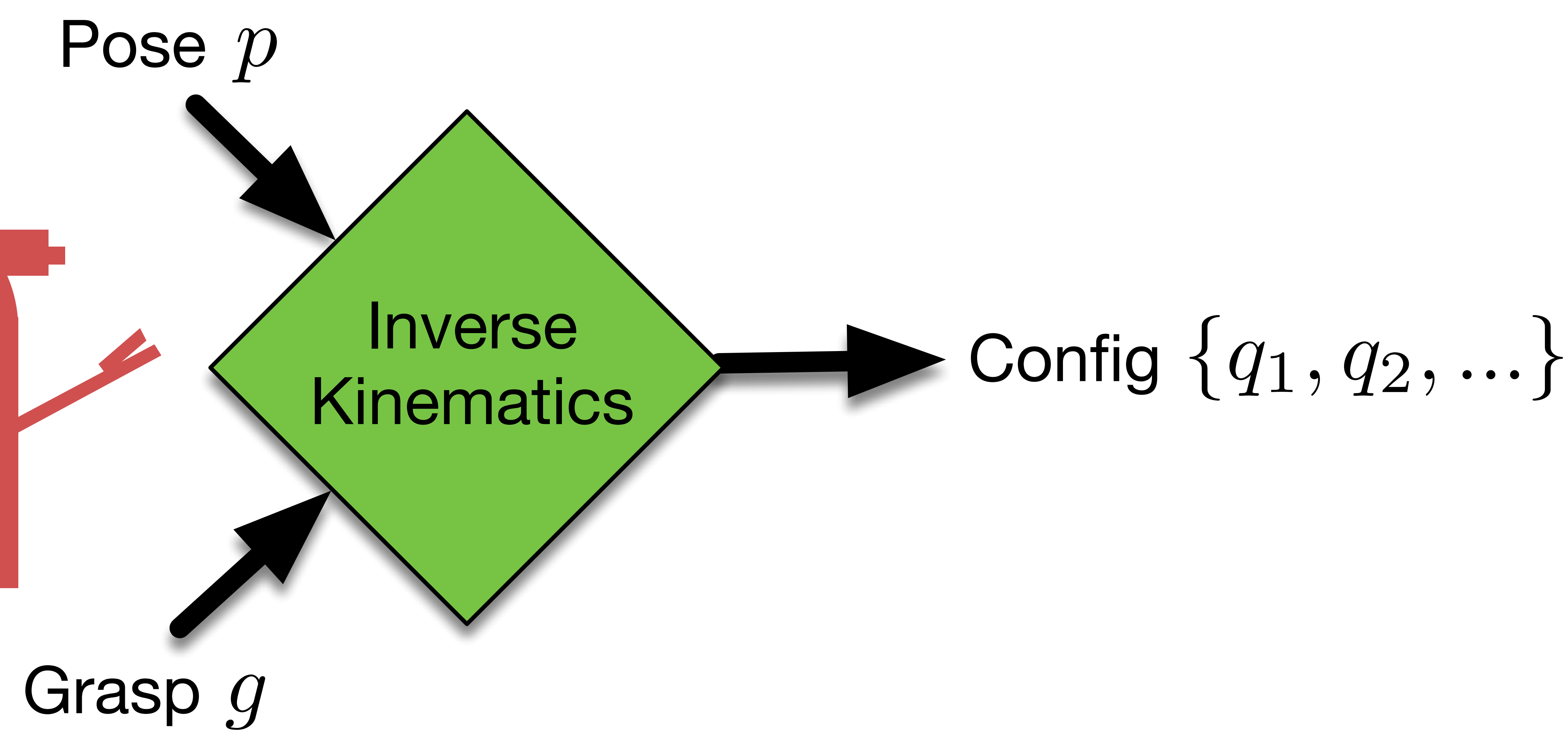
- **Low-dimensional** placement stability constraint (`Contain`)
  - e.g. 1D line embedded in 2D placement space
- Directly **sample values that satisfy the constraint**
- May need **arbitrarily many** samples
  - Gradually enumerate an **infinite sequence**



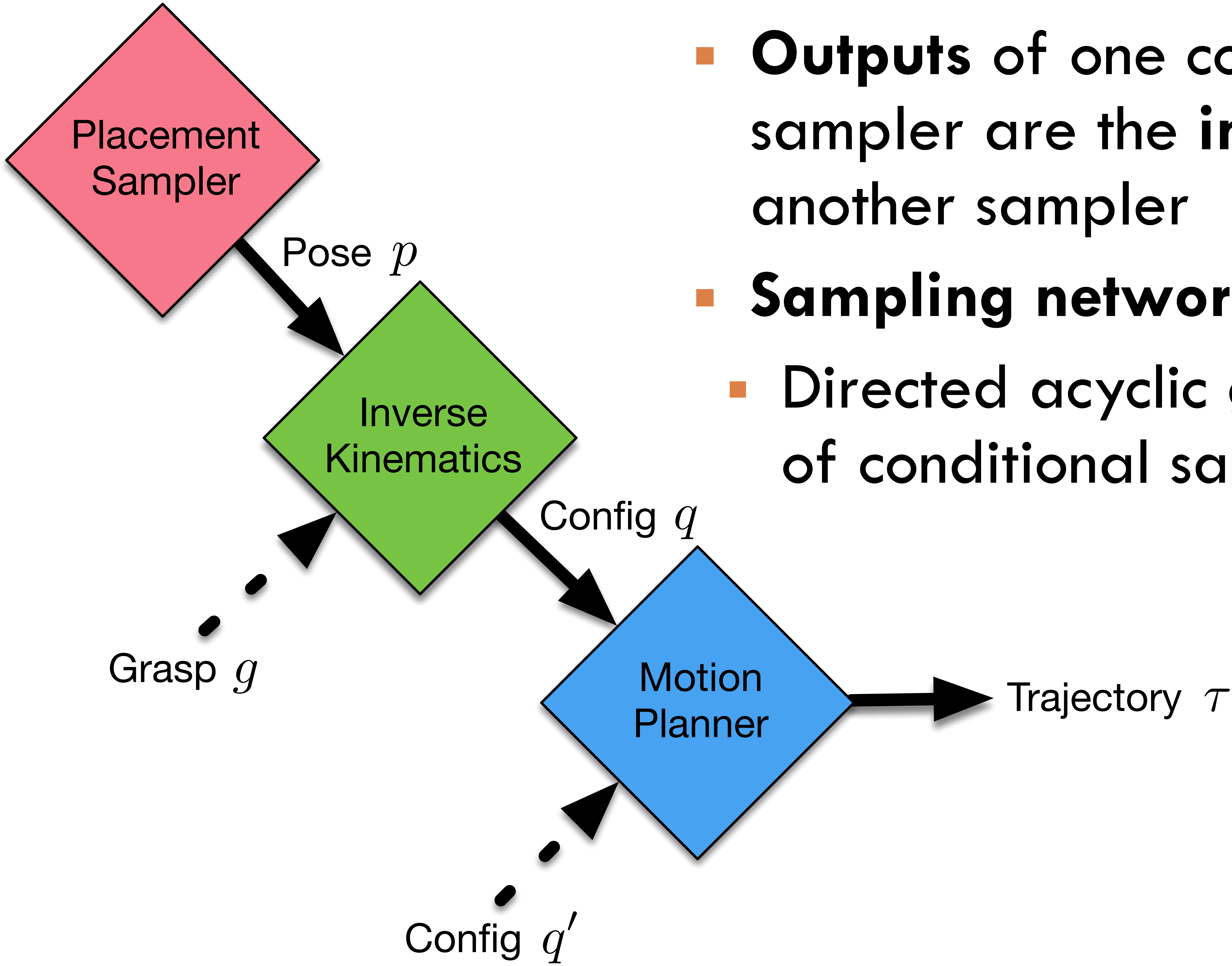
Intersection of Constraints

- **Kinematic constraint** (`Kin`) involves poses, grasps, and configurations
- **Conditional samplers** - function from **input values** to a sampler that generates **output values**





Composing Conditional Samplers



- **Outputs** of one conditional sampler are the **inputs** to another sampler
- **Sampling network**
  - Directed acyclic graph (DAG) of conditional samplers

Stream: Specification for a Sampler

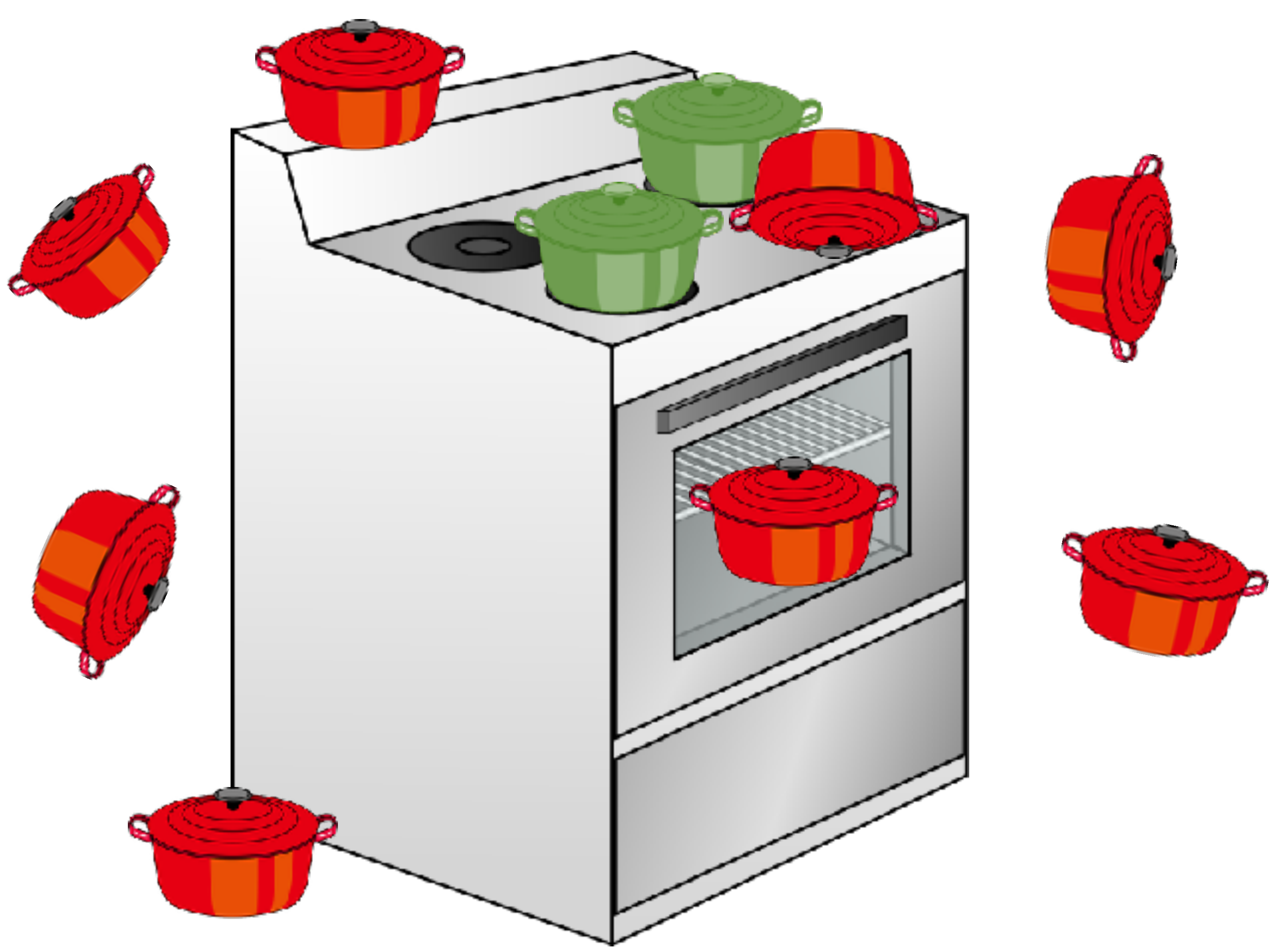
- What do **inputs** & **outputs** represent?
  - Communicate semantics using **predicates** (constraints)

- Declarative stream specification:
  - **Domain facts** - static facts declaring legal **inputs**
    - e.g. only configurations can be motion planner inputs

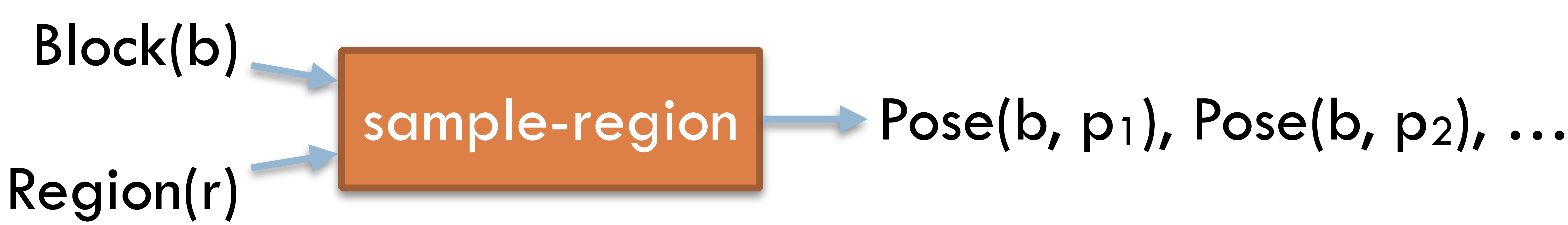
  - **Certified facts** - static facts that all **outputs** are **asserted** to satisfy with their corresponding **inputs**
    - e.g. poses sampled from a region are within it

Sampling Placements in a Region


```
(:stream sample-region
 :inputs (?b, ?r)
 :domain {Block(?b), Region(?r)}
 :outputs (?p)
 :certified {Pose(?b, ?p), Contain(?b, ?p, ?r)})
```



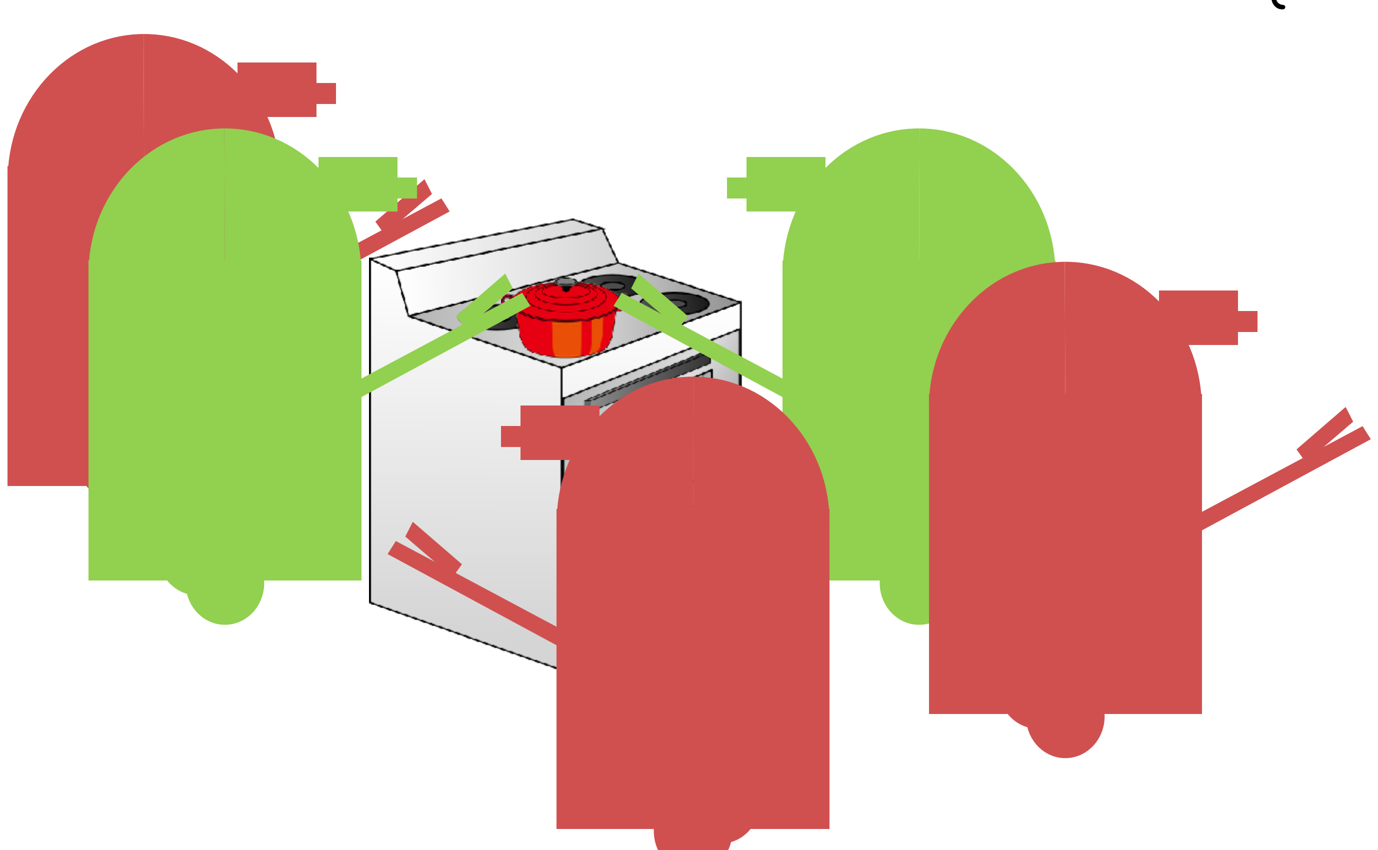
```
def sample_region(b, r):
  x_min, x_max = REGIONS[r]
  w = BLOCKS[b].width
  while True:
      x = random.uniform(x_min + w/2,
                         x_max - w/2)
      p = np.array([x, 0.])
      yield (p,)
```

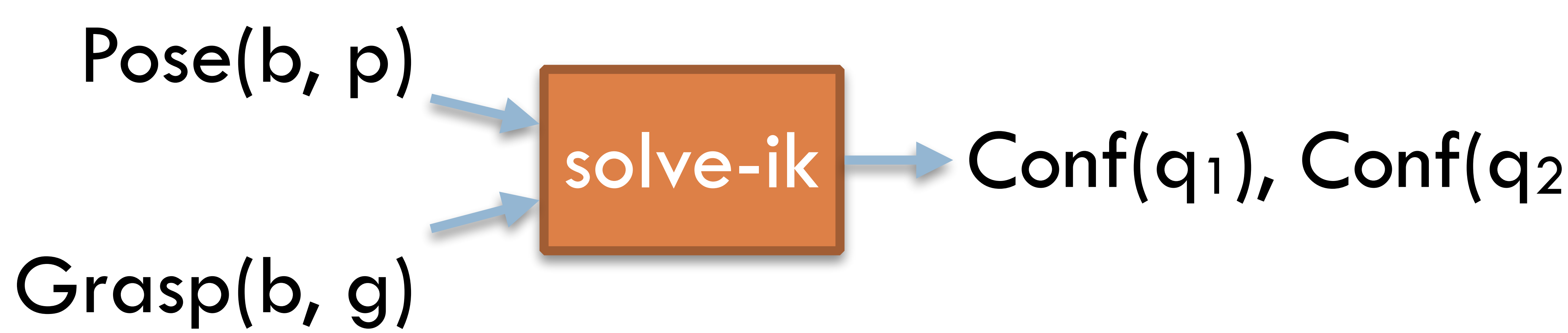


Sampling IK Solutions

- **Inverse kinematics** (IK) to produce robot grasping configurations
- Trivial in 2D, non-trivial in general (e.g. 7-DOF arm)

```
(:stream solve-ik
 :inputs (?b, ?p, ?g)
 :domain {Pose(?b, ?p), Grasp(?b, ?g)}
 :outputs (?q)
 :certified {Conf(?q), Kin(?b, ?p, ?g, ?q)})
```

