CSE 571Robotics

Recap and Discussion

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Goal of this course

- Provide an overview of fundamental problems / techniques in robotics
- Understanding of estimation and decision making in dynamical systems
  - Probabilistic modeling and filtering
  - Deterministic and non-deterministic planning
  - Learning for perception and modeling
- Augment model-based understanding with hands-on experience in deep learning

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Bayesian Filtering, Models

ESTIMATION

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Bayes Filters

z = observation
u = action
x = state

$$\boxed{Bel(x_t)} = P(x_t \mid u_1, z_1 \ldots, u_t, z_t)$$

Bayes $$= \eta \; P(z_t \mid x_t, u_1, z_1, \ldots, u_t) \; P(x_t \mid u_1, z_1, \ldots, u_t)$$

Markov $$= \eta \; P(z_t \mid x_t) \; P(x_t \mid u_1, z_1, \ldots, u_t)$$

Total prob. $$= \eta \; P(z_t \mid x_t) \int P(x_t \mid u_1, z_1, \ldots, u_t, x_{t-1}) \; P(x_{t-1} \mid u_1, z_1, \ldots, u_t) \; dx_{t-1}$$

Markov $$= \eta \; P(z_t \mid x_t) \int P(x_t \mid u_t, x_{t-1}) \; P(x_{t-1} \mid u_1, z_1, \ldots, u_t) \; dx_{t-1}$$

$$\boxed{= \eta \; P(z_t \mid x_t) \int P(x_t \mid u_t, x_{t-1}) \; Bel(x_{t-1}) \; dx_{t-1}}$$

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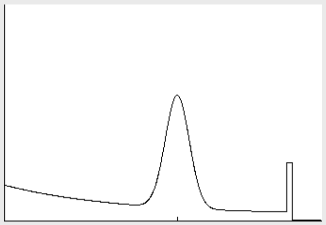
Parametric Sensor Model



$$P(z \mid x,m) = \begin{pmatrix} \alpha_{\text{hit}} \\ \alpha_{\text{unexp}} \\ \alpha_{\max} \\ \alpha_{\text{rand}} \end{pmatrix}^T \cdot \begin{pmatrix} P_{\text{hit}}(z \mid x,m) \\ P_{\text{unexp}}(z \mid x,m) \\ P_{\max}(z \mid x,m) \\ P_{\text{rand}}(z \mid x,m) \end{pmatrix}$$

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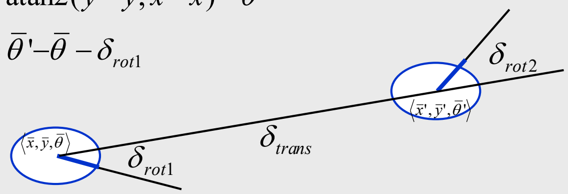
Parametric Kinematics Model

- **Robot moves from** $\langle \bar{x}, \bar{y}, \bar{\theta} \rangle$ **to** $\langle \bar{x}', \bar{y}', \bar{\theta}' \rangle$.
- **Odometry information** $u = \langle \delta_{rot1}, \delta_{rot2}, \delta_{trans} \rangle$.

$$\delta_{trans} = \sqrt{(\bar{x}' - \bar{x})^2 + (\bar{y}' - \bar{y})^2}$$

$$\delta_{rot1} = \operatorname{atan2}(\bar{y}' - \bar{y}, \bar{x}' - \bar{x}) - \bar{\theta}$$

$$\delta_{rot2} = \bar{\theta}' - \bar{\theta} - \delta_{rot1}$$



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The Prediction-Correction-Cycle of Kalman Filters

Prediction

$$bel(x_t) = \begin{cases} \mu_t = \bar{\mu}_t + K_t(z_t - \bar{\mu}_t) \\ \sigma_t^2 = (1 - K_t)\bar{\sigma}_t^2 \end{cases}, K_t = \frac{\bar{\sigma}_t^2}{\bar{\sigma}_t^2 + \bar{\sigma}_{obs,t}^2}$$

$$bel(x_t) = \begin{cases} \mu_t = \bar{\mu}_t + K_t(z_t - C_t\bar{\mu}_t) \\ \Sigma_t = (I - K_t C_t)\bar{\Sigma}_t \end{cases}, K_t = \bar{\Sigma}_t C_t^T (C_t \bar{\Sigma}_t C_t^T + Q_t)^{-1}$$

$$\overline{bel}(x_t) = \begin{cases} \bar{\mu}_t = a_t \mu_{t-1} + b_t u_t \\ \bar{\sigma}_t^2 = a_t^2 \sigma_t^2 + \sigma_{act,t}^2 \end{cases}$$

$$\overline{bel}(x_t) = \begin{cases} \bar{\mu}_t = A_t \mu_{t-1} + B_t u_t \\ \bar{\Sigma}_t = A_t \Sigma_{t-1} A_t^T + R_t \end{cases}$$

