

# Robotics Spring 2023

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# Recap: Course Overview

Filtering/Smoothing Localization

Mapping SLAM

Search Motion Planning

TrajOpt Stability/Certification

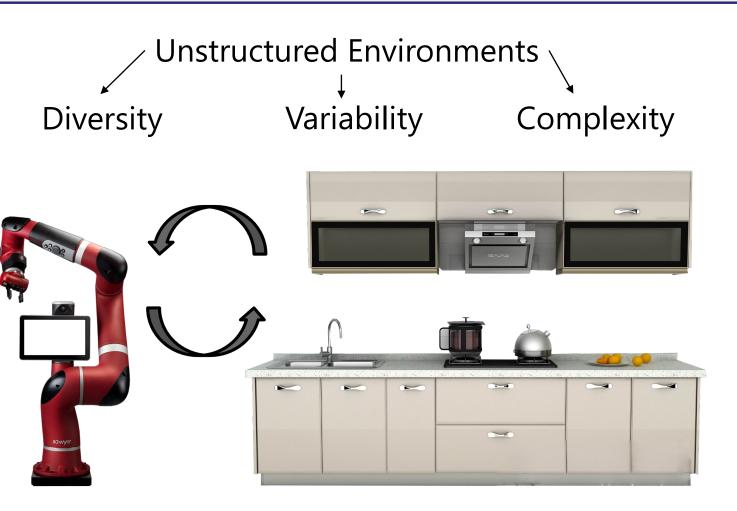
MDPs and RL

Imitation Learning Solving POMDPs

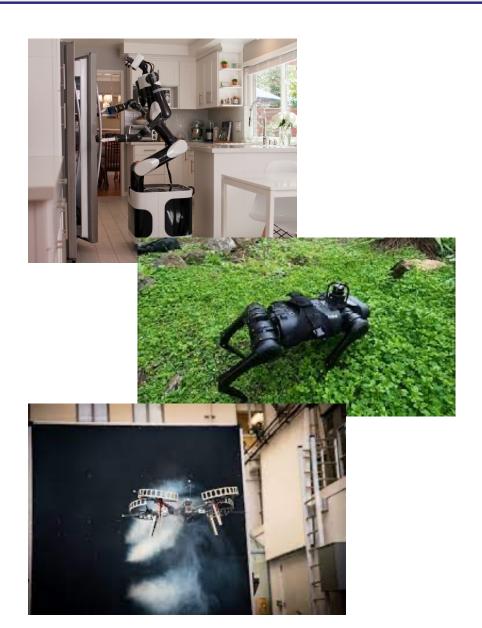
### Lecture Outline

Reinforcement Learning: Motivation **Imitation Learning** Policy Gradient and Beyond

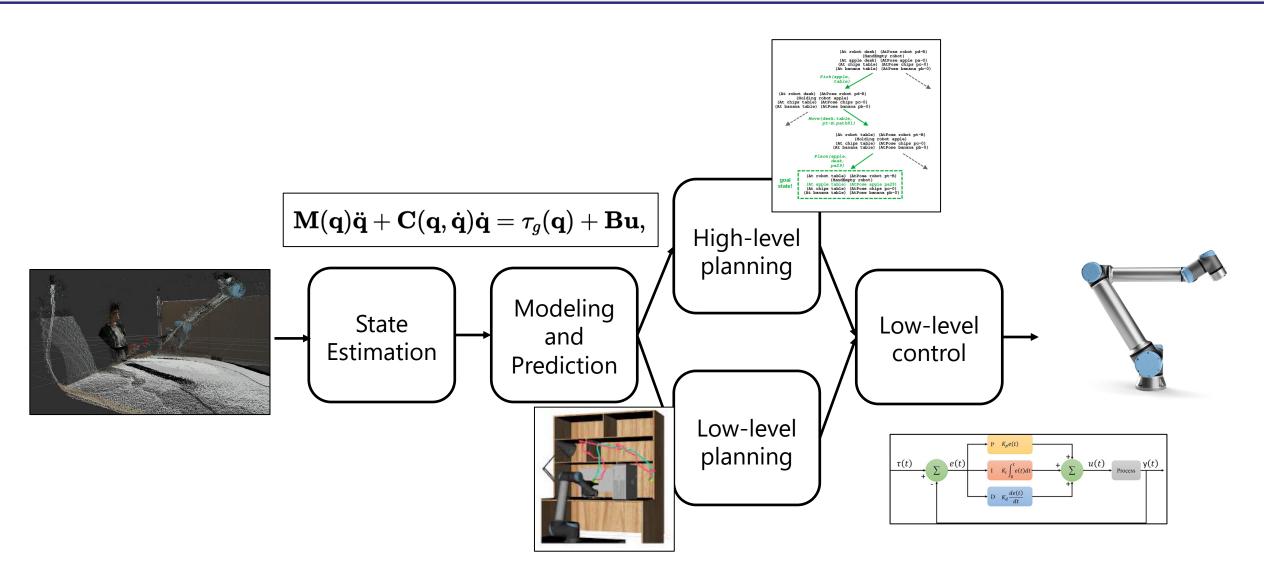
### When is optimal control + planning not enough?



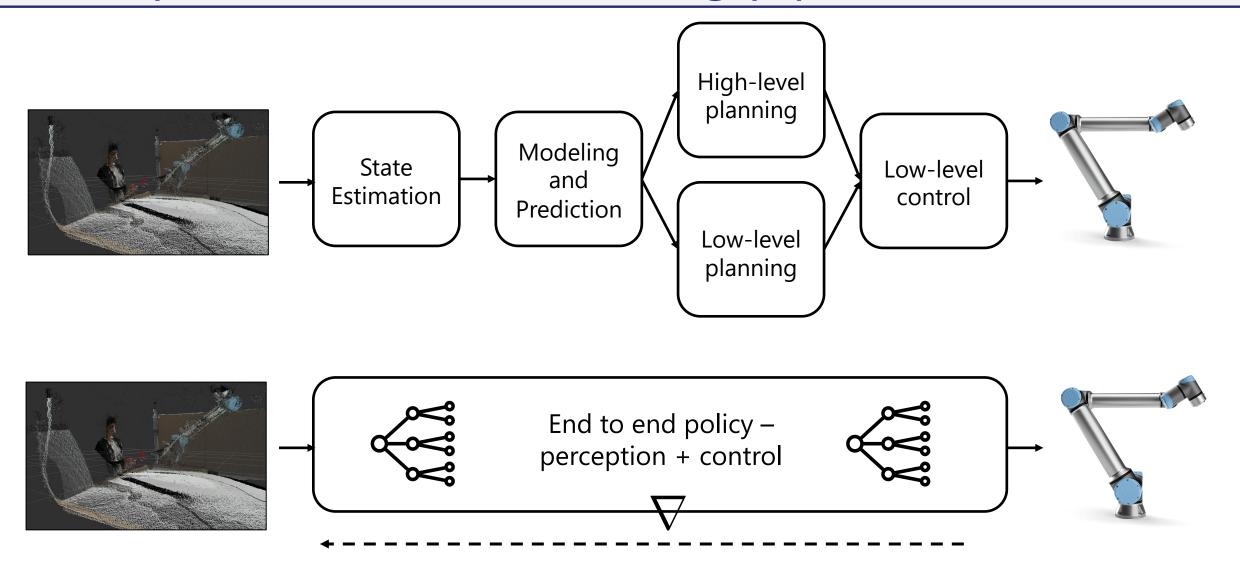
Challenging to model these environments



# How does a typical robotics pipeline look?

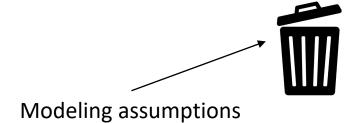


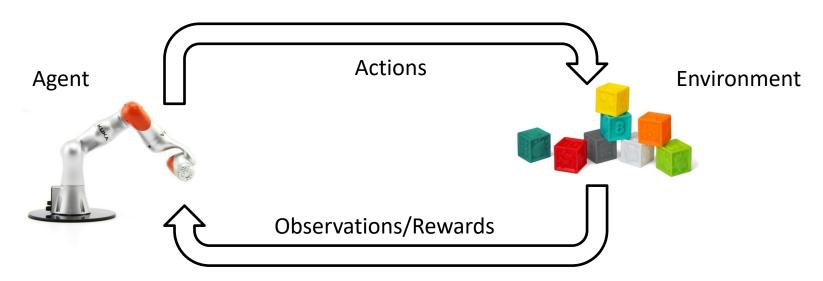
### Deep reinforcement learning pipeline for robotics



### What is reinforcement learning?

Remove assumption for known environment model, learn directly from data





#### Agent has:

- Sensing
- Actuation

#### Environment accepts:

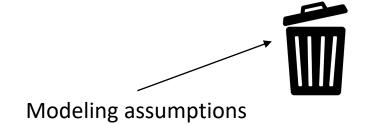
• Actions from agent

#### **Produces**

Observations for sensors(usually unknown)

### Why would you do this?

Remove assumption for known environment model, learn directly from data



### **Pros:**

- Continual improvement on deployment
- 2. Avoid significant modeling assumptions and simulation
- 3. Scale across tasks easily!