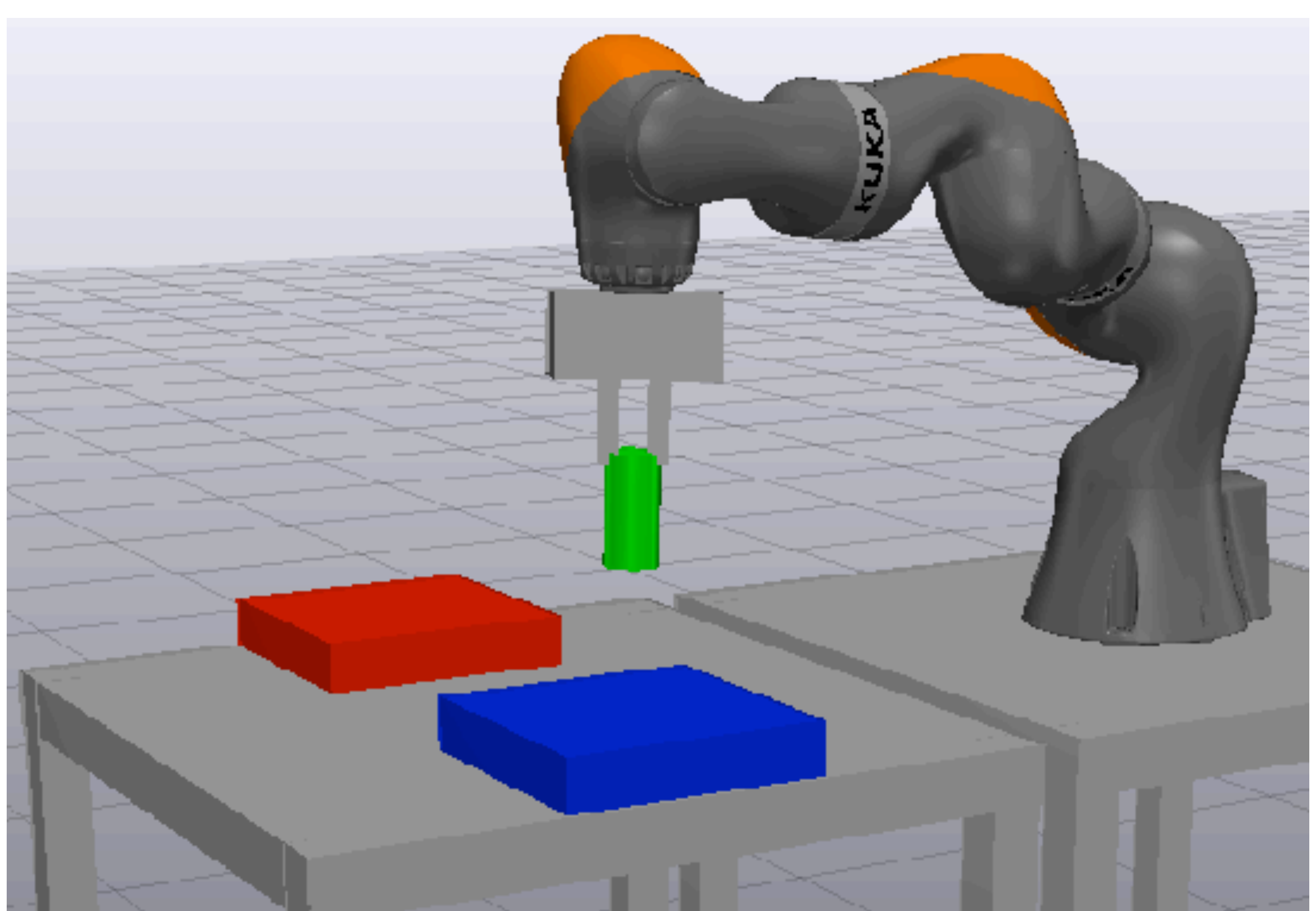


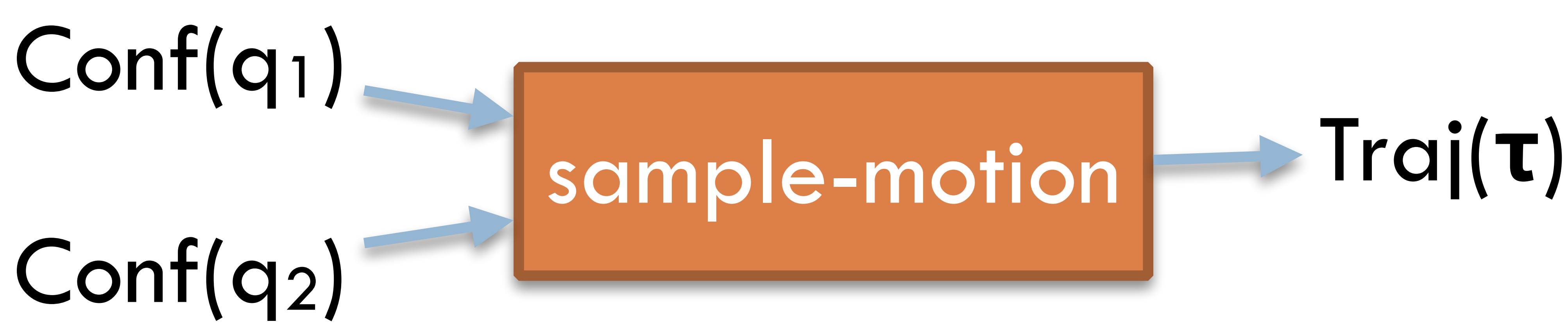


Invoking a Motion Planner

- "Sample" **multi-waypoint** robot **trajectories**
- Use off-the-shelf motion planner (e.g. RRT)

```
(:stream sample-motion
 :inputs (?q1, ?q2)
 :domain {Conf(?q1), Conf(?q2)}
 :outputs (?t)
 :certified {Traj(?t), Motion(?q1, ?t, ?q2)})
```





PDDLStream Algorithms

PDDLStream: Integrating Symbolic Planners and Blackbox Samplers via Optimistic Adaptive Planning. Caelan Reed Garrett, Tomás Lozano-Pérez, Leslie Pack Kaelbling. *International Conference on Automated Planning and Scheduling (ICAPS)*, 2020.

Two PDDLStream Algorithms

- PDDLStream **algorithms decide** which streams to use
- **Reduce** planning to a sequence of PDDL problems
  1. **Search** a finite PDDL problem for plan
  2. **Modify** the PDDL problem (depending on the plan)

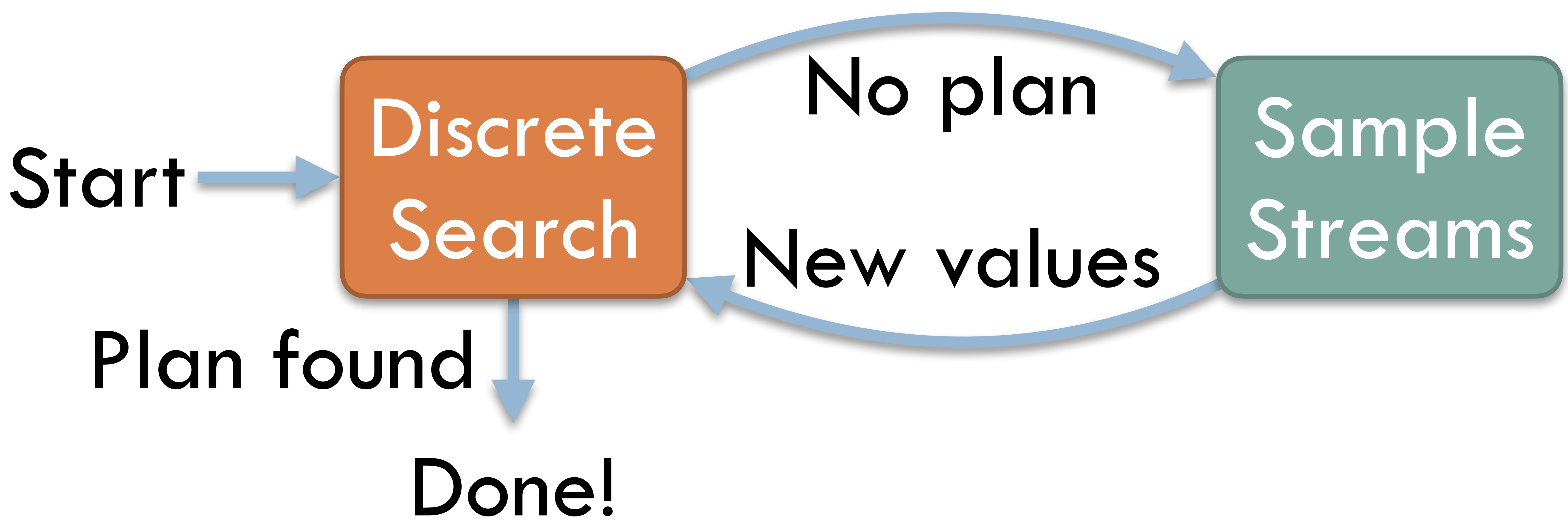
[Garrett 2018]
[Garrett 2020a]

- Implement search using off-the-shelf domain-independent **PDDL planners** (e.g. FastDownward)
  - **Greedy** best-first heuristic search
  - Exploit **factoring** in PDDL for heuristics (e.g. h_{FF})

Incremental Algorithm

- **Incrementally** grow the set of values and facts
- Repeat:

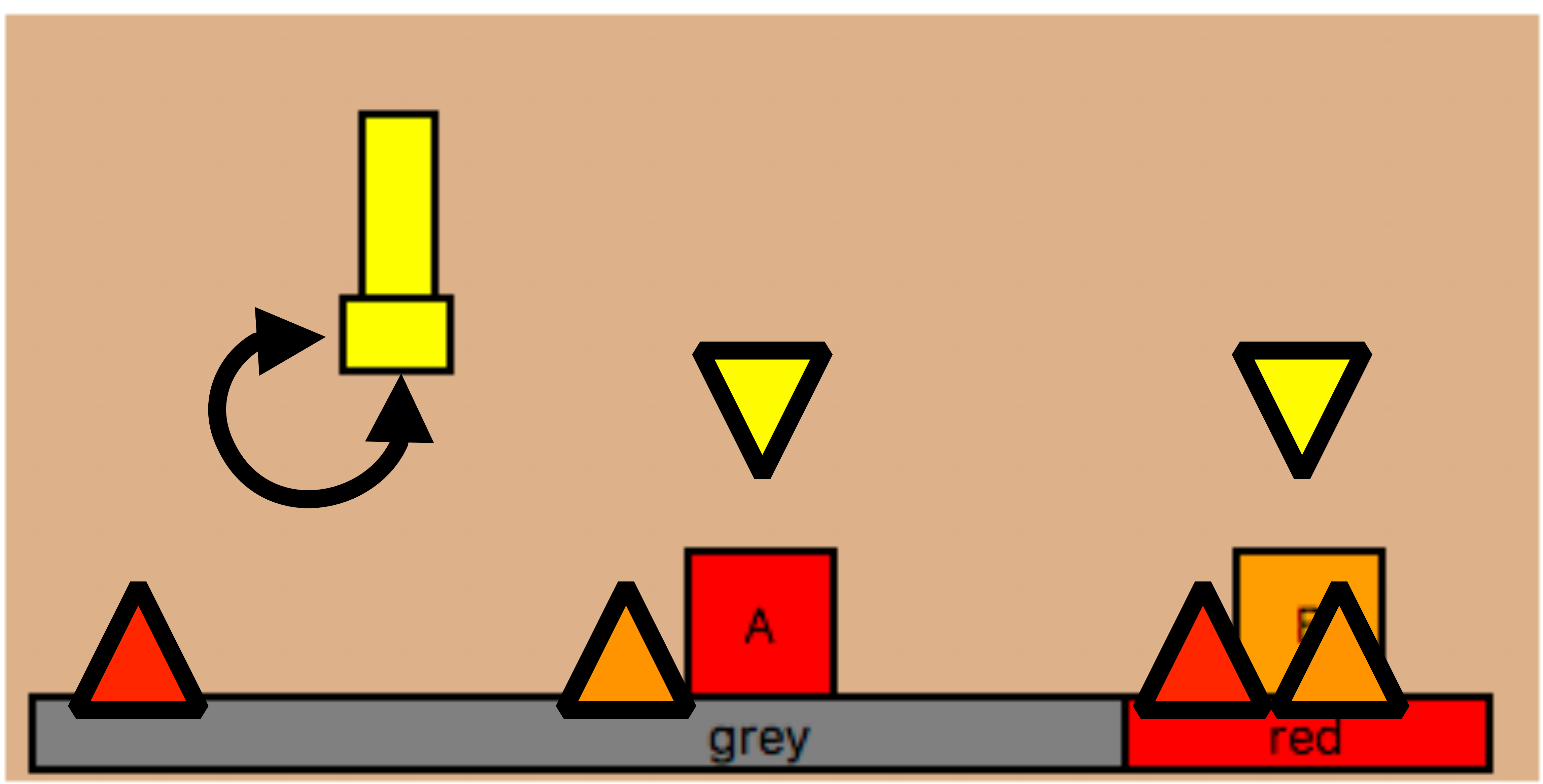
1. **Instantiate** and **sample** streams to generate new values and prove new facts
2. **Search** for a plan using the current values
3. **Return** when a plan is found



[Garrett 2018]
[Garrett 2020a]

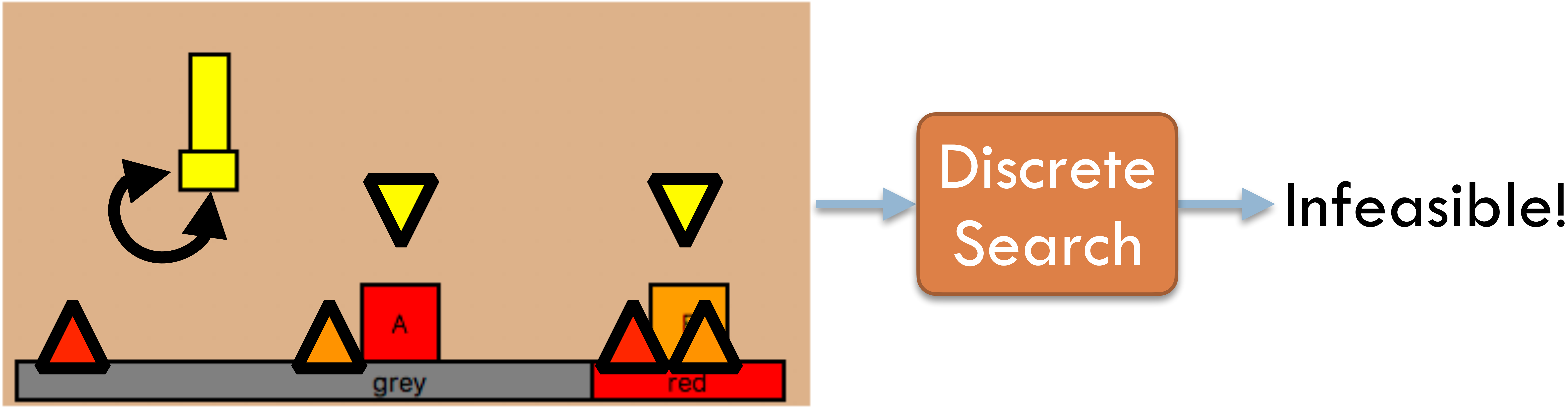
Incremental: Iteration 1 - Sampling

- **Iteration 1** - evaluated 14 streams
- **Sampled:**
  - 4 new block poses:
  - 2 new robot configurations:
  - 2 new trajectories:



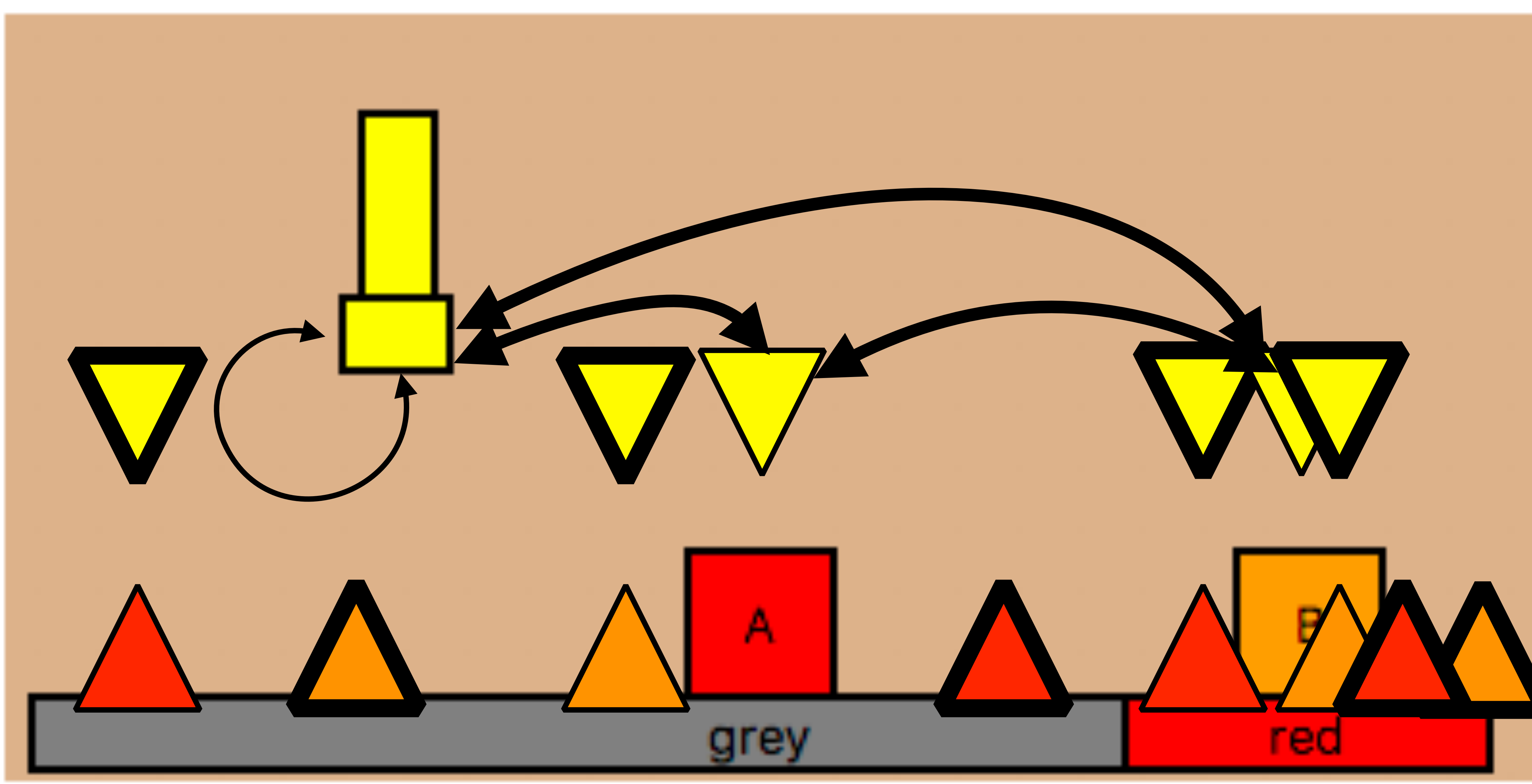
Incremental: Iteration 1 - Search

- Pass current discretization to FastDownward
- If **infeasible**, the current set of samples is insufficient