Correction

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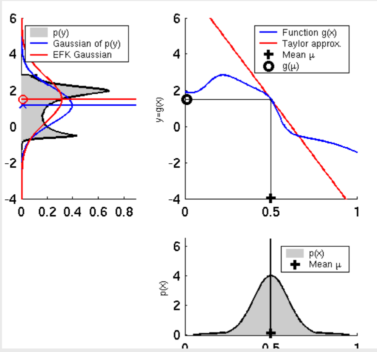
EKF Linearization



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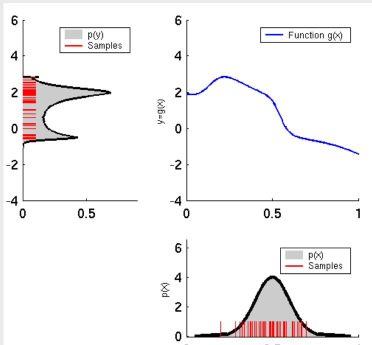
Particle Filter Projection



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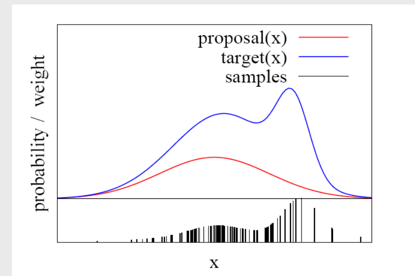
Importance Sampling Principle

- We can use a different distribution g to generate samples from f
- By introducing an importance weight w, we can account for the "differences between g and f"

$$w = f / g$$



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SLAM

ESTIMATION

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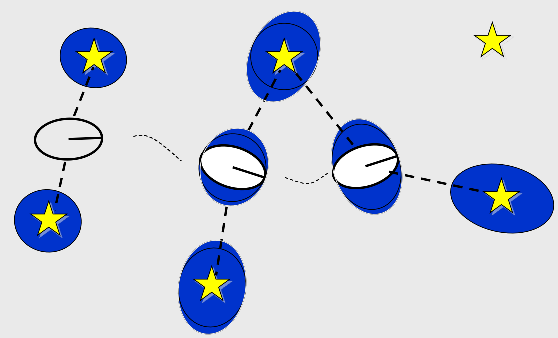
Why is SLAM a hard problem?

•**SLAM: robot path and map are both unknown**



•**Robot path error correlates errors in the map**

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EKF-SLAM

- Map with N landmarks:(3+2N)-dimensional Gaussian

$$
Bel(x_t, m_t) = \left\langle \begin{pmatrix} x \\ y \\ \theta \\ l_1 \\ l_2 \\ \vdots \\ l_N \end{pmatrix}, \begin{pmatrix} \sigma_x^2 & \sigma_{xy} & \sigma_{x\theta} & \sigma_{xl_1} & \sigma_{xl_2} & \cdots & \sigma_{xl_N} \\ \sigma_{xy} & \sigma_y^2 & \sigma_{y\theta} & \sigma_{yl_1} & \sigma_{yl_2} & \cdots & \sigma_{yl_N} \\ \sigma_{x\theta} & \sigma_{y\theta} & \sigma_\theta^2 & \sigma_{\theta l_1} & \sigma_{\theta l_2} & \cdots & \sigma_{\theta l_N} \\ \sigma_{xl_1} & \sigma_{yl_1} & \sigma_{\theta l_1} & \sigma_{l_1}^2 & \sigma_{l_1 l_2} & \cdots & \sigma_{l_1 l_N} \\ \sigma_{xl_2} & \sigma_{yl_2} & \sigma_{\theta l_2} & \sigma_{l_1 l_2} & \sigma_{l_2}^2 & \cdots & \sigma_{l_2 l_N} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \sigma_{xl_N} & \sigma_{yl_N} & \sigma_{\theta l_N} & \sigma_{l_1 l_N} & \sigma_{l_2 l_N} & \cdots & \sigma_{l_N}^2 \end{pmatrix} \right\rangle
$$

