

# Task and Motion Planning (TAMP)

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NVIDIA Research

CSE 571: Robotics

05/24/2022



# (Probable) Roadmap

## 1. Review Background

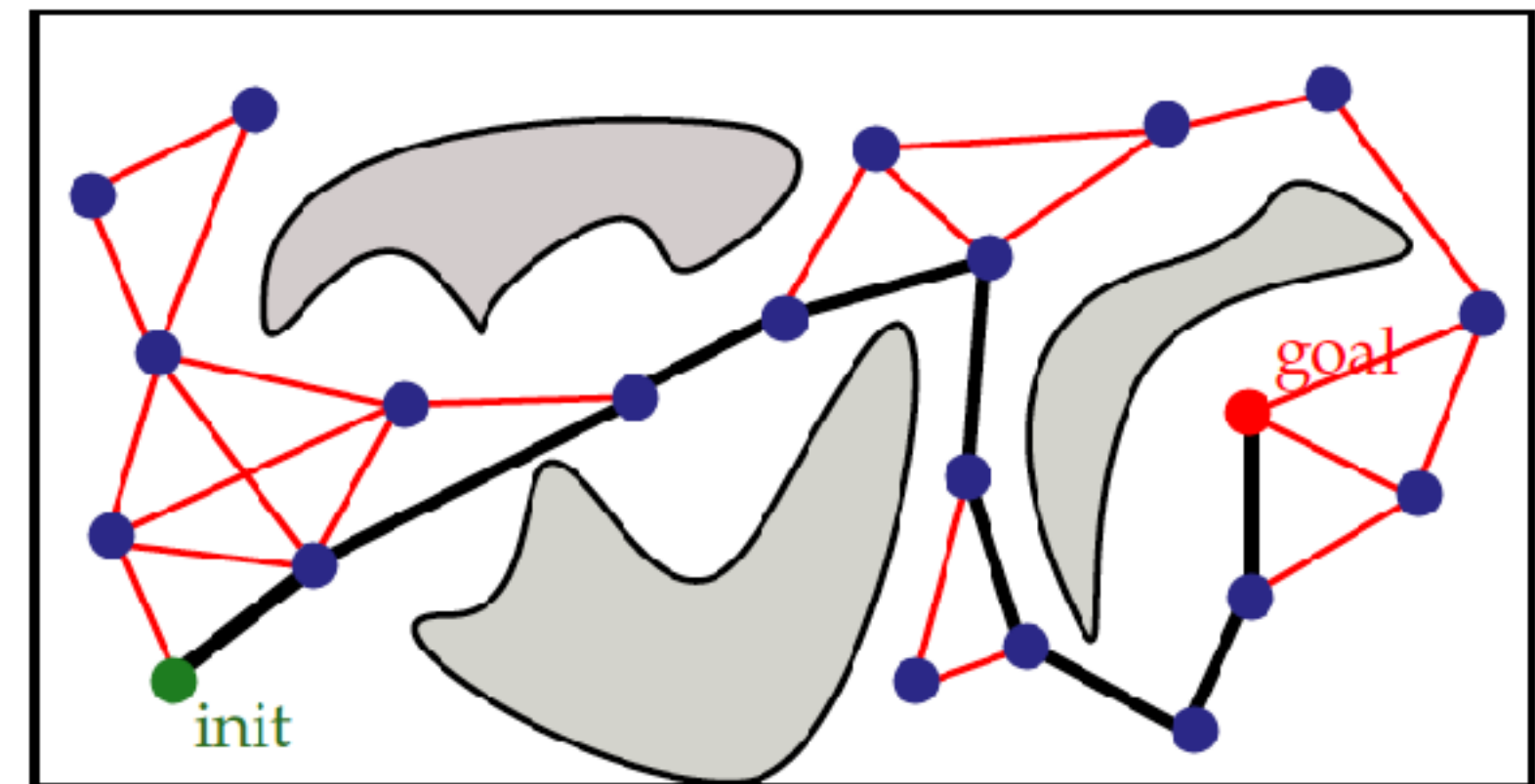
1. Task Planning
2. Motion Planning

## 2. Hybrid Planning

1. Prediscretized & Numeric Planning
2. Multi-Modal Motion Planning
3. Integrated TAMP

## 3. PDDLStream

4. TAMP under Uncertainty
  1. Partially Observable
  2. Unknown Objects



[Fig from Erion Plaku]

# Planning for Autonomous Robots

3

- Robot must select both **high-level** actions & **low-level** controls
- **Application areas:** semi-structured and human environments



Household



Warehouse fulfilment



Food service

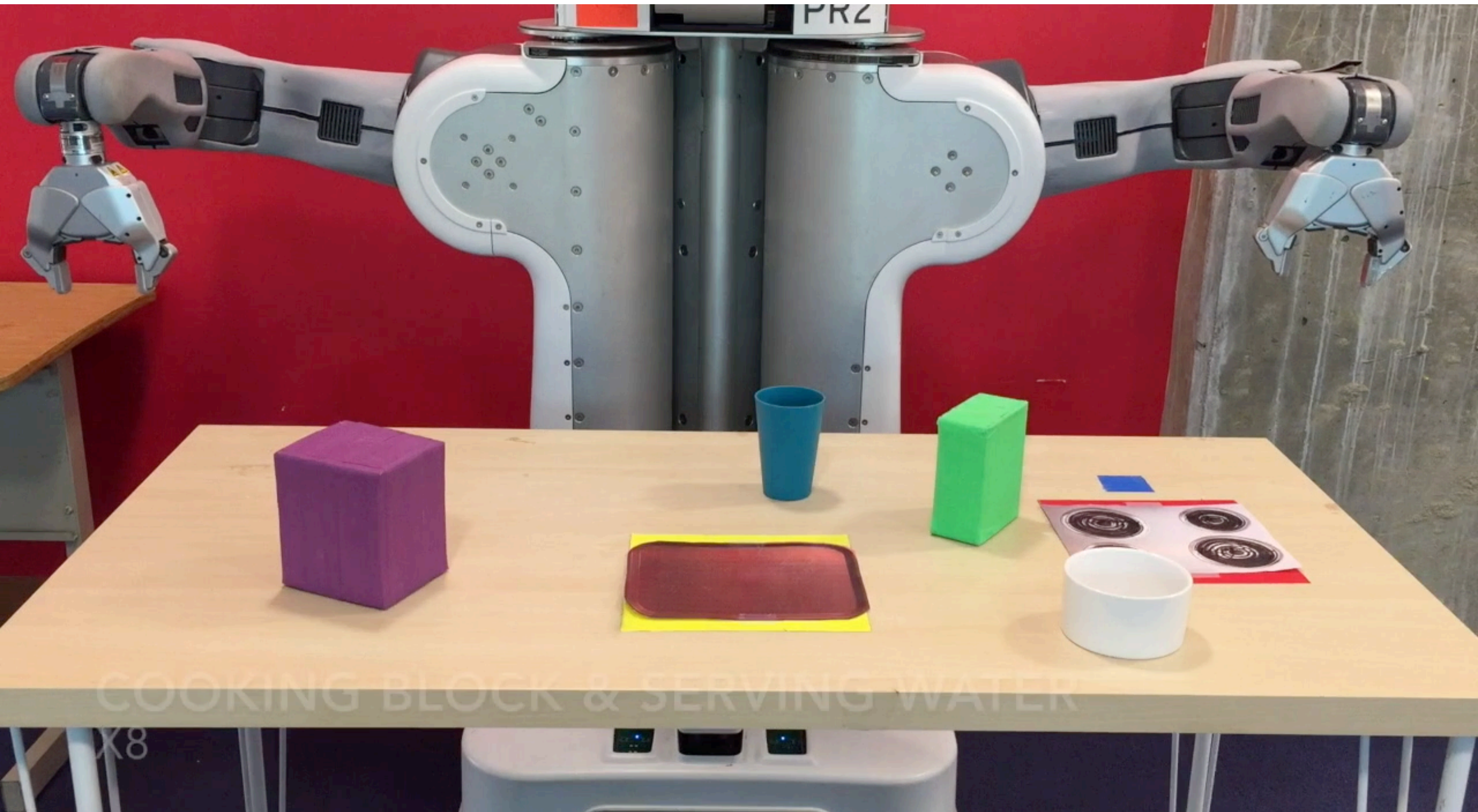


Construction

# Serve Water and “Cooked” Block

4

[[Garrett 2020a](#)]



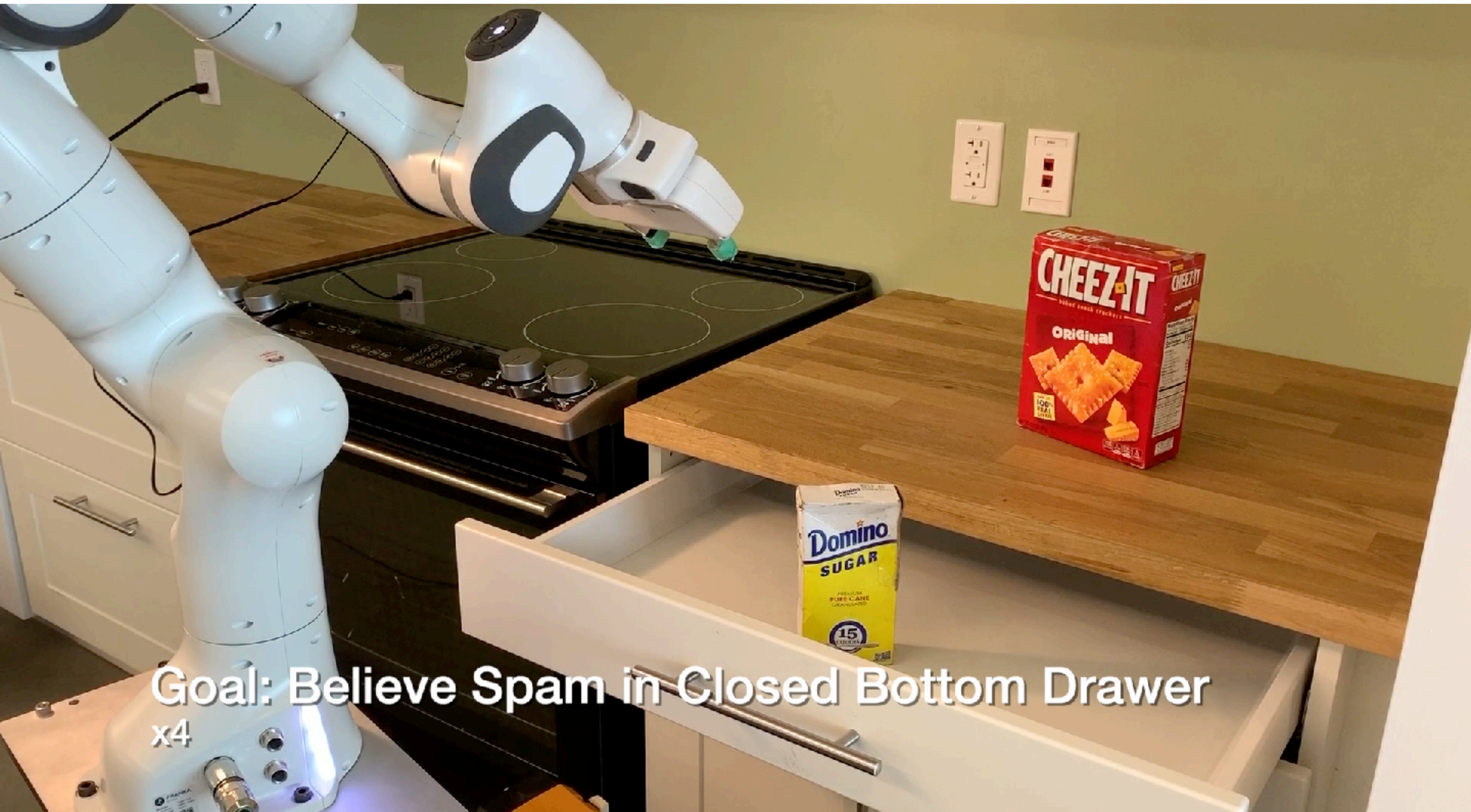
COOKING BLOCK & SERVING WATER

X8

# Localize Spam in Bottom Drawer

5

[Garrett 2020b]

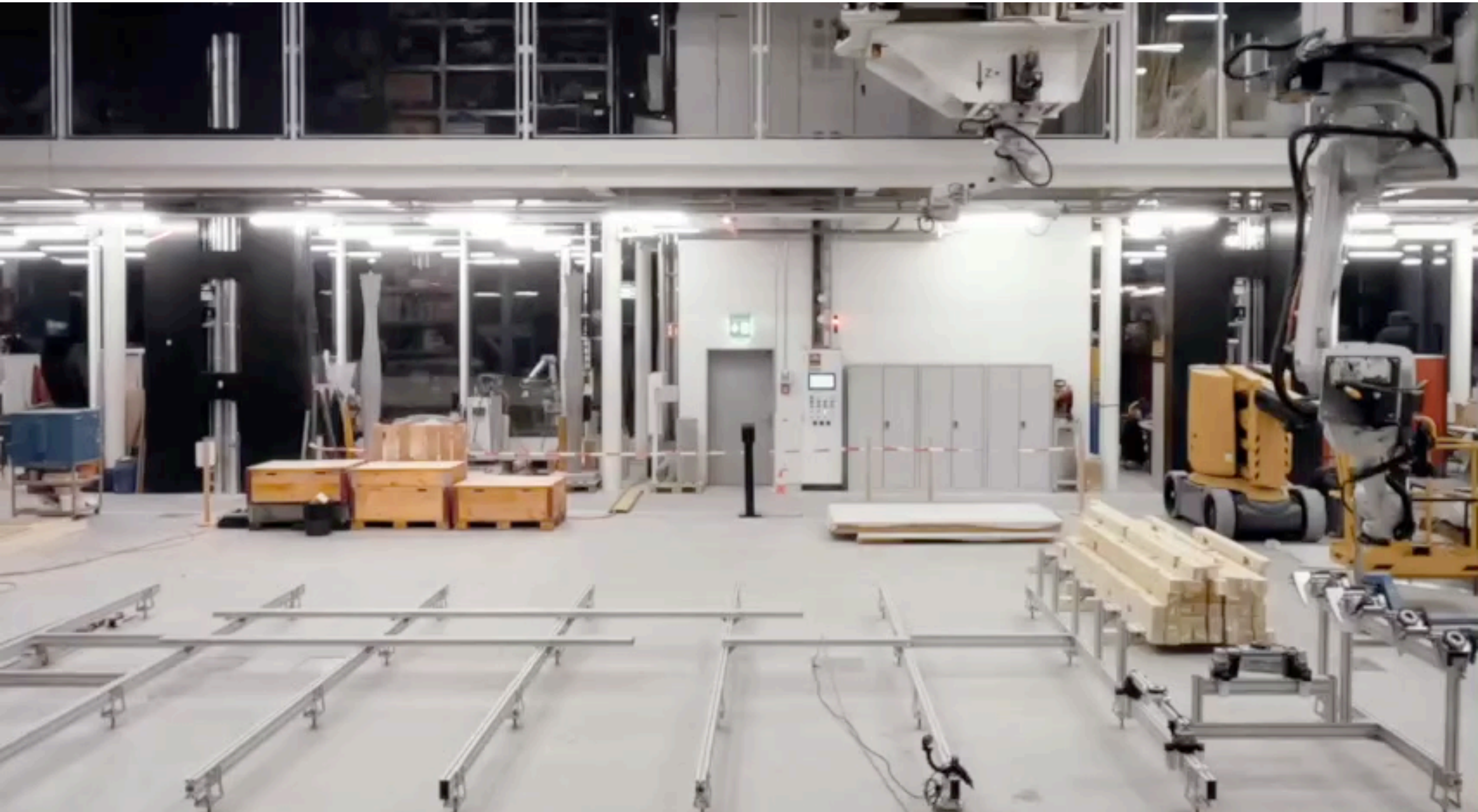


Goal: Believe Spam in Closed Bottom Drawer  
x4

# Assemble Large Timber Structure

6

*[Huang, Leung, Garrett, Gramazio, Kohler, & Mueller 2021]*



# Pouring, Scooping, and Stirring to Prepare “Coffee”

7

[Garrett 2020a]



MAKING COFFEE  
X8

# Problem Class

- **Discrete-time**
  - Plans are finite sequences of controls
- **Deterministic** (for now)
  - Actions always produce the intended effect
  - Solutions are **plans** (instead of policies)
- **Observable** (for now)
  - Access to the full world state
- **Hybrid**
  - States & controls composed of **mixed discrete-continuous variables**



# Task Planning

# Task (Classical, Symbolic) Planning

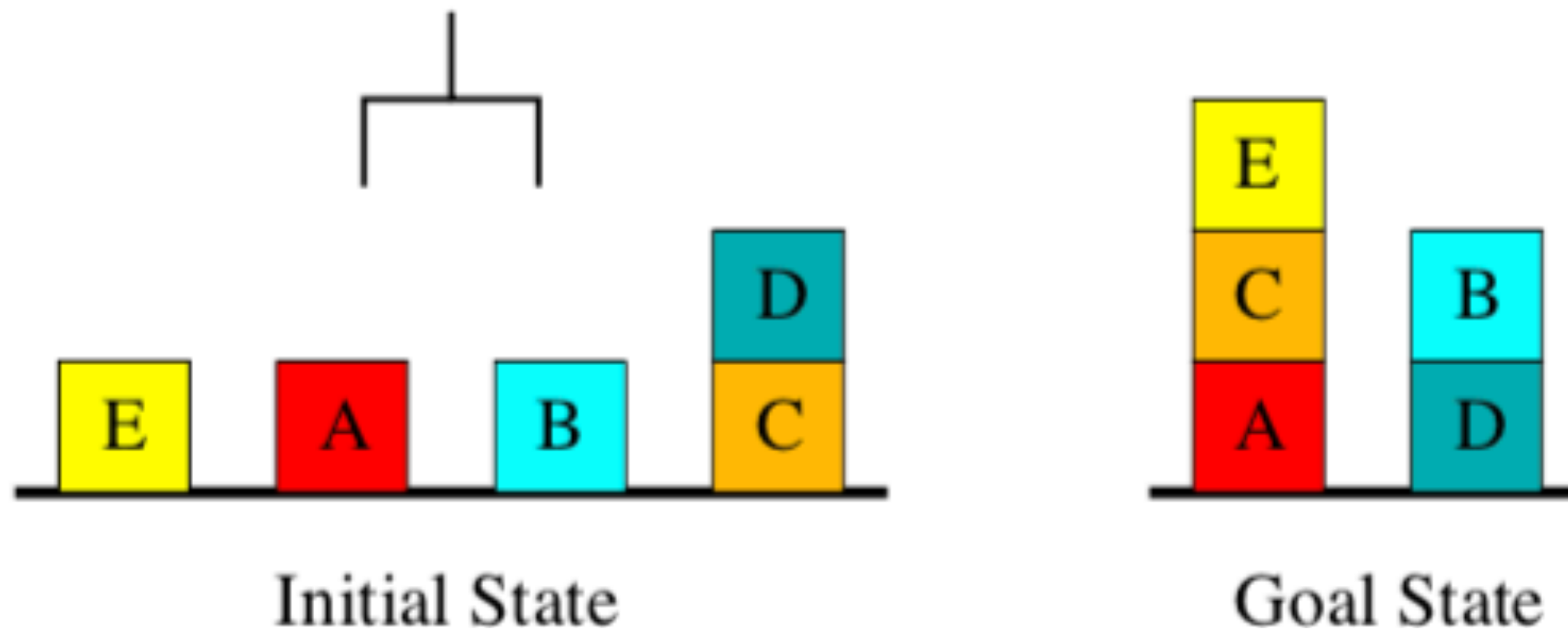
10

- **Discrete** problems with **many variables**
  - Often enormous, but **finite**, state-spaces
- Problems typically described using an **action language**
  - **Propositional Logic (STRIPS)** [Fikes 1971][Aeronautiques 1998]
  - **Planning Domain Description Language (PDDL)**
- Develop **domain-independent** algorithms
  - Can apply to **any problem** expressible using PDDL
- Exploit **factored** and **sparse** structure to develop efficient algorithms

# Classical Planning Representations

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## Blocksworld domain

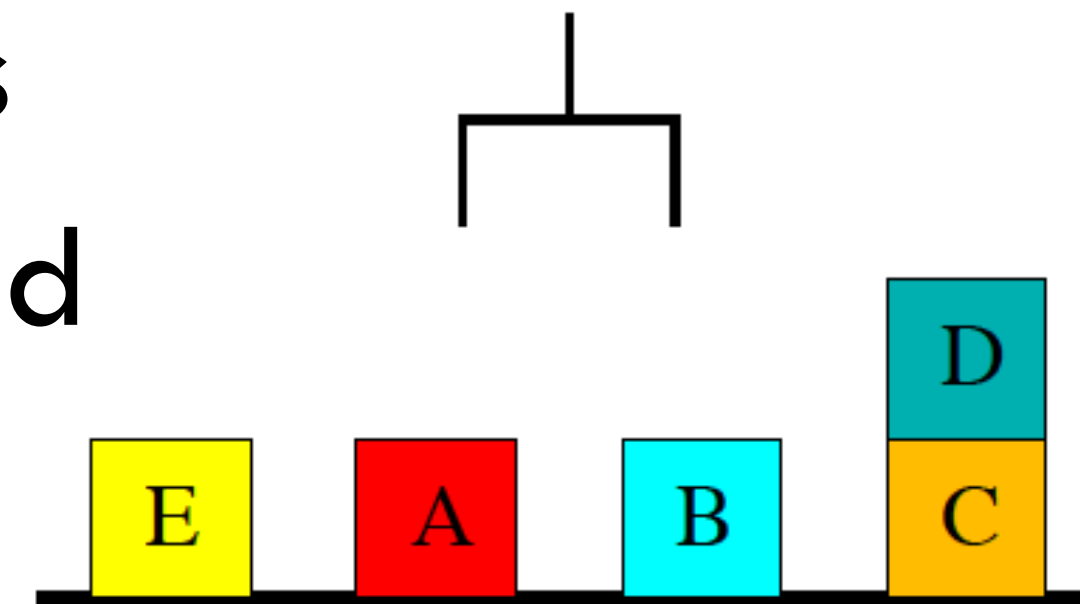


- **Facts:**  $on(x, y)$ ,  $onTable(x)$ ,  $clear(x)$ ,  $holding(x)$ ,  $armEmpty()$ .
- **Initial state:**  $\{onTable(E), clear(E), \dots, onTable(C), on(D, C), clear(D), armEmpty()\}$ .
- **Goal:**  $\{on(E, C), on(C, A), on(B, D)\}$ .
- **Actions:**  $stack(x, y)$ ,  $unstack(x, y)$ ,  $putdown(x)$ ,  $pickup(x)$ .
- **$stack(x, y)?$**   $pre : \{holding(x), clear(y)\}$   
 $add : \{on(x, y), armEmpty()\}$   
 $del : \{holding(x), clear(y)\}$ .