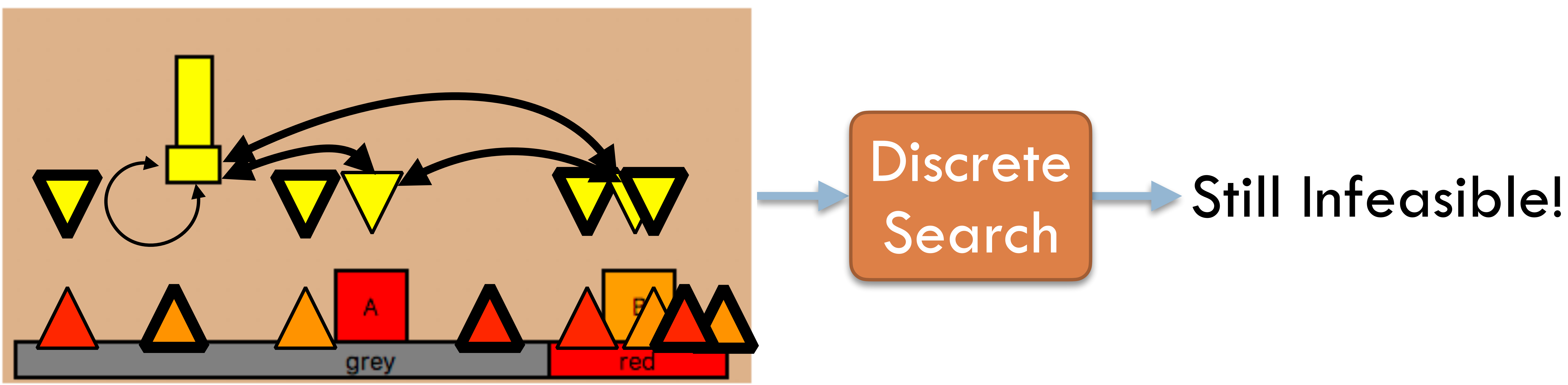
Incremental: Iteration 2 - Sampling

- **Iteration 2** - evaluated 54 streams
- **Sampled:**
  - 4 new block poses:
  - 4 new robot configurations:
  - 10 new trajectories:



Incremental: Iteration 2 - Search

- Pass current discretization to FastDownward
- If **infeasible**, the current set of samples is insufficient



Incremental Example: Iterations 3-4

Iteration 3 - 118 queried streams - infeasible

Iteration 4 - 182 queried streams - **solved!**

Solution:

1. move ([-7.5 5.], τ_1, [7.5 2.5])
2. pick (**B**, [7.5 0.], [0. -2.5], [7.5 2.5])
3. move ([7.5 2.5], τ_2, [10.97 2.5])
4. place(**B**, [10.97 0.], [0. -2.5], [10.97 2.5])
5. move ([10.97 2.5], τ_3, [0. 2.5])
6. pick (**A**, [0. 0.], [0. -2.5], [0. 2.5])
7. move ([0. 2.5], τ_4, [7.65 2.5])
8. place(**A**, [7.65 0.], [0. -2.5], [7.65 2.5])

- **Planner generated** all but the underlined values
- **Drawback** - many unnecessary samples produced

Optimistic Stream Evaluation

- Many TAMP streams are computationally **expensive**
  - Inverse kinematics, collision checking, motion planning
- Only query streams after they are **identified** as useful
  - Plan with **optimistic hypothetical** outputs

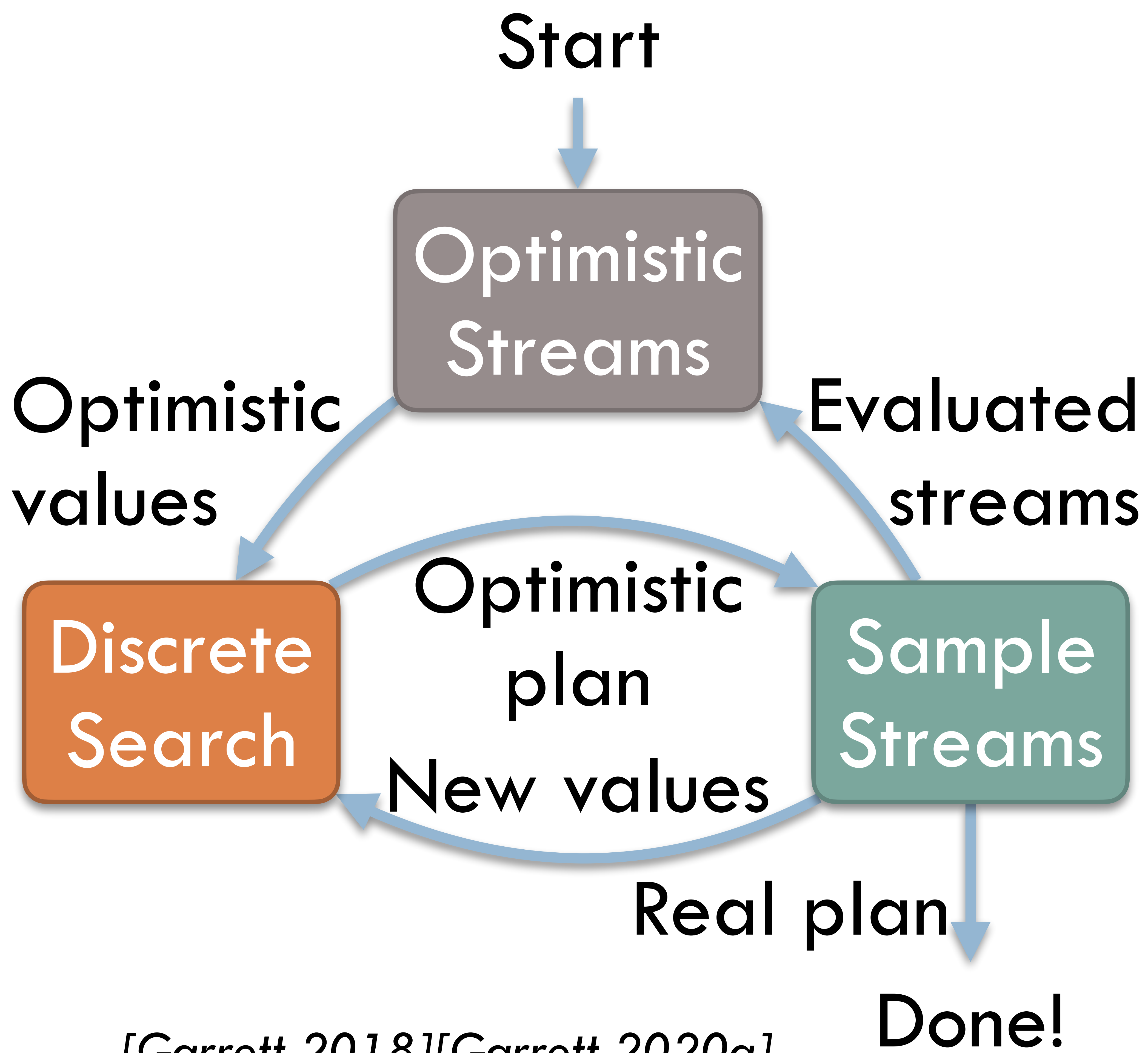
- Inductively create unique **optimistic placeholder values** for each stream output (denoted by prefix #)

1. `s-region(`**`A`**`, `**`red`**`)→#p0`
2. `s-ik(`**`A`**`, [0. 0.], [0. -2.5])→#q0,`
3. `s-ik(`**`A`**`, #p0, [0. -2.5])→#q2,`
4. `s-motion(`**`A`**`, #q0, #q2)→#t0, …`

[Garrett 2018]
[Garrett 2020a]

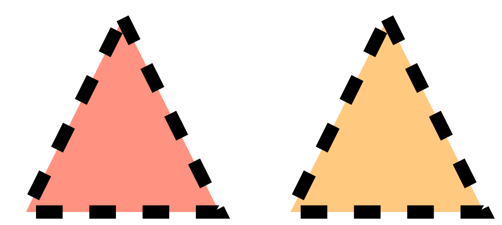
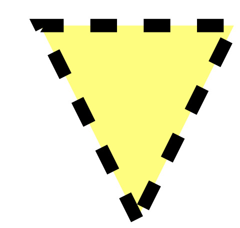
Focused Algorithm

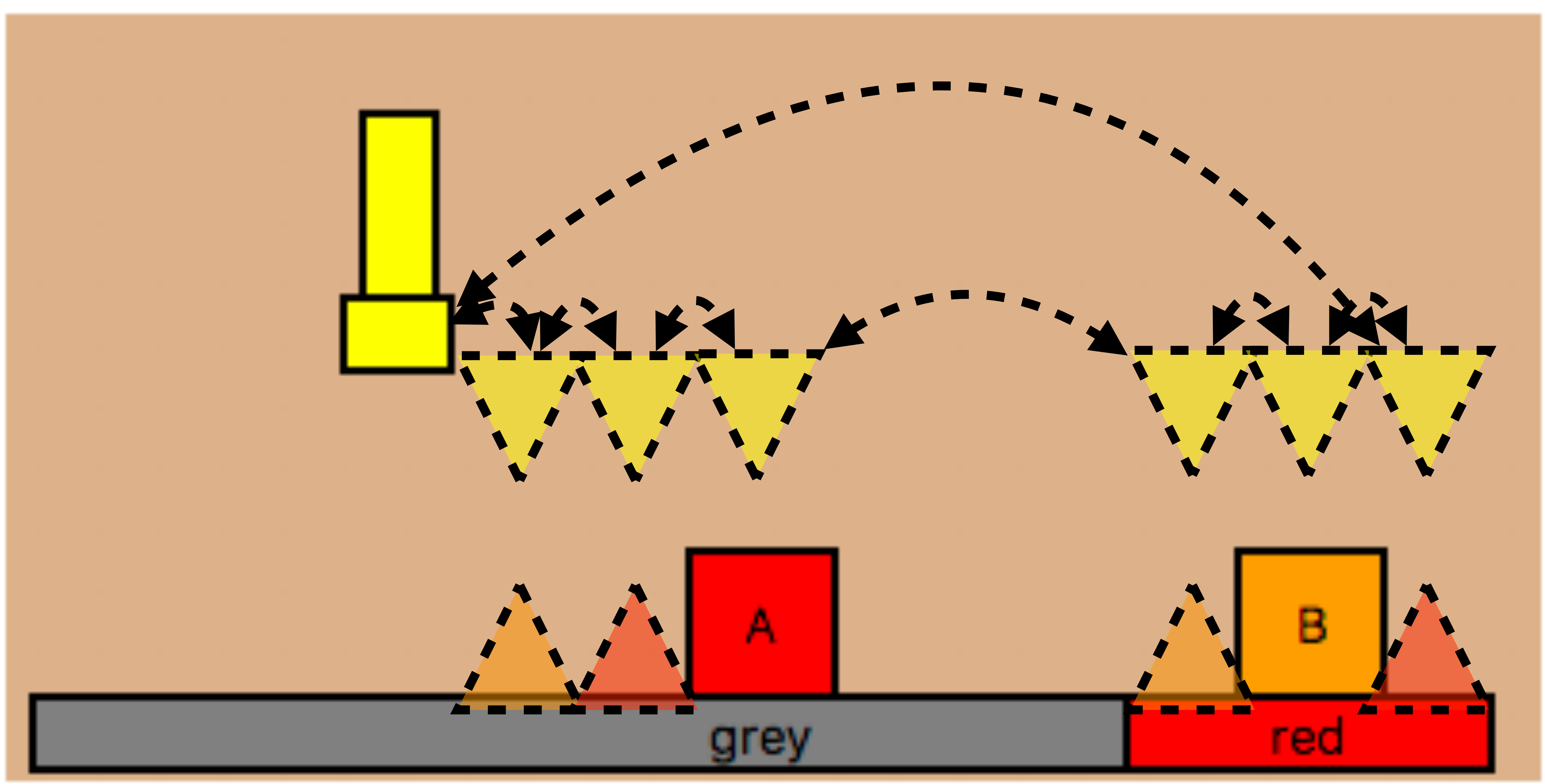
- **Lazily** plan using optimistic values **before** real values
- Repeat:
  1. **Construct** optimistic stream outputs
  2. **Search** with real & optimistic values
  3. **Retrace** and **evaluate** streams
  4. **Replace** optimistic with real if they exist
  5. **Return** if all succeed



[Garrett 2018][Garrett 2020a]

Focused: Iteration 1

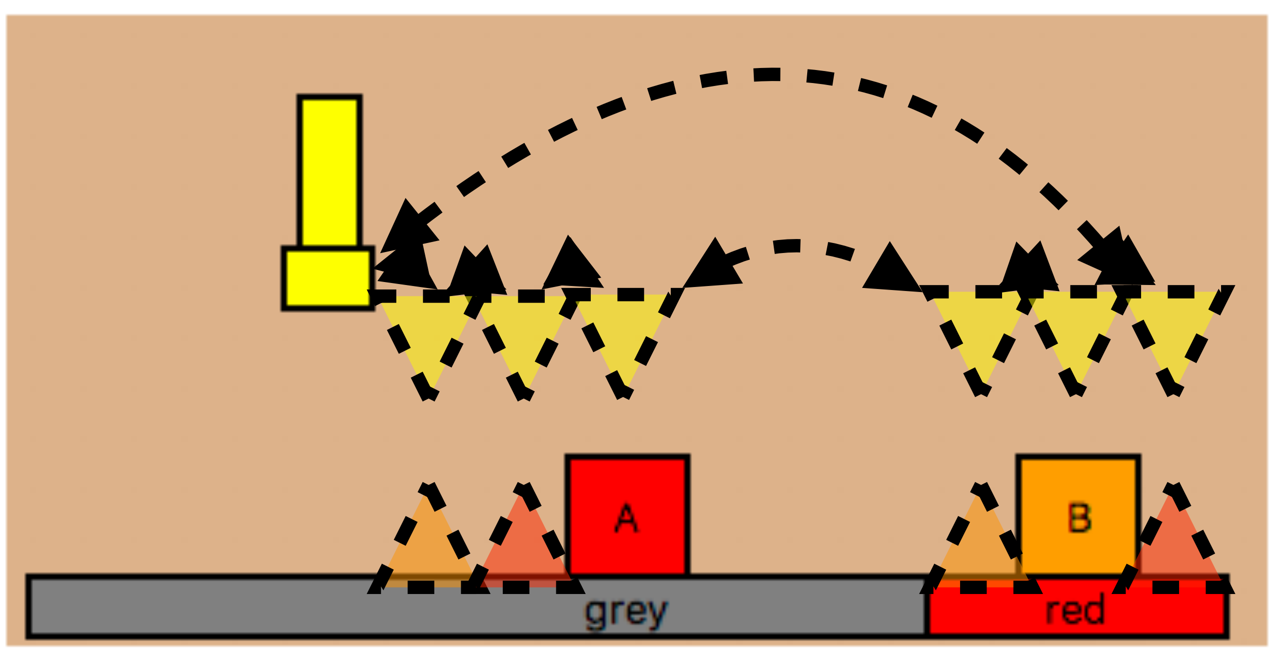
- **Iteration 1** - optimistically evaluated 46 streams
- **Created:**
  - 4 optimistic block poses: 
  - 6 optimistic robot configurations: 
  - 36 optimistic trajectories: 

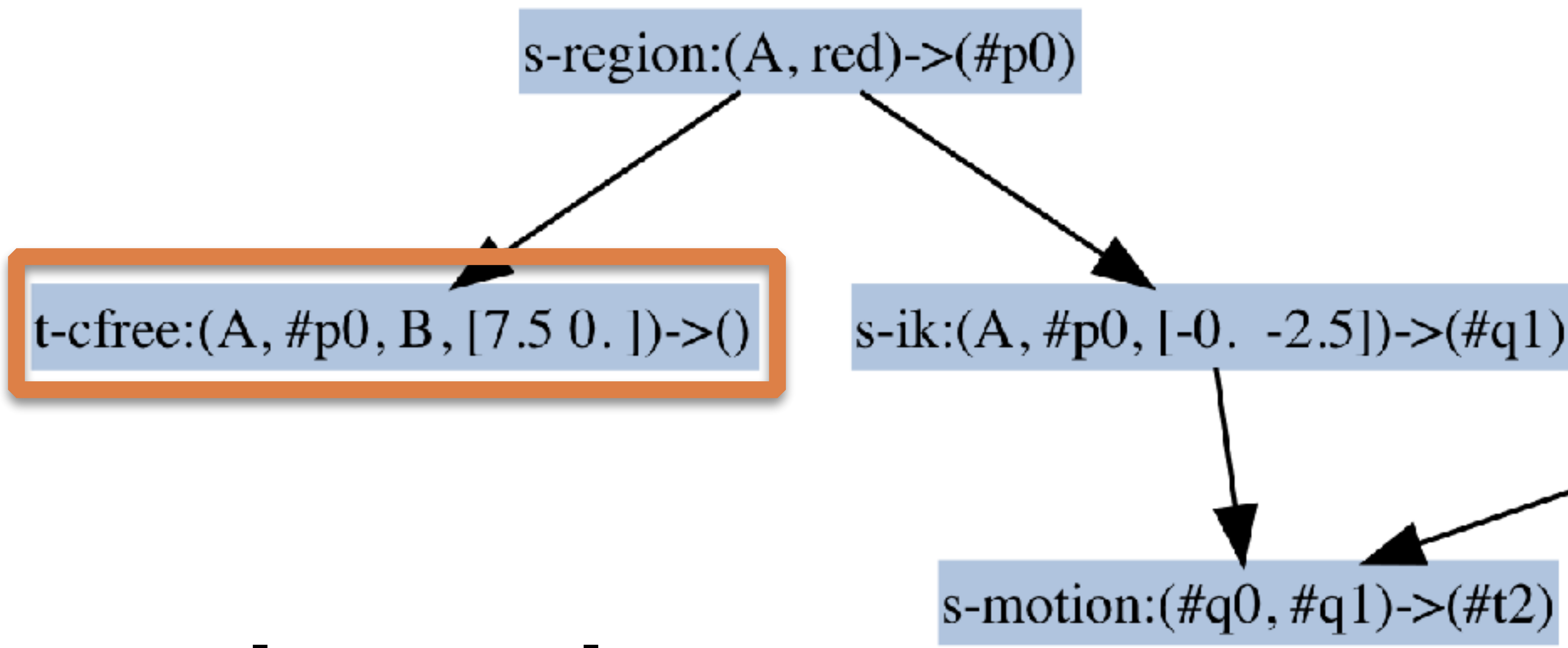


Focused: Iteration 1 - Sampling

Optimistic plan:

[move([-5. 5.],#t0,#q0), pick(**A**,[0. 0.],[0. -2.5],#q0), move(#q0, #t2, #q1), place(**A**,#p0,[0. -2.5],#q1)]





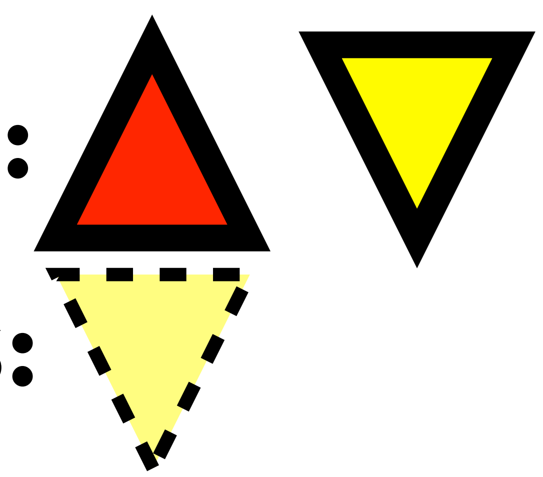
Queried streams:

1. s-region(**A**, **red**)→<u>[8.21 0.]</u>
2. s-ik(**A**, [0. 0.], [0. -2.5])→<u>[0. 2.5]</u>
3. t-cfree(**A**, [8.21 0.], **B**, [7.5 0.])→<u>False</u>

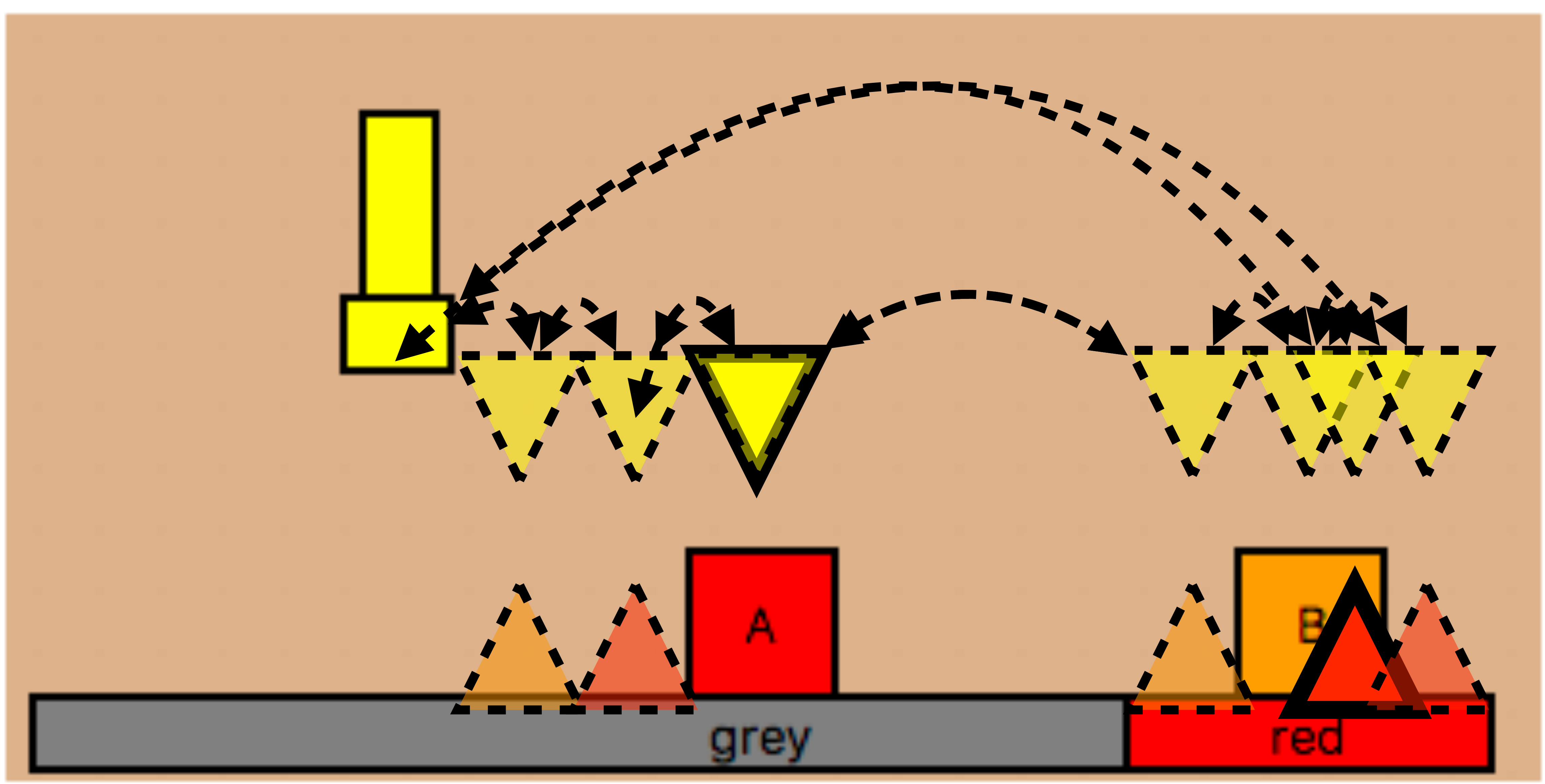
Temporarily **remove** these streams from the next search

Focused: Iteration 2

- **Iteration 2** - optimistically evaluated 42 streams
- Removed **optimistic** pose and configuration
- Added **sampled** pose and configuration:
- Added 1 **optimistic** robot configurations:



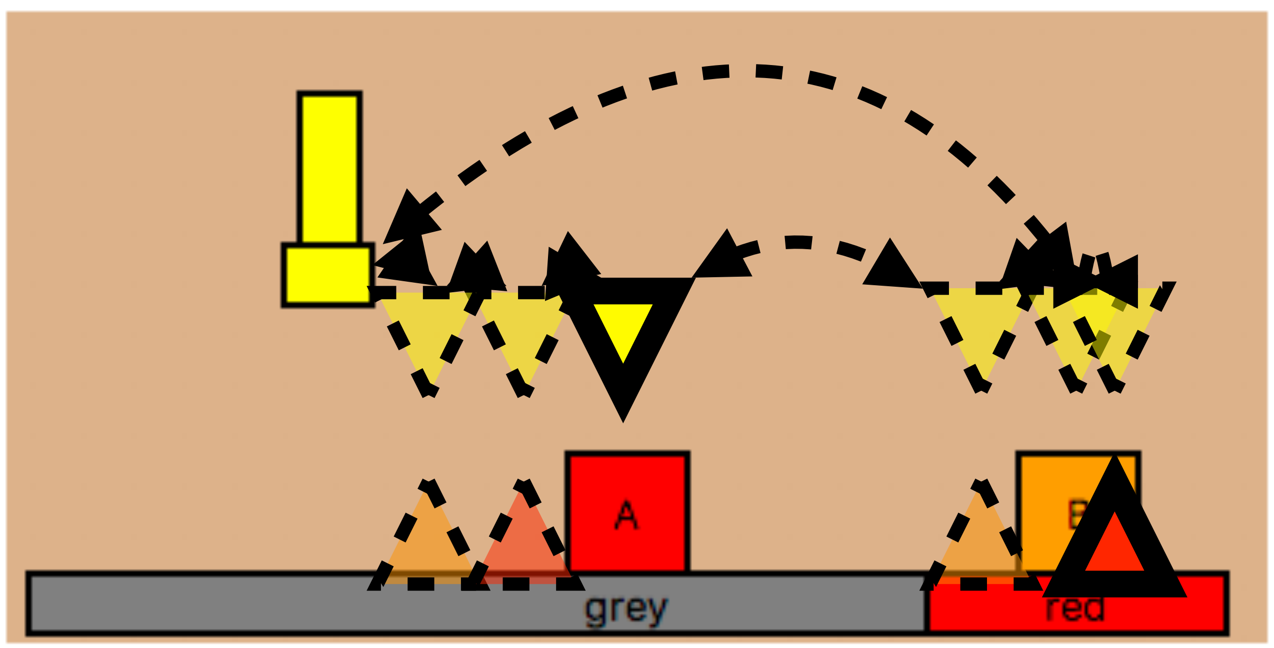
- Added 14 **optimistic** trajectories:



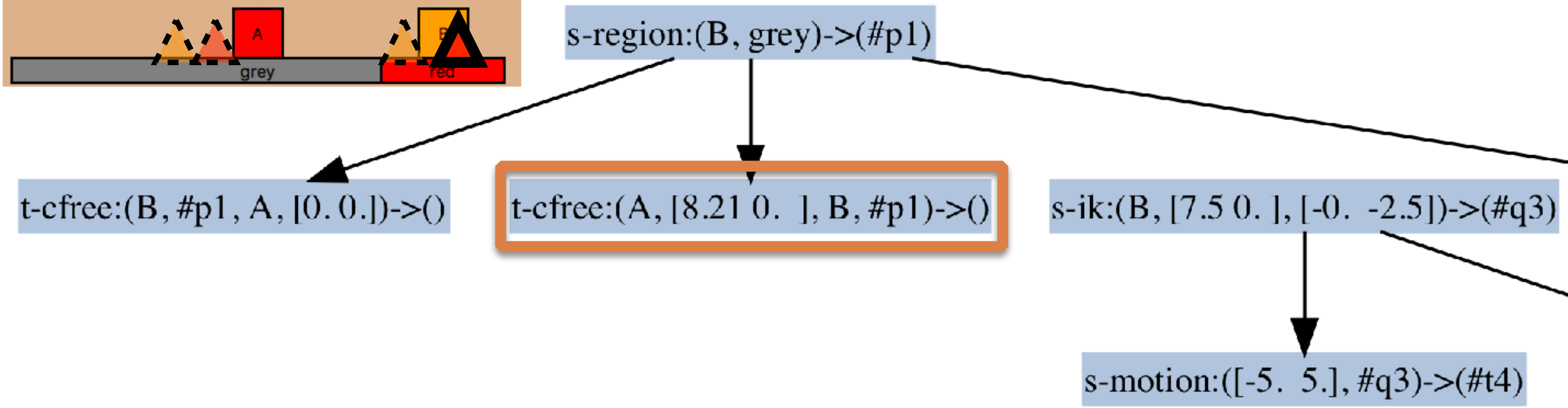
Focused: Iteration 2 - Sampling

New optimistic plan:

```
[move([-5. 5.],#t4,#q2), pick(B,[7.5 0.],[0. -2.5],#q2),
 move(#q2, #t9, #q3),    place(B, #p1, [0. -2.5], #q3),
 move(#q3,#t6,[0. 2.5]), pick(A,[0. 0.],[0. -2.5],[0. 2.5]),
 move([0. 2.5],#t8,#q4), place(A,[8.21 0.],[0. -2.5],#q4)]
```



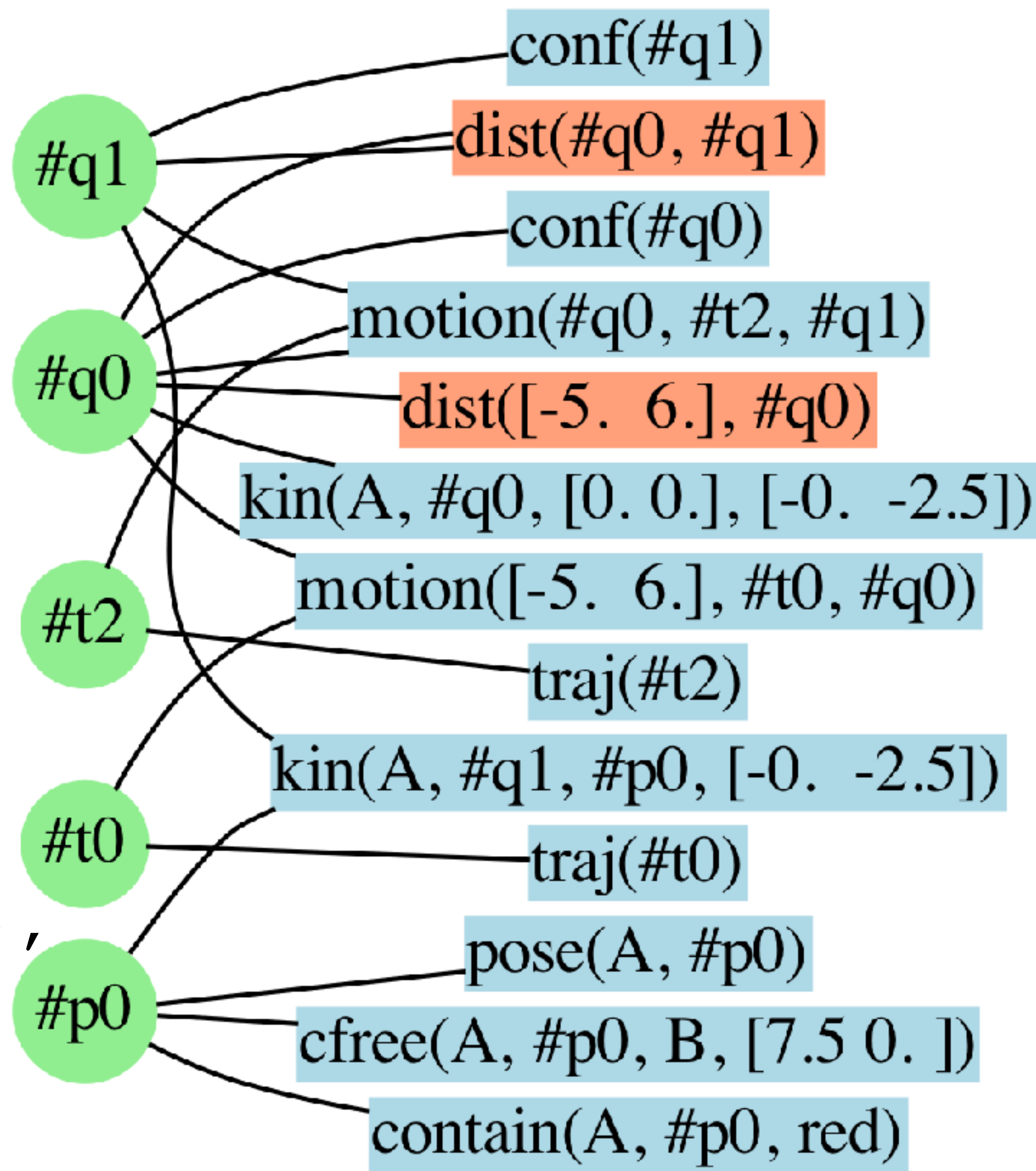
Different stream plan might succeed!



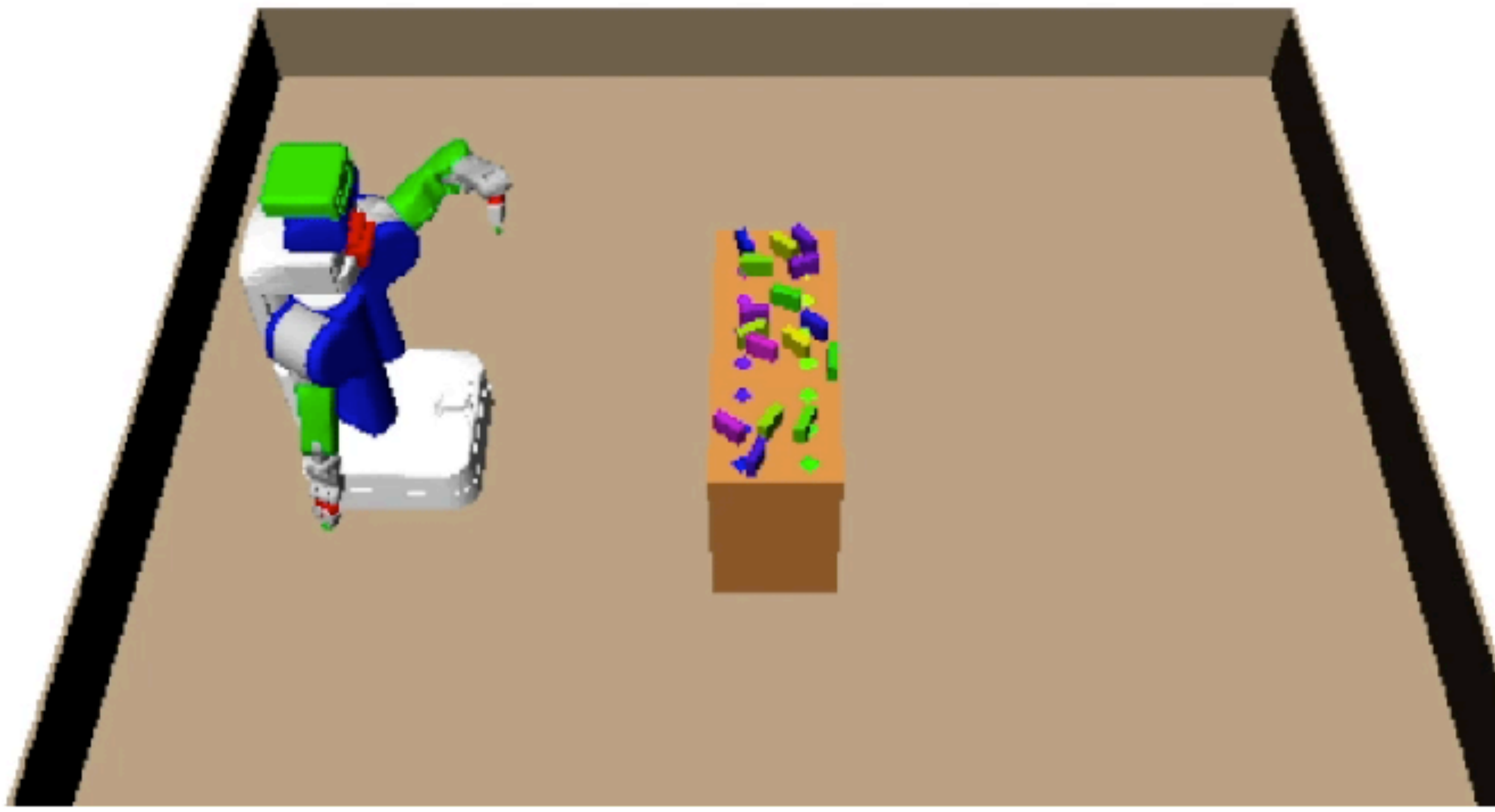
Optimistic Planning with Optimization

- Instead of sampling, directly **optimize** the constraint network
- Non-convex constrained mathematical program **solver** as a **stream**
- Additional PDDLStream algorithms…