### Cons:

- 1. Potentially prohibitive data requirements
- 2. Sometimes unstable, lacks guarantees
- 3. Poor extrapolation

Promising and useful tool in unstructured, dynamic environments

### Connection to Optimal Control

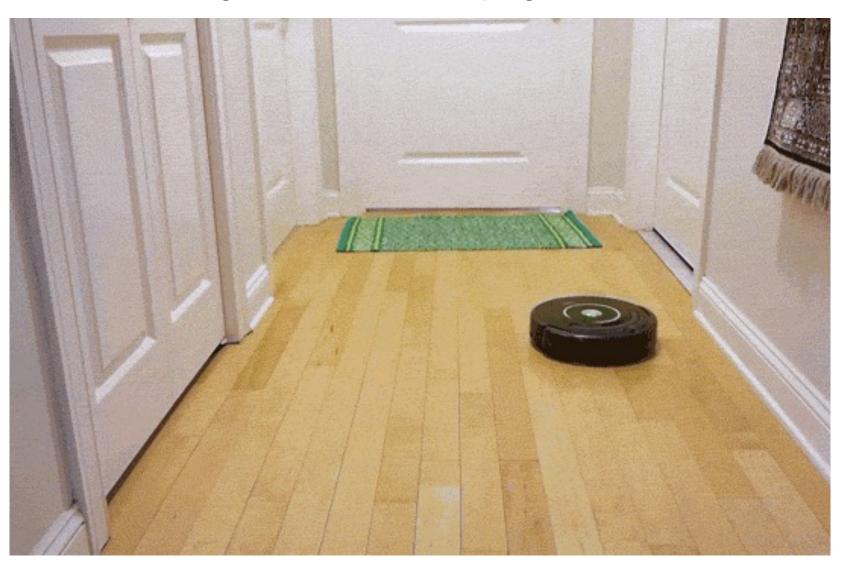
Closely related: typically problem of finding control given a plant



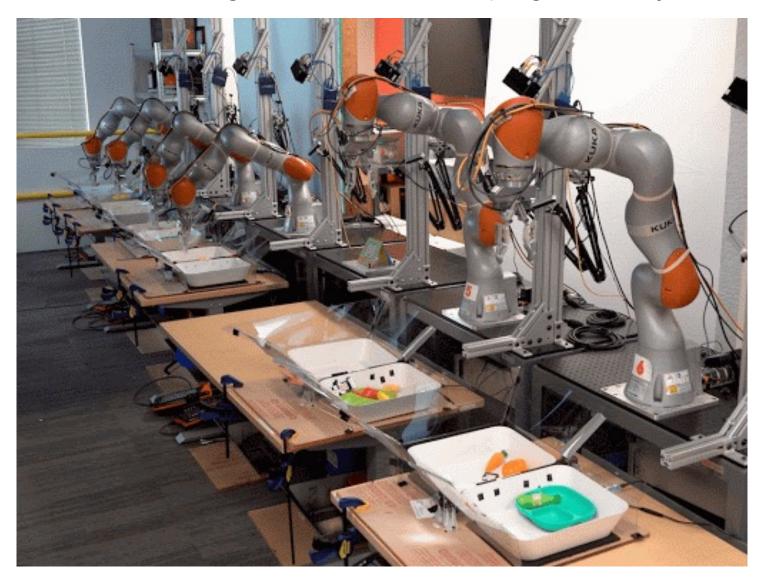


<u>Main difference:</u> model known vs unknown <u>Minor differences:</u> Cost vs reward, discrete vs continuous time

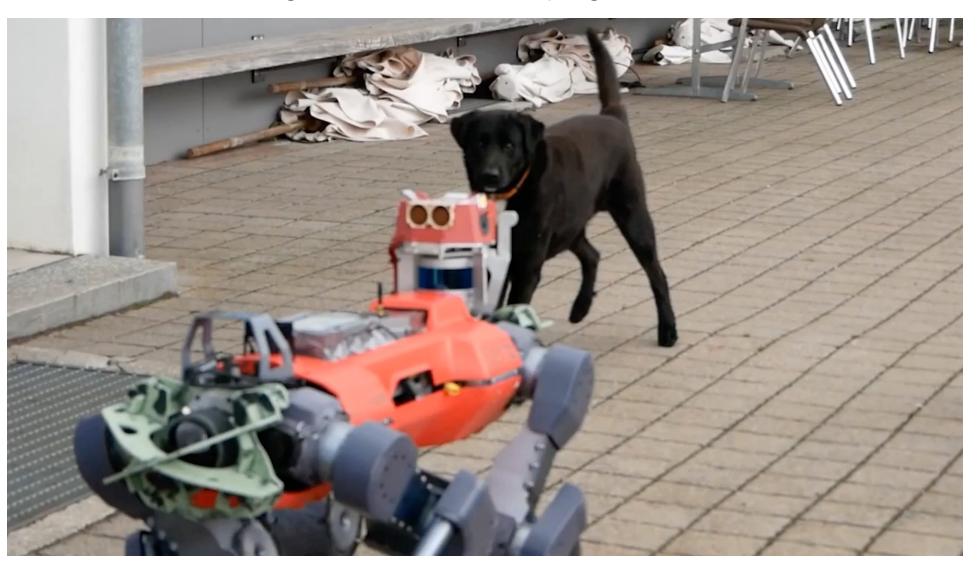
Robots that get better over time, adapting to new **environments** 



Robots that get better over time, adapting to new **objects** 



Robots that get better over time, adapting to new **terrains** 



Robots that get better over time, adapting to new **tasks** 



### Why should we care about RL?

Allows agents to continue improving/adapting on deployment with minimal human effort



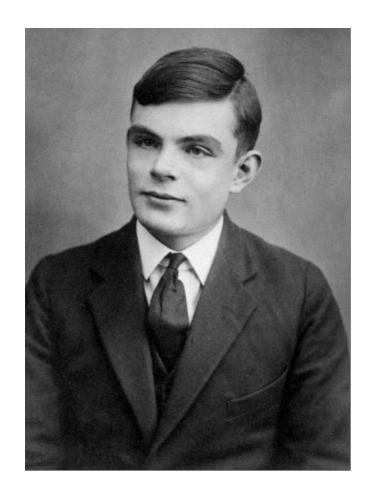




Agents can overfit to domains at test time rather than per-domain human design

### Why should we care about RL?

Hypothesis: By designing algorithms that can improve themselves, we can reach fully intelligent systems

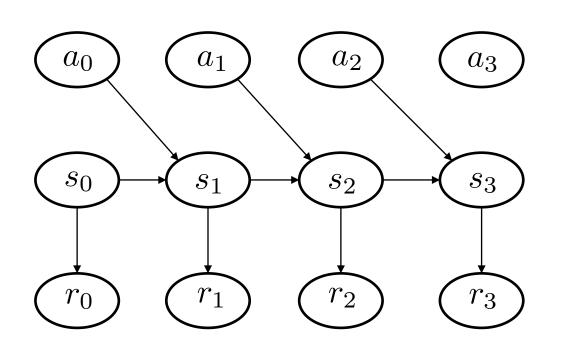


"Instead of trying to produce a programme to simulate the adult mind, why not rather try to produce one which simulates the child's? If this were then subjected to an appropriate course of education one would obtain the adult brain" – Alan Turing