- Can handle hundreds of dimensions

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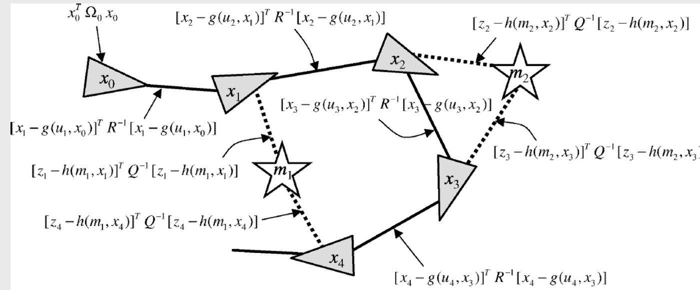
Graph-SLAM Idea



<u>Sum of all constraints:</u>

$$
J_{\text{GraphSLAM}} = x_0^T \Omega_0 x_0 + \sum_t [x_t - g(u_t, x_{t-1})]^T R^{-1} [x_t - g(u_t, x_{t-1})] + \sum_t [z_t - h(m_{c_t}, x_t)]^T Q^{-1} [z_t - h(m_{c_t}, x_t)]
$$

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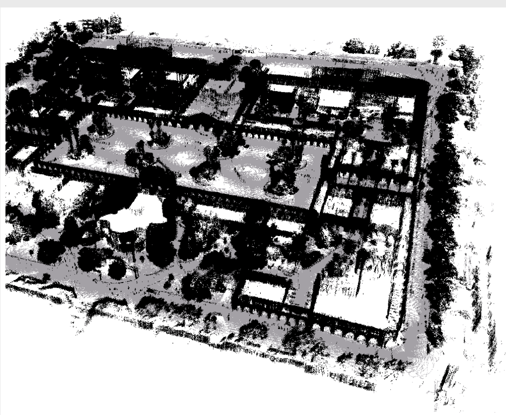
3D Outdoor Mapping



10^8 features, 10^5 poses, only few secs using cg.

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PLANNING / CONTROL

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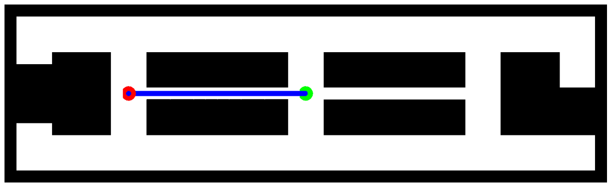
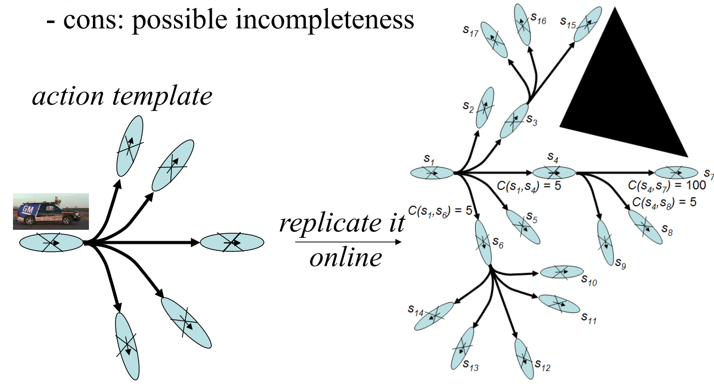
Deterministic, fully observable



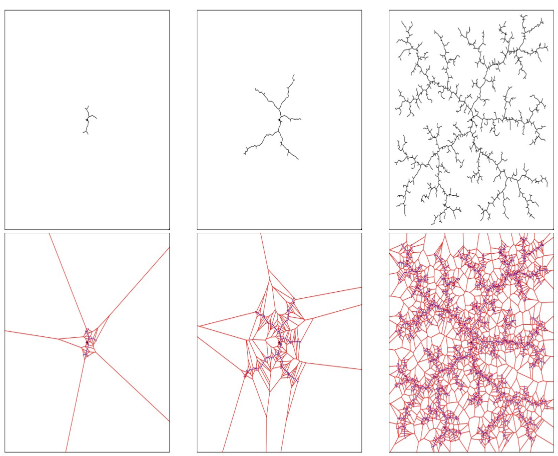
Planning via Cell Decomposition

- Graph construction:
  - lattice graph
  - pros: sparse graph, feasible paths
  - cons: possible incompleteness