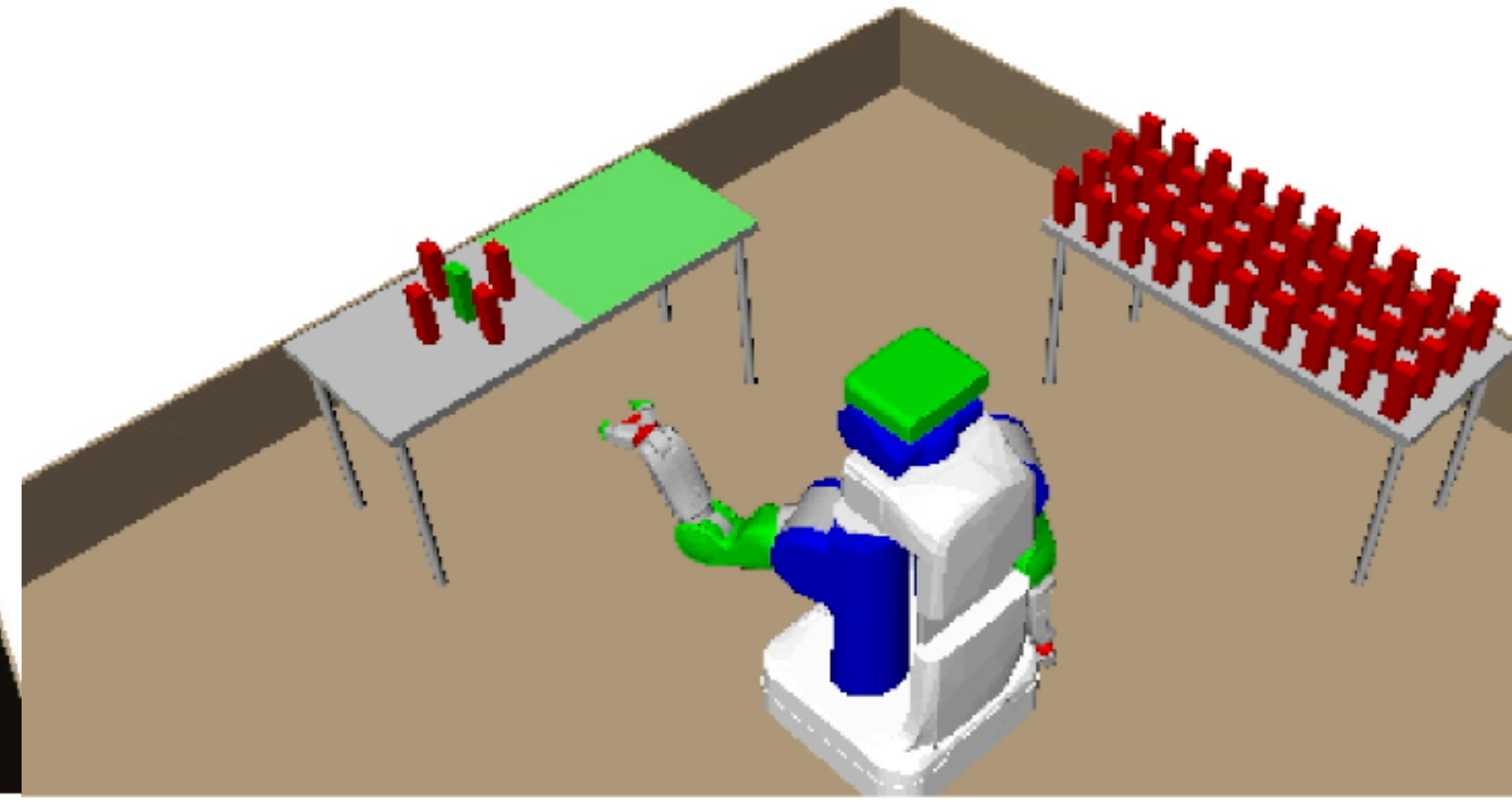
```
[move([-5. 6.], #t0, #q0),
 pick(A,[0. 0.],[0. -2.5],#q0),
 move(#q0, #t2, #q1),
 place(A,#p0,[0. -2.5],#q1)]
```



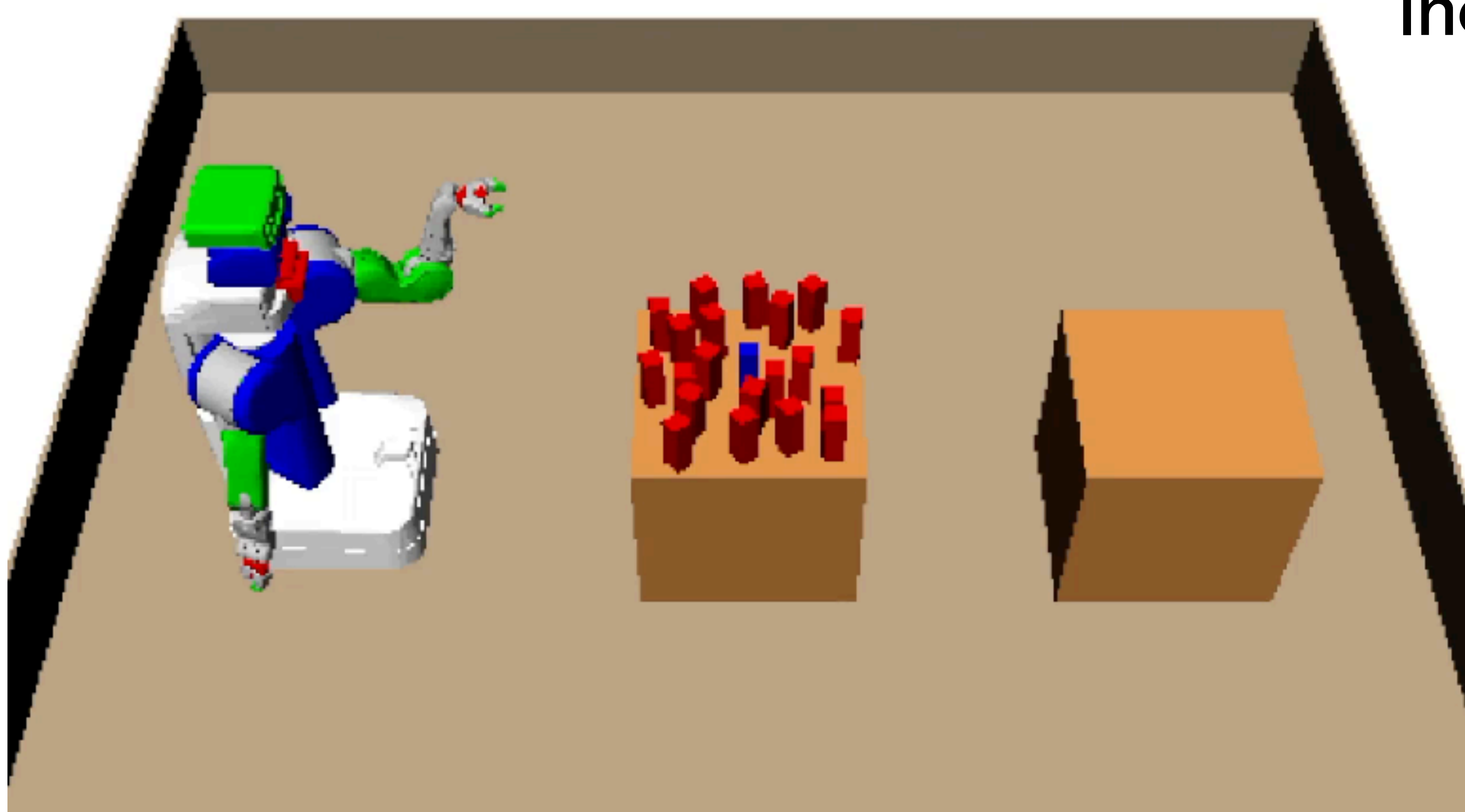
Scaling Experiments



Incremental ~20s
Focused ~10s



Incremental 120+
Focused ~25s

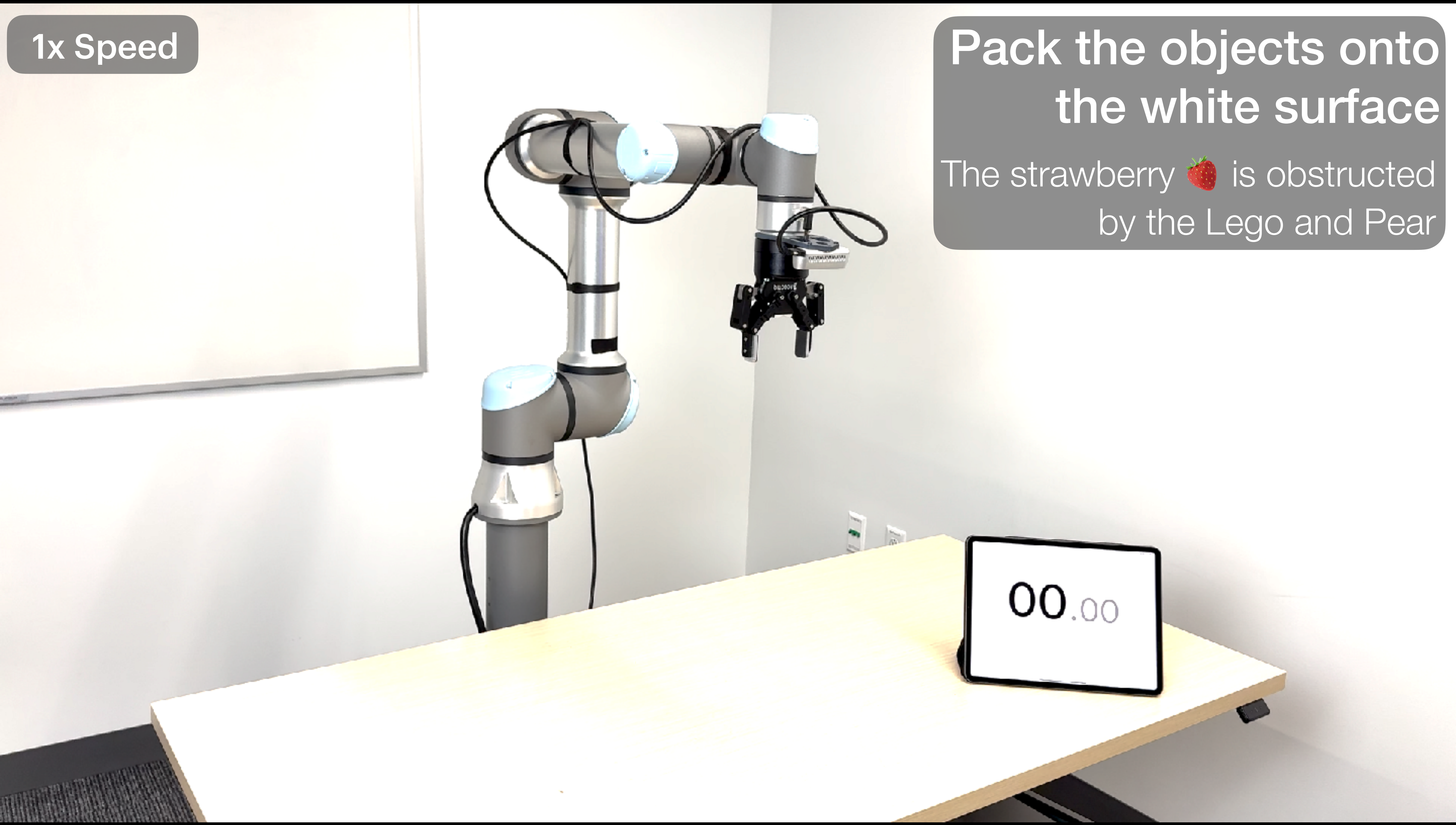


Incremental 120+
Focused ~20s

[Garrett 2018]

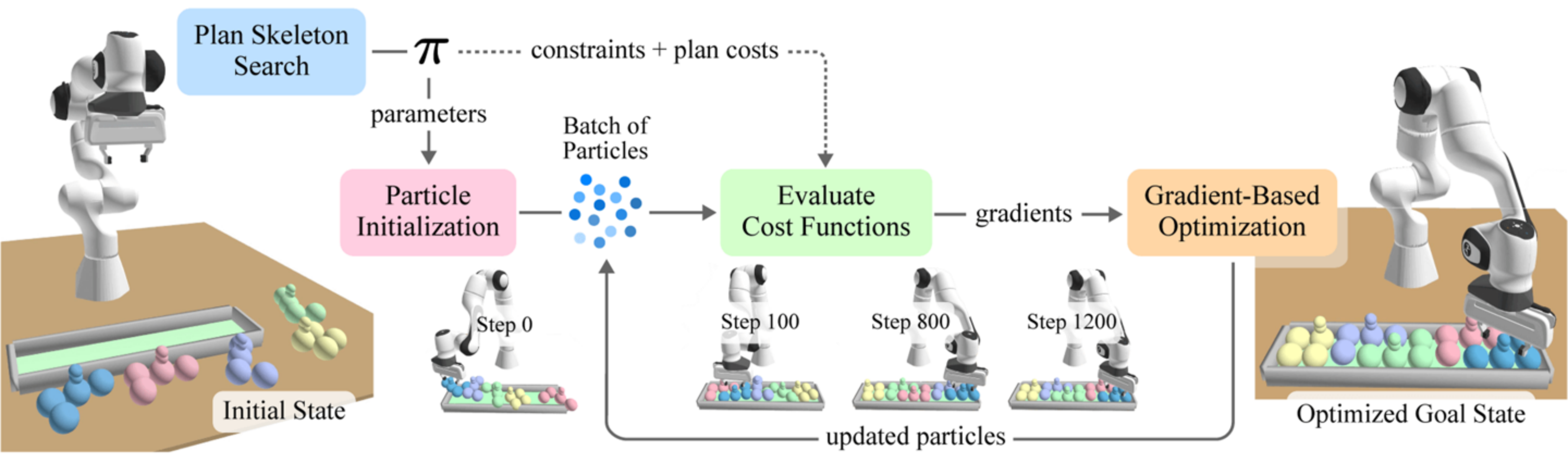
GPU-Parallelized TAMP

Differentiable GPU-Parallelized Task and Motion Planning. William Shen, Caelan Garrett, Nishanth Kumar, Ankit Goyal, Tucker Hermans, Leslie Pack Kaelbling, Tomás Lozano-Pérez, Fabio Ramos. *Robotics: Science and Systems (RSS)*, 2025.


1x Speed
Pack the objects onto the white surface
The strawberry 🍓 is obstructed by the Lego and Pear
00.00

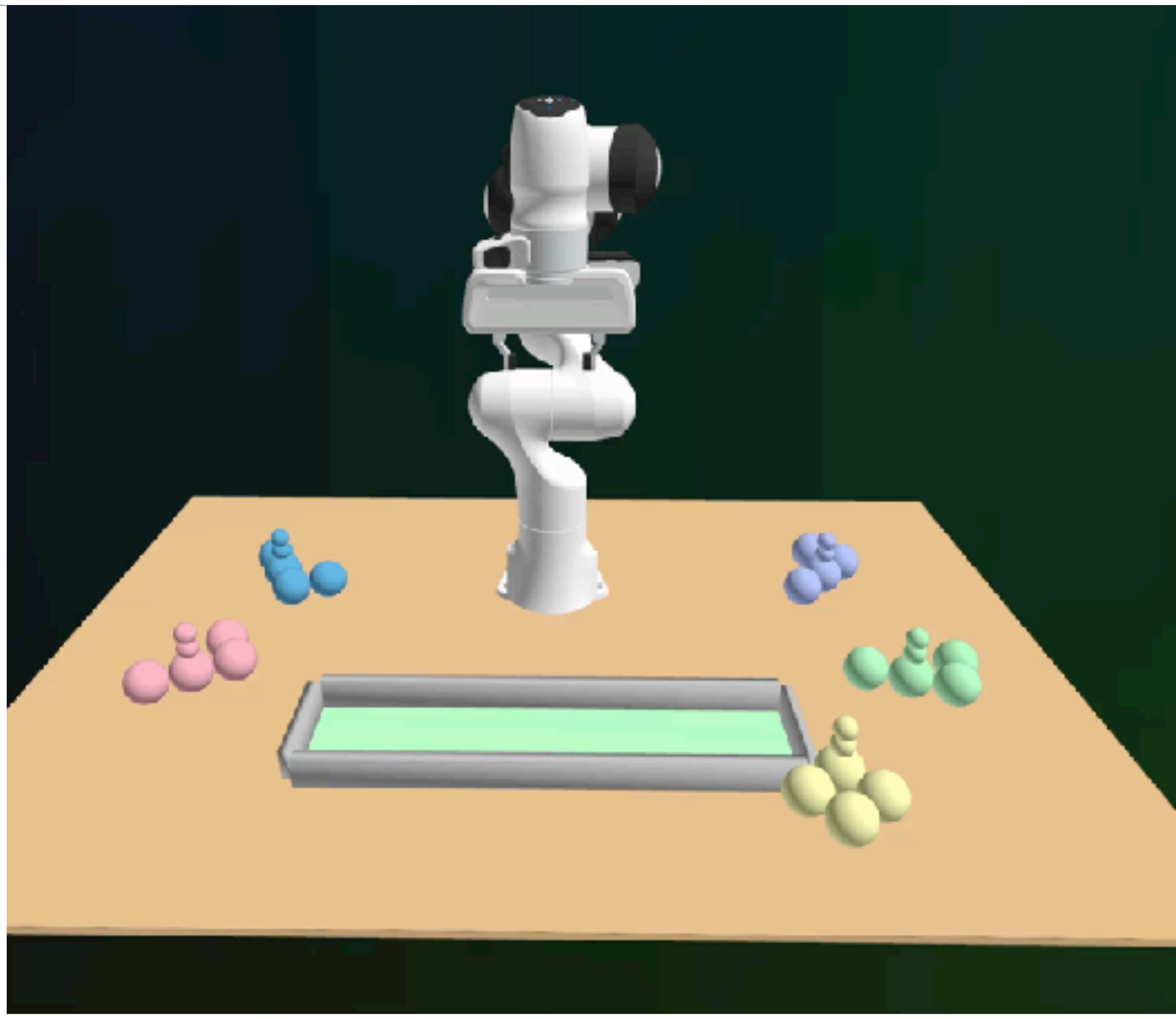
cuTAMP: GPU-Parallelized TAMP

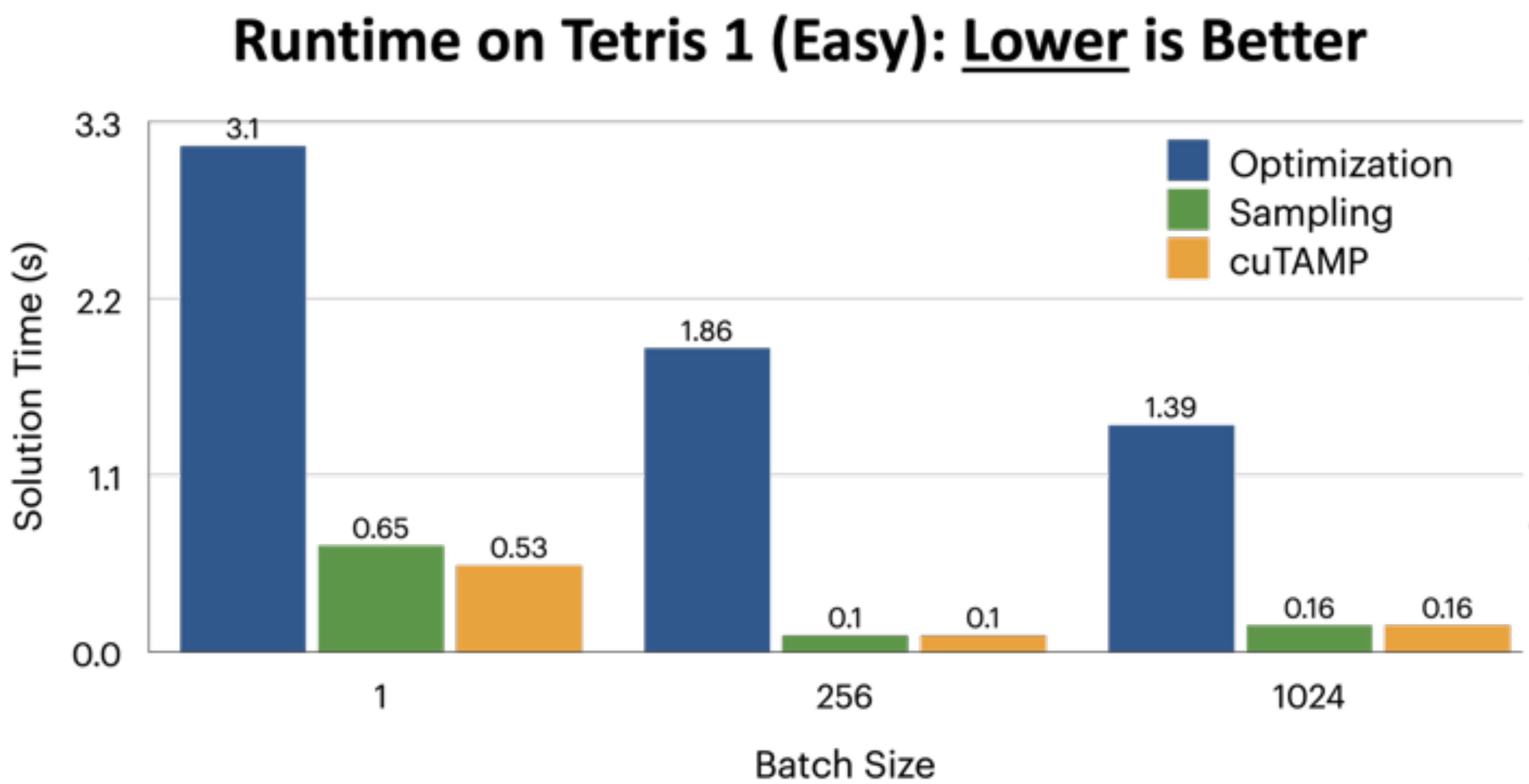
- Like Focused but combine **sampling** & **optimization**
  - Sample ~1000 candidates and optimize in batch
- Leverage **GPU acceleration** during both phases
- Custom CUDA kernels and PyTorch auto differentiation
  - Generalizes **cuRobo** motion planning to TAMP