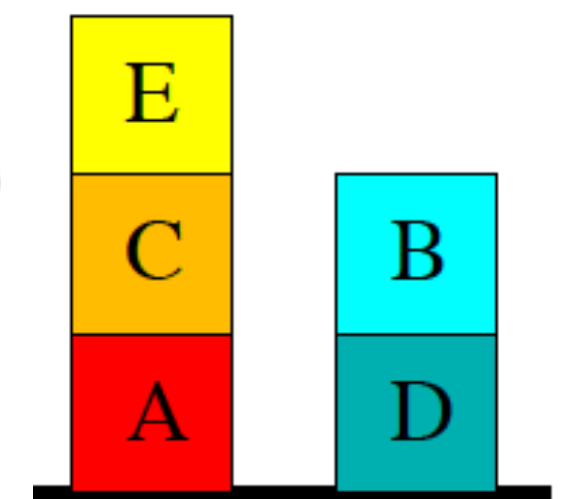
# First-Order Action Languages

12

- **Predicate:** boolean function  $On(?b1, ?b2) = \mathbf{True/False}$
- **Facts (literals):** instantiated predicates  $On(D, C) = \mathbf{True}$
- **State:** set of facts  $\{On(A, B) = \mathbf{False}, On(D, C) = \mathbf{True}, \dots\}$
- Equivalently, boolean state variables
- **Closed-world** assumption: unspecified facts are false



Initial State



Goal State

**Facts:**  $on(x, y)$ ,  $onTable(x)$ ,  $clear(x)$ ,  $holding(x)$ ,  $armEmpty()$ .

**Initial state:**  $\{onTable(E)$ ,  $clear(E)$ ,  $\dots$ ,  $onTable(C)$ ,  $on(D, C)$ ,  $clear(D)$ ,  $armEmpty()\}$ .

**Goal:**  $\{on(E, C)$ ,  $on(C, A)$ ,  $on(B, D)\}$ .

**Actions:**  $stack(x, y)$ ,  $unstack(x, y)$ ,  $putdown(x)$ ,  $pickup(x)$ .

# (Lifted) Action Schema

13

- A tuple of free **parameters**
- A **precondition** formula tests applicability
- An **effect** formula modifies the state
- Logical **conjunctions** enable factoring
- Effects are **deltas**

```
(:action stack
:parameters (?b1, ?b2)
:precondition {
  Holding(?b1), Clear(?b2) }
:effect {ArmEmpty(),
  On(?b1, ?b2),
  Clear(?b1)
¬Holding(?b1),
¬Clear(?b2) }
```

```
(:action unstack
:parameters (?b1, ?b2)
:precondition {ArmEmpty(),
  On(?b1, ?b2),
  Clear(?b1) }
:effect {Holding(?b1),
  Clear(?b2),
  ¬Clear(?b1),
  ¬ArmEmpty(),
  ¬On(?b1, ?b2) }
```

# Planning Approaches

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- **State-space search:** [Bonet 2001] [Hoffman 2001] [Helmert 2006]
  - **Progression** (forward) or regression (backward)
  - **Best-first heuristic search** algorithms
- **Partial-order planning** [Penberthy 1992]
  - Search directly over plans (**plan-space**)
- Planning as **Satisfiability** [Kautz 1999]
  - Compile to **fixed-horizon SAT** instance
  - SAT is **NP-Complete**
  - Planning is **PSPACE-Complete**
  - **Increase horizon if formula unsatisfiable**

# Forward Best-First Search

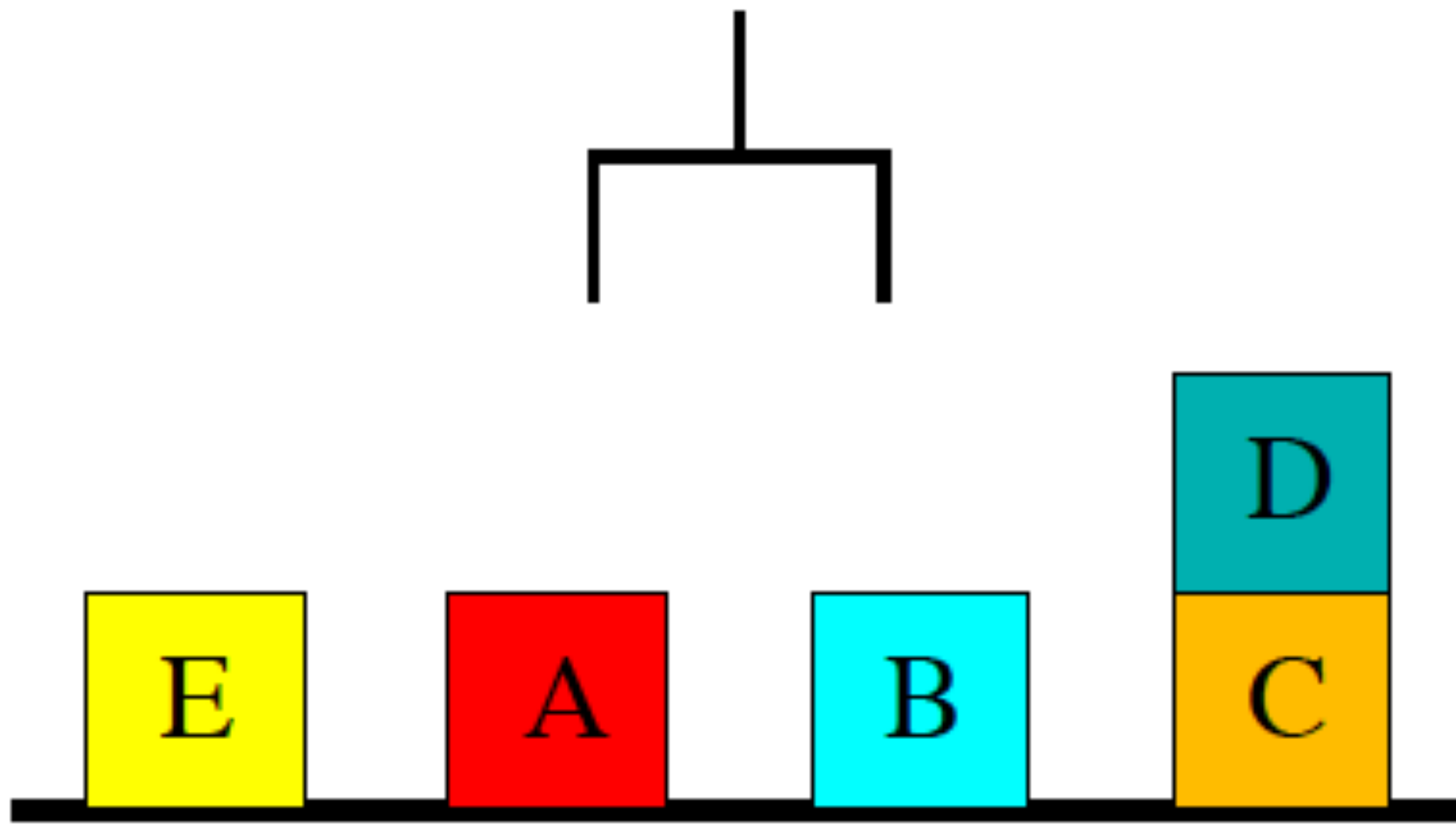
15

- For a state  $s$ 
  - Path **cost**:  $g(s)$
  - **Heuristic estimate**:  $h(s)$
  - Open list **sorted** by priority  $f(s)$
- **Weighted A\***:  $f(s) = g(s) + wh(s)$ 
  - Uniform cost search:  $w = 0 \implies f(s) = g(s)$
  - A\* search:  $w = 1 \implies f(s) = g(s) + h(s)$
  - **Greedy best-first search**:  $w = \infty \implies f(s) = h(s)$
- How do we estimate  $h(s)$ ?
  - No obvious metric (no metric-space embedding)

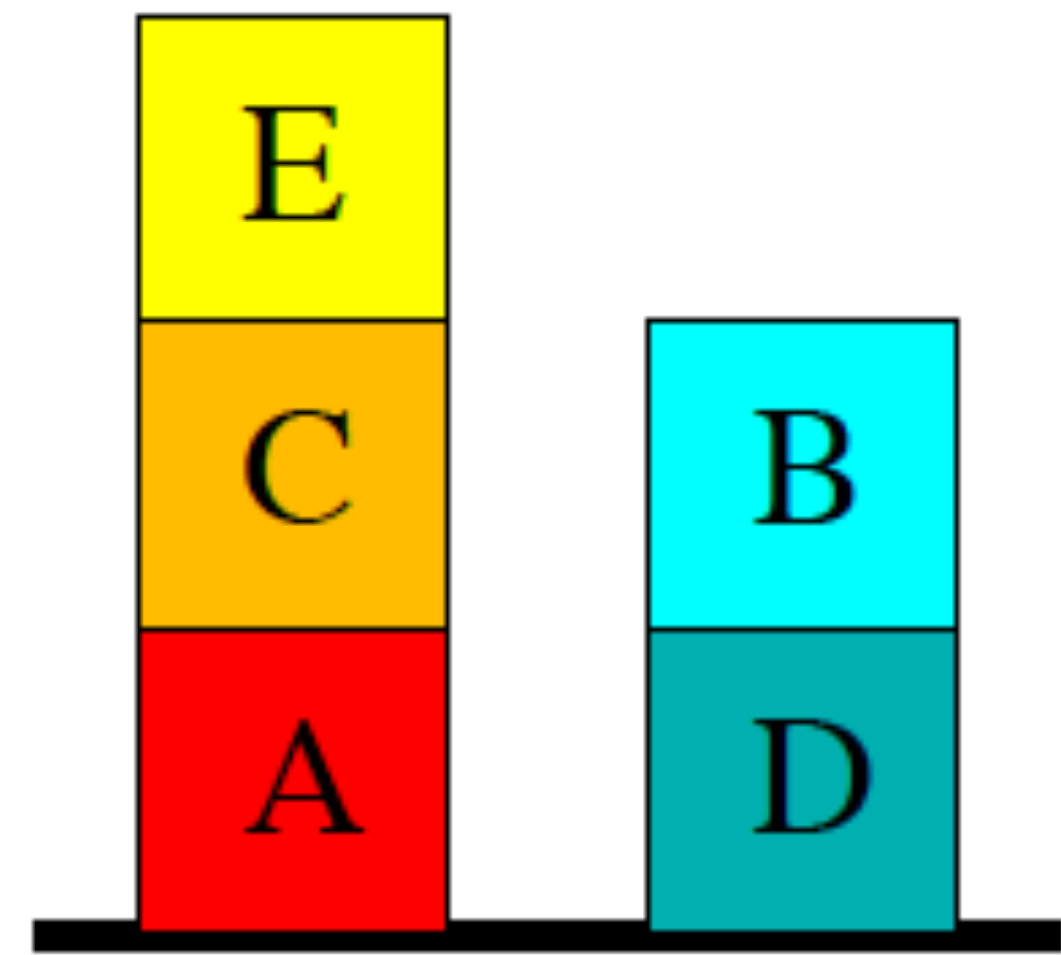
# Predict the Minimum Plan Length

16

- Can stack / unstack anywhere on the ground
- Hint: is an **even** number



Initial State



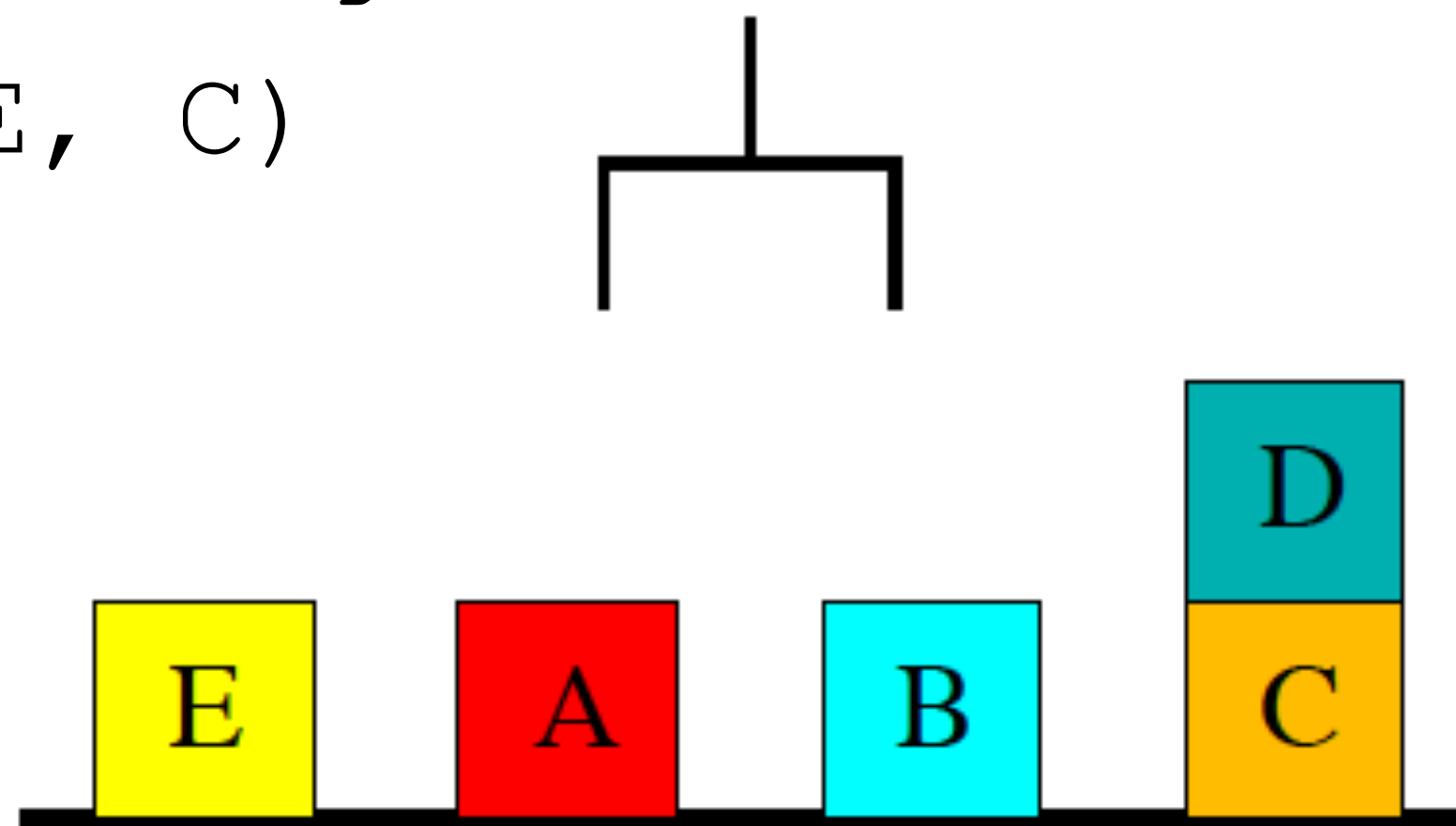
Goal State



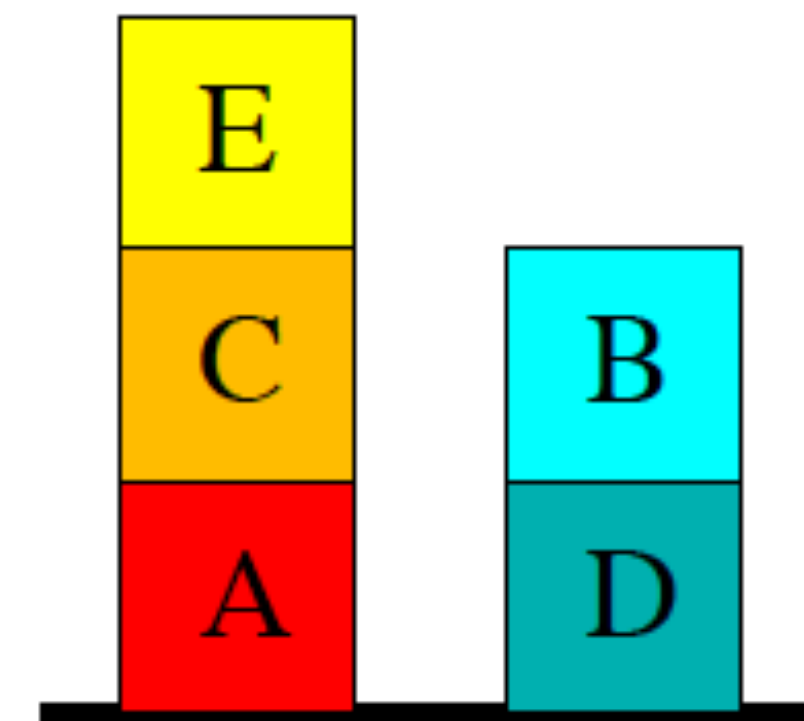
# Predict the Minimum Plan Length

17

- **Solution (length=6):**
  - **unstack** (D, C)
  - **stack** (D, B)
  - **unstack** (C, ground)
  - **stack** (C, A)
  - **unstack** (E, ground)
  - **stack** (E, C)



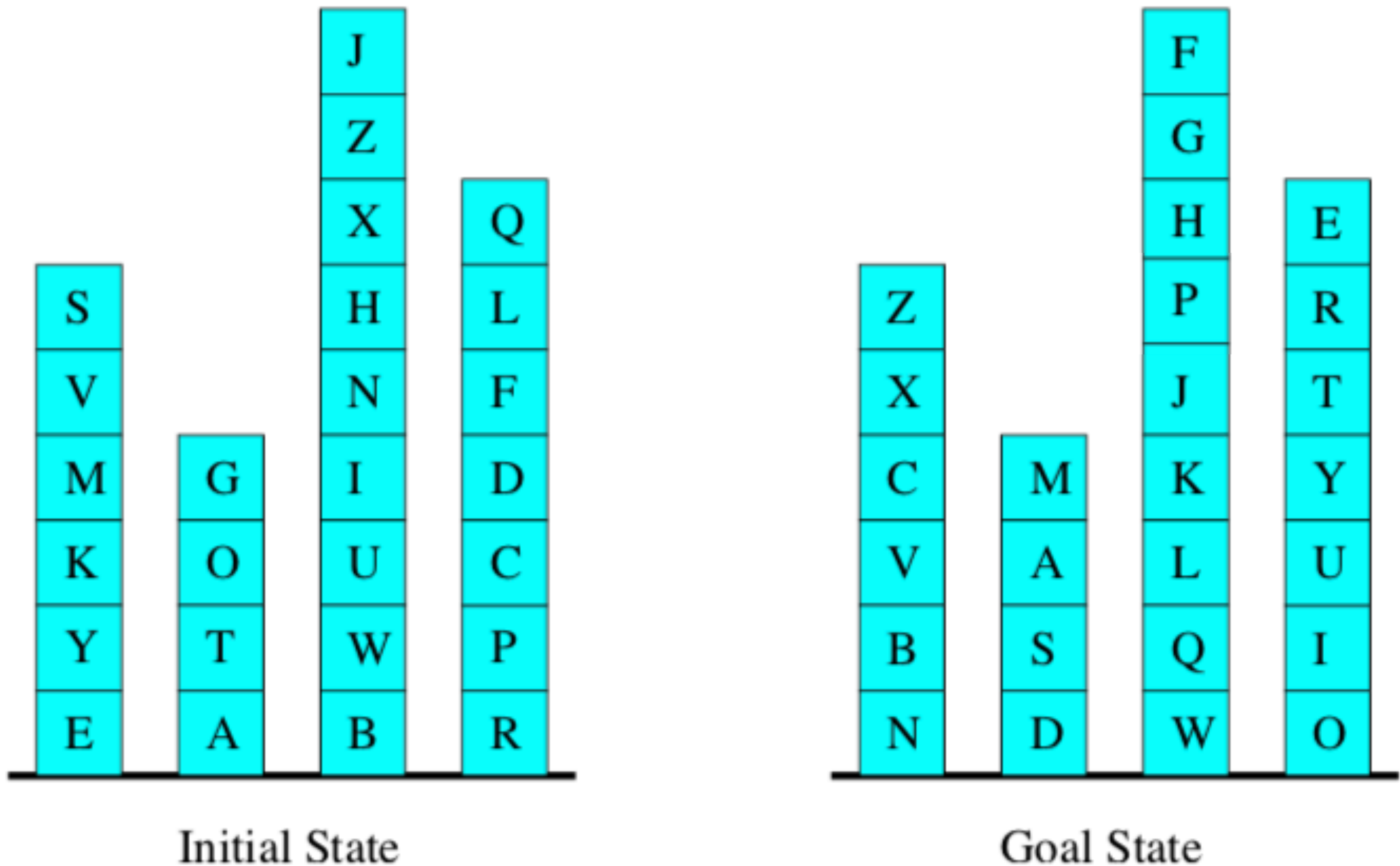
Initial State



Goal State

# Predict the Minimum Plan Length

18



# Domain-Independent Heuristics

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- Estimating  $h(s)$  is **nontrivial**
- Can we do it in an a **domain-independent** manner?
- Solve a related, **approximate** planning problem
  - Primary focus for almost all of classical planning
- Suggestions for how to do this?
  - **Independently** plan for each goal
  - **Remove** some action preconditions [Helmert 2006]
  - Remove negative (**delete**) **effects** [Bonet 2001] [Hoffman 2001]
  - ...