Rather than try to directly replicate behaviors, try to replicate adaptative learning mechanisms

### Markov Decision Processes



States:  $\mathcal{S}$ 

Actions:  $\mathcal{A}$ 

Rewards:  $\mathcal{R}$ 

Transition Dynamics - $p(s_{t+1}|s_t, a_t)$ 

Markov property  $p(s_1,s_2,s_3) = p(s_3|s_2)p(s_2|s_1)p(s_1)$  Trajectory  $au = (s_0,a_0,r_0,s_1,a_1,r_1,\ldots,s_T,a_T,r_T)$ 

### MDPs to the Real World

Task: Place kettle in sink



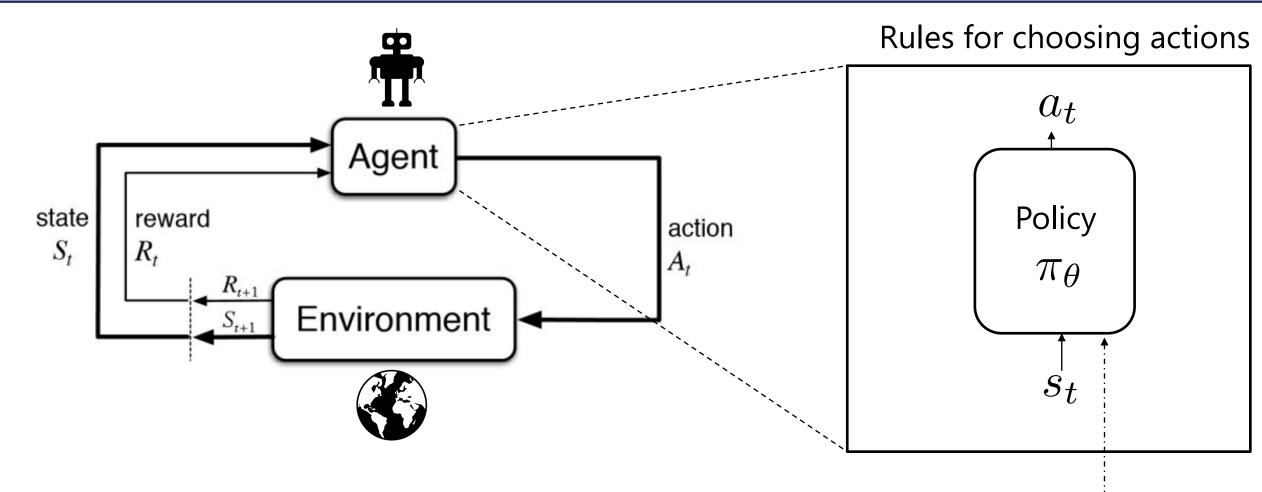
State: Camera Images / Joint Encoders

Action: Joint torques/velocities

Reward: Distance from kettle to sink

Transition: World physics

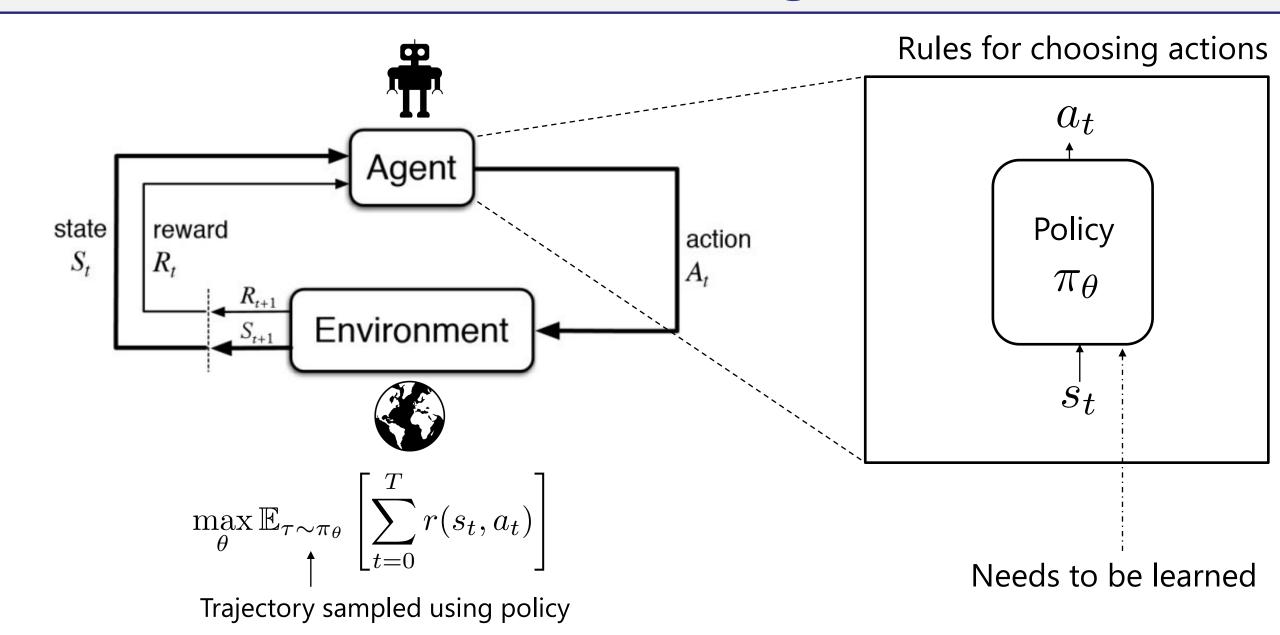
### Reinforcement Learning Formalism



Maximize the sum of expected rewards under policy

Needs to be learned

### Reinforcement Learning Formalism



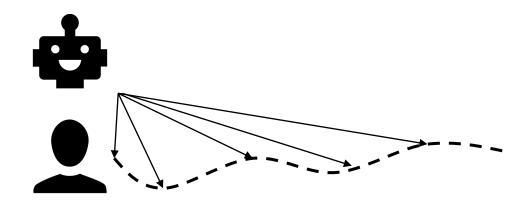
# Why is this not just supervised learning?

### **Supervised Learning**

$$\max_{\theta} \mathbb{E}_{(x,y) \sim \mathcal{D}} \left[ \log \hat{p}_{\theta}(y|x) \right]$$

Sampling from expert

$$D_{\mathrm{KL}}(p^*||p_{\theta})$$
 IID

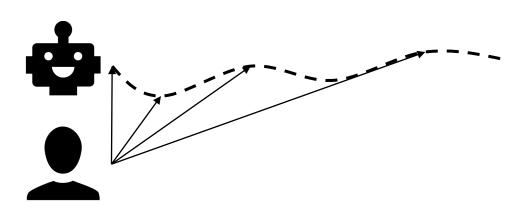


### Reinforcement Learning

$$\max_{\theta} \mathbb{E}_{\tau \sim \pi_{\theta}} \left[ \sum_{t=0}^{T} r(s_t, a_t) \right]$$

Sampling from policy

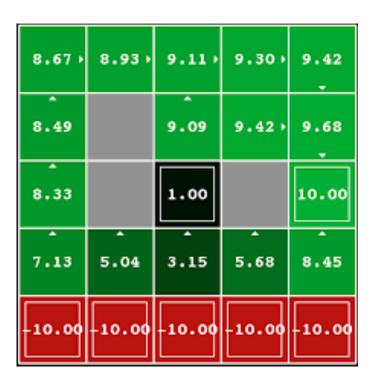
$$D_{\mathrm{KL}}(p_{\theta}||p^*)$$
 Non-IID