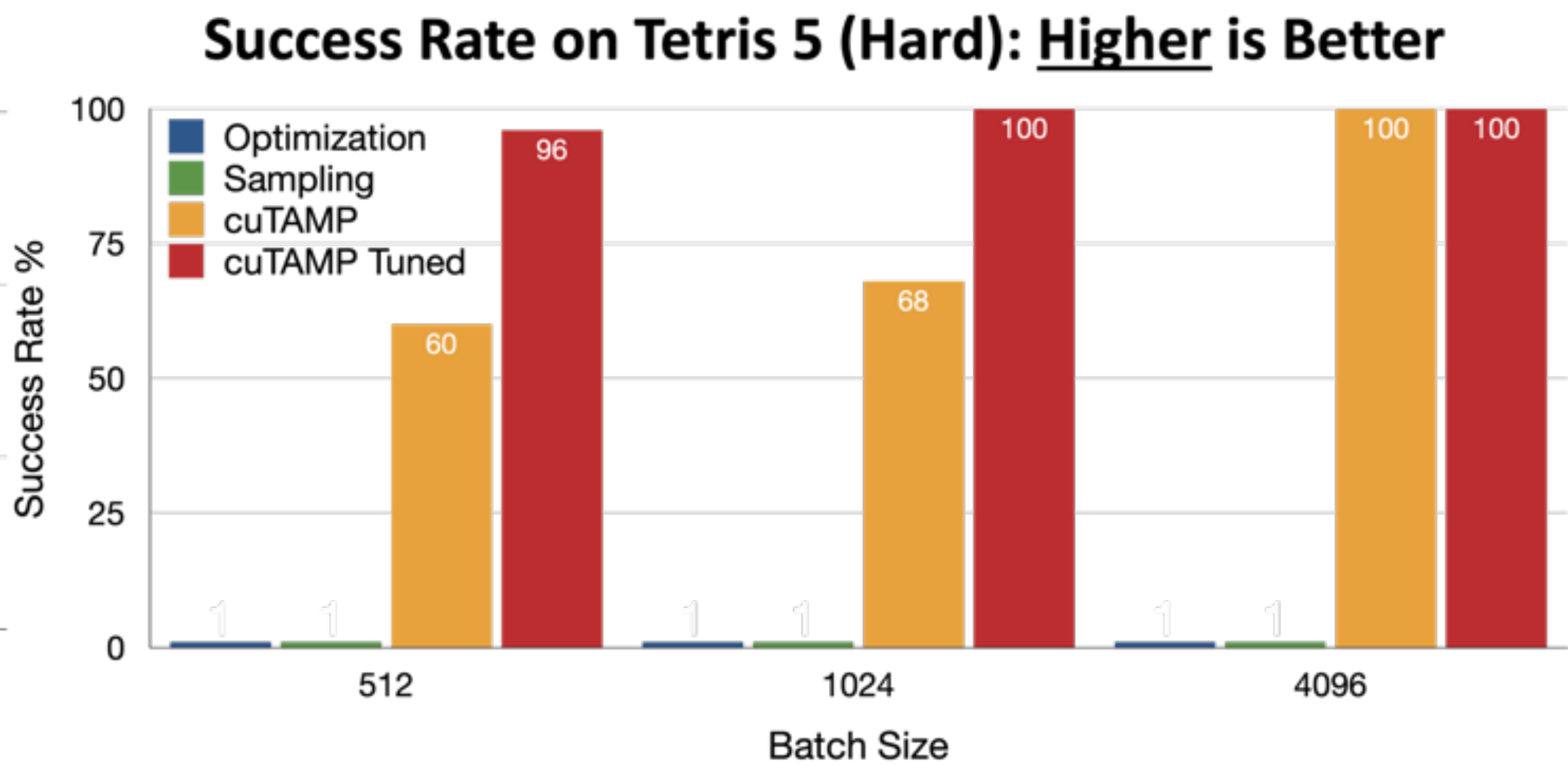


cuTAMP Experimental Results

- cuTAMP's combines **sampling** & **optimization** to outperform each individually
- Larger **GPU batch sizes** increase success rates and decrease runtime

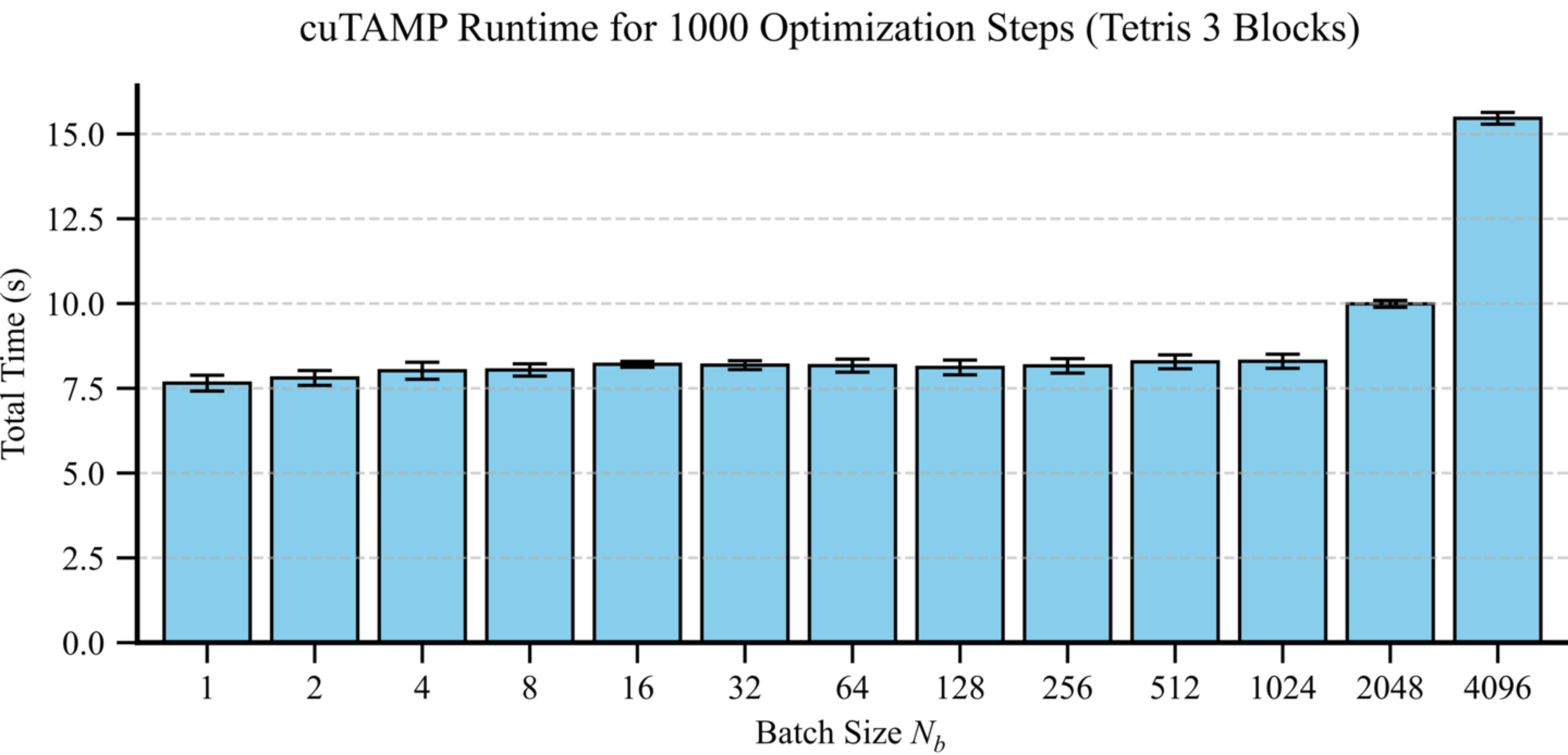


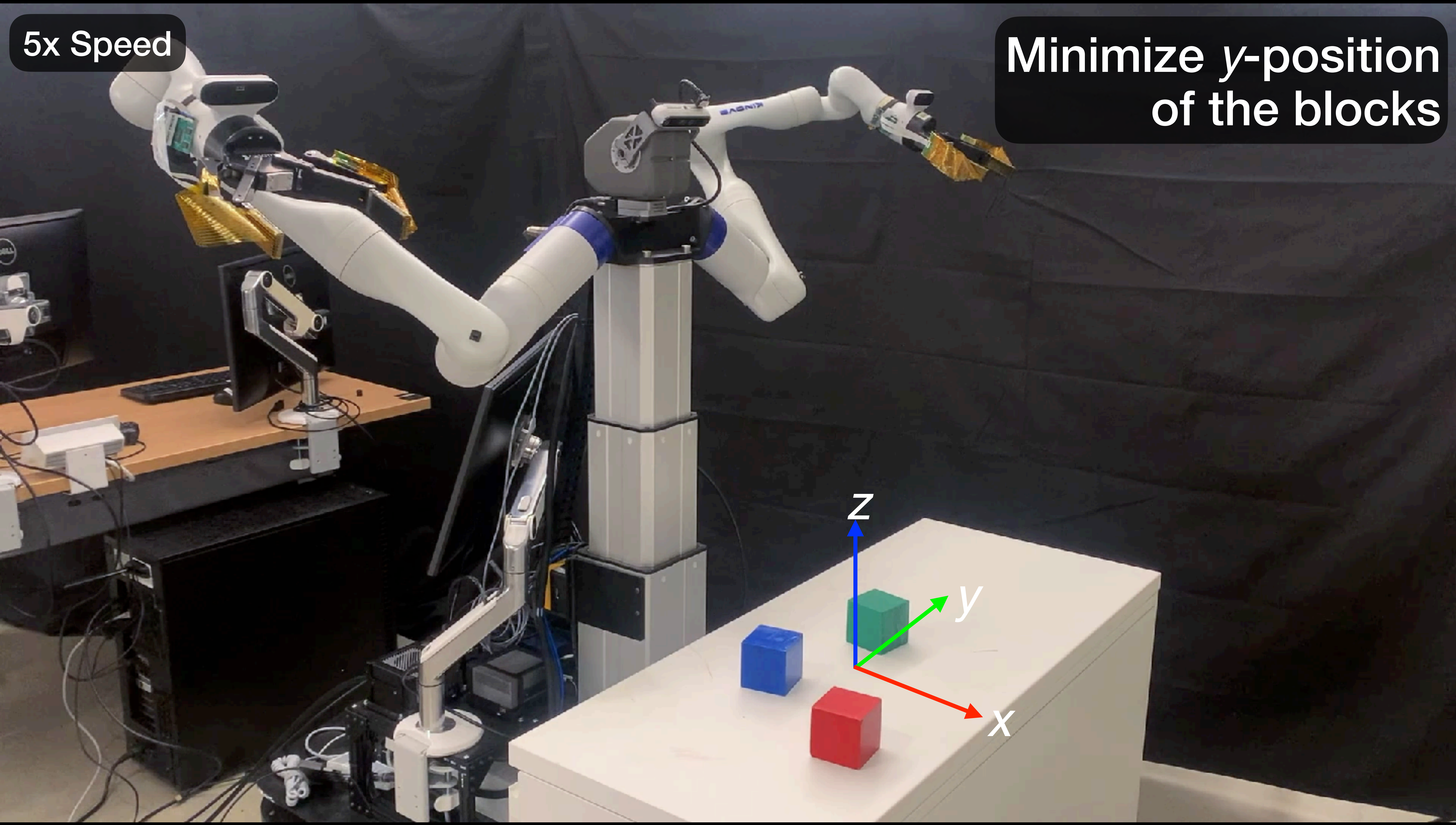




cuTAMP Scales Sublinearly*

- *For up to ~1000 Particles (GPU memory limit)




5x Speed
Minimize y-position
of the blocks
z
y
x

TAMP Under Uncertainty

Online Replanning in Belief Space for Partially Observable Task and Motion Problems. Caelan Reed Garrett, Chris Paxton, Tomás Lozano-Pérez, Leslie Pack Kaelbling, Dieter Fox. *IEEE International Conference on Robotics and Automation (ICRA)*, 2020.

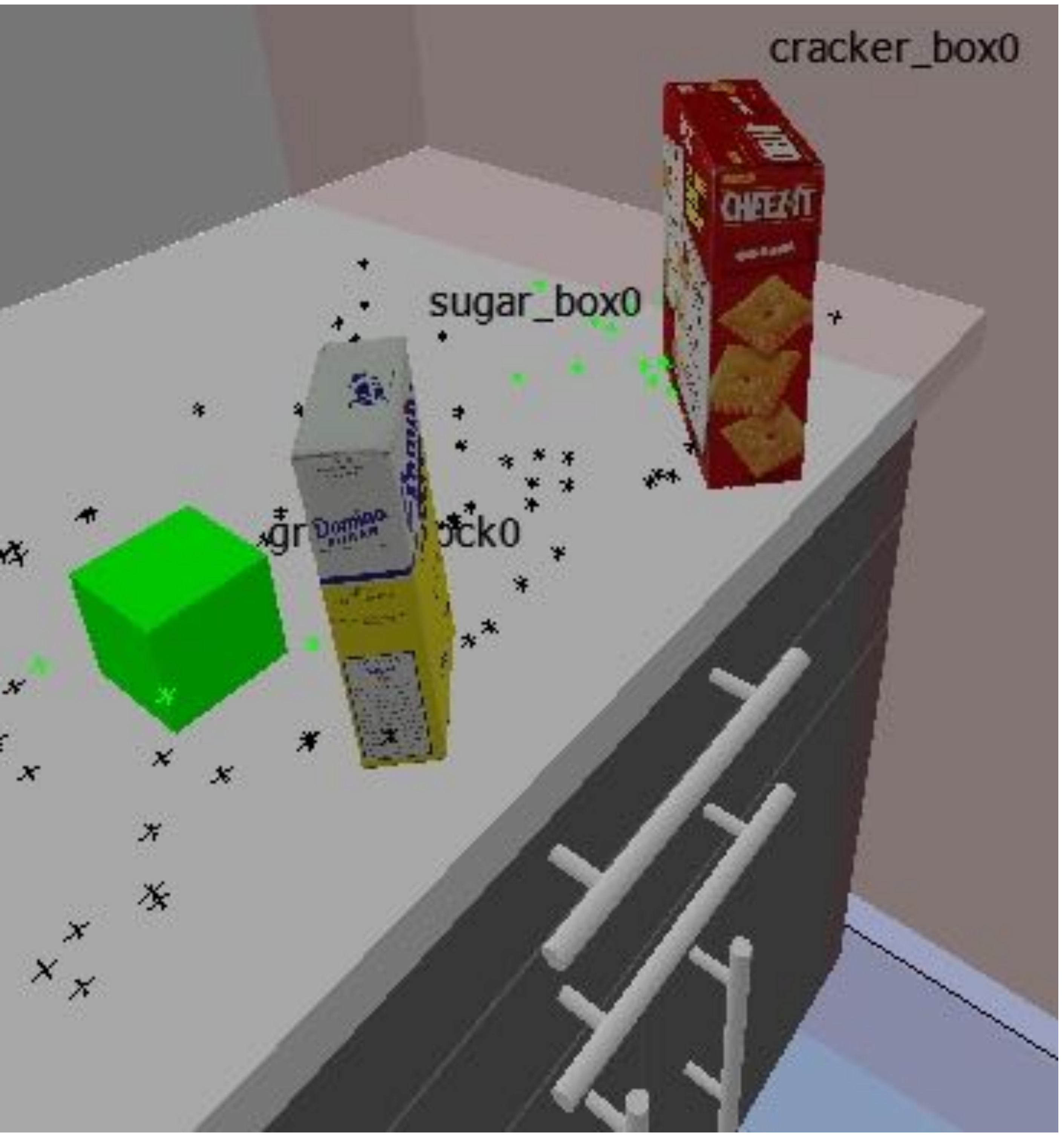
Long-Horizon Manipulation of Unknown Objects via Task and Motion Planning with Estimated Affordances. Aidan Curtis*, Xiaolin Fang*, Leslie Pack Kaelbling, Tomás Lozano-Pérez, Caelan Reed Garrett. *IEEE International Conference on Robotics and Automation (ICRA)*, 2022.