# Delete-Relaxation Heuristics

20

- Remove all negative ( $\neg$ ) effects
- Solving optimally is **NP-Complete**
- Can greedily find a short plan in polynomial time
- Basis for both **admissible** and **greedier**, non-admissible heuristics

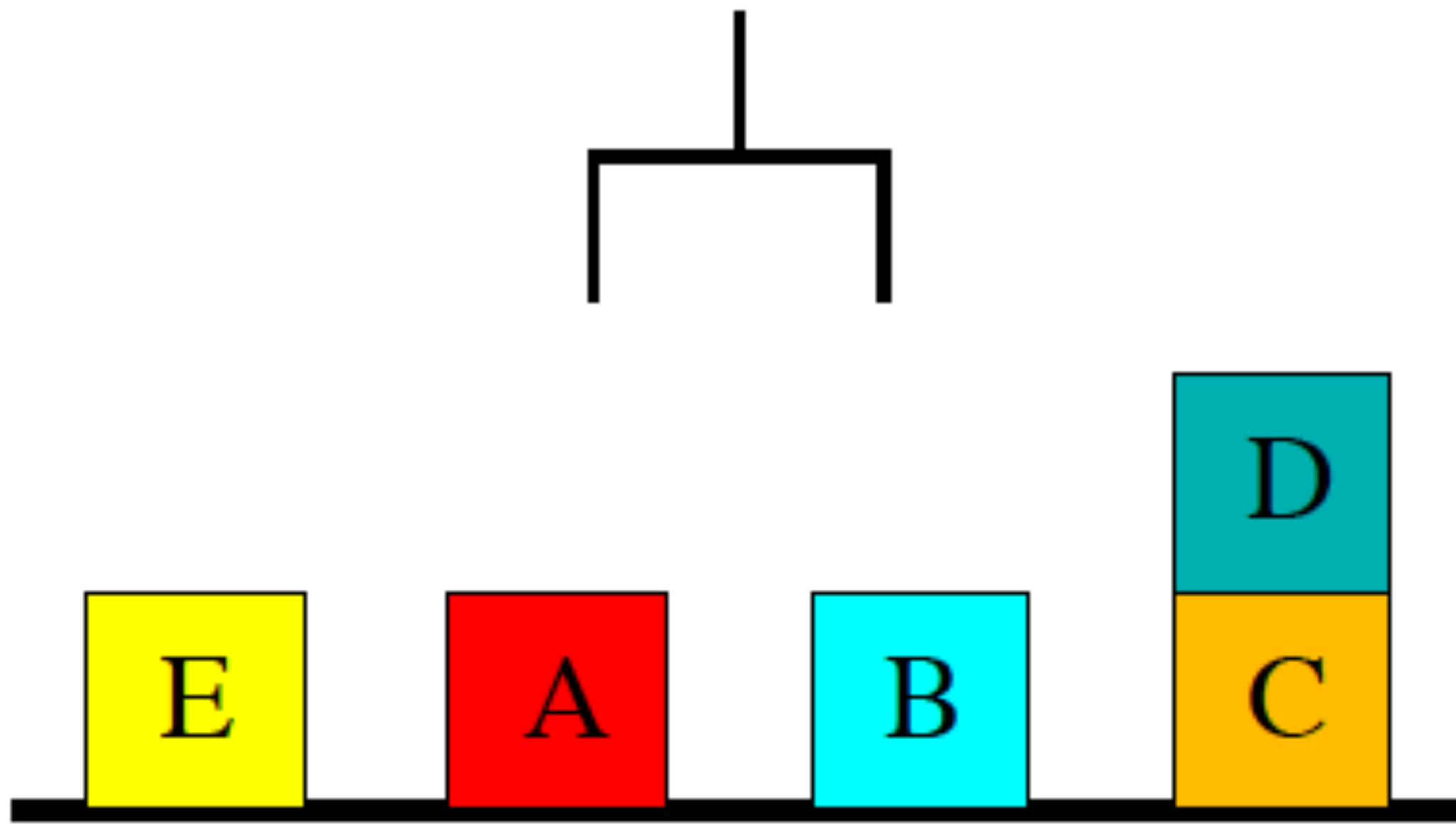
```
(:action stack
:parameters (?b1, ?b2)
:precondition {
  Holding(?b1), Clear(?b2) }
:effect {ArmEmpty(),
  On(?b1, ?b2),
  Clear(?b1)
¬Holding(?b1),
¬Clear(?b2)}
```

```
(:action unstack
:parameters (?b1, ?b2)
:precondition {ArmEmpty(),
  On(?b1, ?b2),
  Clear(?b1) }
:effect {Holding(?b1),
  Clear(?b2),
¬Clear(?b1),
¬ArmEmpty(),
¬On(?b1, ?b2)}
```

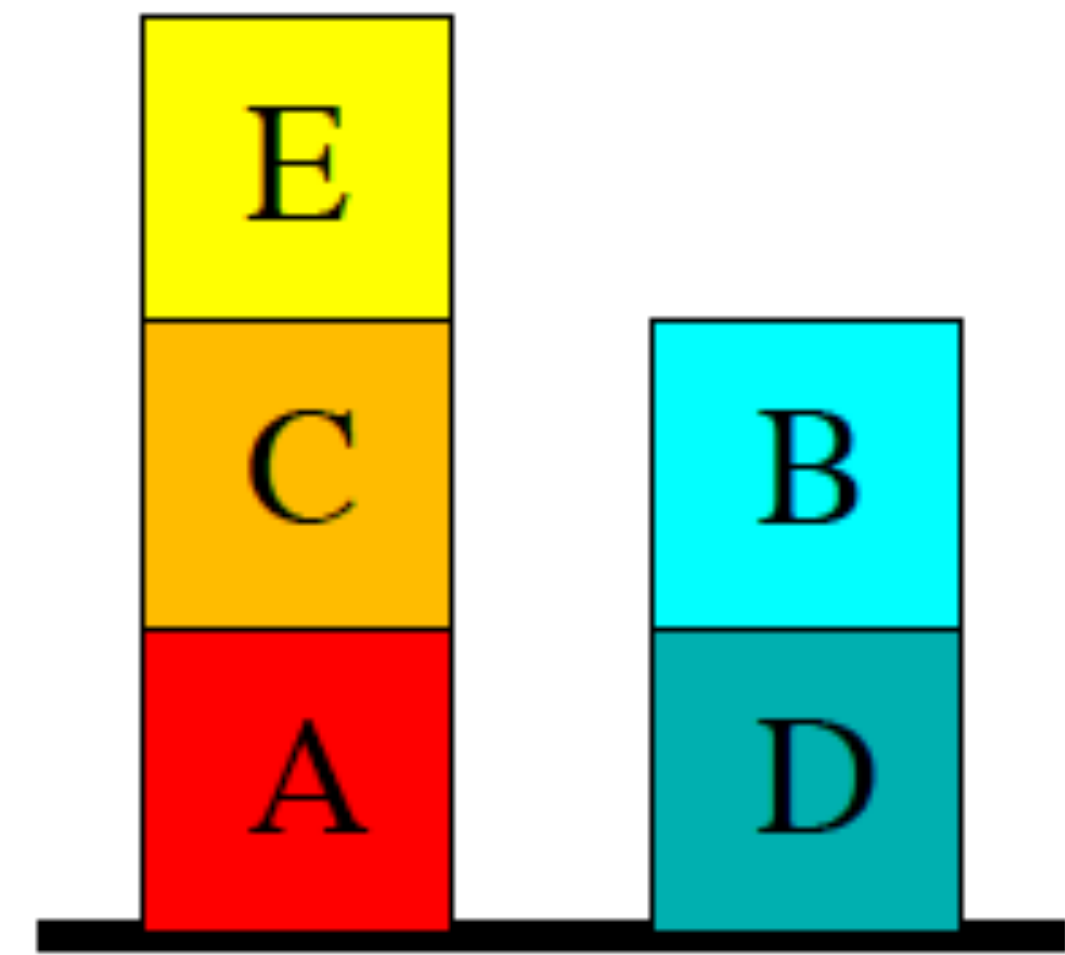
# Predict the Minimum Delete-Relaxed Plan Length

21

- Can stack / unstack anywhere on the ground
- Hint: is **no greater** than 6



Initial State

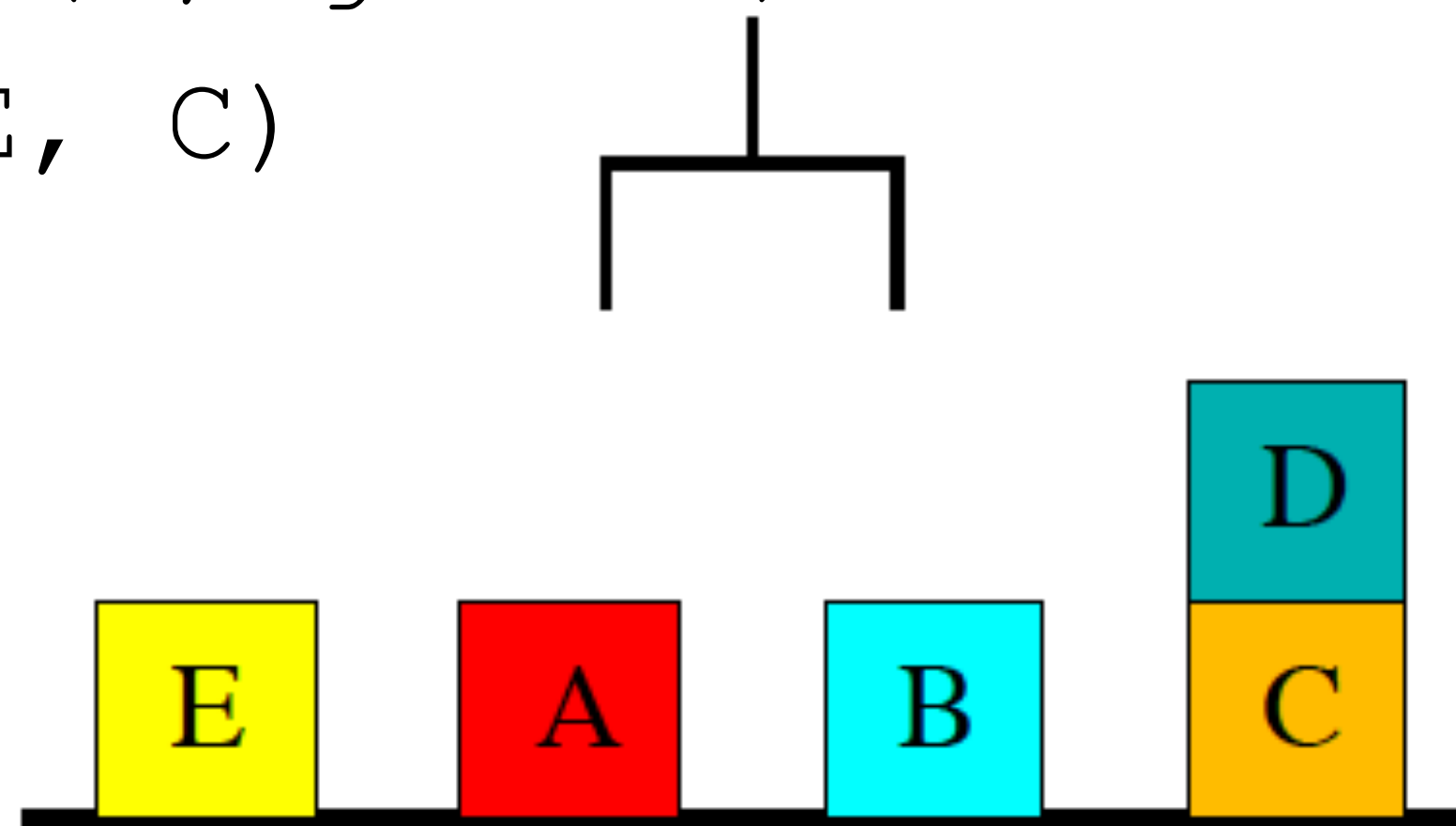


Goal State

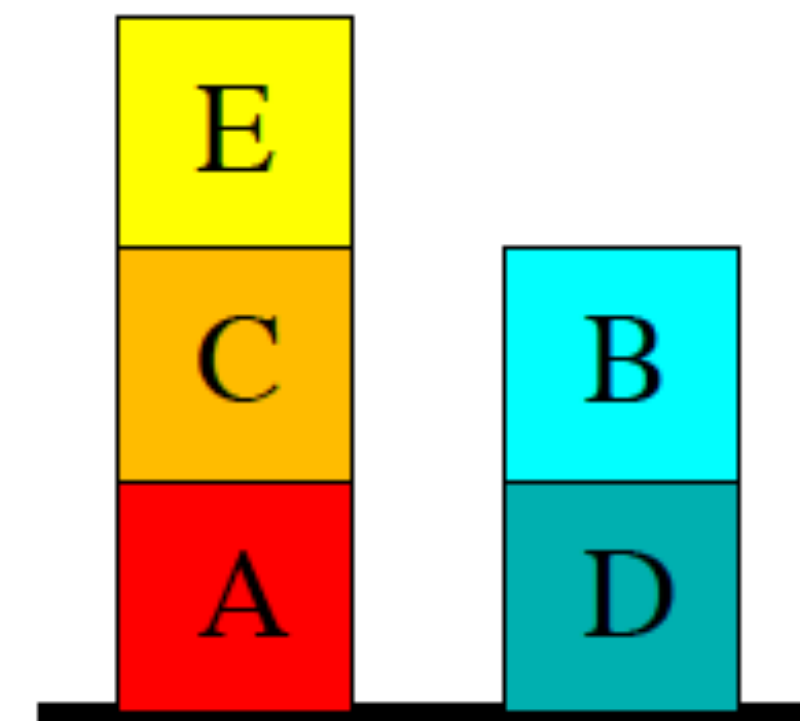
# Predict the Minimum Delete-Relaxed Plan Length

22

- **Solution (length=6):**
  - **unstack** (D, C)
  - **stack** (D, B)
  - **unstack** (C, ground)
  - **stack** (C, A)
  - **unstack** (E, ground)
  - **stack** (E, C)



Initial State

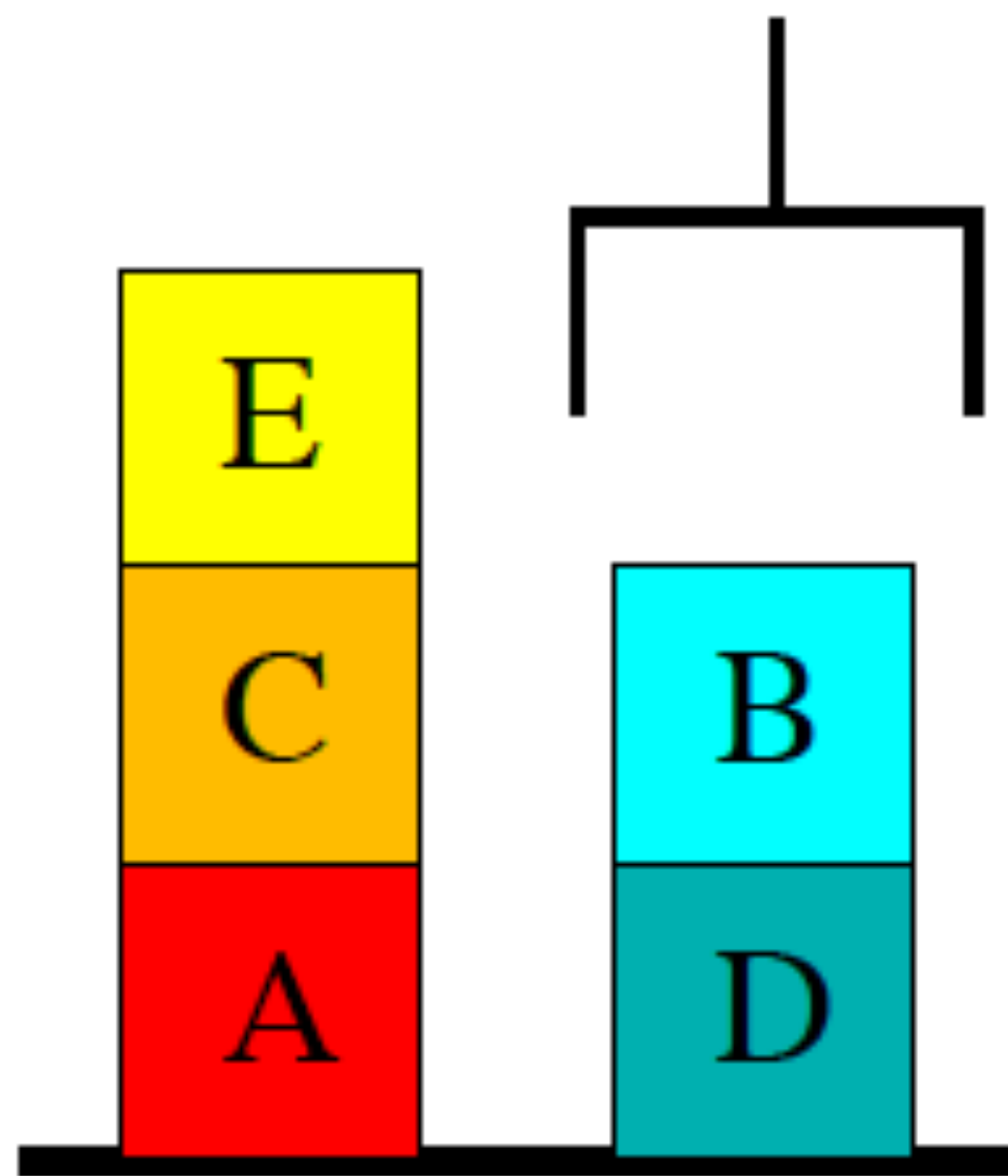


Goal State

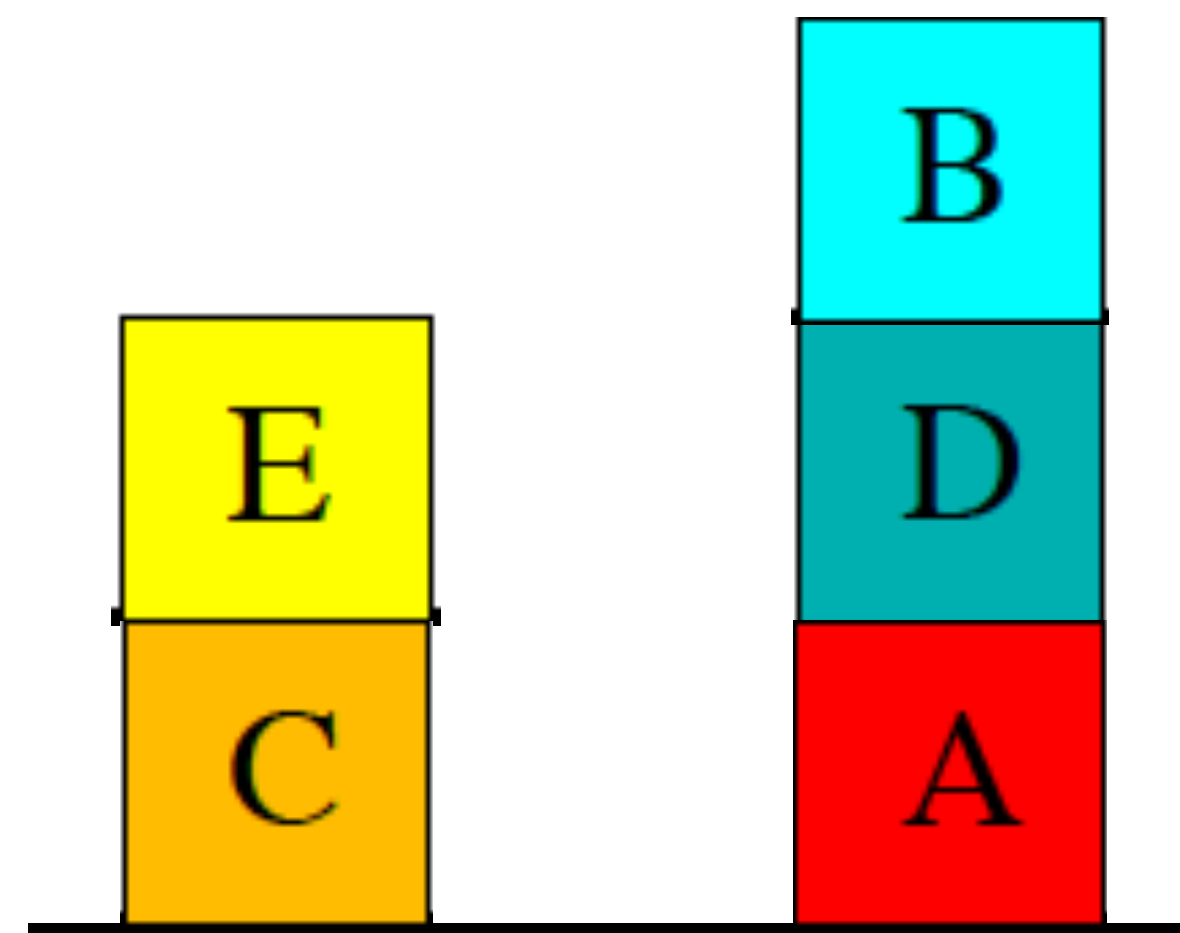
# Predict the Minimum Plan Length

23

- Can **stack** / **unstack** anywhere on the ground
- Hint: is an **even** number