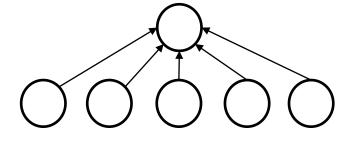
### Main thing to learn - Policies

### Policies are mappings from states to optimal actions

#### Tabular

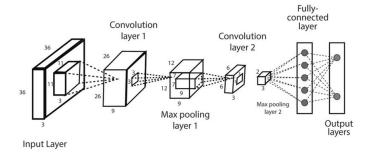


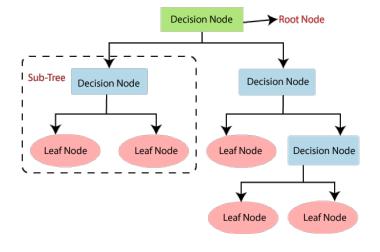
#### <u>Linear</u>



$$\pi(a|s) = \langle \phi(s,a), w \rangle$$

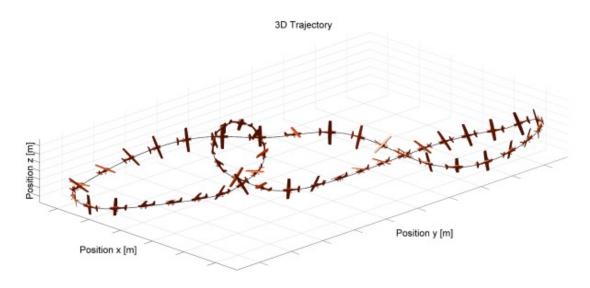
#### **Arbitrary function approx**





### Where is Reinforcement Learning not useful?

Not the right call for very safety-critical, repetitive applications







### Where is Reinforcement Learning "potentially" useful?

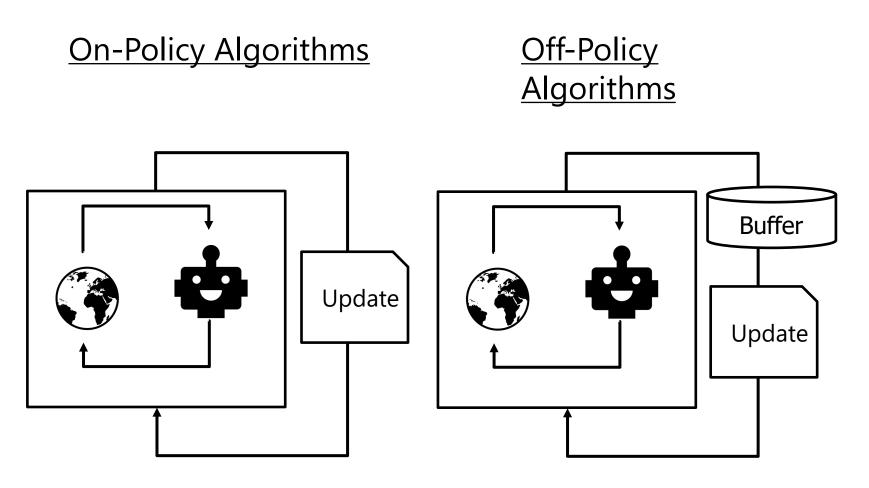
Domains which have high diversity, yet relatively cheap autonomous data collection





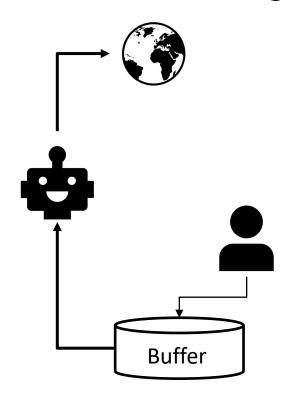
But these domains are not as simple as just running RL algorithms!

### Learning Algorithms for Robotics



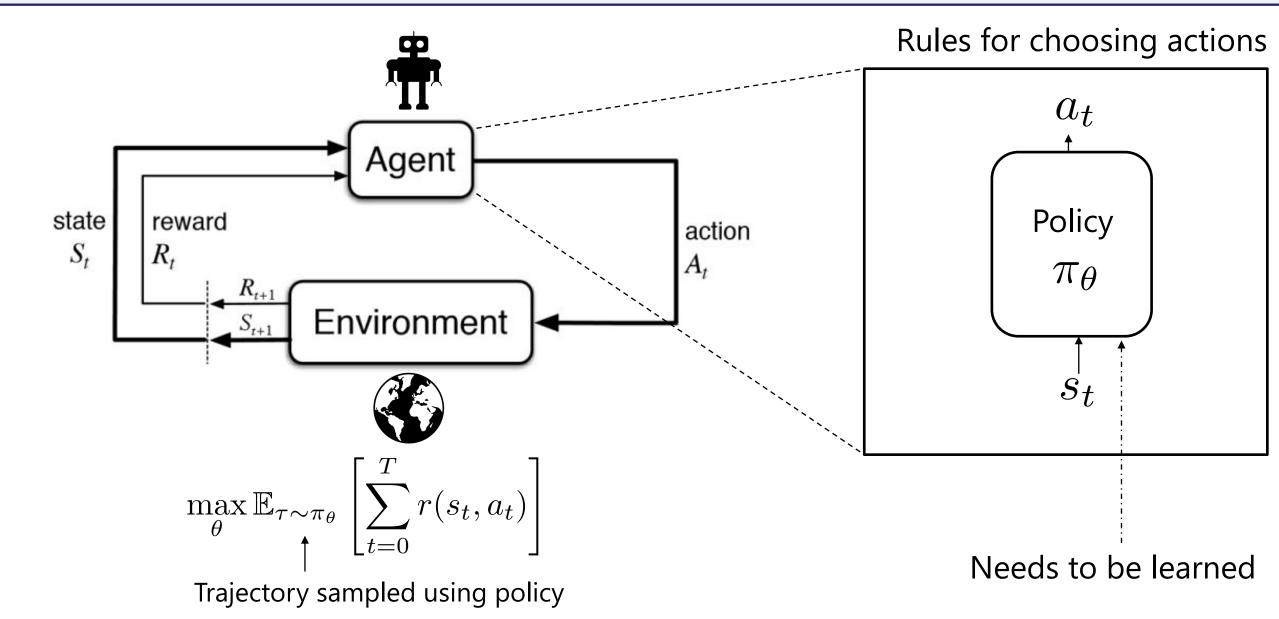
Simple, performant, Data inefficient Data-efficient, sometimes unstable

**Imitation Learning** 

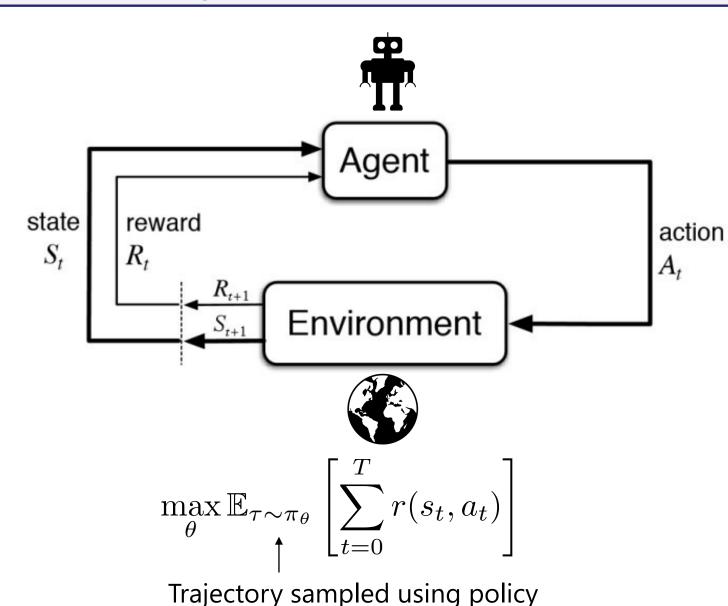


Performant, efficient, but compounding error and expensive data collection

## Objective of Reinforcement Learning



## Objective of Reinforcement Learning



### Assumptions:

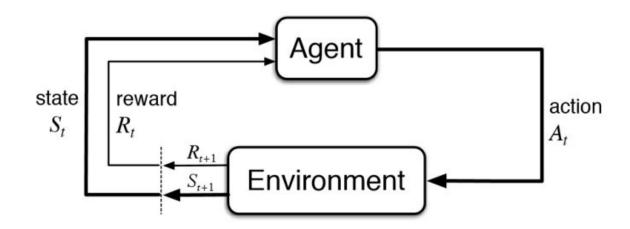
- 1. Rewards are additive
- 2. Dynamics can be sampled from, but functional form is unknown
- 3. Rewards are provided as every state is visited, functional form is unknown

### Lecture Outline

**Reinforcement Learning: Motivation** 

**Imitation Learning** Policy Gradient and Beyond

### What makes RL hard?



Exploration: Find the right data

Exploitation: Using the exploration data

Exploration is challenging in itself in large state spaces, and exploration + exploitation is particularly tough

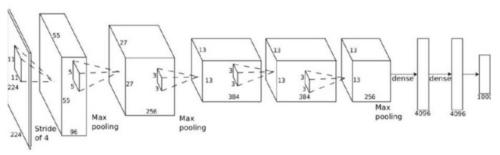
→ let's avoid for now!

