CSE-571
Robotics

Planning and Control:

Markov Decision Processes

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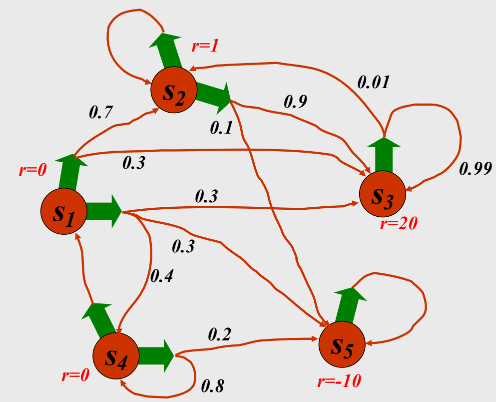
Markov Decision Process (MDP)



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Markov Decision Process (MDP)

- **Given:**
- States x
- Actions u
- Transition probabilities $p(x'|u,x)$
- Reward / payoff function $r(x,u)$

- **Wanted:**
- Policy $\pi(x)$ that maximizes the future expected reward

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Rewards and Policies

- Policy (general case):

$$\pi: \quad z_{1:t-1}, u_{1:t-1} \rightarrow \; u_t$$

- Policy (fully observable case):

$$\pi: \quad x_t \rightarrow \; u_t$$

- Expected cumulative payoff:

$$R_T = E\left[\sum_{\tau=1}^{T} \gamma^{\tau} r_{t+\tau}\right]$$

  - T=1: greedy policy
  - T>1: finite horizon case, typically no discount
  - T=infty: infinite-horizon case, finite reward if discount < 1

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Policies contd.

- Expected cumulative payoff of policy:

$$R_T^\pi(x_t) = E\left[\sum_{\tau=1}^{T} \gamma^\tau r_{t+\tau} \mid u_{t+\tau} = \pi\left(z_{1:t+\tau-1} u_{1:t+\tau-1}\right)\right]$$

- Optimal policy:

$$\pi^* = \underset{\pi}{\operatorname{argmax}} \quad R_T^\pi(x_t)$$

- 1-step optimal policy:

$$\pi_1(x) = \underset{u}{\operatorname{argmax}} \quad r(x,u)$$

- Value function of 1-step optimal policy:

$$V_1(x) = \gamma \max_u r(x,u)$$

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2-step Policies

- Optimal policy:

$$\pi_2(x) = \underset{u}{\operatorname{argmax}} \quad \left[r(x,u) + \int V_1(x') p(x' \mid u,x) dx'\right]$$

- Value function:

$$V_2(x) = \gamma \max_u \quad \left[r(x,u) + \int V_1(x') p(x' \mid u,x) dx'\right]$$

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T-step Policies

- Optimal policy:

$$\pi_T(x) = \underset{u}{\operatorname{argmax}} \quad \left[r(x,u) + \int V_{T-1}(x') p(x' \mid u,x) dx'\right]$$

- Value function:

$$V_T(x) = \gamma \max_u \quad \left[r(x,u) + \int V_{T-1}(x') p(x' \mid u,x) dx'\right]$$

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Infinite Horizon

- Optimal policy:

$$V_\infty(x) = \gamma \max_u \quad \left[r(x,u) + \int V_\infty(x') p(x' \mid u,x) dx'\right]$$

- Bellman equation
- Fix point is optimal policy
- Necessary and sufficient condition

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Value Iteration

- for all x do

  $\hat{V}(x) \leftarrow r_{\min}$
- endfor

- repeat until convergence
  - for all x do

    $$\hat{V}(x) \leftarrow \gamma \max_u \left[r(x,u) + \int \hat{V}(x') p(x' \mid u,x) dx' \right]$$
  - endfor
- endrepeat

$$\pi(x) = \underset{u}{\operatorname{argmax}} \left[r(x,u) + \int \hat{V}(x') p(x' \mid u,x) dx' \right]$$

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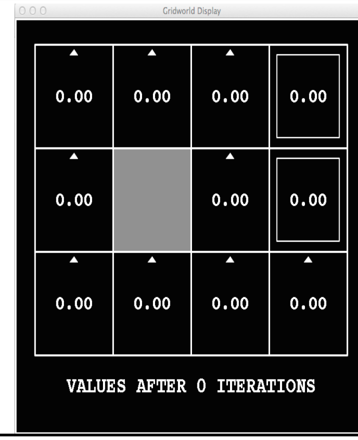
k=0



