CSE-571
Sampling-Based Motion Planning

Slides based on those from Pieter Abbeel, Zoe McCarthy
Many images from Lavalle, Planning Algorithms

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Motion/Path Planning

- Task:
  find a feasible (and cost-minimal) path/motion from the current configuration of the robot to its goal configuration (or one of its goal configurations)

- Two types of constraints:
  environmental constraints (e.g., obstacles)
  dynamics/kinematics constraints of the robot

- Generated motion/path should (objective):
  be any feasible path
  minimize cost such as distance, time, energy, risk, …

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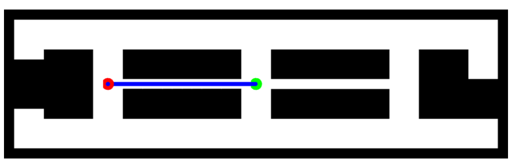
Deterministic, fully observable



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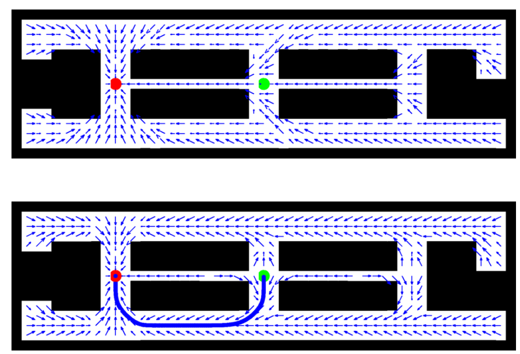
Stochastic, Fully Observable



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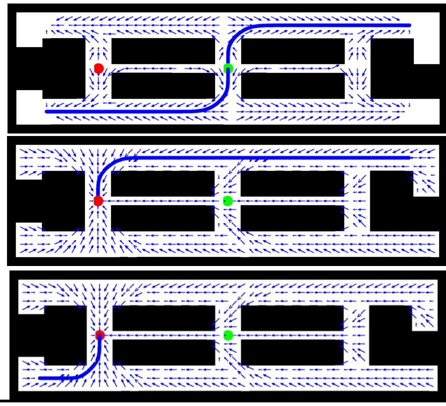
Stochastic, Partially Observable



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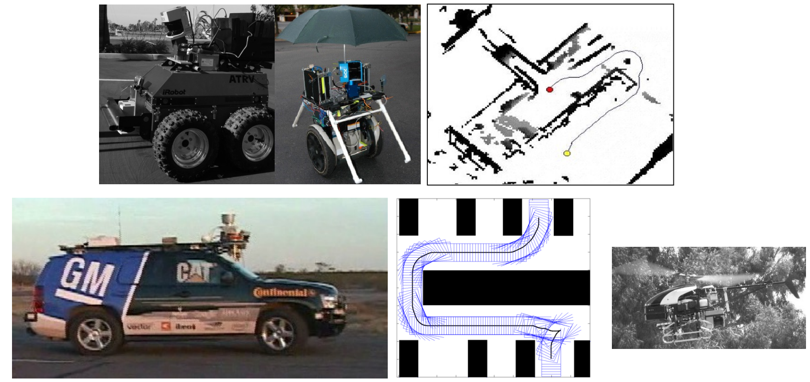
Motion/Path Planning

Examples (of what is usually referred to as path planning):



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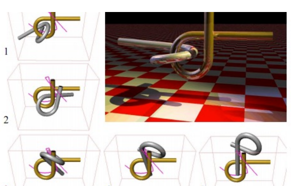
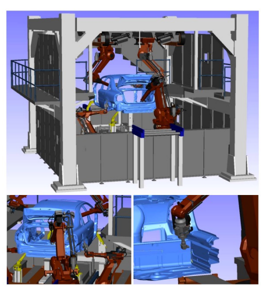
Examples