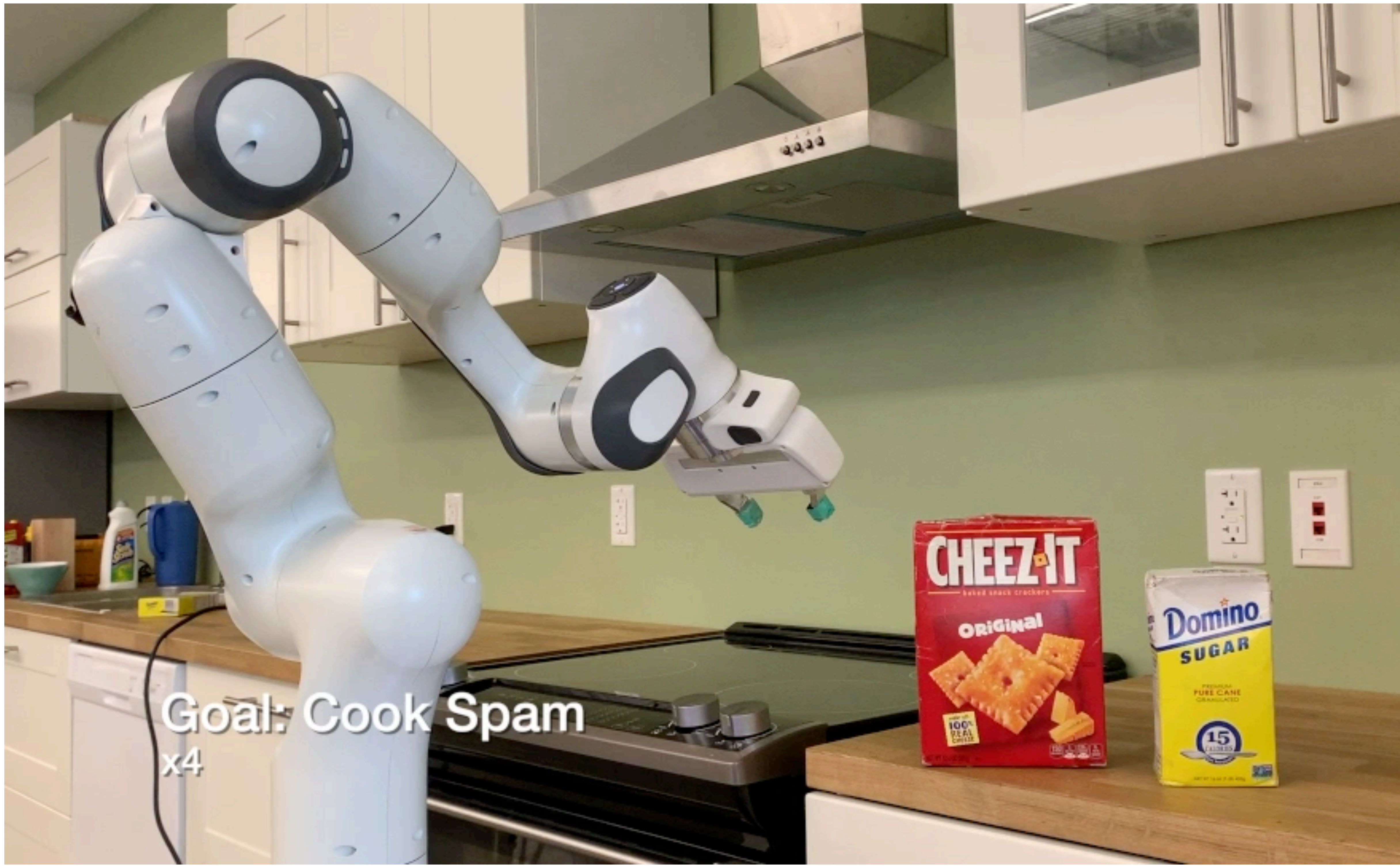
MDP: Stochastic Action Effects

- Approximate as cost-sensitive **deterministic** problem
- **Policy** evaluated online via **replanning**

POMDP: Partially Observable State

- Update a **belief** (probability distribution) over states
- Plan in **belief space** using **inference** streams & actions
- **Observation** actions reduce state variable **uncertainty**



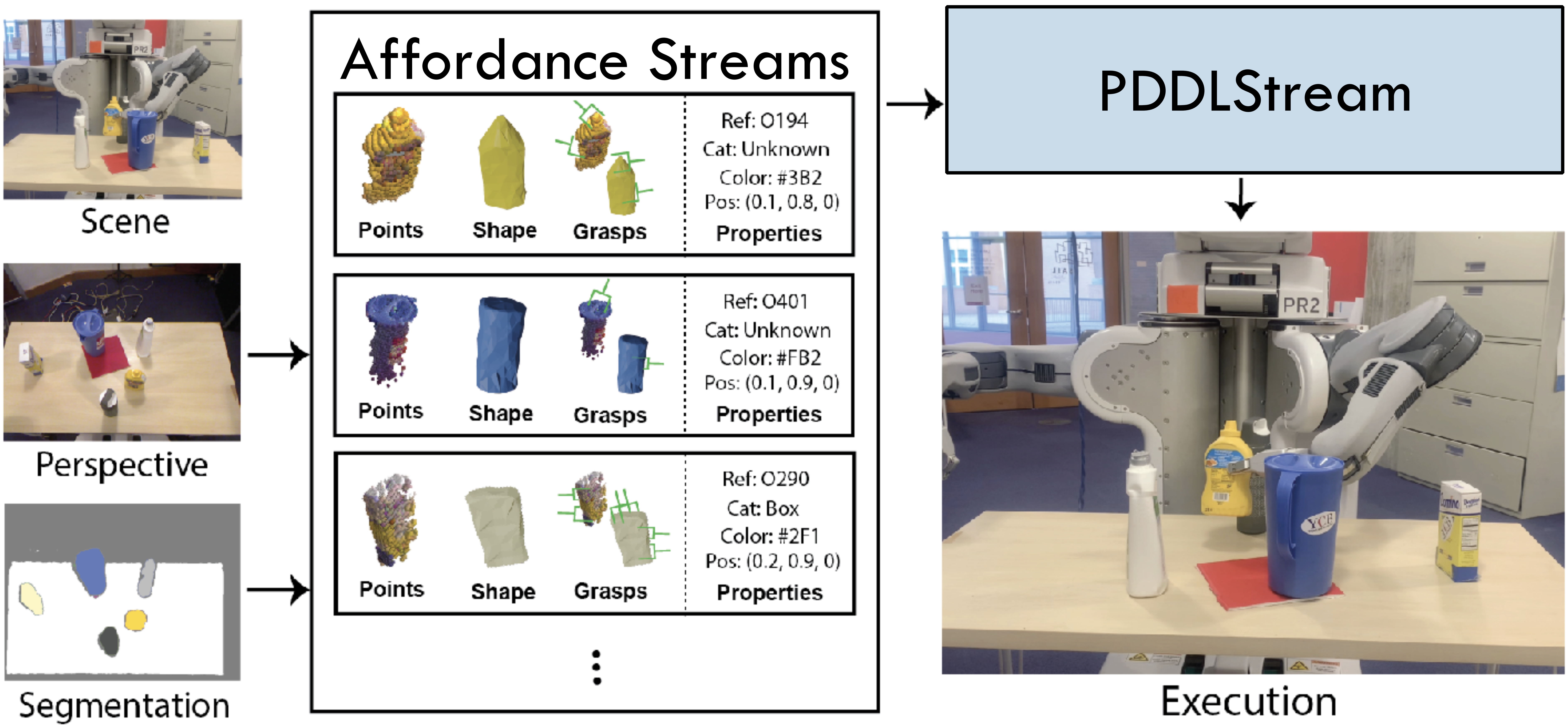


Probabilistic & Geometric Constraints

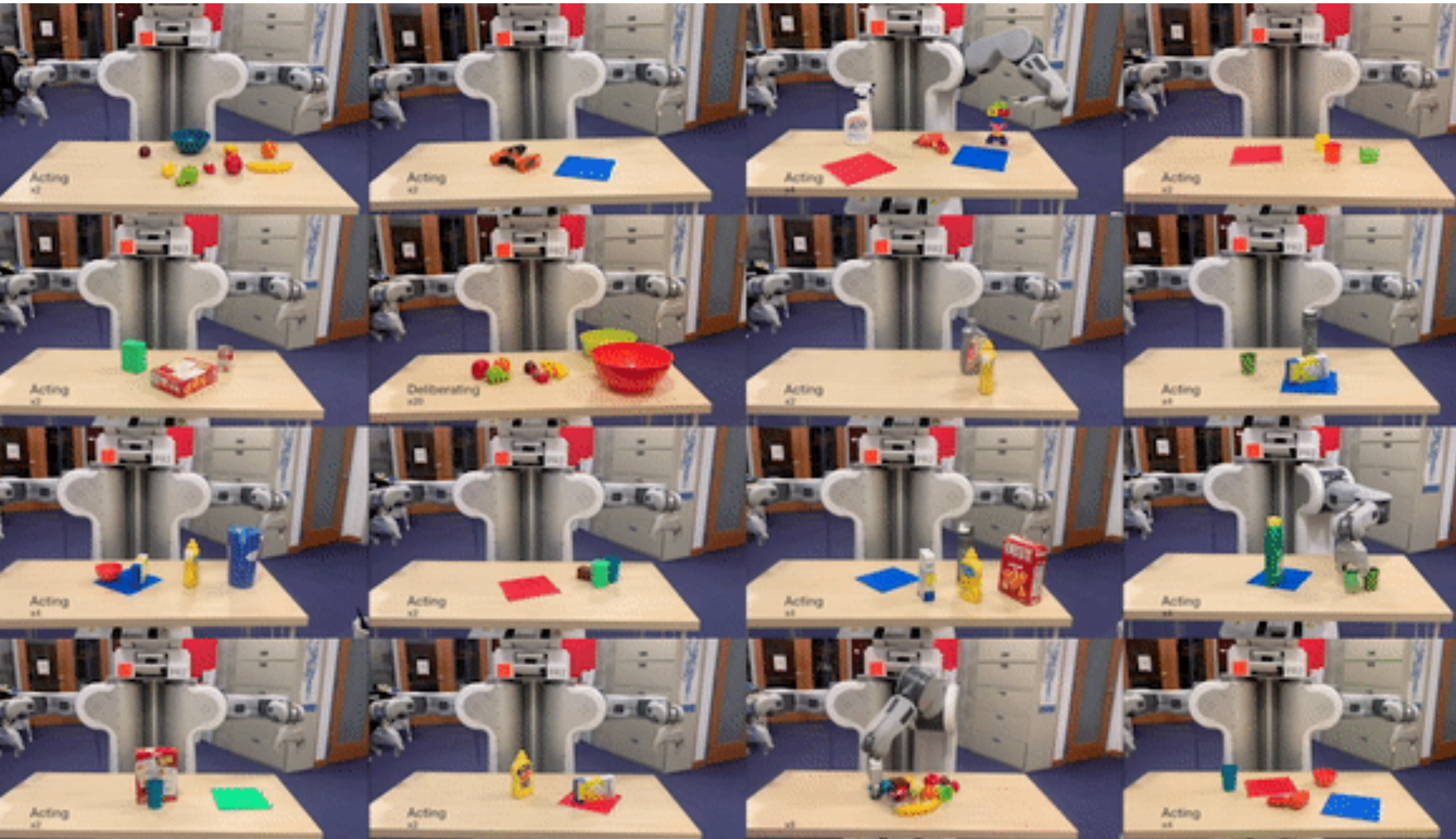
Unknown Objects via Learned Streams

Plan using Estimated Affordances

- **Learned** segmentation, grasp prediction, collision checking
- Streams call perceptual modules using object **point clouds**



Single System Generalizes Across Novel Objects, Initial States, & Goals

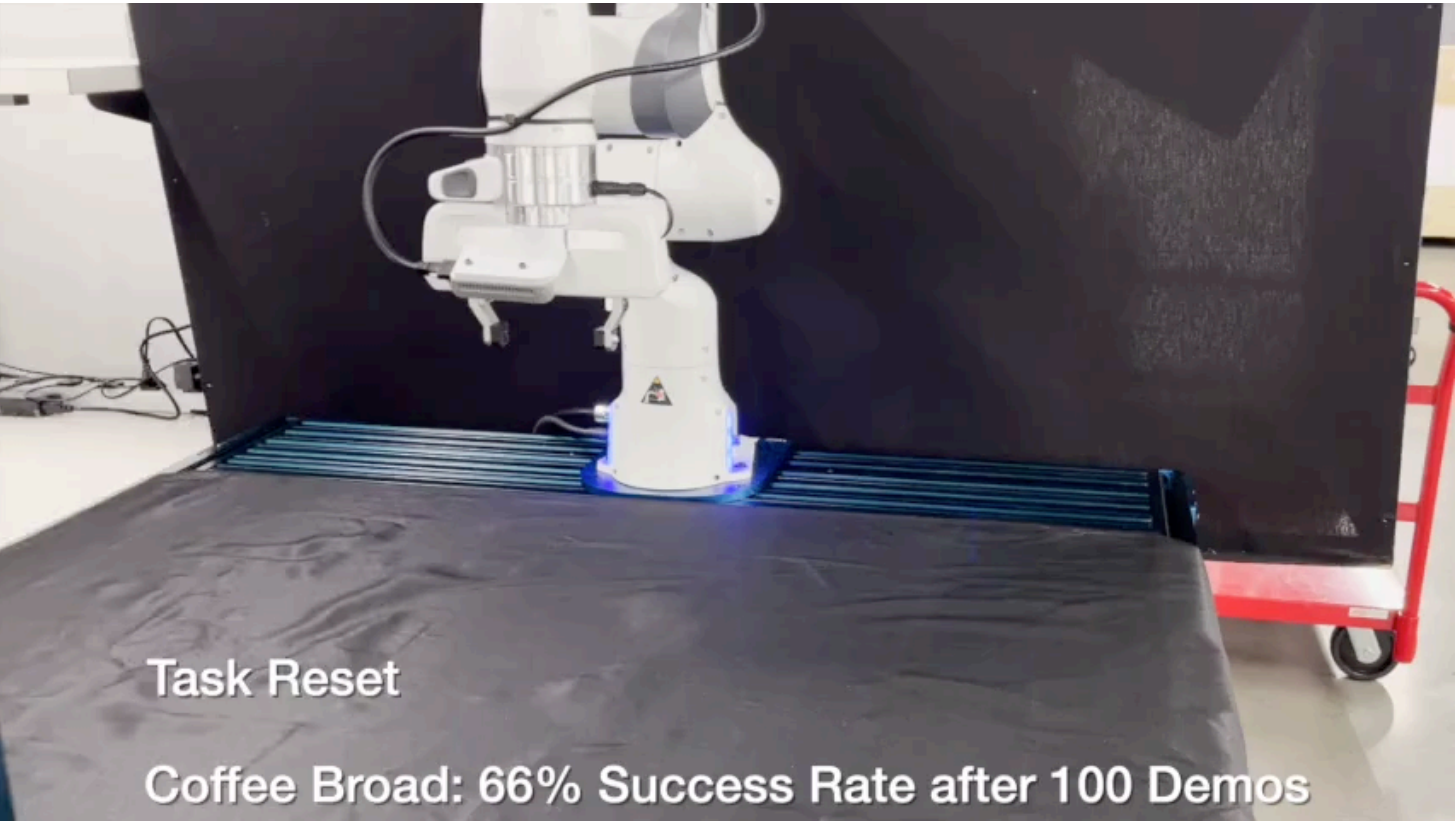


TAMP + Imitation Learning

Human-In-The-Loop Task and Motion Planning for Imitation Learning. Ajay Mandlekar*, Caelan Garrett*, Danfei Xu, Dieter Fox. *Conference on Robot Learning (CoRL)*, 2023.

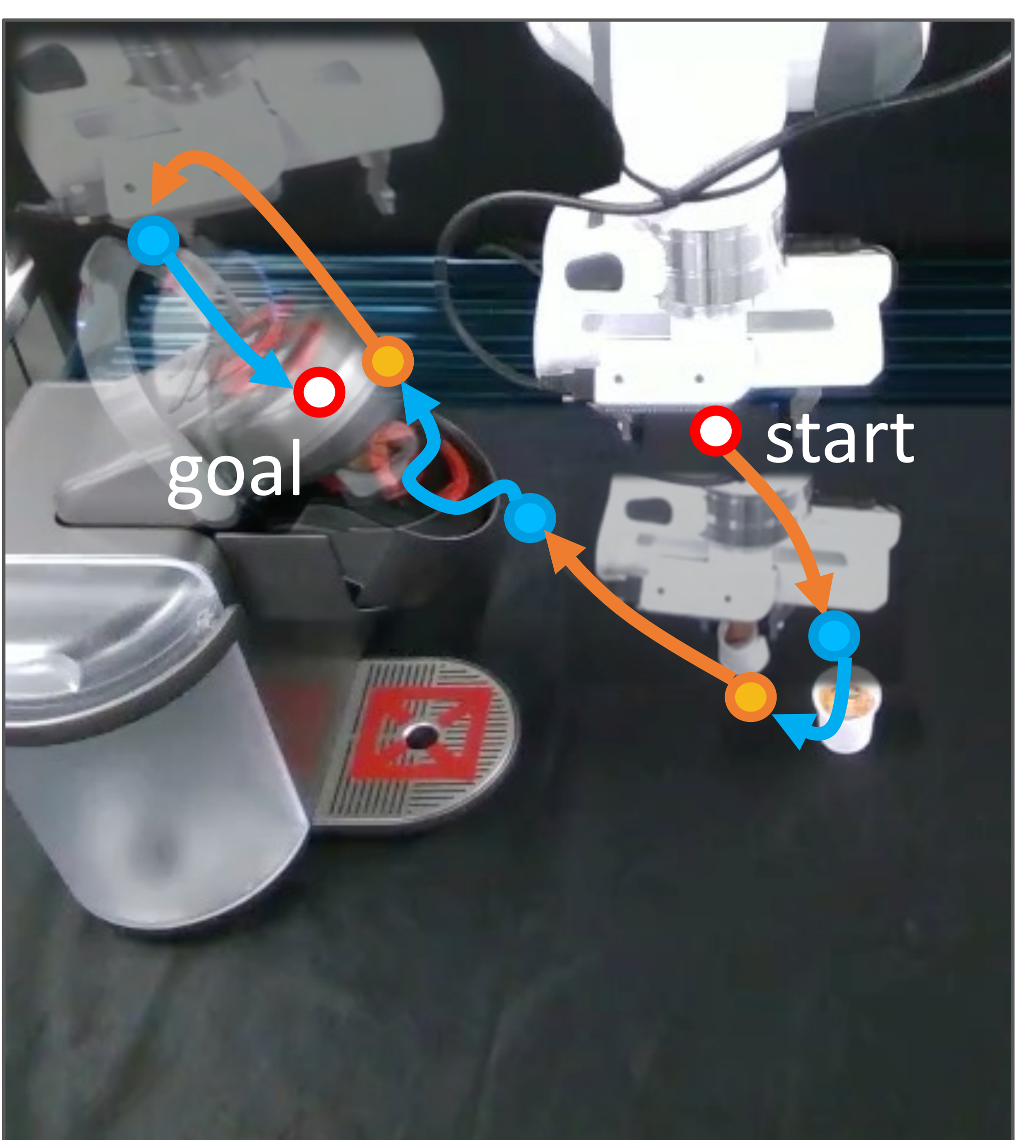
Imitating Task and Motion Planning with Visuomotor Transformers. Murtaza Dalal, Ajay Mandlekar*, Caelan Garrett*, Ankur Handa, Ruslan Salakhutdinov, Dieter Fox. *Conference on Robot Learning (CoRL)*, 2023.