Noise = 0.2
Discount = 0.9
Living reward = 0

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k=1

Noise = 0.2
Discount = 0.9
Living reward = 0

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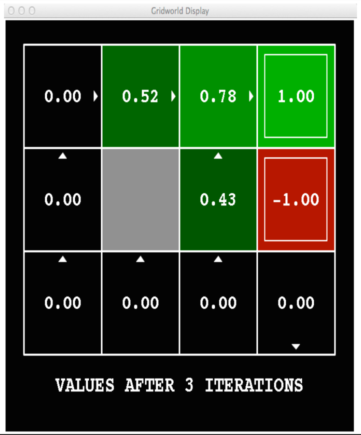
k=2

12

k=3



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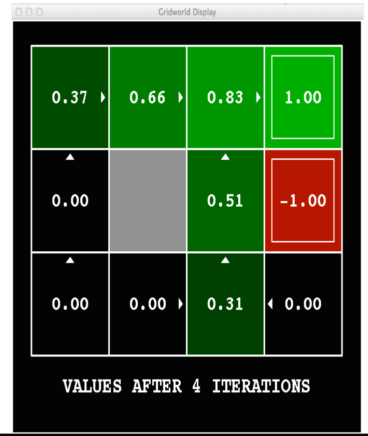
k=4



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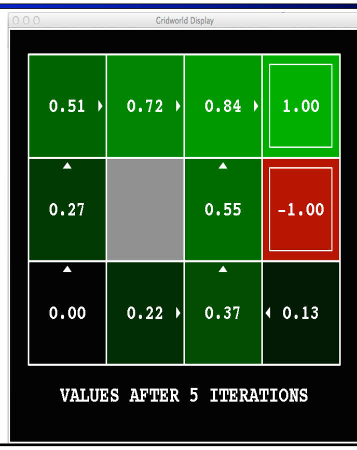
k=5



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k=6

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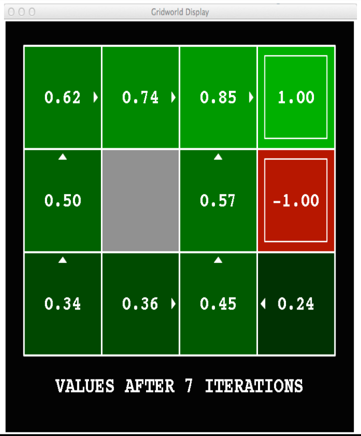
k=7



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k=8

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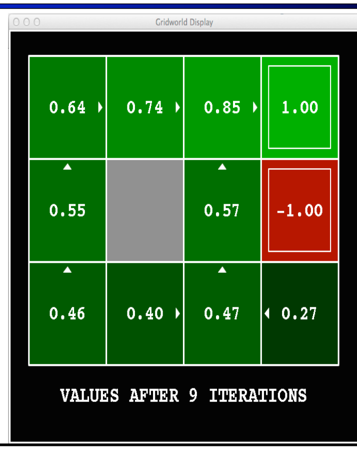
k=9



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k=10

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k=11

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k=12

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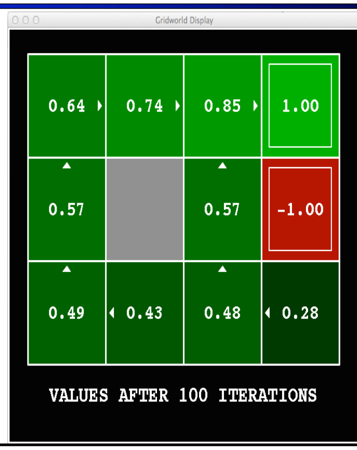
k=100



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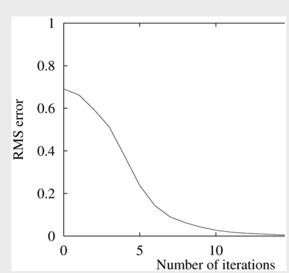
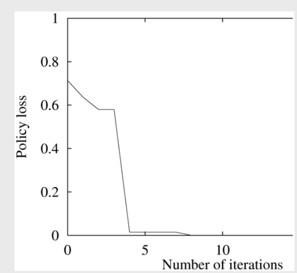
Value Function and Policy

- Each step takes O(|A| |S| |S|) time.
- Number of iterations required is polynomial in |S|, |A|, 1/(1-gamma)