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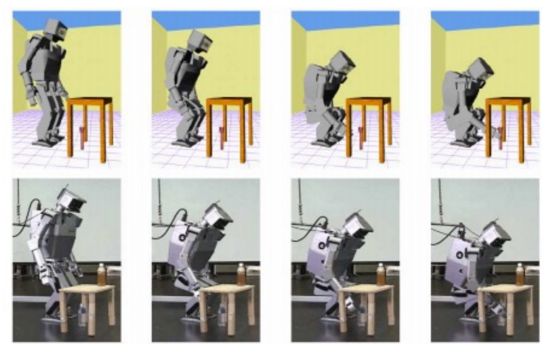
Examples



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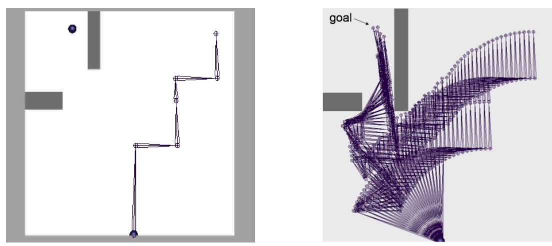
Motion/Path Planning

Examples (of what is usually referred to as motion planning):



Planned motion for a 6DOF robot arm

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Motion Planning: Outline

- Configuration Space
- Probabilistic Roadmap
- Rapidly-exploring Random Trees (RRTs)
- Extensions
- Smoothing

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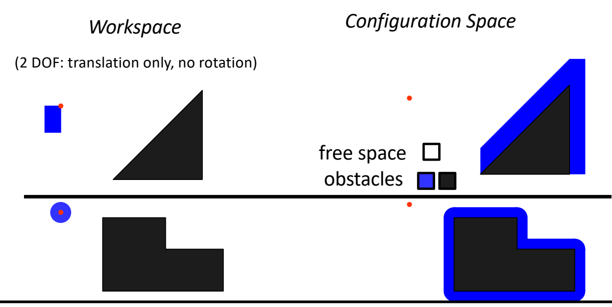
Configuration Space (C-Space)

= { x | x is a pose of the robot}

- obstacles → configuration space obstacles



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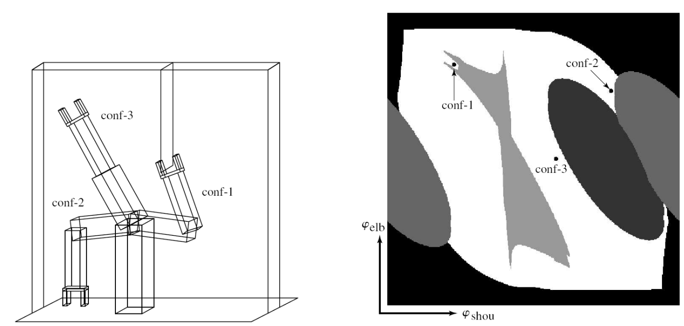
Motion planning



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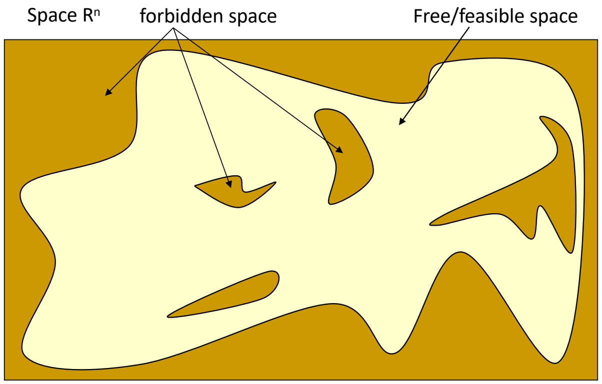
Probabilistic Roadmap (PRM)



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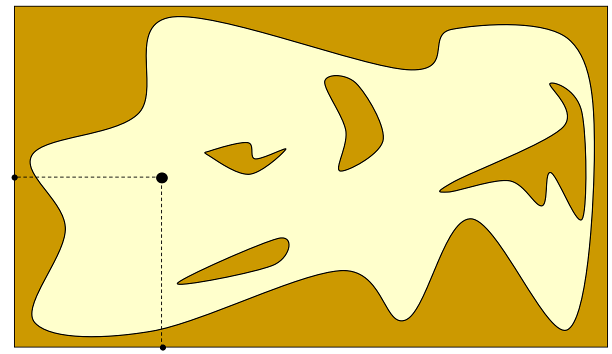
Probabilistic Roadmap (PRM)