### Imitation Learning (IL) in a Nutshell

**Given** Demonstration

**Goal** a policy to mimic the demonstrator







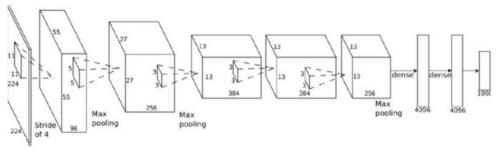
# **Behavior Cloning**

**Given** demonstration  $D^* = s_1^*, a_1^*, s_2^*, a_2^*, ...$ 

**Optimize** arg min  $E_{(s_i^*,a_i^*)\sim D^*} L(a_i^* \mid \pi(a|s_i^*))$ 

e.g. 
$$(\pi(s_i^*) - a_i^*)^2$$







 $S_t$ 

 $\pi_{\theta}(a_t|s_t)$ 

 $a_t$ 

### ALVINN: <u>Autonomous Land Vehicle In a Neural Network</u>

#### What's Hidden in the Hidden Layers?

The contents can be easy to find with a geometrical problem, but the hidden layers have yet to give up all their secrets

David S. Touretzky and Dean A. Pomerleau

AUGUST 1989 • B Y T E 231

Road Intensity
Feedback Unit
Output Units

29
Hidden
Units

8x32 Range Finder
Input Retina

30x32 Video Input Retina

Figure 1: ALVINN Architecture

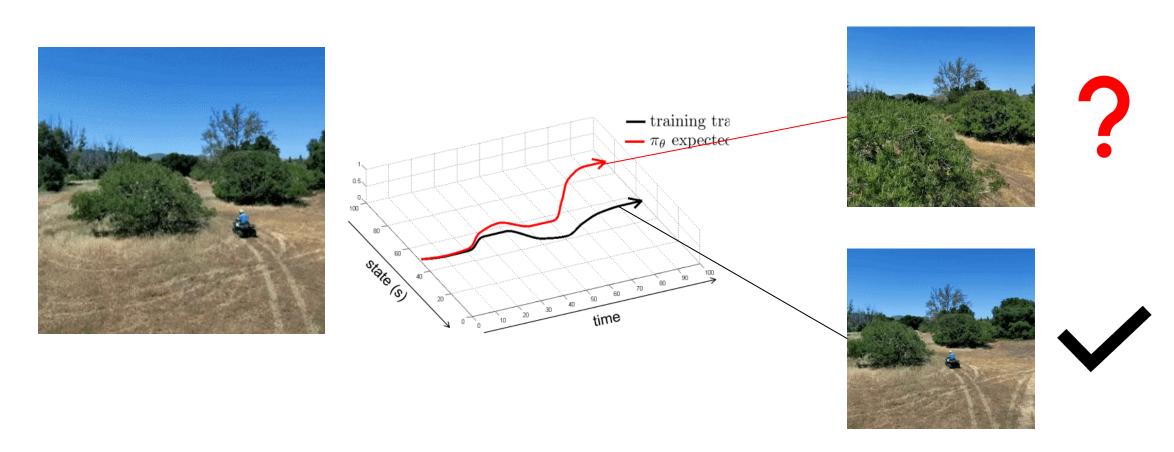


Photo 1: The NAVLAB autonomous navigation test-bed vehicle and the road used for trial runs.



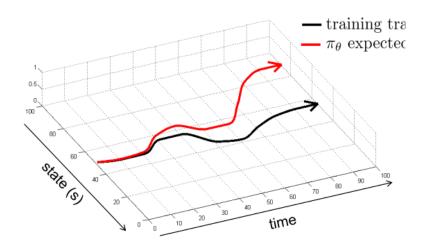
### **Covariate Shift**

Imitation Learning ≠ Supervised Learning



### **Covariate Shift**

Imitation Learning ≠ Supervised Learning



$$\arg\min E_{(s_i^*,a_i^*)\sim D^*} L(a_i^* \mid \pi(a|s_i^*))$$

### Learn to Recover from Mistakes





Is there a more general purpose solution?

# DAgger

Reformulate Imitation Learning as an Online Learning problem  $p_{ heta}(s_t) o p_{ ext{train}}(s_t)$ 

### DAgger: Dataset Aggregation, 2011

- Initialize  $\hat{\pi}_1$  to any policy in  $\Pi$ .
- 2 for i = 1 to N do
- Sample trajectories using  $\hat{\pi}_i$ .
- Query expert for labels:  $\mathcal{D}_i = \{(s, \pi^*(s))\}$  of visited states
- Aggregate datasets:  $\mathcal{D} \leftarrow \mathcal{D} \bigcup \mathcal{D}_i$ .
- 7 Train classifier  $\hat{\pi}_{i+1}$  on  $\mathcal{D}$ .

Combats covariate shift by bringing  $p_{\theta}$  and  $p_{train}$  together





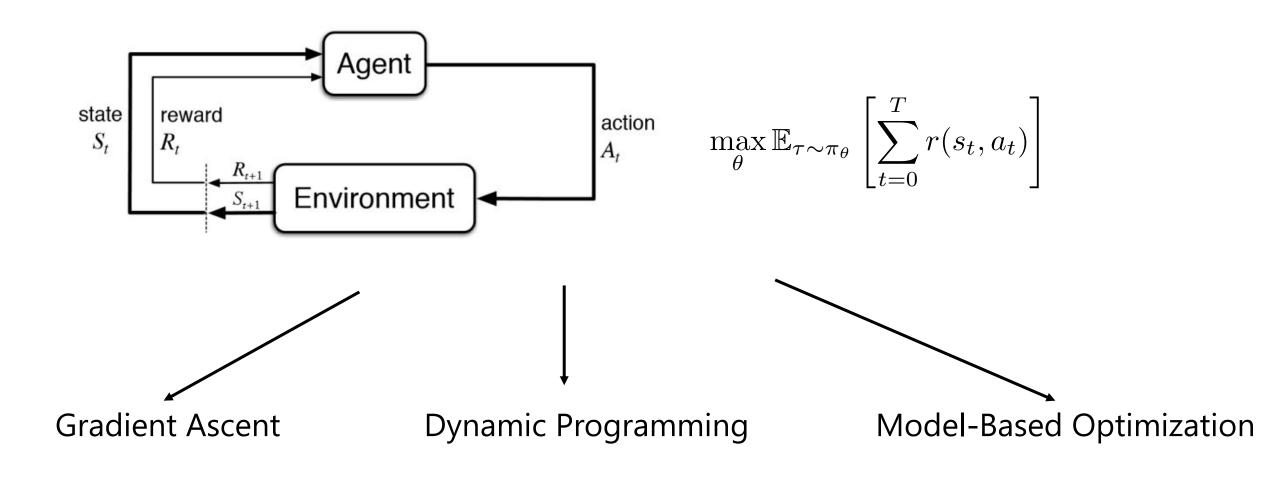
## Lecture Outline

**Reinforcement Learning: Motivation** 

**Imitation Learning** 

Policy Gradient and Beyond

# Let's revisit the overall RL problem



Each method has it's own +/-

# What if we just performed gradient ascent?

$$\max_{\theta} \mathbb{E}_{\tau \sim \pi_{\theta}} \left[ \sum_{t=0}^{T} r(s_t, a_t) \right]$$
$$= \int p_{\theta}(\tau) R(\tau) d\tau$$

Standard gradient descent (supervised learning)

$$\nabla_{\theta} \mathbb{E}_{x \sim g(x)} \left[ f_{\theta}(x) \right]$$

REINFORCE gradient descent (RL)

$$\nabla_{\theta} \mathbb{E}_{x \sim p_{\theta}(x)} [f(x)]$$

Gradient wrt expectation variable, not of integrand!