Initial State



Goal State

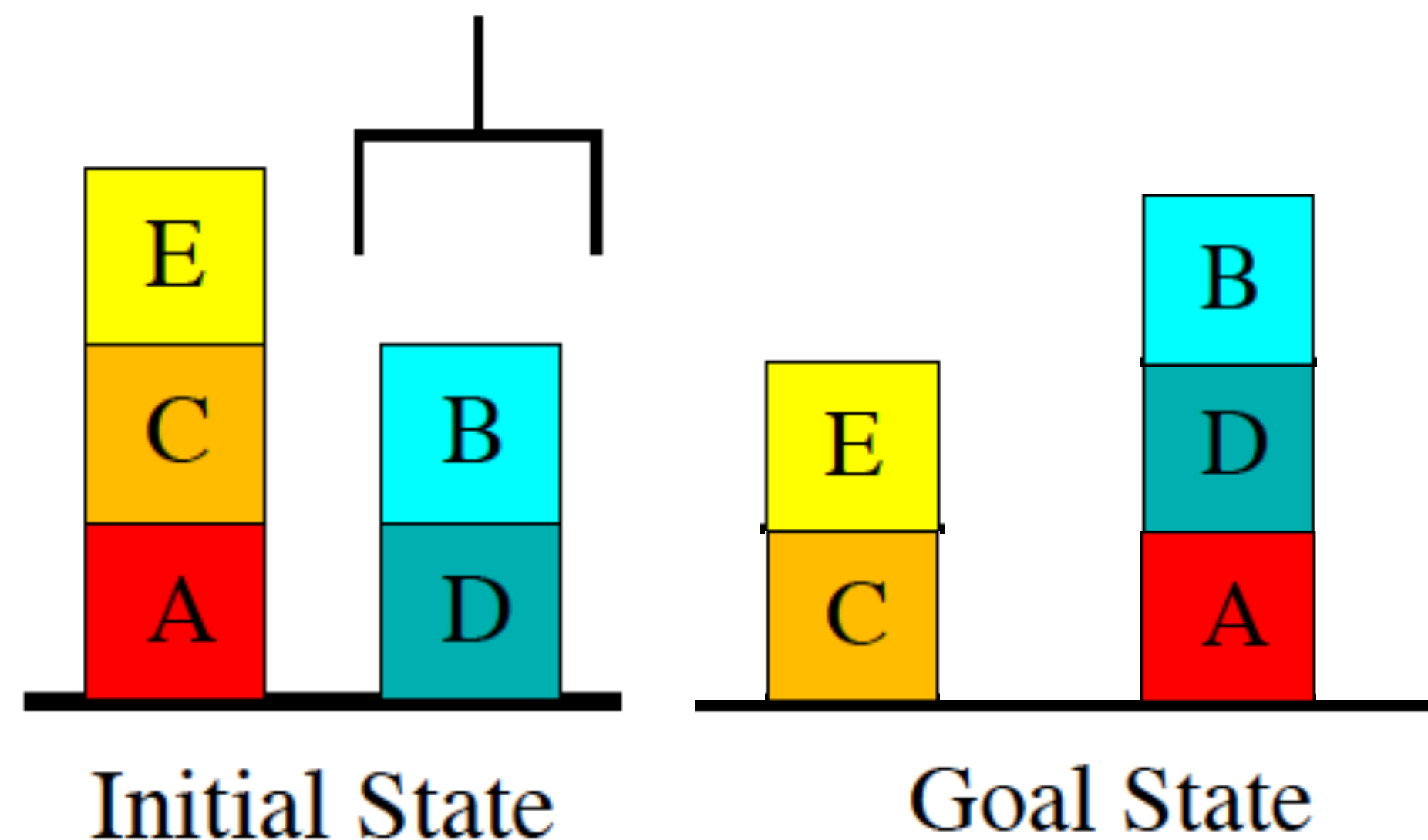
# Predict the Minimum Plan Length

24

## ■ Solution (length=12):

- **unstack** (E, C)
- **stack** (E, ground)
- **unstack** (C, A)
- **stack** (C, ground)
- **unstack** (E, ground)
- **stack** (E, C)
- **unstack** (B, D)
- **stack** (B, ground)

- **unstack** (D, ground)
- **stack** (D, A)
- **unstack** (B, ground)
- **stack** (B, D)

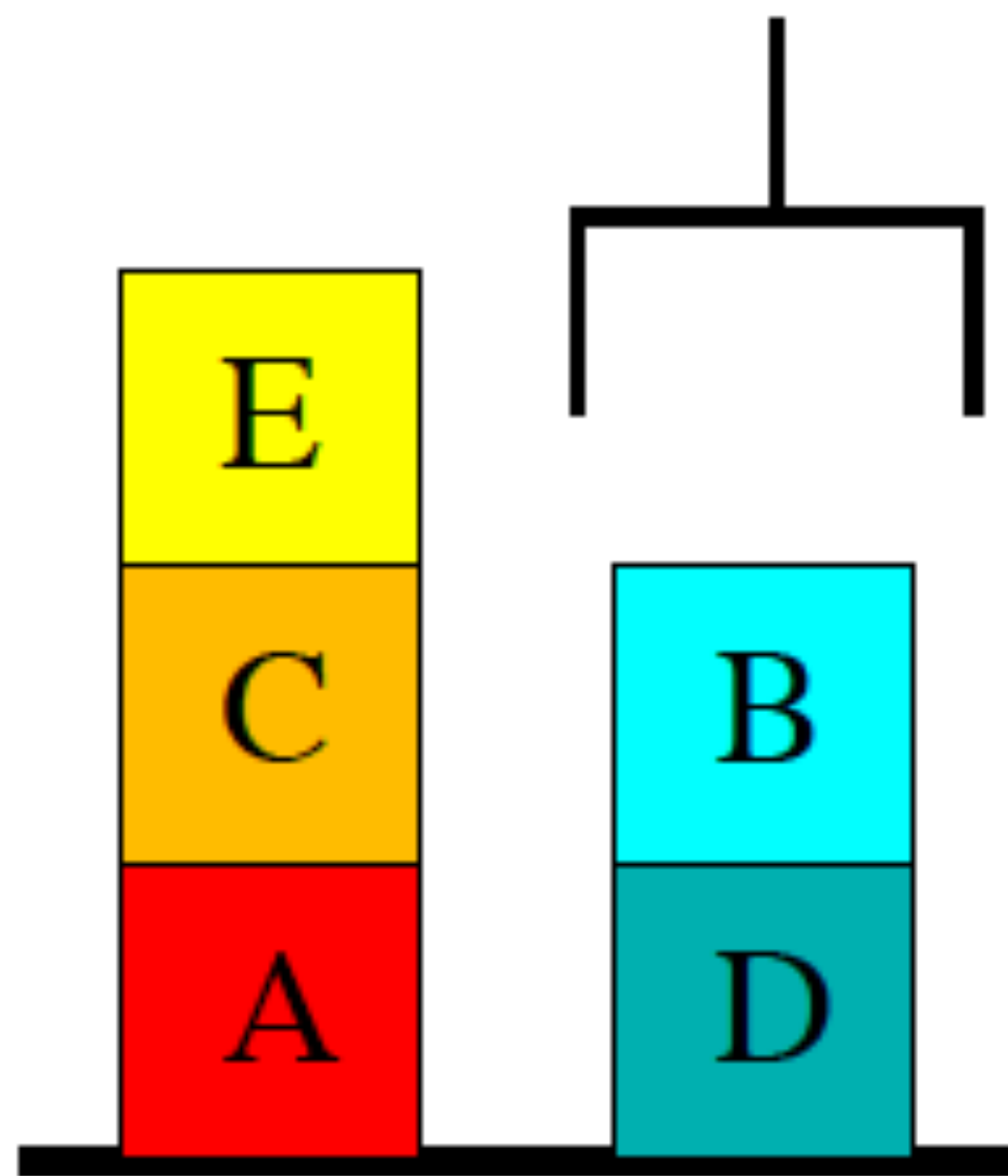




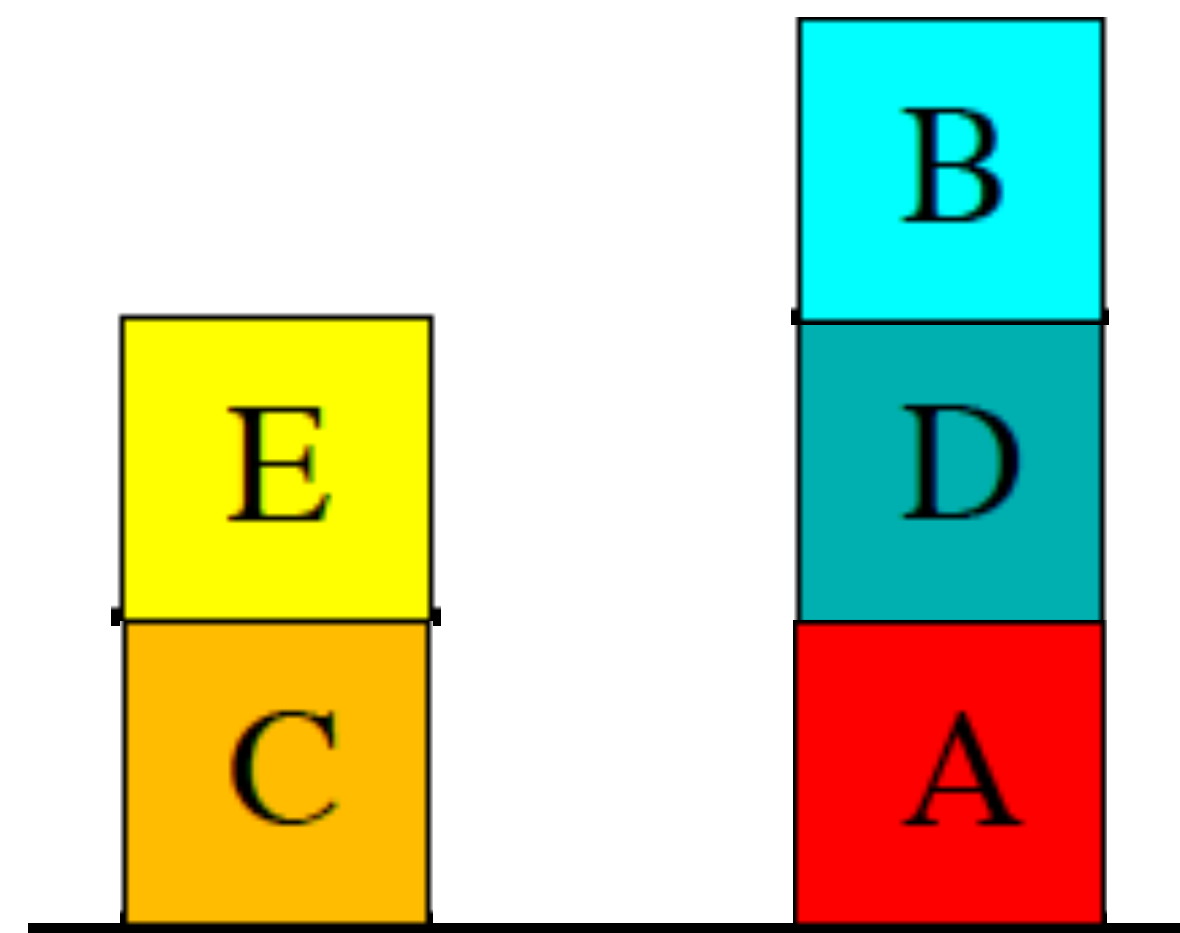
# Predict the Minimum Delete-Relaxed Plan Length

25

- Can stack / unstack anywhere on the ground
- Hint: is **no greater** than 12



Initial State

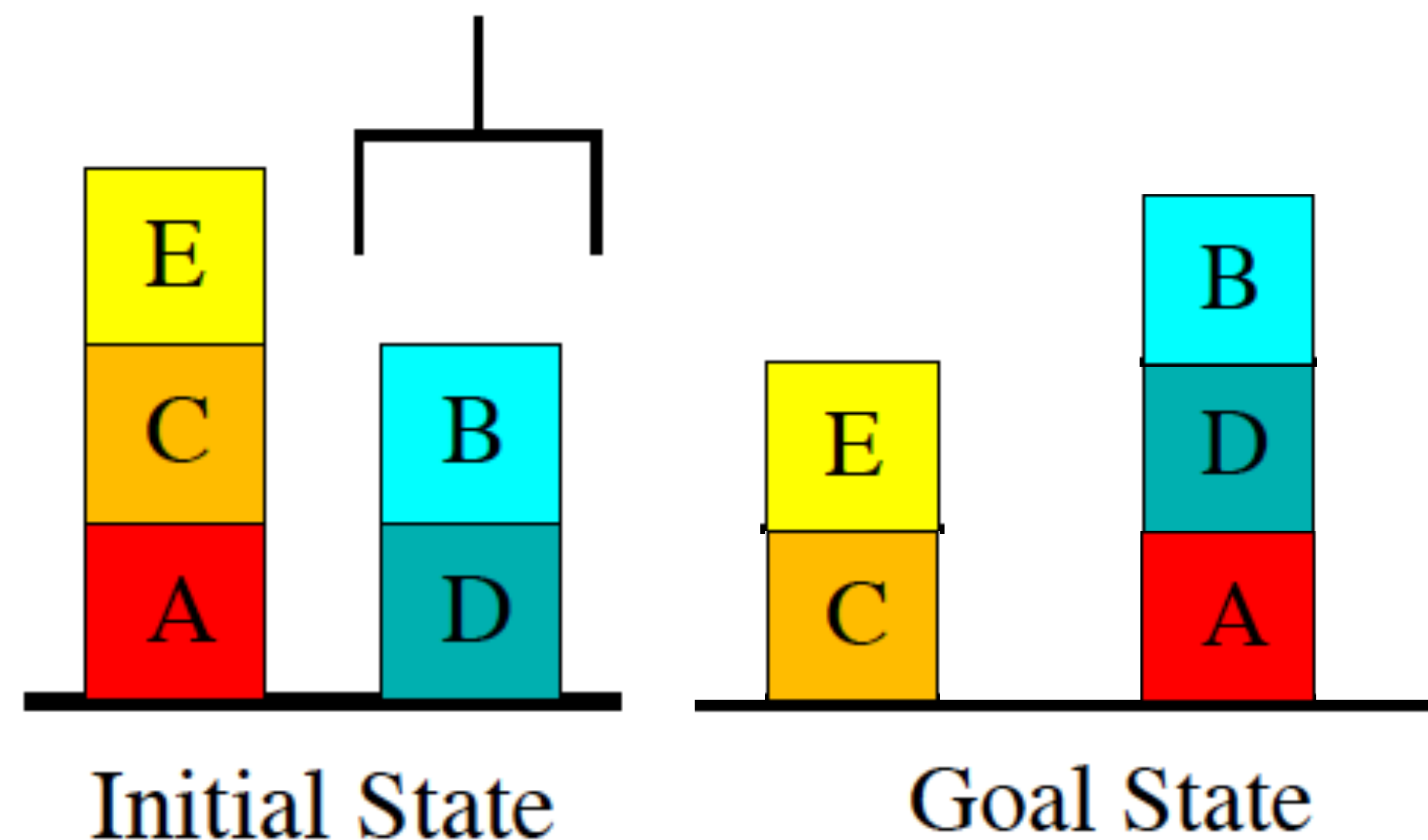


Goal State

# Predict the Minimum Delete-Relaxed Plan Length

26

- **Solution (length=5):**
  - **unstack** (E, C)
  - **unstack** (C, A)
  - **unstack** (B, D)
  - **unstack** (D, ground)
  - **stack** (D, A)



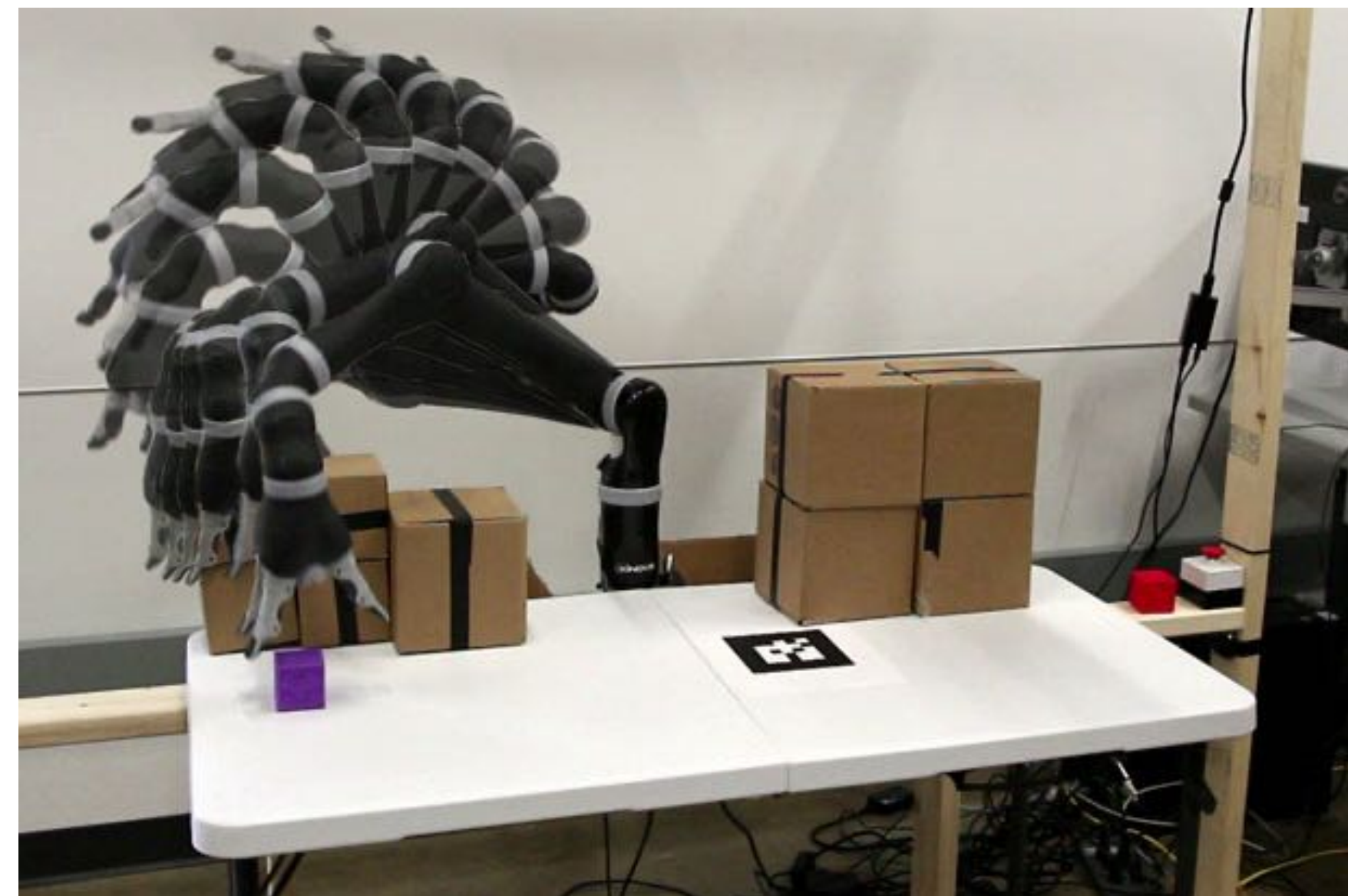


# Motion Planning

# Review: Motion Planning

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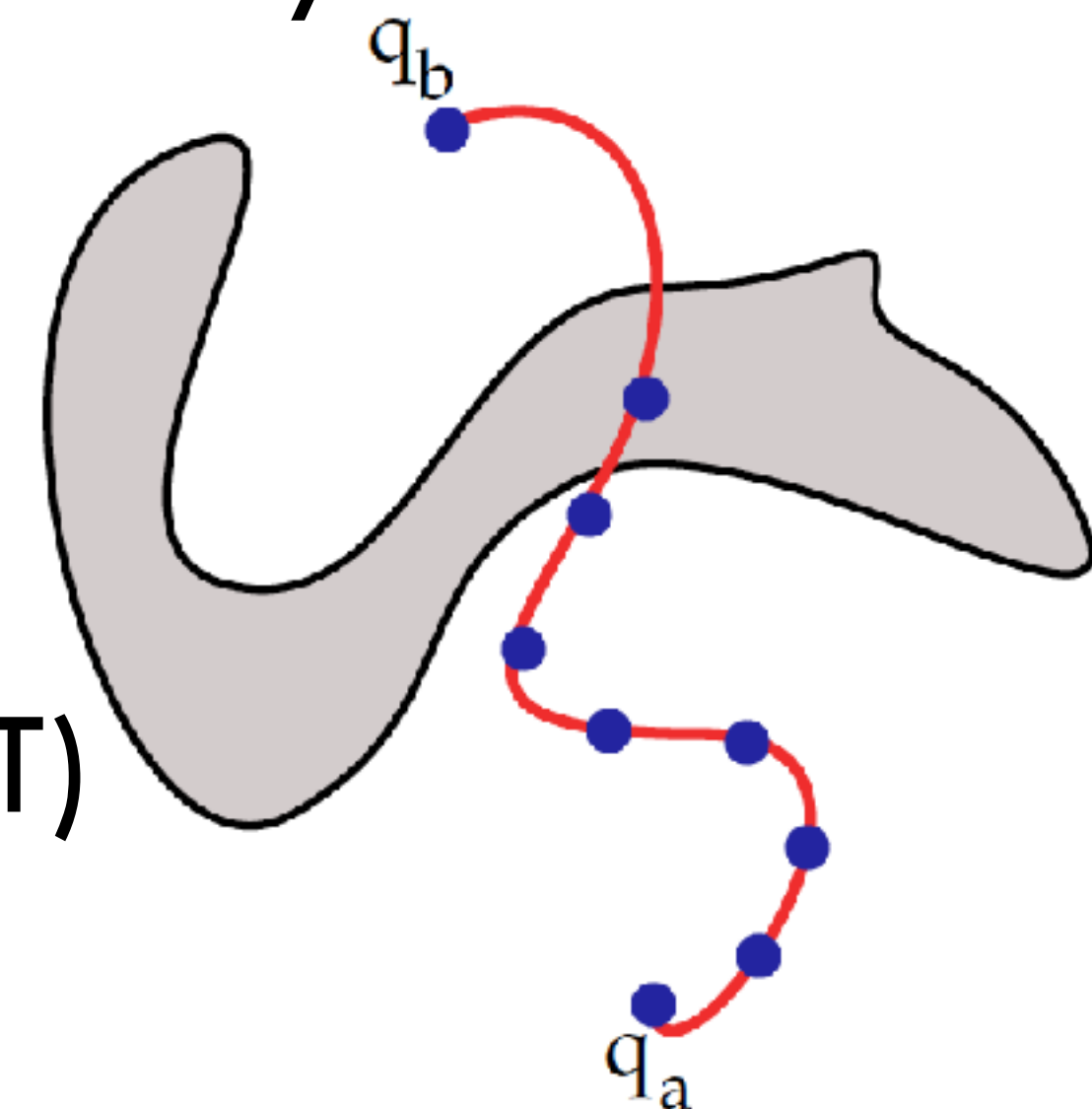
- Plan a **path** for a robot from an initial configuration to a goal configuration that **avoids obstacles**
  - Sequence of **continuous** configurations
  - Configurations often are **high-dimensional**
    - Example: 7 DOFs
- High-level approaches:
  - Geometric decomposition
  - **Sampling-based**
  - Optimization-based