Configurations are sampled by picking coordinates at random



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Probabilistic Roadmap (PRM)

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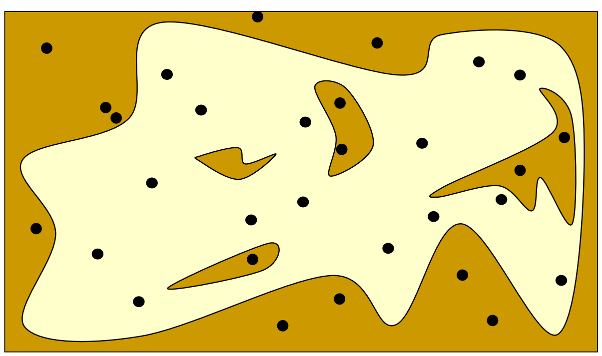

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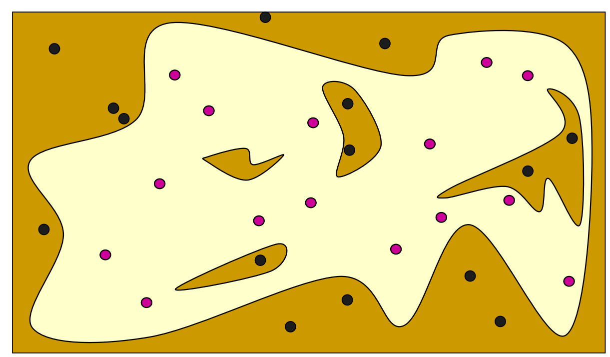
Probabilistic Roadmap (PRM)

Sampled configurations are tested for collision



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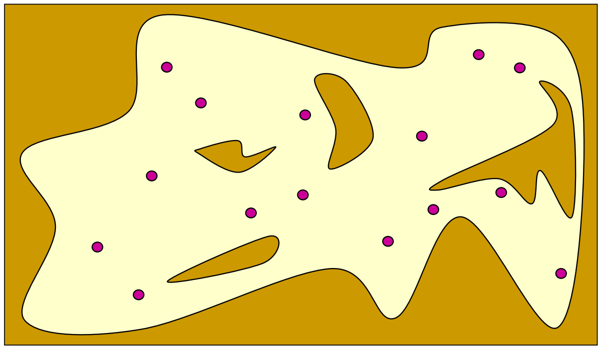
Probabilistic Roadmap (PRM)

The collision-free configurations are retained as milestones



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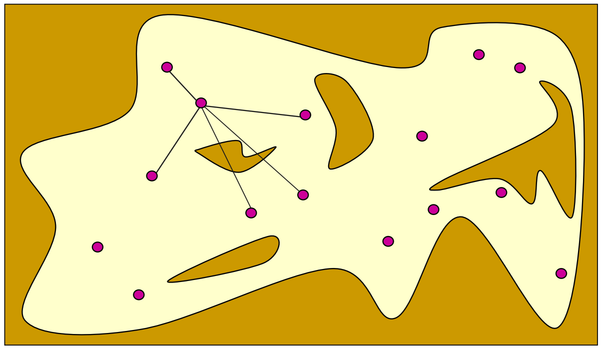
Probabilistic Roadmap (PRM)

Each milestone is linked by straight paths to its nearest neighbors



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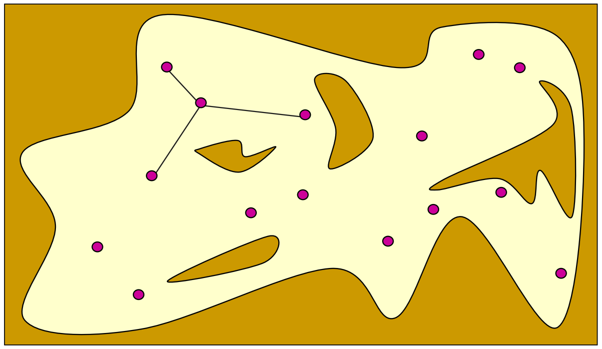
Probabilistic Roadmap (PRM)