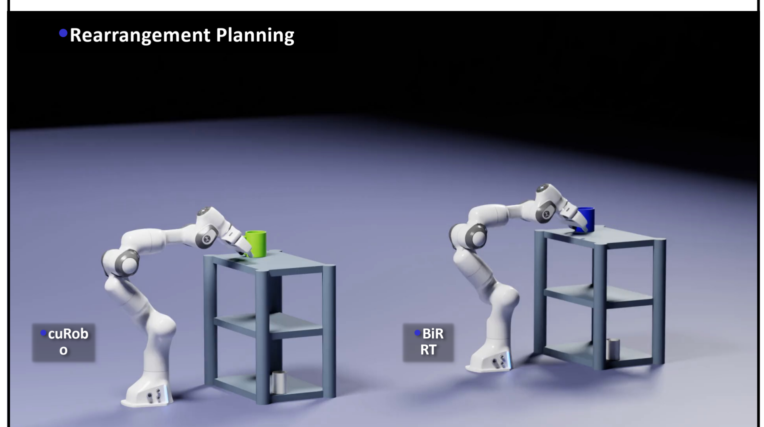


CSE-571: Courtesy of Maxim Likhachev, CMU

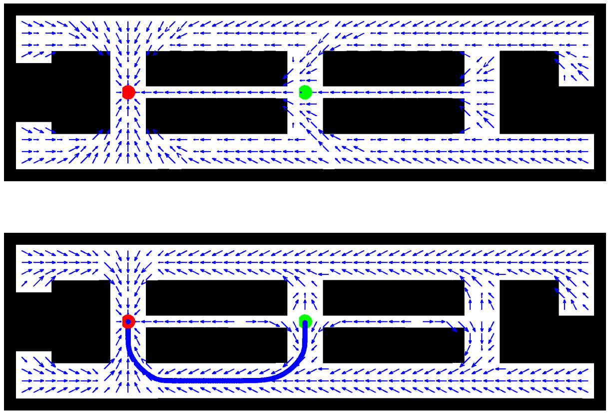
Rapidly exploring Random Tree (RRT)



Source: LaValle and Kuffner 01



Stochastic, Fully Observable



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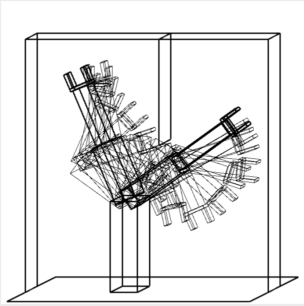
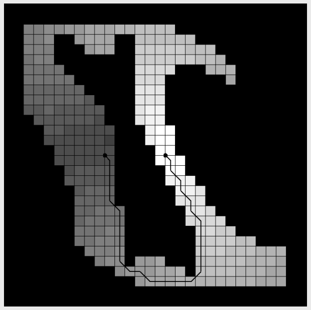
Manipulator Control Path



Work space



Configuration space

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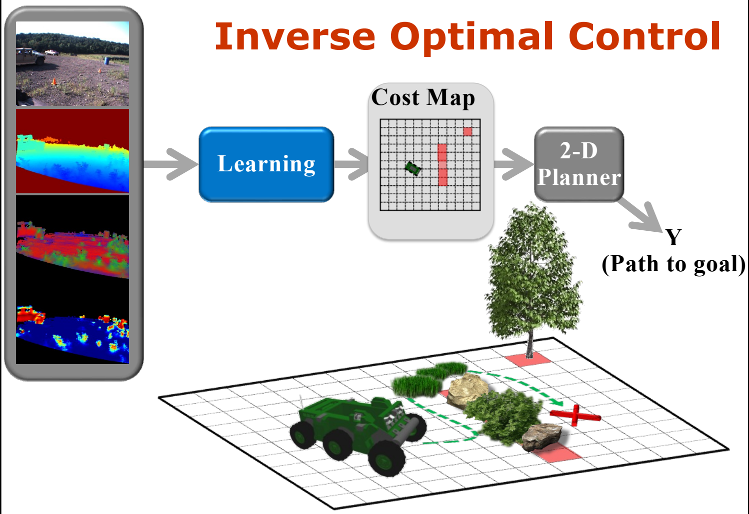
Inverse Optimal Control



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Where is RobotGPT?

Learning at scale enables powerful models for images, videos, and language, but truly capable RobotGPT still has a way to go

Leveraging VLMs for robotics provides exciting initial capabilities but quickly hits ceiling

Robotics must play active role in developing open-world vision-language-action models by

- driving the development of models that have robust understanding of 3D geometric reasoning, physical interaction, planning, and the combination of high-level reasoning and low-level control
- developing benchmarks and test protocols geared toward real world robotics relevance

Simulation has the potential to

- generate the kind of data necessary to train foundation manipulation capabilities (diversity and scaling via Gen-AI-based asset, scene, and task generation; privileged information enables generation of demonstrations via TAMP, control, RL)
- benchmark models and techniques, enabling community to make measurable progress
- broadening community participation due to manageable cost
- **Open issues**: assets, sim2real, real2sim, non-rigid objects and complex interactions

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