# Sampling-Based Motion Planning

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- **Discretize** configuration space by **sampling**
  - Sampling be deterministic or **random**
- **Implicitly** represent the collision-free configuration space using an blackbox **collision checker**
- **Abstracts away** complex robot geometry
- Algorithms
  - **Probabilistic Roadmap (PRM)**
  - Rapidly-Exploring Random Tree (RRT)
  - Bidirectional RRT (BiRRT)

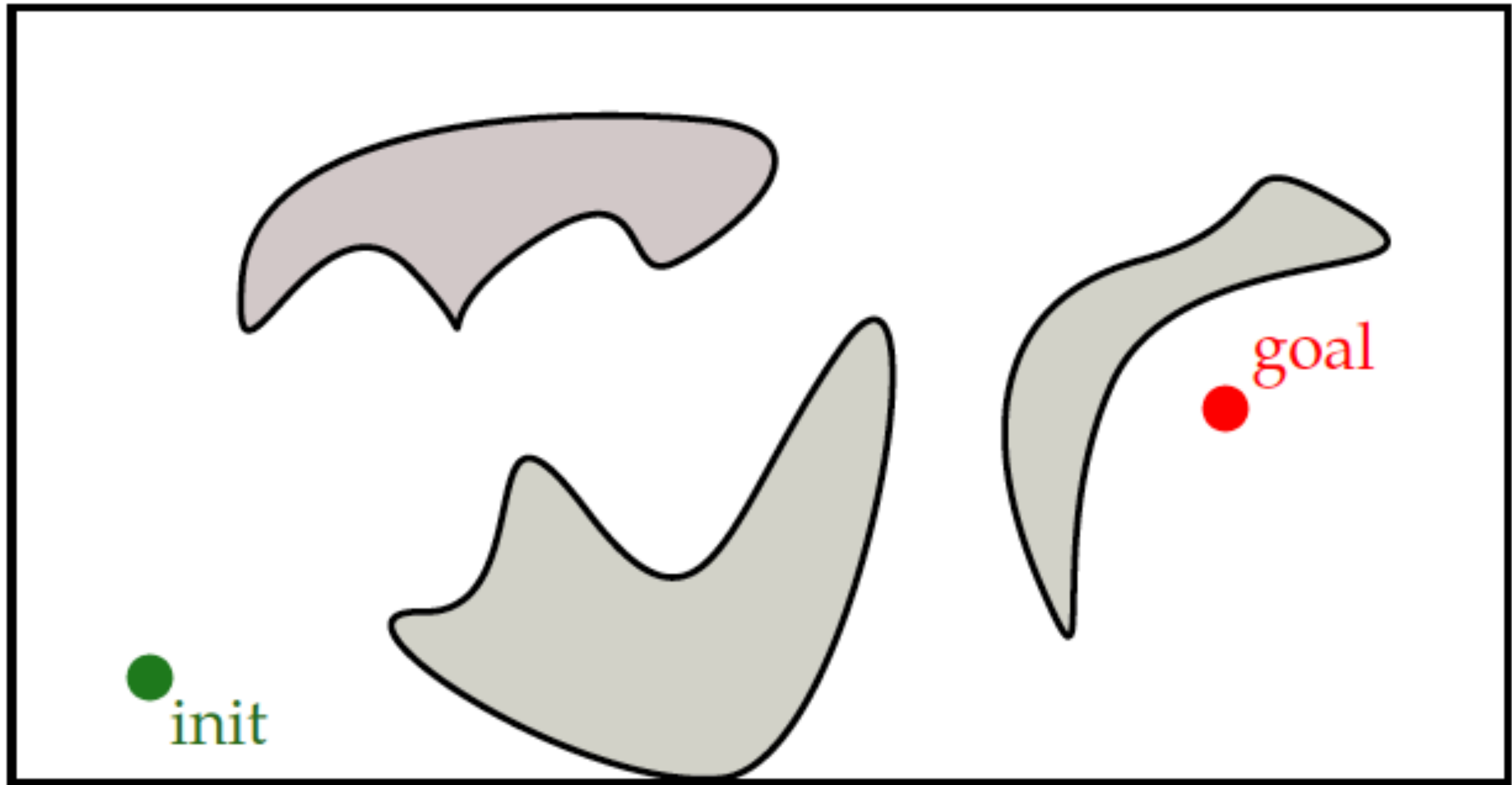


[Kavraki 1994][Kuffner 2000][LaValle 2006]

[Fig from Erion Plaku]

# Probabilistic Roadmap (1 / 7)

30

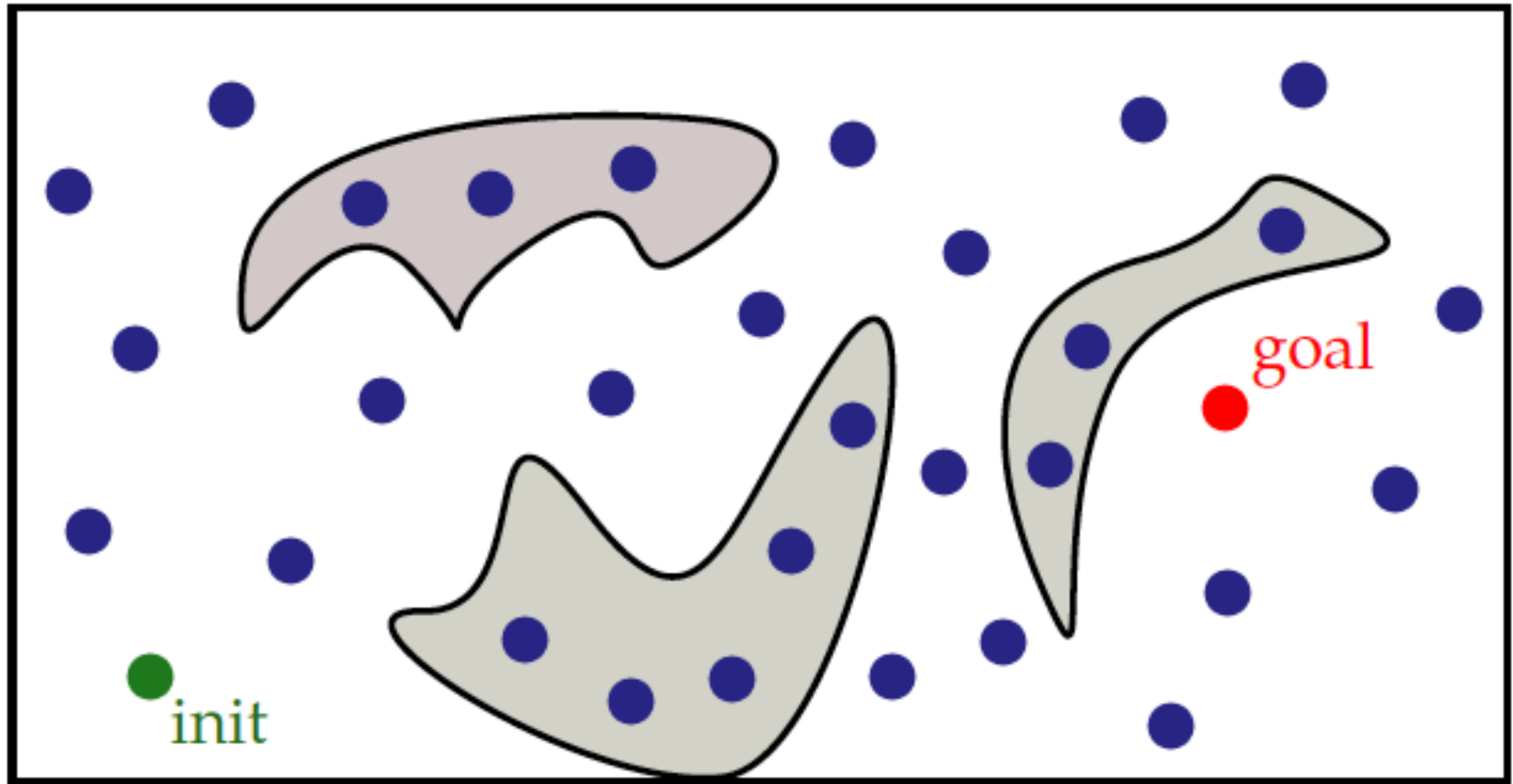


[Fig from Erion Plaku]

Find a path from **init** to **goal** that avoids the obstacles

# Probabilistic Roadmap (2/7)

31

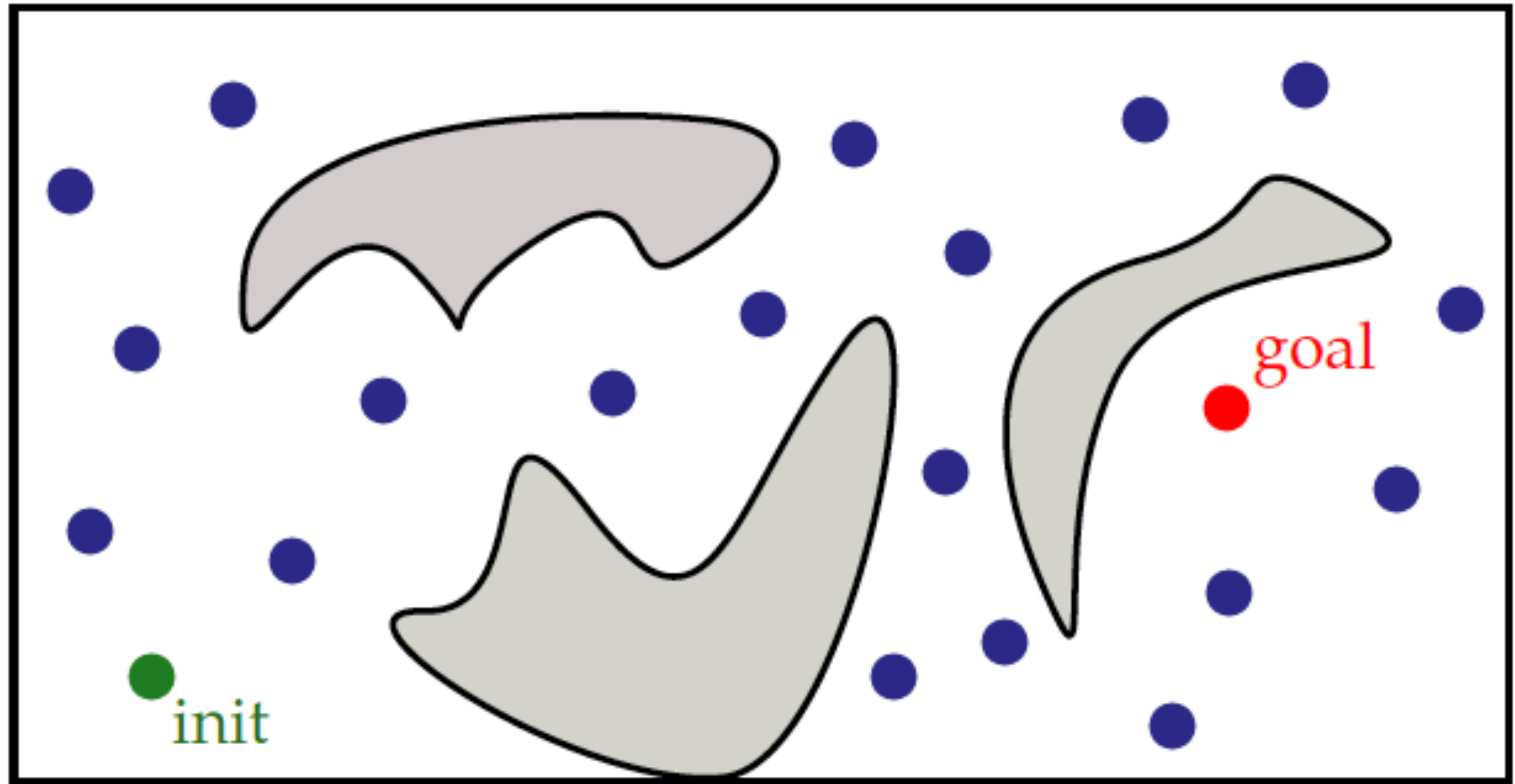


[Fig from Erion Plaku]

Sample a set of **configurations**

# Probabilistic Roadmap (3/7)

32



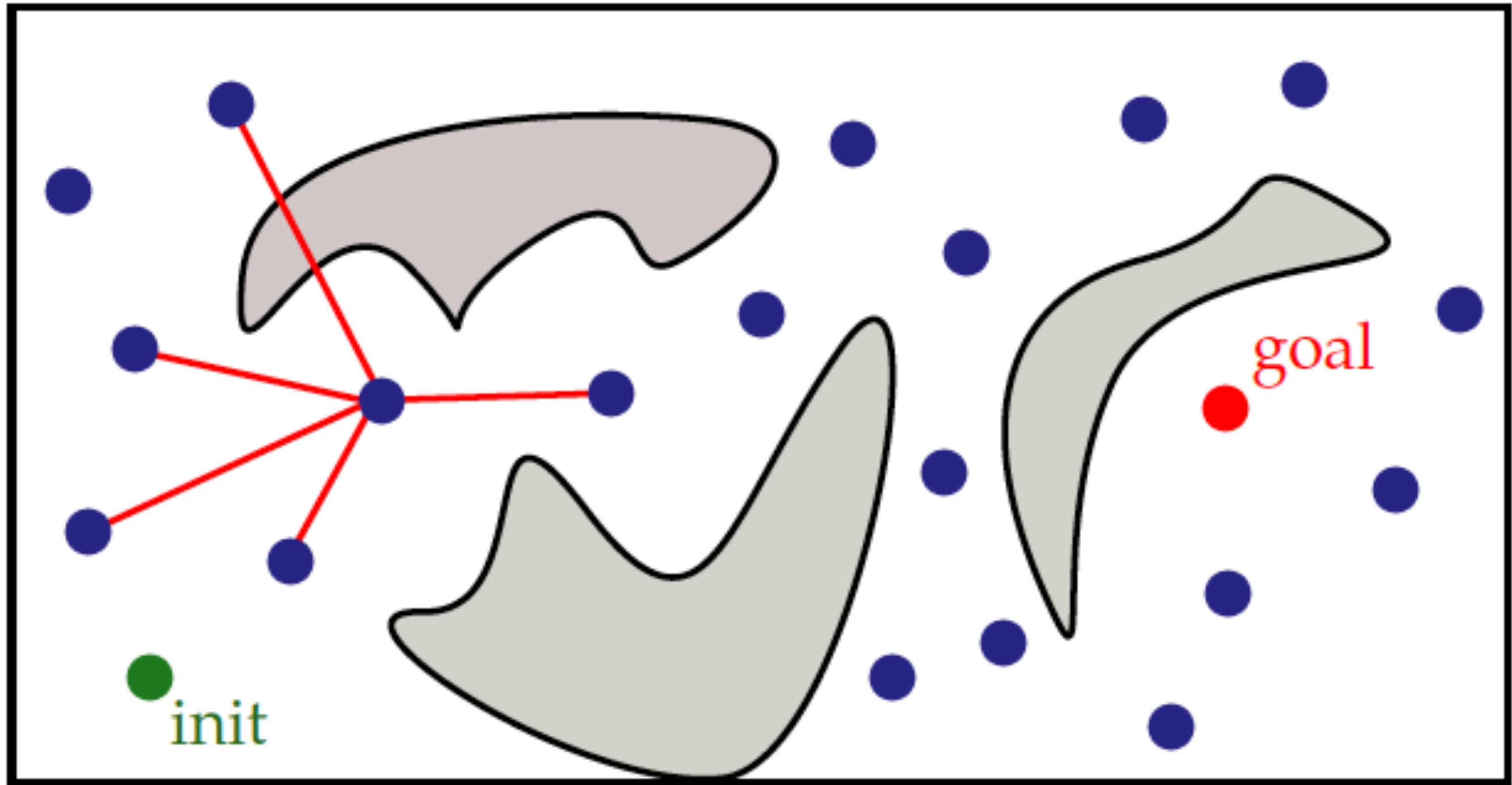
[Fig from Erion Plaku]

Remove configurations that collide with the obstacles



# Probabilistic Roadmap (4/7)

33

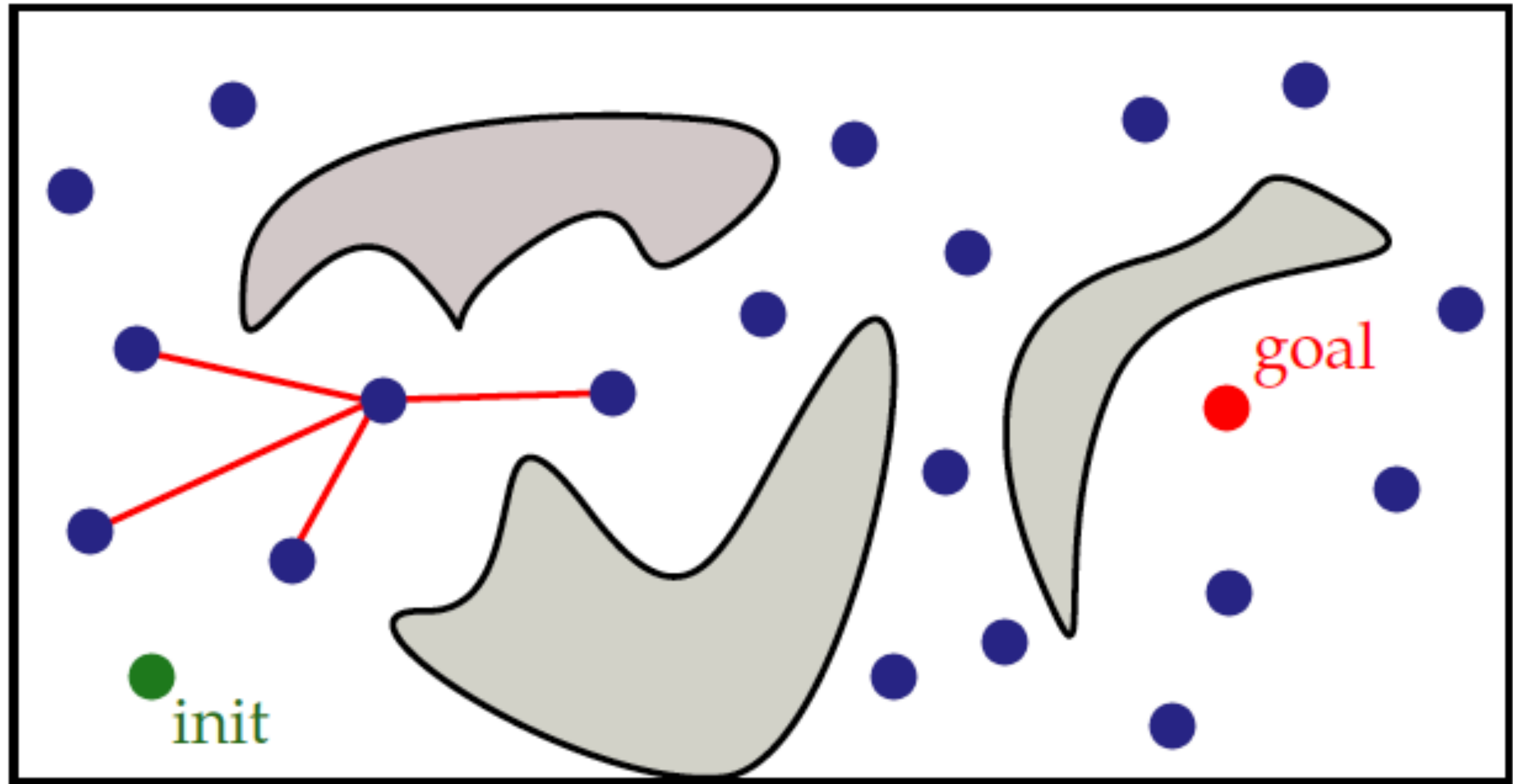


[Fig from Erion Plaku]