Each milestone is linked by straight paths to its nearest neighbors



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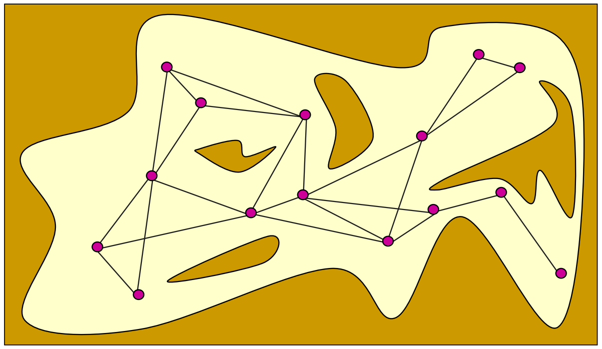
Probabilistic Roadmap (PRM)

The collision-free links are retained as local paths to form the PRM



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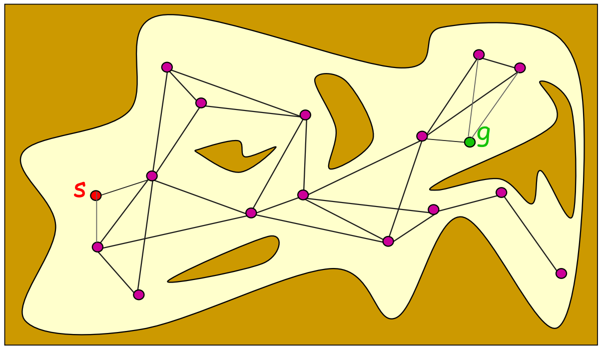
Probabilistic Roadmap (PRM)

The start and goal configurations are included as milestones



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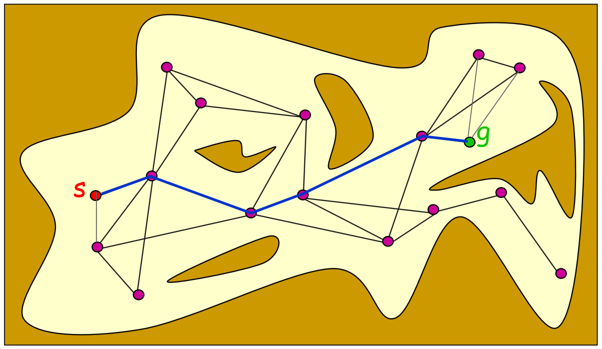
Probabilistic Roadmap (PRM)

The PRM is searched for a path from s to g



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Probabilistic Roadmap

- Initialize set of points with x_S and x_G
- Randomly sample points in configuration space
- Connect nearby points if they can be reached from each other
- Find path from x_S to x_G in the graph
  - Alternatively: keep track of connected components incrementally, and declare success when x_S and x_G are in same connected component

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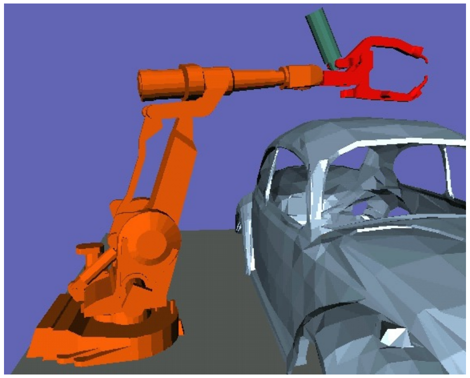
PRM Example 1



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PRM Example 2

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PRM's Pros and Cons

- Pro:
  - Probabilistically complete: i.e., with probability one, if run for long enough the graph will contain a solution path if one exists.
- Cons:
  - Required to solve 2-point boundary value problem
  - Build graph over state space but no focus on generating a path