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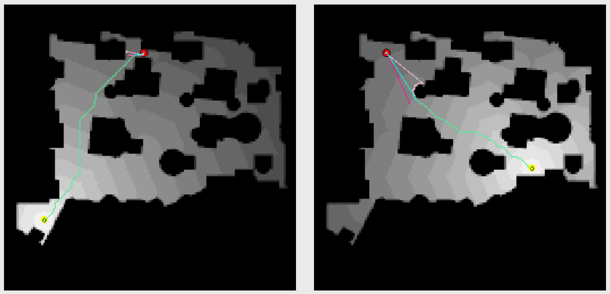
Value Iteration for Motion Planning
(assumes knowledge of robot's location)



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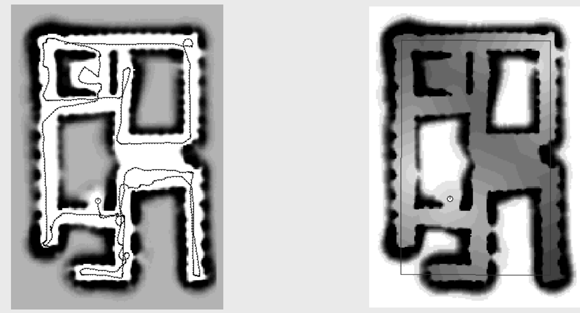
Frontier-based Exploration

- Every unknown location is a target point.



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POMDPs

- In POMDPs we apply the very same idea as in MDPs.
- Since the **state is not observable**, the agent has to **make its decisions based on the belief state** which is a posterior distribution over states.
- For finite horizon problems, the resulting value functions are piecewise linear and convex.
- In each iteration the **number of linear constraints grows exponentially**.
- Full fledged POMDPs have only been applied to very small state spaces with small numbers of possible observations and actions.
- **Approximate solutions are becoming more and more capable**.

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CSE 571
Inverse Optimal Control
(Inverse Reinforcement Learning)

Many slides by Drew Bagnell
Carnegie Mellon University

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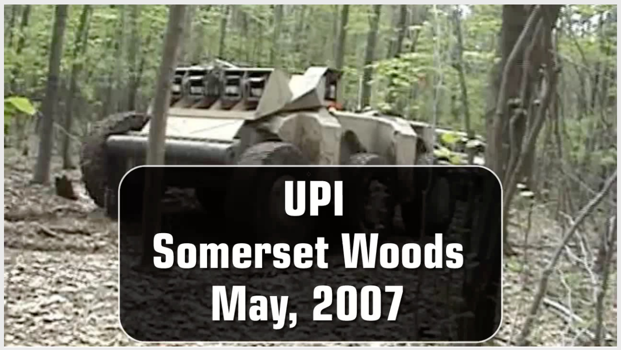
Autonomous Navigation



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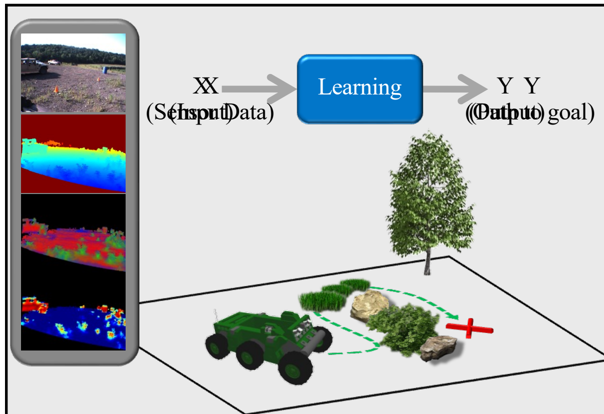

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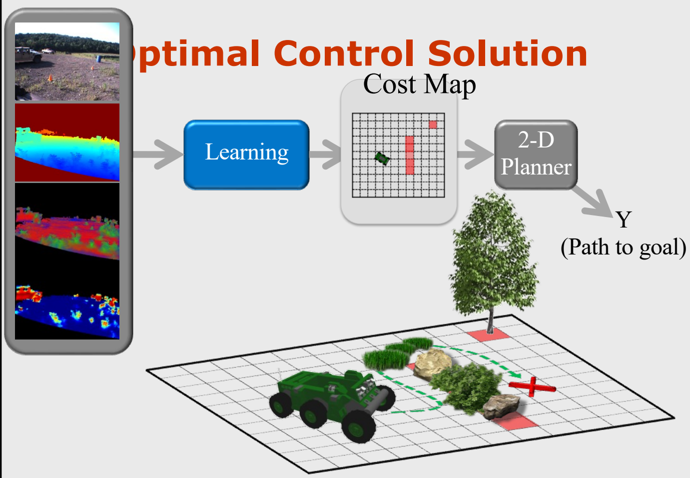
ptimal Control Solution