## Taking the gradient of sum of rewards

$$J(\theta) = \int p_{\theta}(\tau)R(\tau)d(\tau)$$

$$\nabla_{\theta}J(\theta) = \nabla_{\theta}\int p_{\theta}(\tau)R(\tau)d(\tau)$$

$$= \int \nabla_{\theta}p_{\theta}(\tau)R(\tau)d(\tau) = \int \frac{p_{\theta}(\tau)}{p_{\theta}(\tau)}\nabla_{\theta}p_{\theta}(\tau)R(\tau)d(\tau)$$

$$= \int p_{\theta}(\tau)\nabla_{\theta}\log p_{\theta}(\tau)R(\tau)d(\tau) = \mathbb{E}_{p_{\theta}(\tau)}\left[\nabla_{\theta}\log p_{\theta}(\tau)R(\tau)\right]$$

**REINFORCE** trick

## Taking the gradient of return

Initial State Dynamics Policy  $p_{\theta}(\tau) = p(s_0) \prod_{t=0}^{T-1} p(s_{t+1}|s_t, a_t) \pi(a_t|s_t)$  $\log p_{\theta}(\tau) = \log p(s_0) + \sum \log p(s_{t+1}|s_t, a_t) + \log \pi(a_t|s_t)$ t=0 $\nabla_{\theta} \log p_{\theta}(\tau) = \nabla_{\theta} \log p(s_0) + \sum_{t=0}^{T-1} \nabla_{\theta} \log p(s_{t+1}|s_t, a_t) + \nabla_{\theta} \log \pi(a_t|s_t)$  $\nabla_{\theta} \log p_{\theta}(\tau) = \sum \nabla_{\theta} \log \pi(a_t | s_t)$ Model Free!!

## Taking the gradient of return

$$\nabla_{\theta} J(\theta) = \mathbb{E}_{\tau \sim p_{\theta}(\tau)} \left[ \nabla_{\theta} \log p_{\theta}(\tau) \sum_{t=0}^{T} r(s_t, a_t) \right]$$

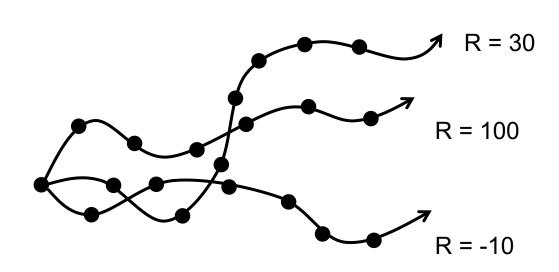
$$\nabla_{\theta} J(\theta) = \mathbb{E}_{\substack{s_0 \sim p(s_0) \\ s_{t+1} \sim p(s_{t+1}|s_t, a_t) \\ a_t \sim \pi(a_t|s_t)}} \left[ \sum_{t=0}^{T} \nabla_{\theta} \log \pi_{\theta}(a_t|s_t) \sum_{t'=0}^{T} r(s_t, a_t) \right]$$

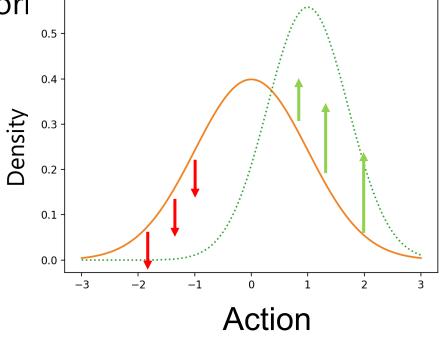
$$\approx \frac{1}{N} \sum_{i=0}^{N} \sum_{t=0}^{T} \nabla_{\theta} \log \pi_{\theta}(a_t^i | s_t^i) \sum_{t'=0}^{T} r(s_{t'}^i, a_{t'}^i) \text{ (approximating using samples)}$$

#### What does this mean?

$$\nabla_{\theta} J(\theta) = \int p_{\theta}(\tau) \nabla_{\theta} \log p_{\theta}(\tau) d\tau \approx \frac{1}{N} \sum_{i=0}^{N} \sum_{t=0}^{T} \nabla_{\theta} \log \pi_{\theta}(a_t^i | s_t^i) \sum_{t'=0}^{T} r(s_{t'}^i, a_{t'}^i)$$

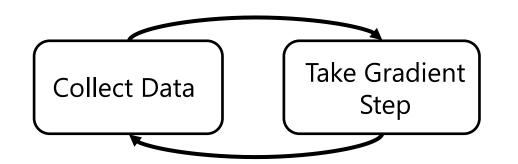
Increase the likelihood of actions in high return trajectori





## Resulting Algorithm (REINFORCE)

$$\nabla_{\theta} J(\theta) = \int p_{\theta}(\tau) \nabla_{\theta} \log p_{\theta}(\tau) d\tau$$



#### REINFORCE algorithm:

On-policy \_\_\_\_

- On-policy 1. sample  $\{\tau^i\}$  from  $\pi_{\theta}(\mathbf{a}_t|\mathbf{s}_t)$  (run it on the robot)
  - 2.  $\nabla_{\theta} J(\theta) \approx \sum_{i} \left( \sum_{t} \nabla_{\theta} \log \pi_{\theta}(\mathbf{a}_{t}^{i} | \mathbf{s}_{t}^{i}) \right) \left( \sum_{t} r(\mathbf{s}_{t}^{i}, \mathbf{a}_{t}^{i}) \right)$ 
    - 3.  $\theta \leftarrow \theta + \alpha \nabla_{\theta} J(\theta)$

### How is this related to supervised learning?

#### Reinforcement Learning

$$\nabla_{\theta} J(\theta) = \int p_{\theta}(\tau) \nabla_{\theta} \log p_{\theta}(\tau) d\tau$$

$$pprox rac{1}{N} \sum_{i=0}^{N} \sum_{t=0}^{T} 
abla_{ heta} \log \pi_{ heta}(a_{t}^{i}|s_{t}^{i}) \sum_{t'=0}^{T} r(s_{t'}^{i}, a_{t'}^{i})$$

#### Supervised Learning

$$\max_{\theta} \mathbb{E}_{(x,y) \sim \mathcal{D}} \left[ \log p_{\theta}(y|x) \right]$$

$$pprox rac{1}{N} \sum_{i} \nabla_{\theta} \log p_{\theta}(y^{i}|x^{i})$$

PG = select good data + increase likelihood of selected data

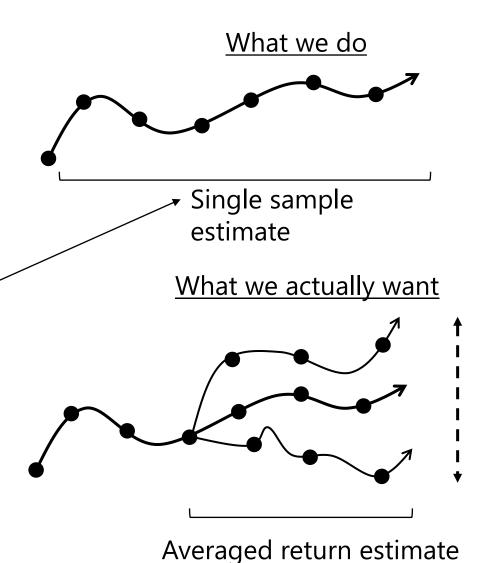
### What makes policy gradient challenging?

$$\nabla_{\theta} J(\theta) = \int p_{\theta}(\tau) \nabla_{\theta} \log p_{\theta}(\tau) d\tau$$

$$\approx \frac{1}{N} \sum_{i=0}^{N} \sum_{t=0}^{T} \nabla_{\theta} \log \pi_{\theta}(a_{t}^{i} | s_{t}^{i}) \sum_{t'=0}^{T} r(s_{t'}^{i}, a_{t'}^{i})$$