**Connect** nearby configurations

# Probabilistic Roadmap (5/7)

34

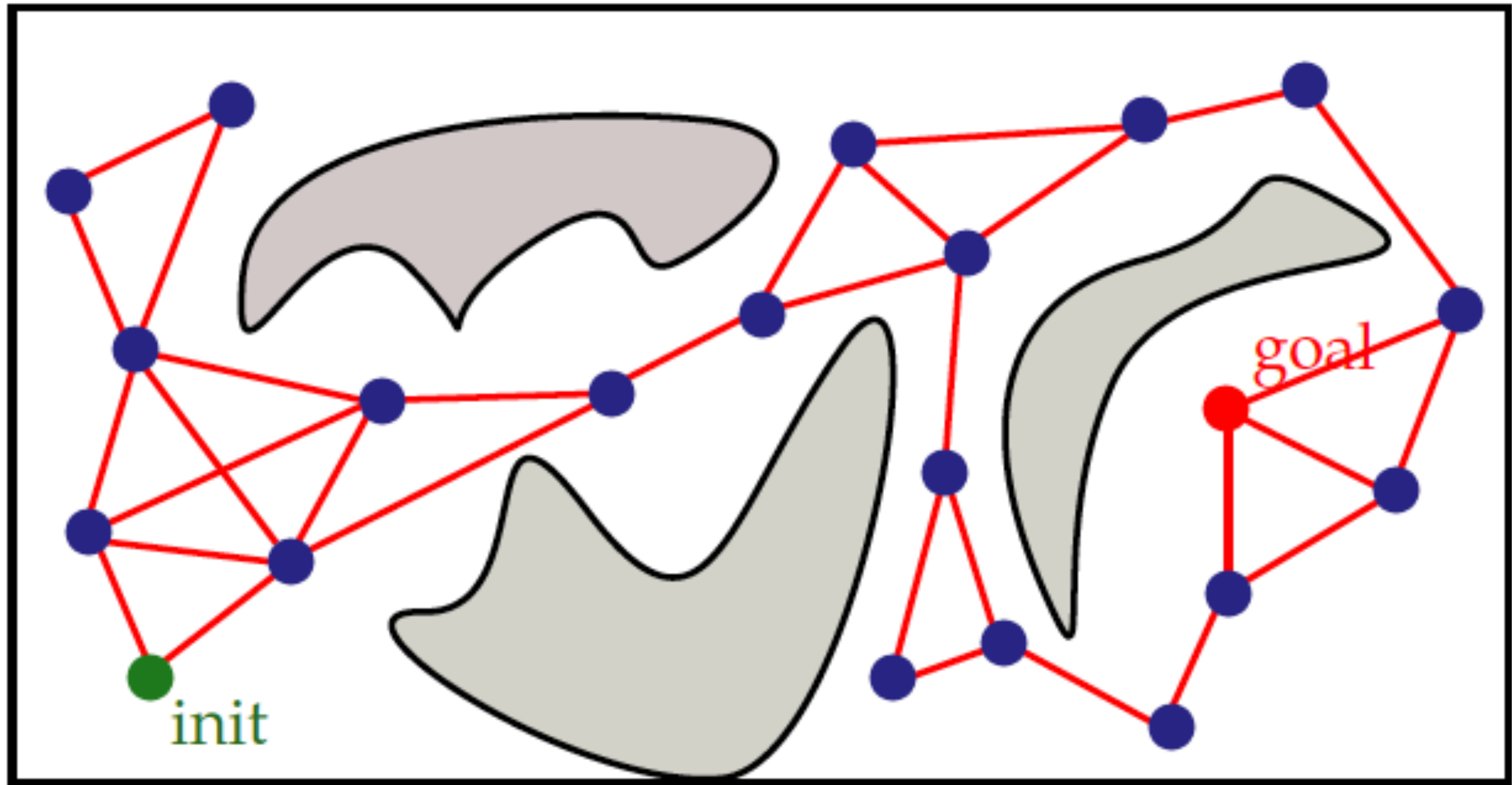


[Fig from Erion Plaku]

Prune **connections** that collide with the obstacles

# Probabilistic Roadmap (6/7)

35

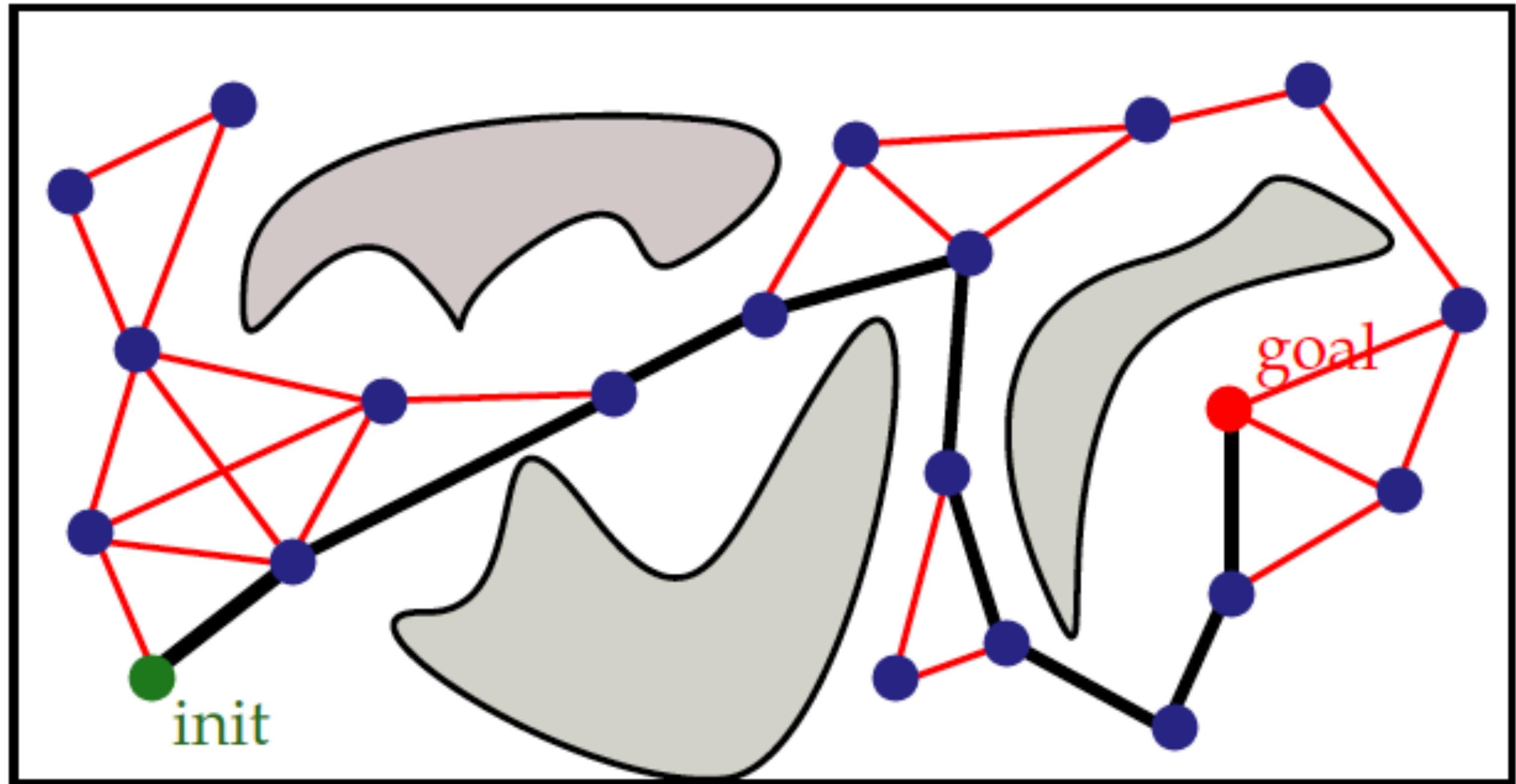


[Fig from Erion Plaku]

The resulting structure is a finite roadmap (graph)

# Probabilistic Roadmap (7/7)

36



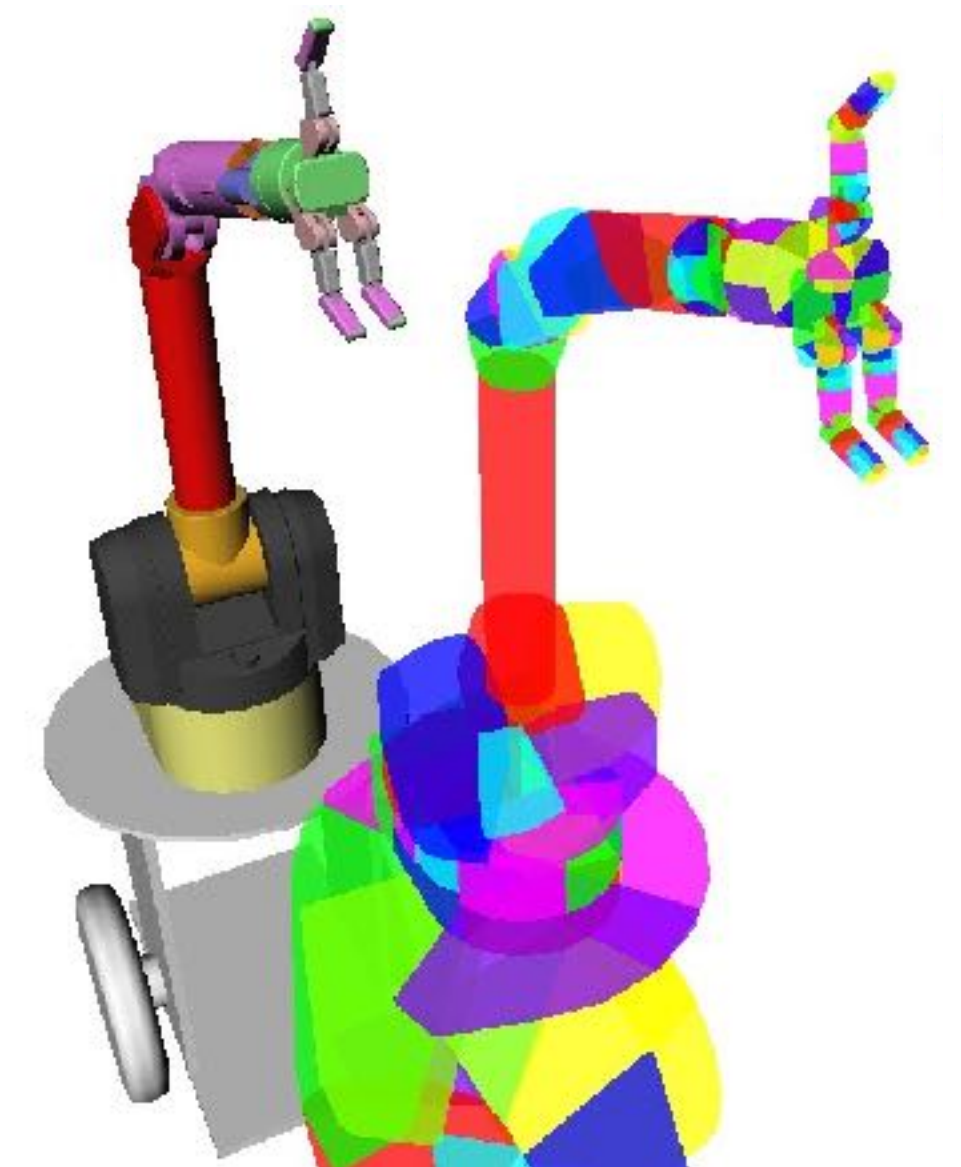
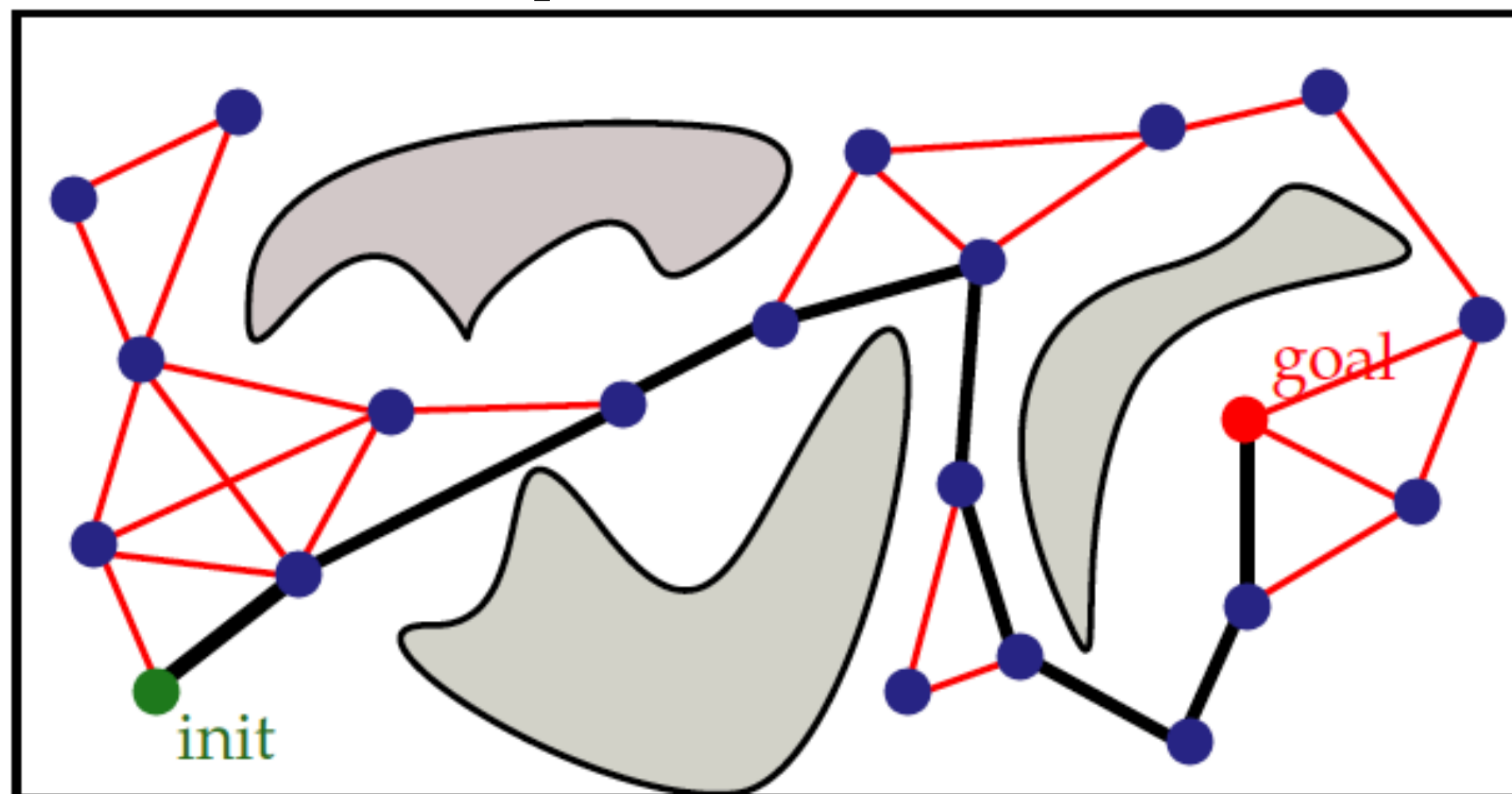
[Fig from Erion Plaku]

Search for the shortest-path on the roadmap

# Collision Checking is Expensive

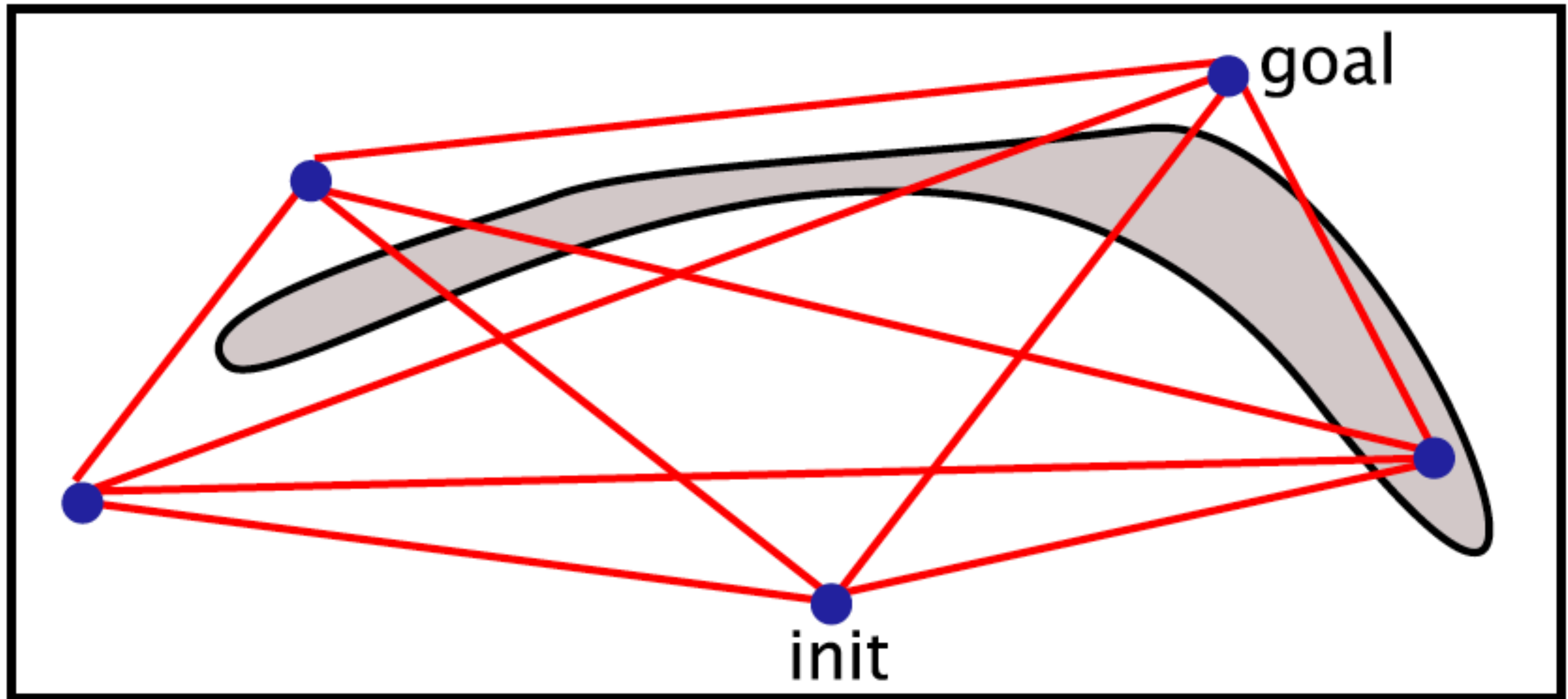
37

- Collision checking **dominates** runtime
  - **Complex geometries & fine resolutions** (for safety)
- Many edges clearly do not lie on a low-cost path
- **Optimistically plan without collisions**
- Check collisions **lazily** only by only evaluating **candidate plans**



# Lazy PRM (1/10)

38

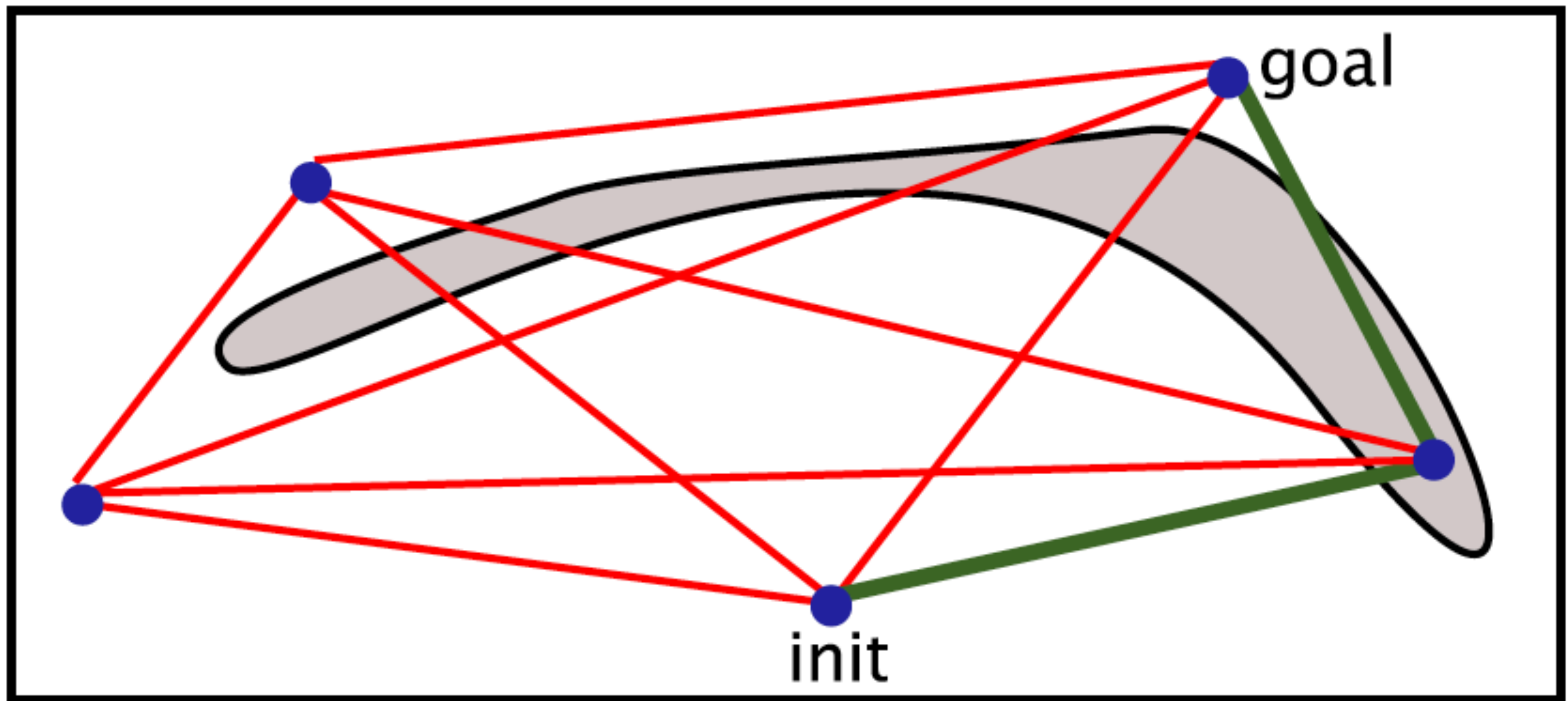


[Fig from Erion Plaku]