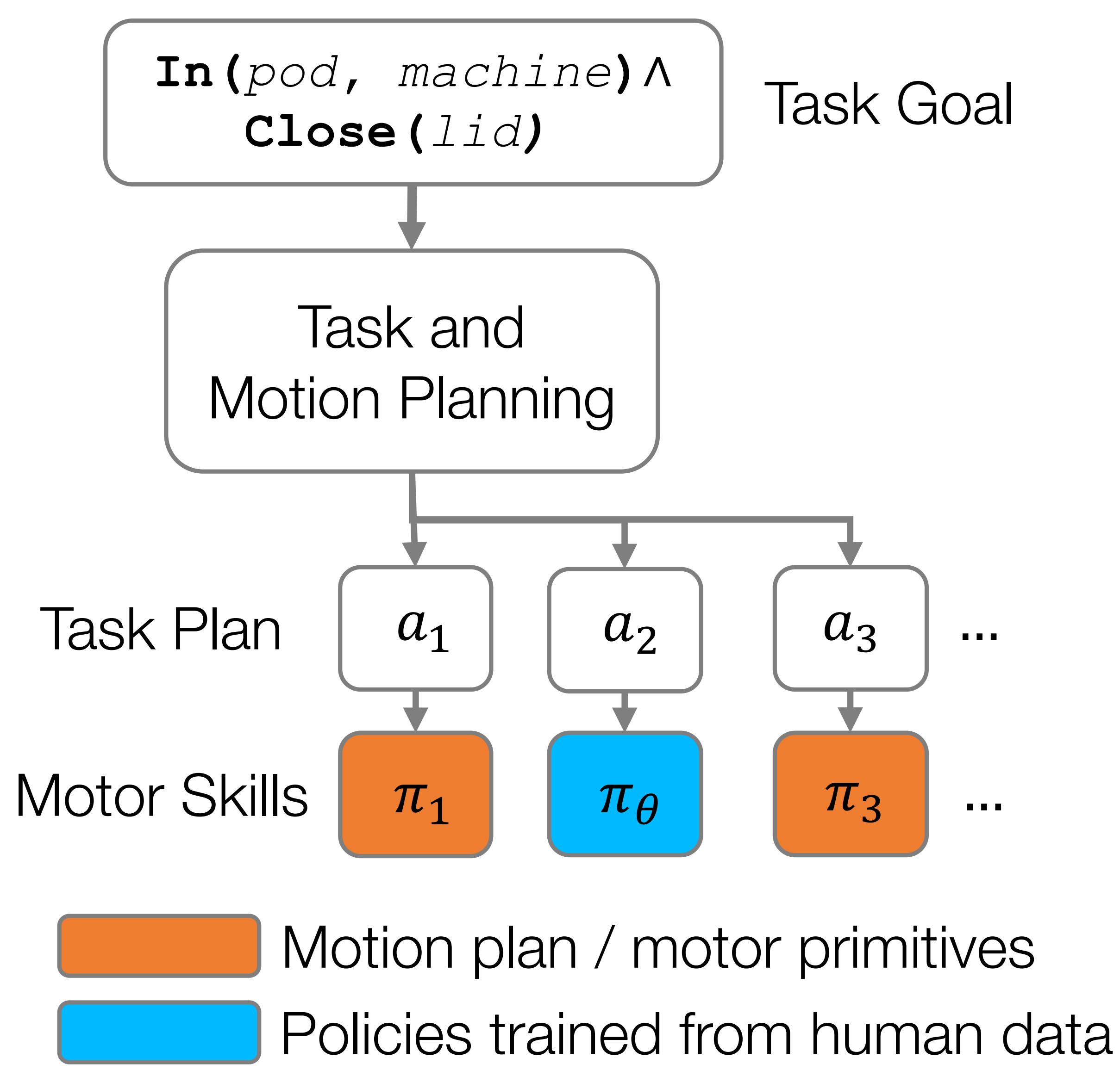
Planning with Contact-Rich Actions



Actions Learned from Human Demos

- Assume human **teleoperated** skill demonstrations
- Train image => control policy using **behavior cloning** (5/22)
- **Stochastic** actions within TAMP

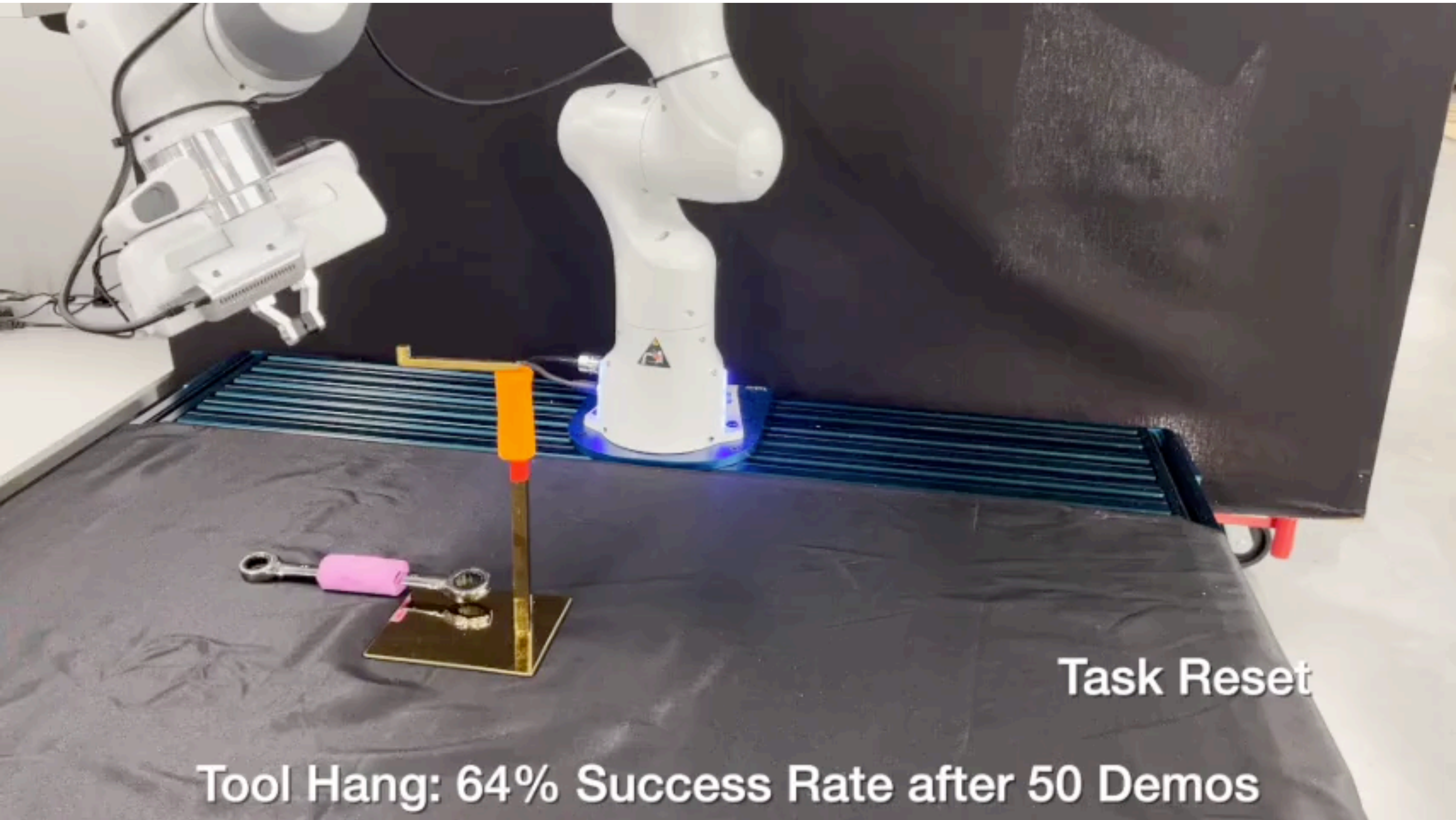




Alternating TAMP & Learned Control

- TAMP plans when to **deploy** which learned policies
- Can also use planning for **data generation** (5/22)

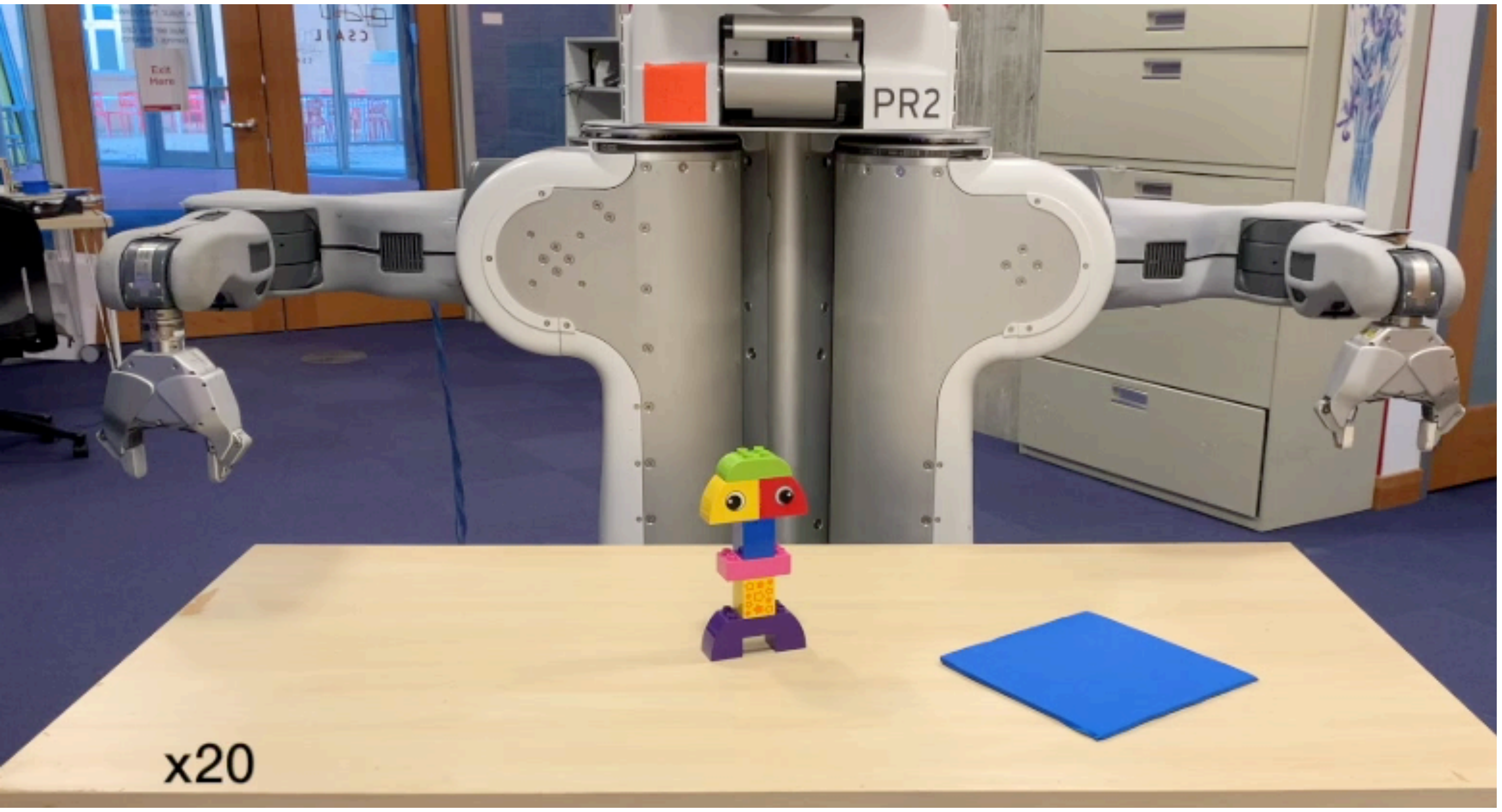
TAMP as a Learning Inductive Bias

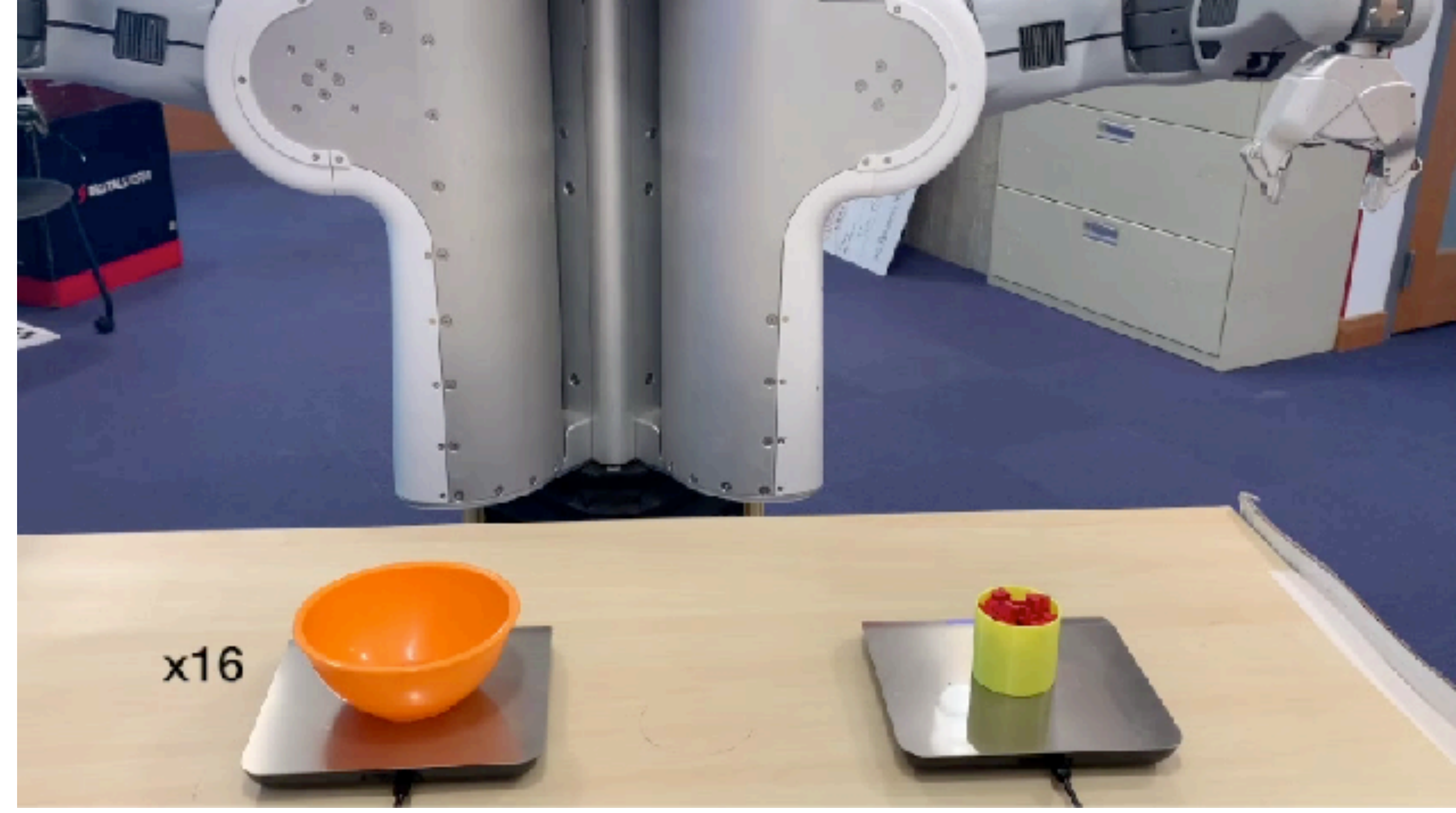


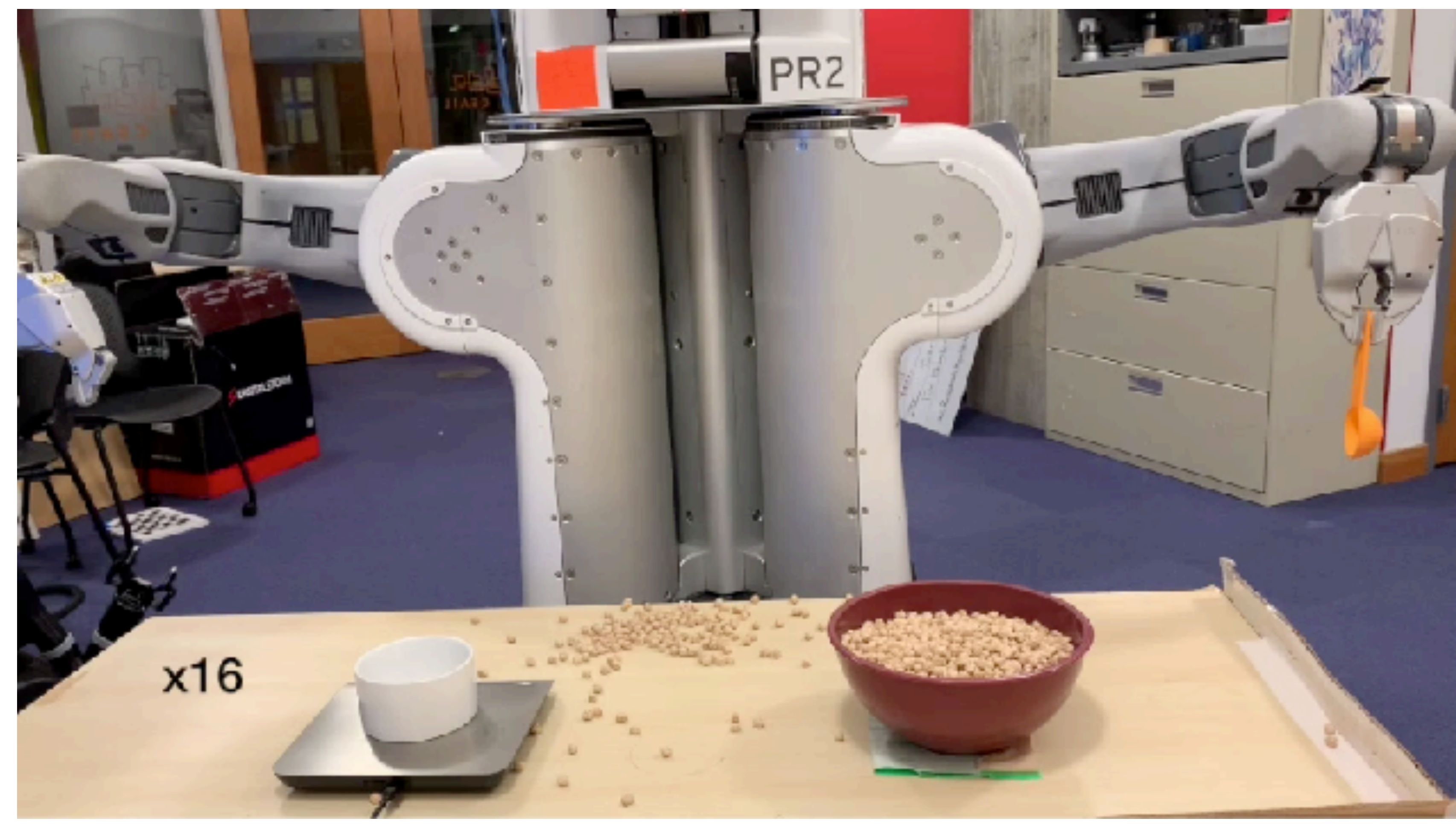
Takeaways

- **Task and Motion Planning (TAMP):** hybrid planning where continuous constraints affect discrete decisions
- **Sampling** & **optimization** for continuous satisfaction
- **PDDLStream:** planning language that supports **sampling procedures** as blackbox streams
  - **Domain-independent** algorithms
  - Efficient **lazy/optimistic** planning (focused algorithm)
- **Ongoing work:** GPU-accelerated, probabilistic & partially observable, learning-assisted TAMP

Questions?







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