**High variance estimator!**!

Hard to tell what matters without many samples

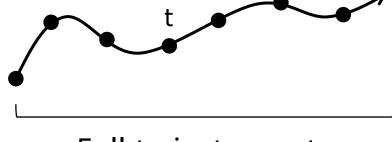


### Variance Reduction with Causality

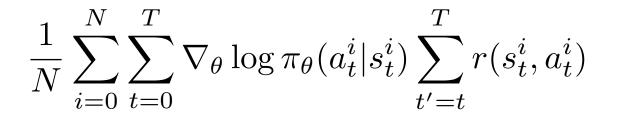
Idea: Trajectory returns depend on past and future, but we only care about the future, since actions cannot affect the past. Instead, consider <u>"return-to-go"</u>

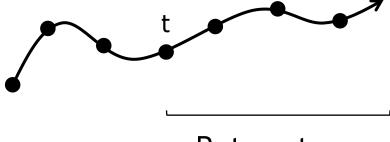
$$\approx \frac{1}{N} \sum_{i=0}^{N} \sum_{t=0}^{T} \nabla_{\theta} \log \pi_{\theta}(a_{t}^{i} | s_{t}^{i}) \sum_{t'=0}^{T} r(s_{t'}^{i}, a_{t'}^{i})$$

Ignore past terms



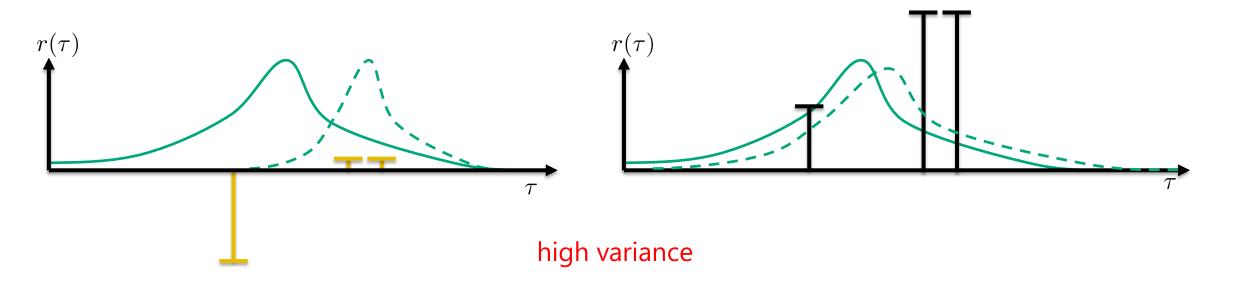
Full trajectory return





Return to go

### Can we reduce variance further?



Arbitrarily uncentered, scaled returns can lead to huge variance:

- → Imagine all rewards were positive, every action would be pushed up, some more than others
- → What if instead, we pushed down some actions and pushed up some others (even if rewards are pos

### Variance Reduction with a Baseline

Idea: We can reduce variance by subtracting a current state dependent function from the policy gradient return

$$\frac{1}{N} \sum_{i=1}^{N} \sum_{t=0}^{T} \nabla_{\theta} \log \pi_{\theta}(a_t^i | s_t^i) \left[ \sum_{t'=t}^{T} r(s_{t'}^i a_{t'}^i) - b(s_t) \right]$$

Baseline: Centers the returns, reduces variance

But does this increase bias??

### Variance Reduction with a Baseline

$$\int_{\mathcal{S}} \int_{\mathcal{A}} p(s_t, a_t) \nabla_{\theta} \log \pi_{\theta}(a_t | s_t) \left[ \sum_{t'=t}^{T} r(s_{t'}, a_{t'}) - b(s_t) \right] ds_t da_t$$

$$\int_{\mathcal{S}} \int_{\mathcal{A}} p(s_t, a_t) \nabla_{\theta} \log \pi_{\theta}(a_t | s_t) \left[ \sum_{t'=t}^{T} r(s_{t'}, a_{t'}) \right] ds_t da_t - \left[ \int_{\mathcal{S}} \int_{\mathcal{A}} p(s_t, a_t) \nabla_{\theta} \log \pi_{\theta}(a_t | s_t) b(s_t) ds_t da_t \right]$$

Let us show this is 0!

### Variance Reduction with a Baseline

$$\int \int p(s_t, a_t) \nabla_{\theta} \log \pi_{\theta}(a_t | s_t) \left[ b(s_t) \right] ds_t da_t = \int \int p(s_t) \pi_{\theta}(a_t | s_t) \nabla_{\theta} \log \pi_{\theta}(a_t | s_t) \left[ b(s_t) \right] ds_t da_t$$

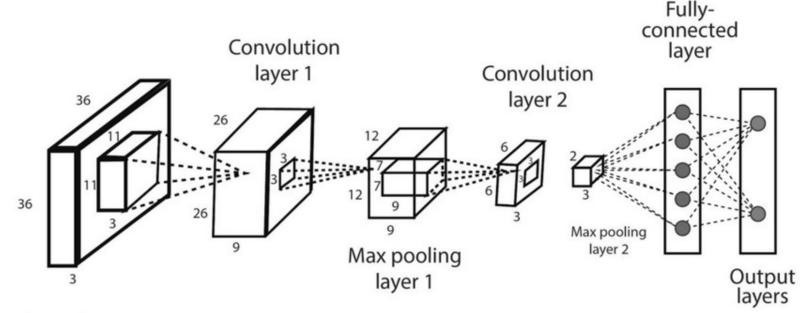
$$= \int p(s_t)b(s_t) \int \pi_{\theta}(a_t|s_t) \nabla_{\theta} \log \pi_{\theta}(a_t|s_t) da_t ds_t$$
$$= \int p(s_t)b(s_t) \int \nabla_{\theta} \pi_{\theta}(a_t|s_t) da_t ds_t$$

$$= \int p(s_t)b(s_t)\nabla_{\theta} \int \pi_{\theta}(a_t|s_t)da_tds_t = \int p(s_t)b(s_t)\nabla_{\theta}(1)ds_t = 0$$

**Unbiased!** 

### Learning Baselines

Baselines are typically learned as deep neural nets from  $R^s \rightarrow R^1$ 



Input Layer

$$\frac{1}{N} \sum_{j=1}^{N} \|\hat{V}(s_t^j, a_t^j) - \sum_{t=1}^{H} r(s_t^j, a_t^j)\|$$

Minimize with Monte-carlo regression at every iteration, club with policy loss

$$A(s_t, a_t) = \sum_{t'=t}^{T} r(s'_t, a'_t) - V(s_t)$$

Allows us to define advantages

## Lecture Outline

**Reinforcement Learning: Motivation Imitation Learning** 

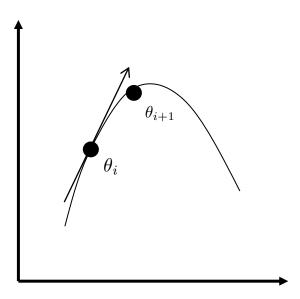
**Policy Gradient and Beyond** 

### Take a deeper look at REINFORCE

$$\nabla_{\theta} J(\theta) = \int p_{\theta}(\tau) \nabla_{\theta} \log p_{\theta}(\tau) d\tau \approx \frac{1}{N} \sum_{i=0}^{N} \sum_{t=0}^{T} \nabla_{\theta} \log \pi_{\theta}(a_t^i | s_t^i) \sum_{t'=0}^{T} r(s_{t'}^i, a_{t'}^i)$$

Gradient descent is steepest descent on linear approximation under the Euclidean metric

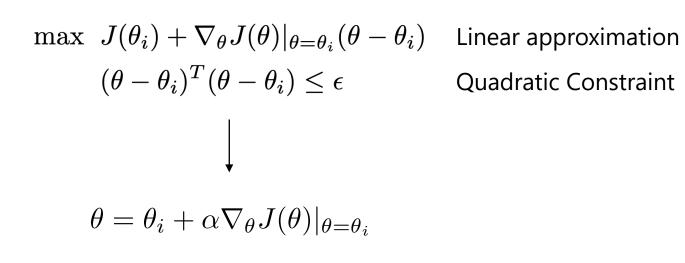
$$\max_{\theta} \mathbb{E}_{\tau \sim \pi_{\theta}} \left[ \sum_{t=0}^{T} r(s_t, a_t) \right]$$
$$= J(\theta)$$