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Rapidly exploring Random Tree (RRT)

Steve LaValle (98)

- Basic idea:
  - Build up a tree through generating "next states" in the tree by executing random controls
  - However: not exactly above to ensure good coverage

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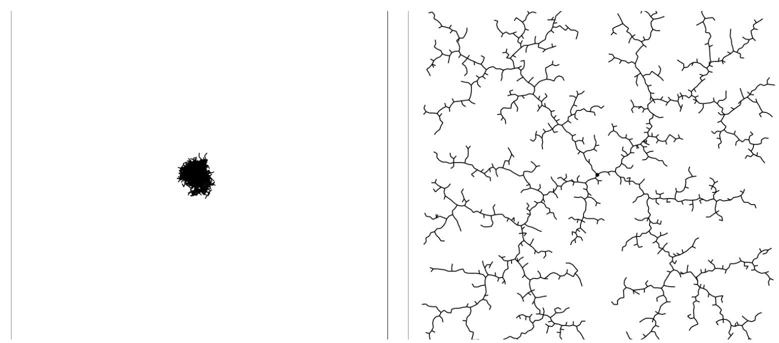
How to Sample



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Rapidly exploring Random Tree (RRT)

- Select random point, and expand nearest vertex towards it
  - Biases samples towards largest Voronoi region

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Rapidly exploring Random Tree (RRT)

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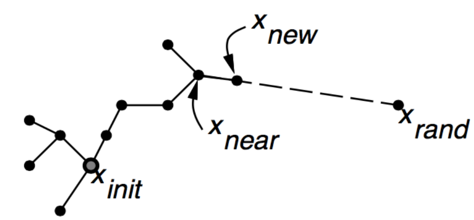

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Rapidly exploring Random Tree (RRT)

GENERATE_RRT($x_{init}, K, \Delta t$)

$\mathcal{T}$.init(x_{init});
for $k = 1$ **to** K **do**
  $x_{rand} \leftarrow$ RANDOM_STATE();
  $x_{near} \leftarrow$ NEAREST_NEIGHBOR($x_{rand}, \mathcal{T}$);
  $u \leftarrow$ SELECT_INPUT(x_{rand}, x_{near});
  $x_{new} \leftarrow$ NEW_STATE($x_{near}, u, \Delta t$);
  $\mathcal{T}$.add_vertex(x_{new});
  $\mathcal{T}$.add_edge(x_{near}, x_{new}, u);
Return $\mathcal{T}$

RANDOM_STATE(): often uniformly at random over space with probability 99%, and the goal state with probability 1%, this ensures it attempts to connect to goal semi-regularly

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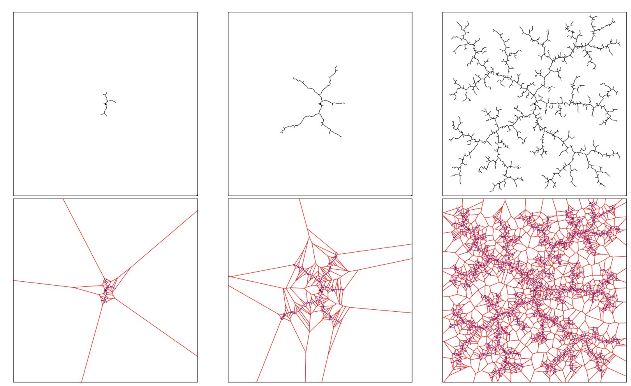
Rapidly exploring Random Tree (RRT)



Source: LaValle and Kuffner 01

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RRT Practicalities

- NEAREST_NEIGHBOR(x_{rand}, T): need to find (approximate) nearest neighbor efficiently
  - KD Trees data structure (upto 20-D) [e.g., FLANN]
  - Locality Sensitive Hashing
- SELECT_INPUT(x_{rand}, x_{near})
  - Two point boundary value problem
    - If too hard to solve, often just select best out of a set of control sequences. This set could be random, or some well chosen set of primitives.