Construct a PRM ignoring collisions

# Lazy PRM (2/10)

39

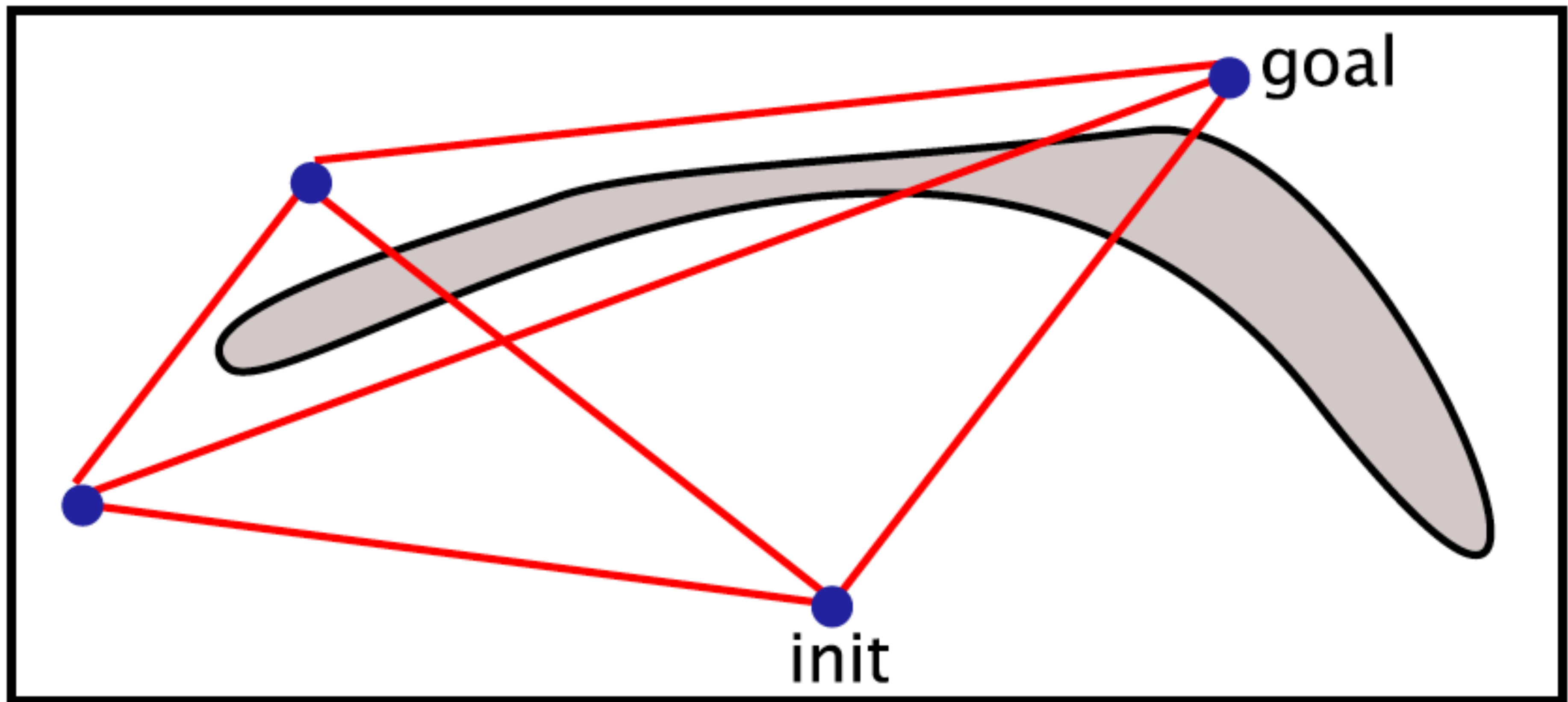


[Fig from Erion Plaku]

Search for the **shortest-path** on the roadmap

# Lazy PRM (3/10)

40



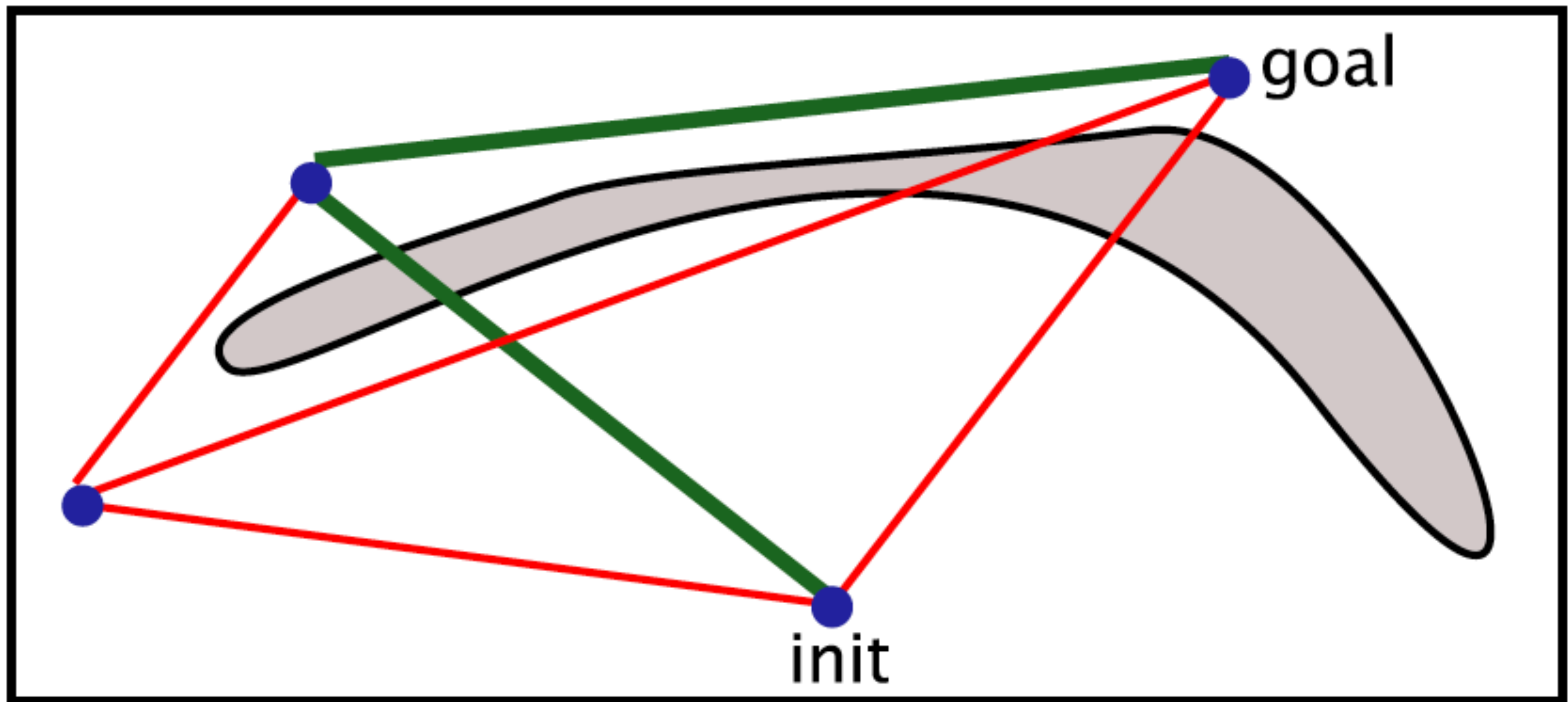
[Fig from Erion Plaku]

Remove plan edges that collide with obstacles



# Lazy PRM (4/10)

41

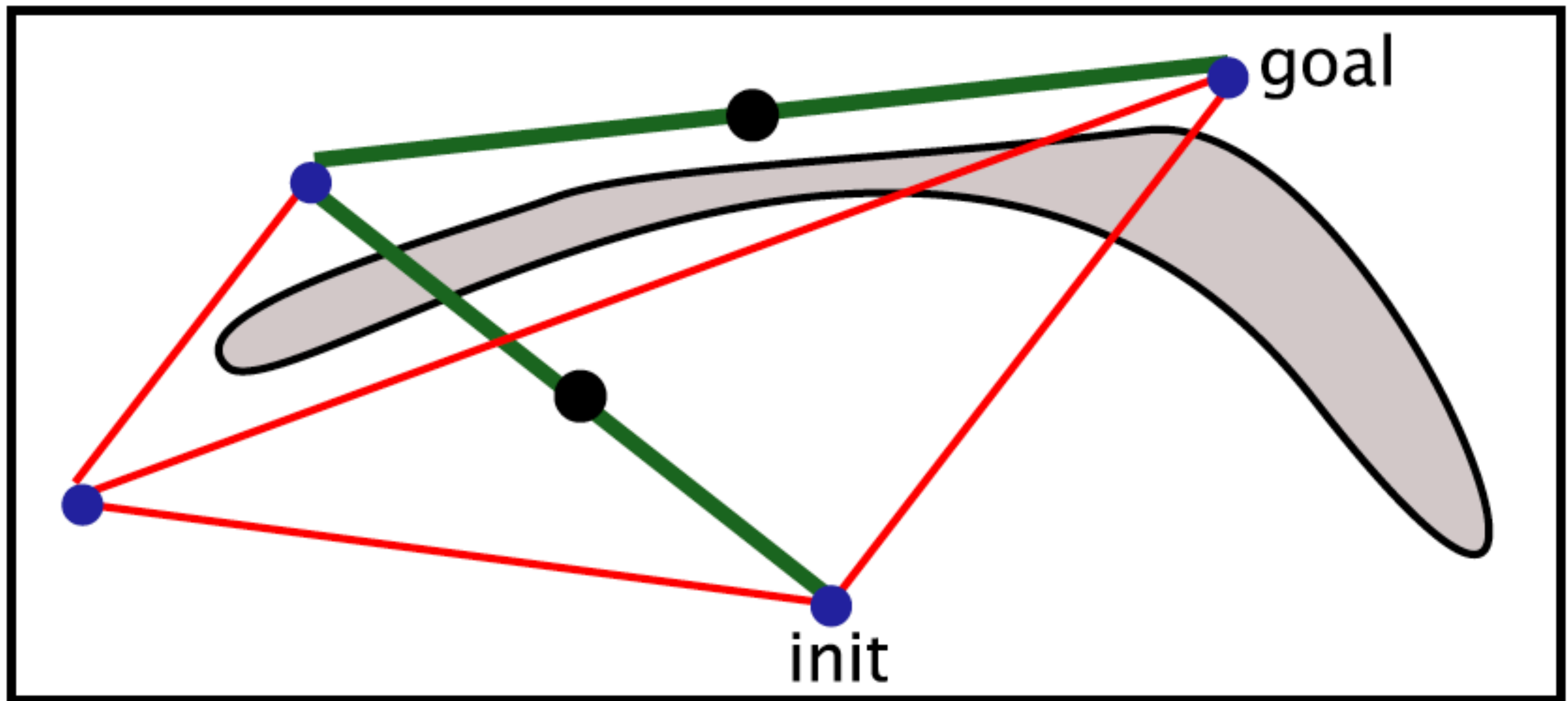


[Fig from Erion Plaku]

Search for the new **shortest-path** on the roadmap

# Lazy PRM (5/10)

42

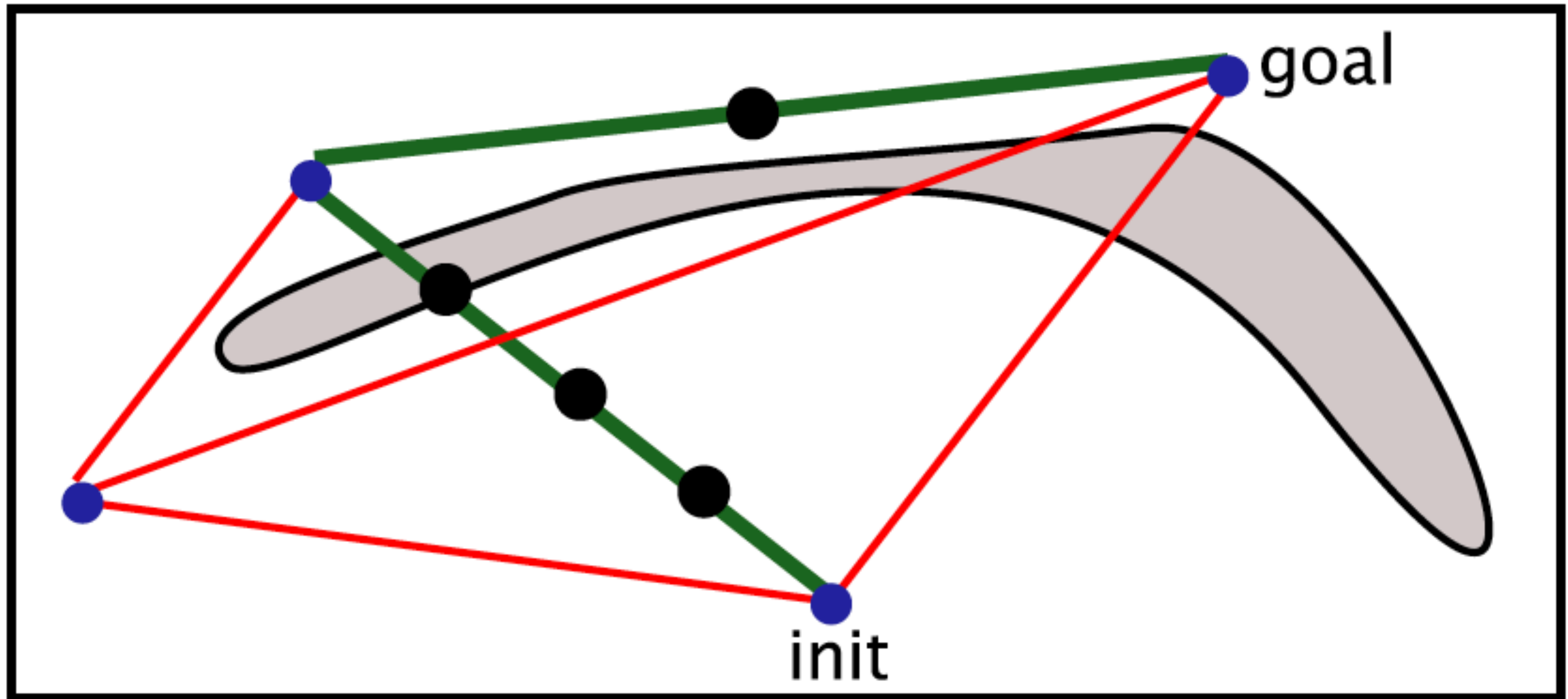


[Fig from Erion Plaku]

Check the edges on the plan for collisions

# Lazy PRM (6/10)

43

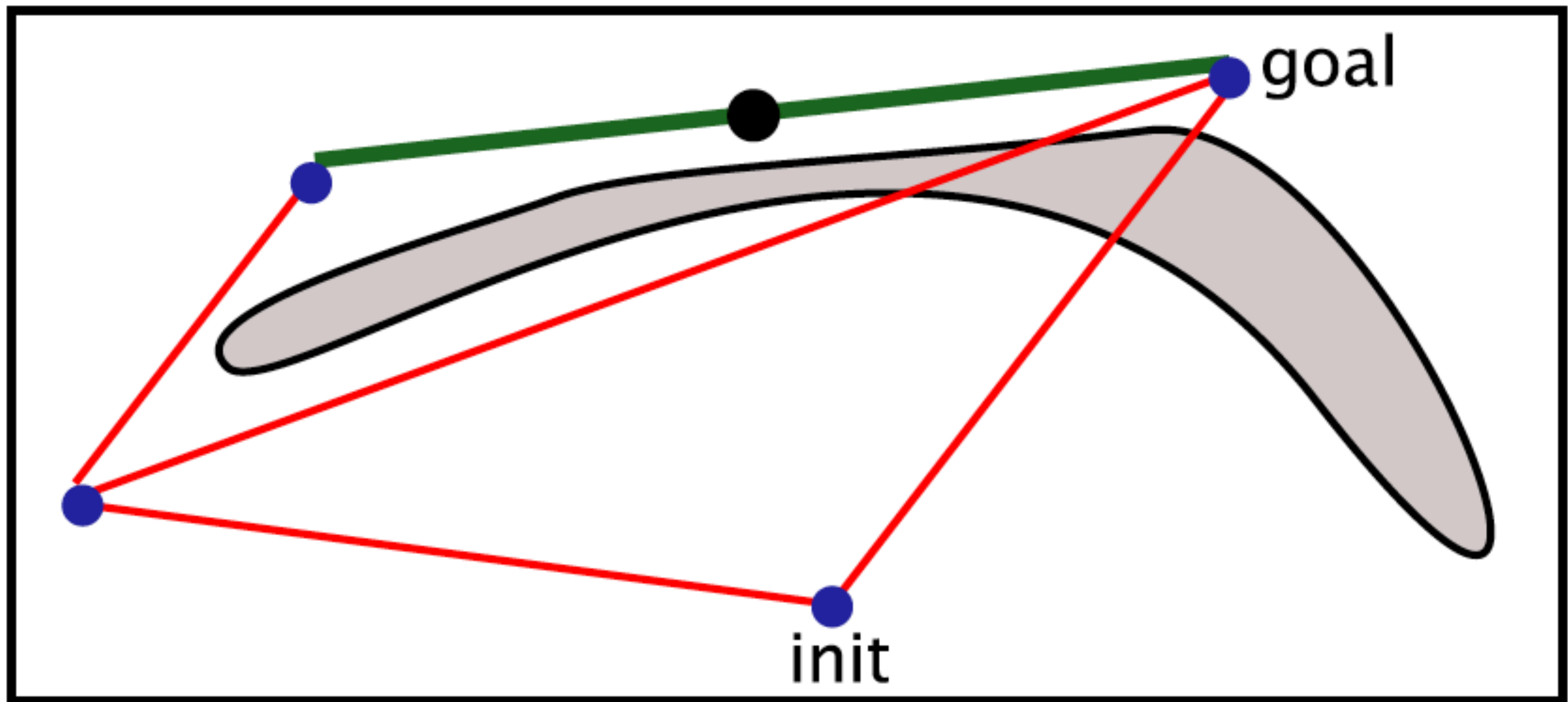


[Fig from Erion Plaku]

Check the edges on the plan for collisions  
(with increased resolution)

# Lazy PRM (7/10)

44

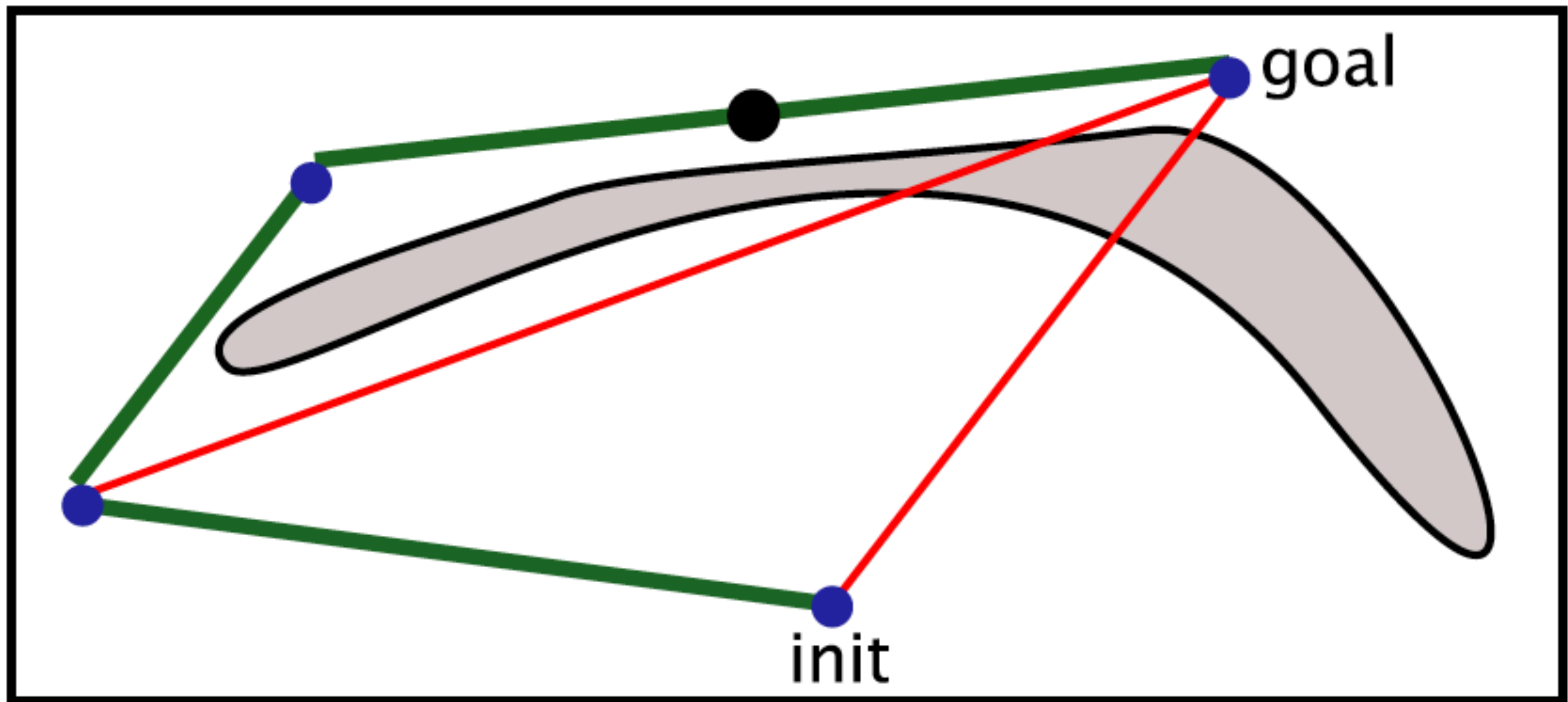


[Fig from Erion Plaku]