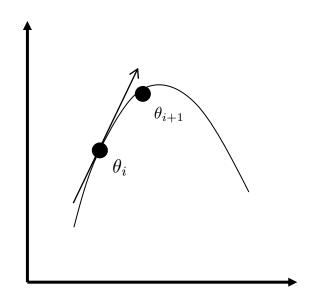


### Take a deeper look at REINFORCE

$$\nabla_{\theta} J(\theta) = \int p_{\theta}(\tau) \nabla_{\theta} \log p_{\theta}(\tau) d\tau \approx \frac{1}{N} \sum_{i=0}^{N} \sum_{t=0}^{T} \nabla_{\theta} \log \pi_{\theta}(a_t^i | s_t^i) \sum_{t'=0}^{T} r(s_{t'}^i, a_{t'}^i)$$

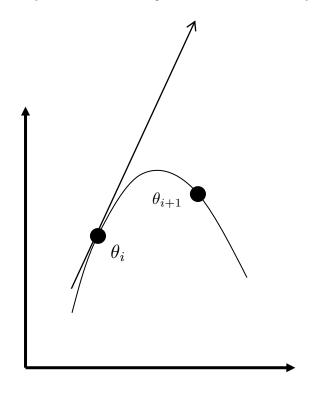
Gradient descent is steepest descent on linear approximation under the Euclidean metric





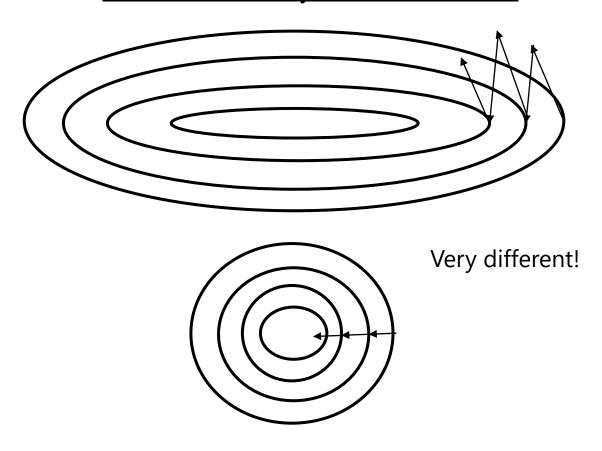
### When might this fail?

Large step sizes may cause collapse



Must use very small step sizes, slow!

Sensitive to Policy Parameterization



Can struggle for a deep neural network!

### Parameterization dependence of PG

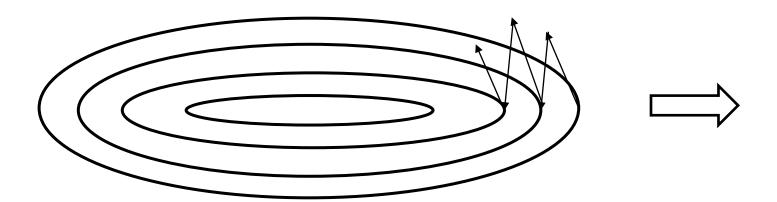
#### Sensitive to Policy Parameterization

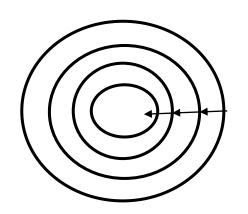
$$L(\theta) = \theta_1 + \theta_2 \qquad \qquad L(\phi) = \phi_1^{0.5} + \phi_2^{-1} \\ \phi_1 = \theta_1^2 \\ \phi_2 = \theta_2^{-1} \\ \nabla_{\theta_1} L = 1 \\ \nabla_{\theta_2} L = 1 \qquad \qquad \text{Not covariant!} \qquad \begin{array}{c} \nabla_{\phi_1} L = 0.5 \phi_1^{-0.5} = 0.5 \theta_1^{-1} \\ \nabla_{\phi_2} L = -\phi_2^{-2} = -\theta_2^2 \end{array}$$

### Modified Constraint on Policy Gradient

$$\max |J(\theta_i) + \nabla_{\theta} J(\theta)|_{\theta = \theta_i} (\theta - \theta_i)$$
$$(\theta - \theta_i)^T (\theta - \theta_i) \le \epsilon$$

$$\max |J(\theta_i) + \nabla_{\theta} J(\theta)|_{\theta = \theta_i} (\theta - \theta_i)$$
$$(\theta - \theta_i)^T G(\theta - \theta_i) \le \epsilon$$





$$\theta_{i+1} = \theta_i + \alpha G^{-1} \nabla_{\theta} J(\theta)|_{\theta = \theta_i}$$
 Rescales according to G-1

Adaptive choice of G can avoid sensitivity to policy parameterization!

### Covariant Policy Gradient Updates

$$\max J(\theta_i) + \nabla_{\theta} J(\theta)|_{\theta = \theta_i} (\theta - \theta_i)$$
$$(\theta - \theta_i)^T G(\theta - \theta_i) \le \epsilon$$

What should G be?

$$\max J(\theta_i) + \nabla_{\theta} J(\theta)|_{\theta = \theta_i} (\theta - \theta_i)$$
$$D_{\text{KL}}(\pi_{\theta} || \pi_{\theta_i}) \le \epsilon$$

Let us use the constraint as KL divergence on the policy (2<sup>nd</sup> order Taylor expansion)

Measures functional distance, not parameter distance

### Resulting "Natural" Policy Gradient

$$\max J(\theta_i) + \nabla_{\theta} J(\theta)|_{\theta = \theta_i} (\theta - \theta_i)$$
$$D_{\text{KL}}(\pi_{\theta} || \pi_{\theta_i}) \le \epsilon$$

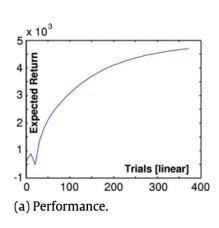
2<sup>nd</sup> order approximation of KL → Fisher Information Metric

$$F = \mathbb{E}_{\pi_{\theta}} \left[ (\nabla_{\theta} \log \pi_{\theta}) (\nabla_{\theta} \log \pi_{\theta})^{T} \right]$$

$$\max |J(\theta_i) + \nabla_{\theta} J(\theta)|_{\theta = \theta_i} (\theta - \theta_i)$$
$$(\theta - \theta_i)^T F(\theta - \theta_i) \le \epsilon$$

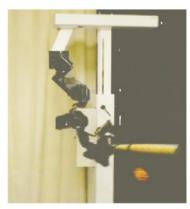
Resulting update  $heta_{i+1} = heta_i + lpha F^{-1} 
abla_{ heta} J( heta)|_{ heta= heta_i}$  Covariant to parameterization

### Natural Policy Gradient in Action





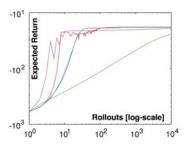




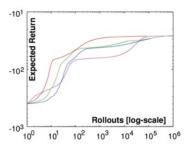
(c) Initial reproduction.



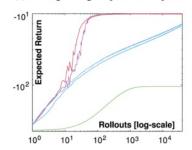
(d) After reinforcement learning.



(b) Minimum motor command with motor primitives



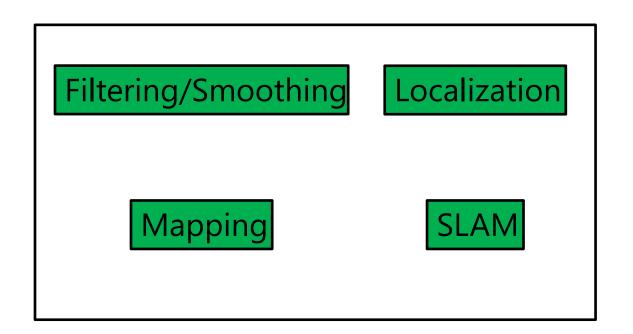
(c) Passing through a point with splines



(d) Passing through a point with motor primitives

Finite Difference Gradient
Vanilla Policy Gradient with constant baseline
Vanilla Policy Gradient with time-variant baseline
Episodic Natural Actor-Critic with single offset basis functions
Episodic Natural Actor-Critic with time-variant offset basis functions

# Recap: Course Overview



Search Motion Planning

TrajOpt Stability/Certification

MDPs and RL

Imitation Learning Solving POMDPs