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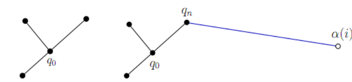
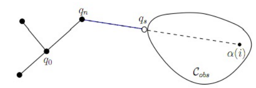
RRT Extension

- No obstacles, holonomic:



- With obstacles, holonomic:



- Non-holonomic: approximately (sometimes as approximate as picking best of a few random control sequences) solve two-point boundary value problem

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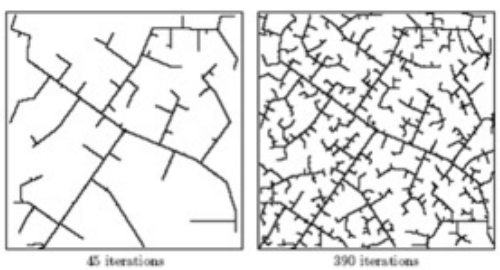
Growing RRT



Demo: http://en.wikipedia.org/wiki/File:Rapidly-exploring_Random_Tree_(RRT)_500x373.gif

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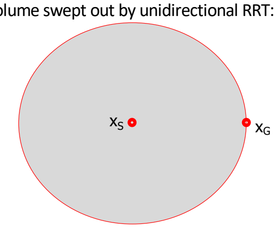
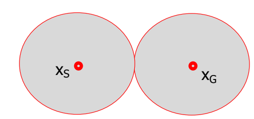
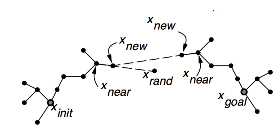
Bi-directional RRT

- Volume swept out by unidirectional RRT:



- Volume swept out by bi-directional RRT:





- Difference more and more pronounced as dimensionality increases

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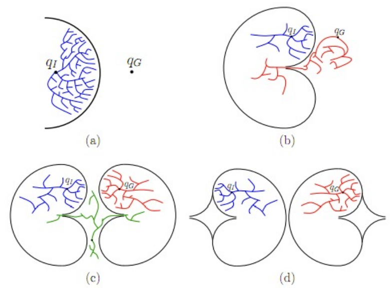
Multi-directional RRT

- Planning around obstacles or through narrow passages can often be easier in one direction than the other



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RRT*

- Asymptotically optimal
- Main idea:
  - Swap new point in as parent for nearby vertices who can be reached along shorter path through new point than through their original (current) parent

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RRT*

Algorithm 6: RRT*

$V \leftarrow \{x_{\text{init}}\};\ E \leftarrow \emptyset;$
for $i = 1, \ldots, n$ **do**
  $x_{\text{rand}} \leftarrow \texttt{SampleFree}_i;$
  $x_{\text{nearest}} \leftarrow \texttt{Nearest}(G = (V, E), x_{\text{rand}});$
  $x_{\text{new}} \leftarrow \texttt{Steer}(x_{\text{nearest}}, x_{\text{rand}})\ ;$
  if $\texttt{ObtacleFree}(x_{\text{nearest}}, x_{\text{new}})$ **then**
    $X_{\text{near}} \leftarrow \texttt{Near}(G = (V, E), x_{\text{new}}, \min\{\gamma_{\text{RRT}^*}(\log(\text{card}\,(V))/\,\text{card}\,(V))^{1/d}, \eta\})\ ;$
    $V \leftarrow V \cup \{x_{\text{new}}\};$
    $x_{\text{min}} \leftarrow x_{\text{nearest}};\ c_{\text{min}} \leftarrow \texttt{Cost}(x_{\text{nearest}}) + c(\texttt{Line}(x_{\text{nearest}}, x_{\text{new}}));$
    foreach $x_{\text{near}} \in X_{\text{near}}$ **do** // Connect along a minimum-cost path
      if $\texttt{CollisionFree}(x_{\text{near}}, x_{\text{new}}) \wedge \texttt{Cost}(x_{\text{near}}) + c(\texttt{Line}(x_{\text{near}}, x_{\text{new}})) < c_{\text{min}}$ **then**
        $x_{\text{min}} \leftarrow x_{\text{near}};\ c_{\text{min}} \leftarrow \texttt{Cost}(x_{\text{near}}) + c(\texttt{Line}(x_{\text{near}}, x_{\text{new}}))$
    $E \leftarrow E \cup \{(x_{\text{min}}, x_{\text{new}})\};$
    foreach $x_{\text{near}} \in X_{\text{near}}$ **do** // Rewire the tree
      if $\texttt{CollisionFree}(x_{\text{new}}, x_{\text{near}}) \wedge \texttt{Cost}(x_{\text{new}}) + c(\texttt{Line}(x_{\text{new}}, x_{\text{near}})) < \texttt{Cost}(x_{\text{near}})$
      then $x_{\text{parent}} \leftarrow \texttt{Parent}(x_{\text{near}});$
      $E \leftarrow (E \setminus \{(x_{\text{parent}}, x_{\text{near}})\}) \cup \{(x_{\text{new}}, x_{\text{near}})\}$
return $G = (V, E);$