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Mode 1: Training example

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Mode 1: Training example

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Mode 1: Learned behavior

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Mode 1: Learned behavior

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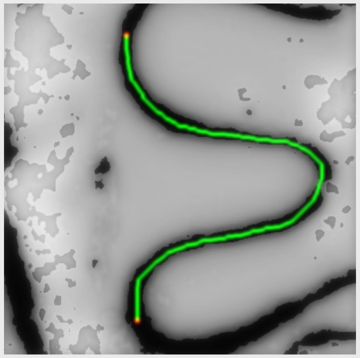
Mode 1: Learned cost map



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Mode 2: Training example

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Mode 2: Training example

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Mode 2: Learned behavior

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Mode 2: Learned behavior

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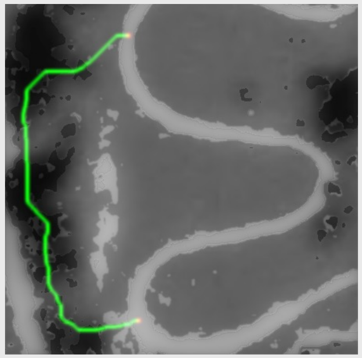
Mode 2: Learned cost map



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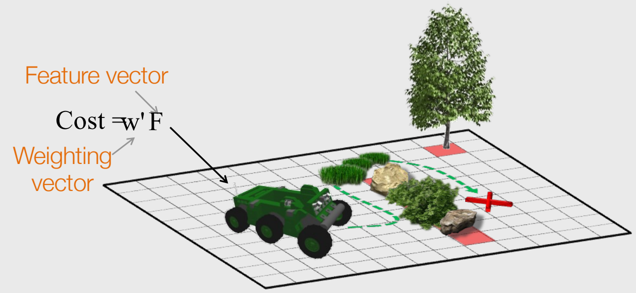

Ratliff, Bagnell, Zinkevich 2005
Ratliff, Bradley, Bagnell, Chestnutt, NIPS 2006
Silver, Bagnell, Stentz, RSS 2008

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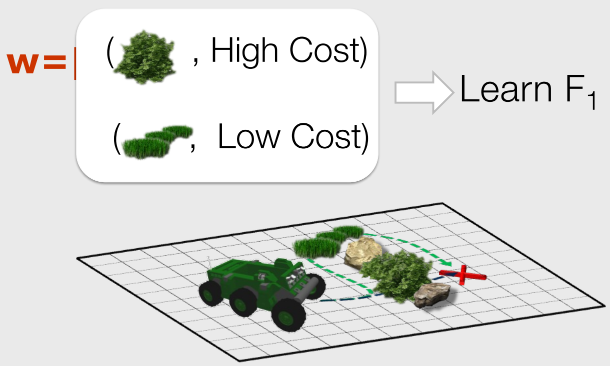

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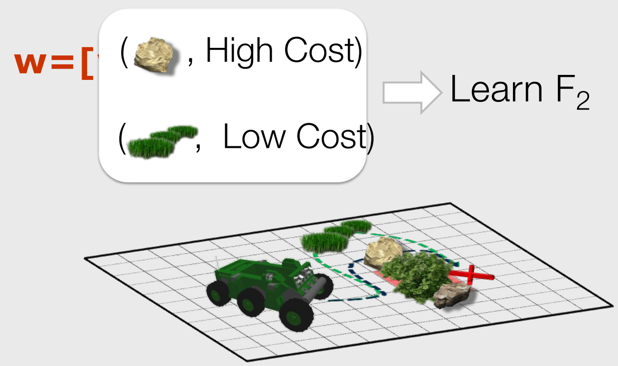

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Readings

- Max-Ent IRL (Ziebart, Bagnell): http://www.cs.cmu.edu/~bziebart/
- CIOC (Levine) http://graphics.stanford.edu/projects/cioc/cioc.pdf
- Manipulation (Byravan/Fox): https://rse-lab.cs.washington.edu/papers/graph-based-IOC-ijcai-2015.pdf
- Imitation learning (Ermon): https://cs.stanford.edu/~ermon/
- Human/manipulation (Dragan): https://people.eecs.berkeley.edu/~anca/research.html

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