Source: Karaman and Frazzoli

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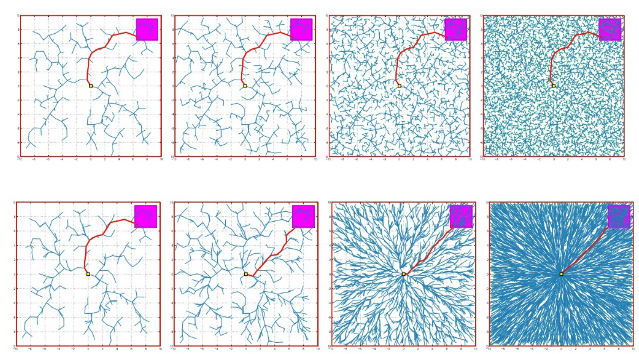
RRT*

RRT

RRT*



Source: Karaman and Frazzoli

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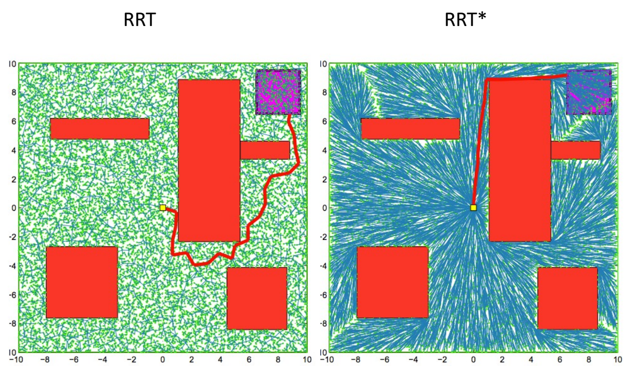
RRT*



Source: Karaman and Frazzoli

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Smoothing

Randomized motion planners tend to find not so great paths for execution: very jagged, often much longer than necessary.

→ In practice: do smoothing before using the path

- Shortcutting:
  - along the found path, pick two vertices x_{t1}, x_{t2} and try to connect them directly (skipping over all intermediate vertices)
- Nonlinear optimization for optimal control
  - Allows to specify an objective function that includes smoothness in state, control, small control inputs, etc.

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Additional Resources

- Marco Pavone (http://asl.stanford.edu/):
  - Sampling-based motion planning on GPUs: https://arxiv.org/pdf/1705.02403.pdf
  - Learning sampling distributions: https://arxiv.org/pdf/1709.05448.pdf
- Sidd Srinivasa (https://personalrobotics.cs.washington.edu/)
  - Batch informed trees: https://robotic-esp.com/code/bitstar/
  - Expensive edge evals: https://arxiv.org/pdf/2002.11853.pdf
- Adam Fishman / Dieter Fox (https://rse-lab.cs.washington.edu/)
  - Motion Policy Networks: https://mpinets.github.io/
- Lydia Kavraki (http://www.kavrakilab.org/)
  - Motion in human workspaces: http://www.kavrakilab.org/nsf-nri-1317849.html

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