Remove plan edges that collide with obstacles

# Lazy PRM (8/10)

45

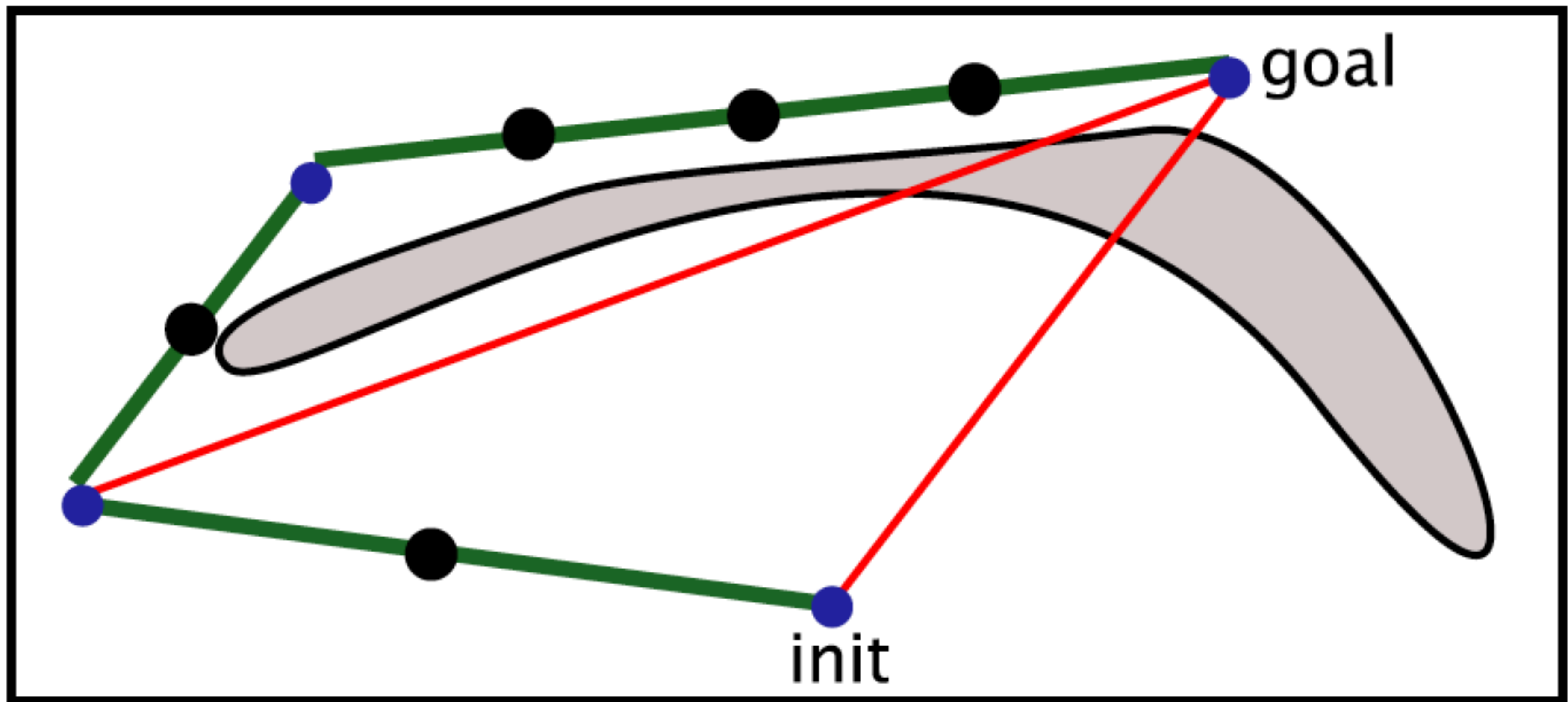


[Fig from Erion Plaku]

Search for the new **shortest-path** on the roadmap

# Lazy PRM (9/10)

46

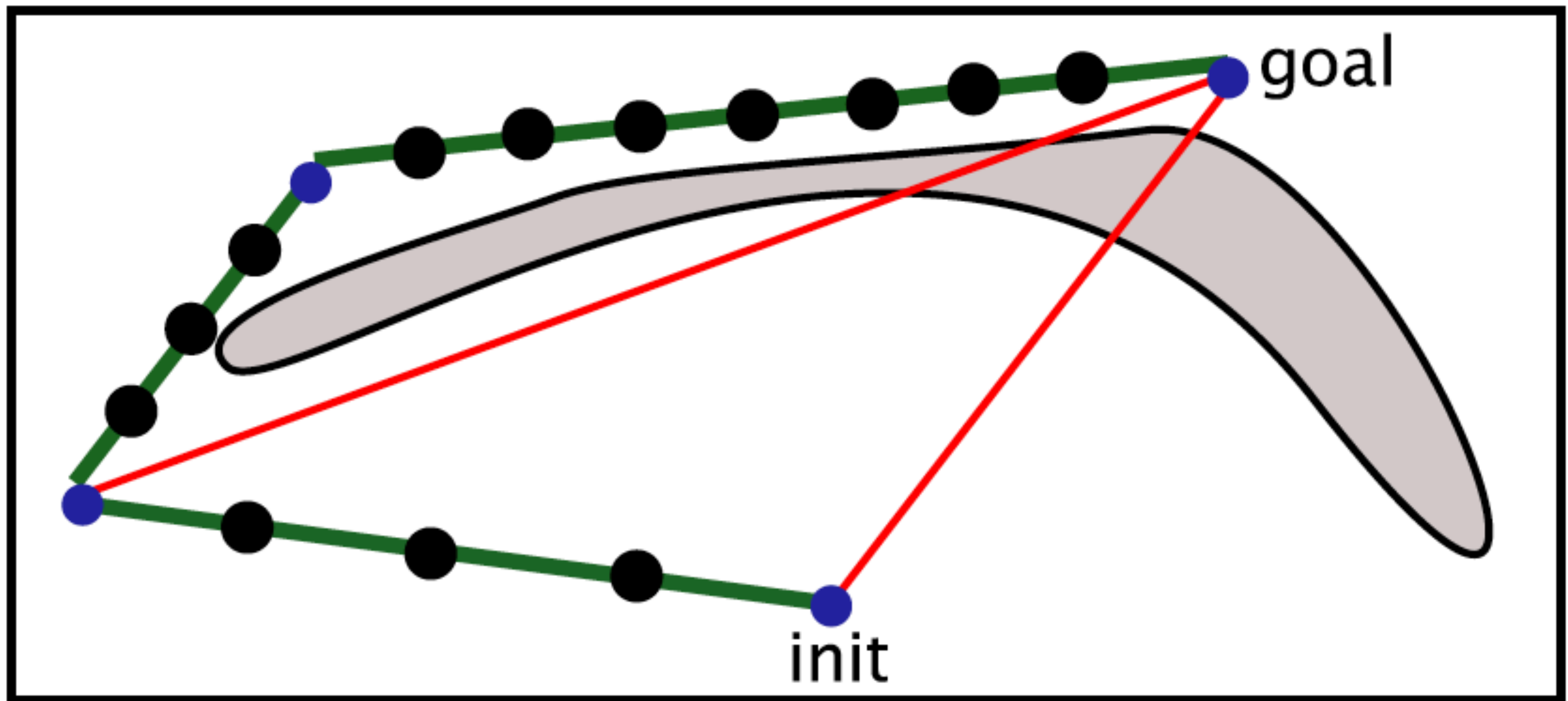


[Fig from Erion Plaku]

Check the edges on the plan for collisions

# Lazy PRM (10/10)

47

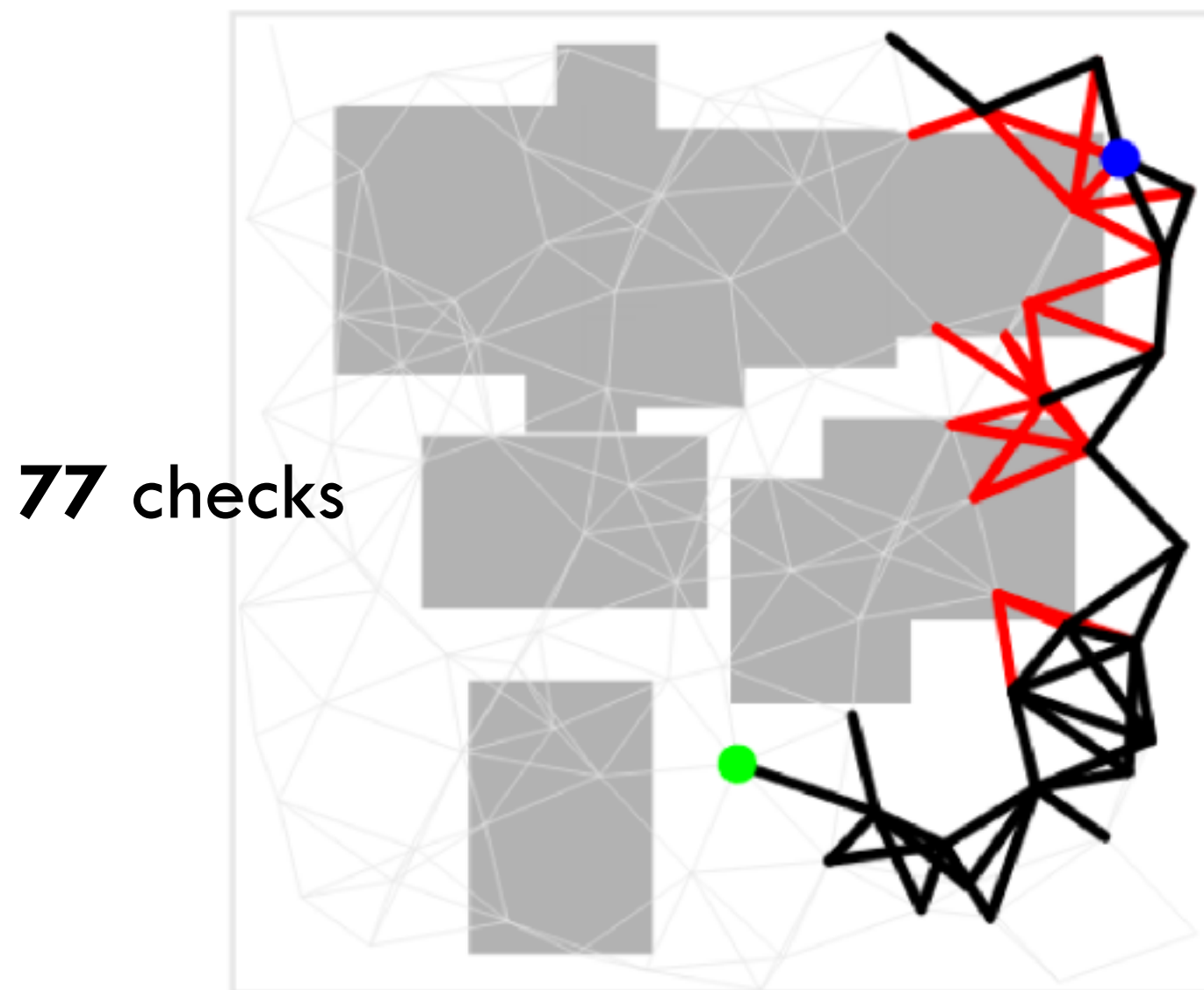


[Fig from Erion Plaku]

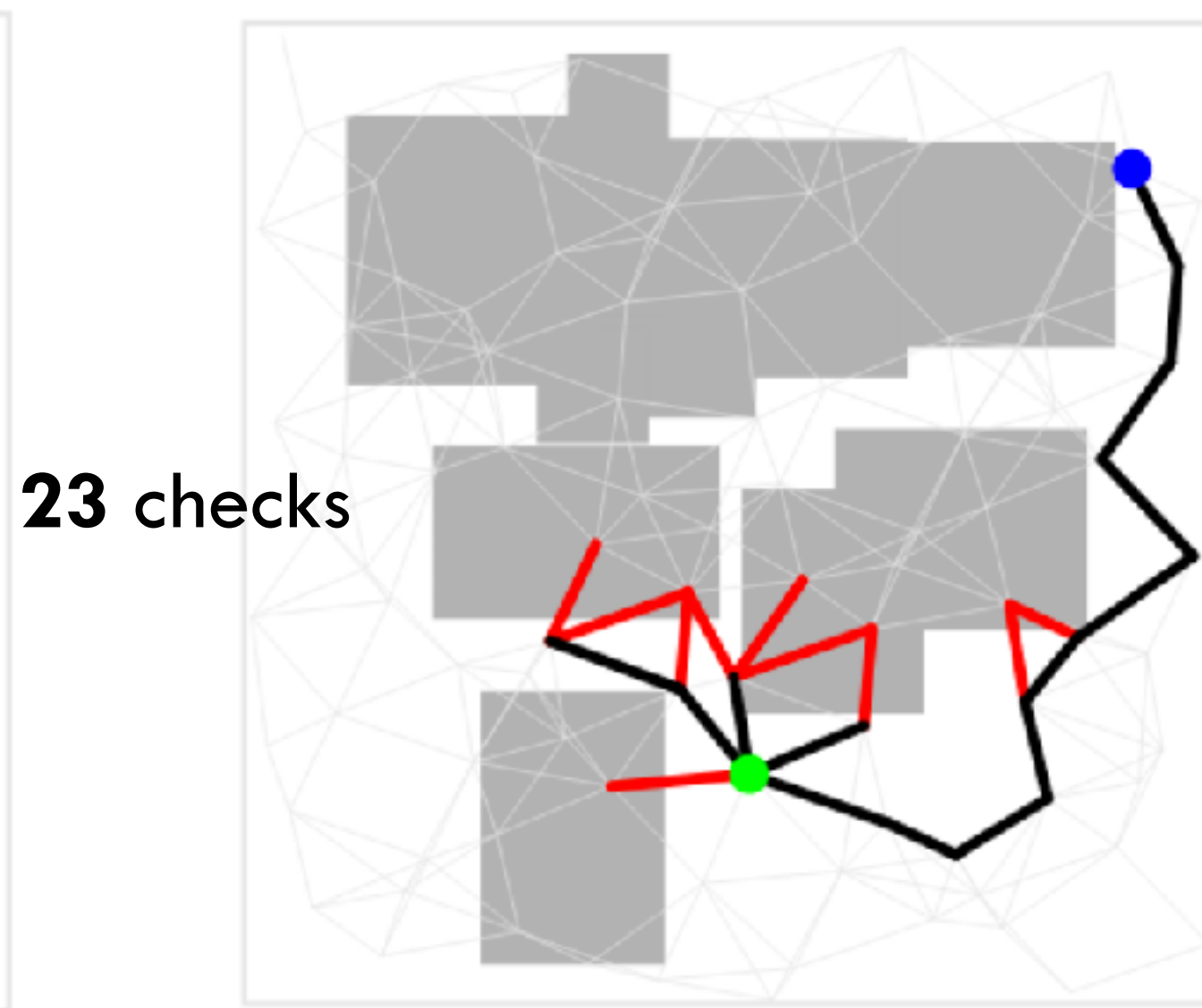
Return the **current path** as a solution

# Lazy Motion Planning

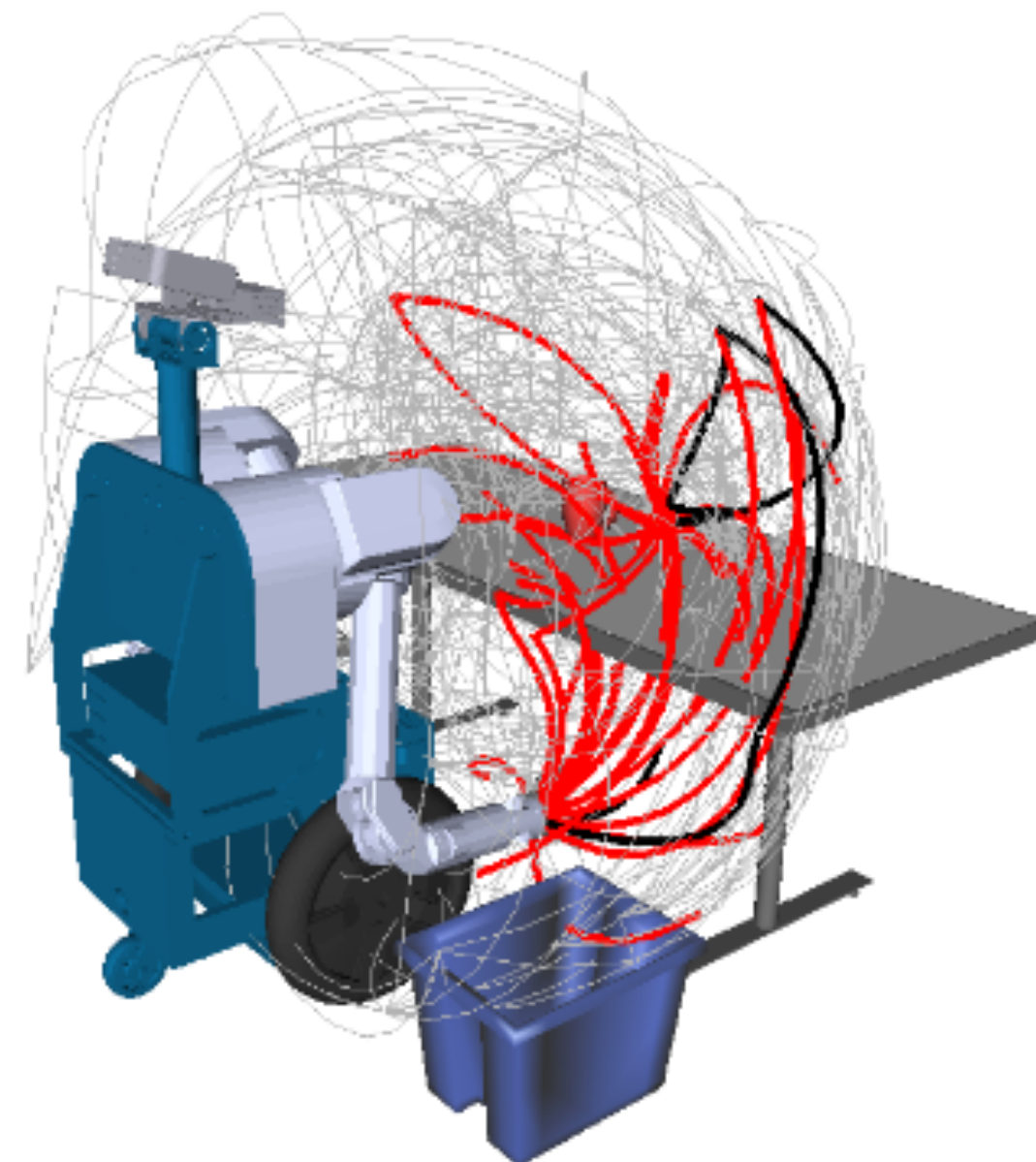
- **Defer** collision checking until a path is found
- **Remove** colliding edges path from the roadmap
- **Repeat** this process with a new path
- **Terminate** when a collision-free path is found



**Eager** (during search)



**Lazy**

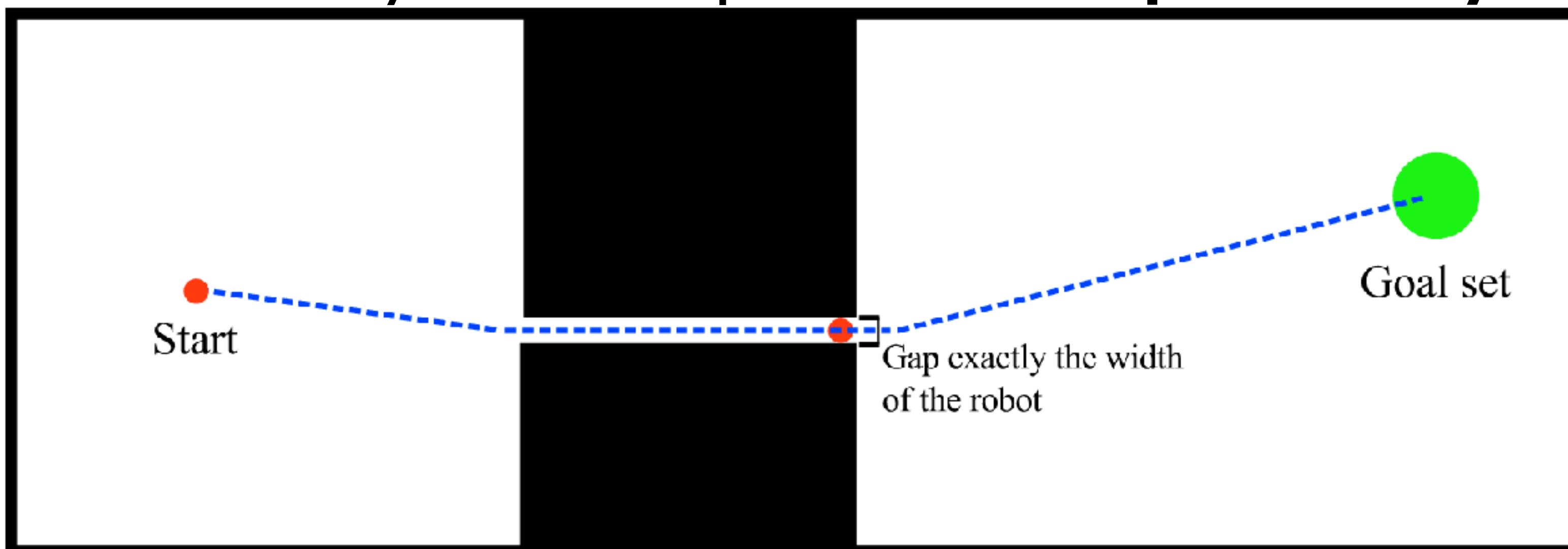




# Theoretical Properties

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- Sampling-based algorithms cannot prove **infeasibility** nor even solve every **feasible problem**
- **Robustly feasible**: a problem that admits a solution for which all **local perturbations** are also solutions
- **Probabilistic complete**: an algorithm that solves any robustly feasible problem with **probability 1**



[Fig from  
Jenny Barry]

# Trajectory Optimization

50

- Frame motion planning as a **non-convex constrained optimization** problem & solve for **local minima**

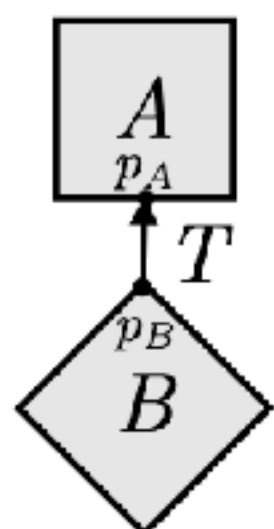
minimize  $f(\mathbf{x})$

subject to

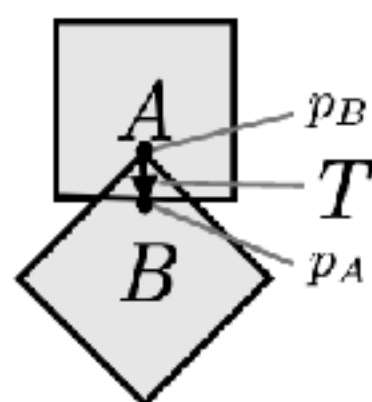
$$g_i(\mathbf{x}) \leq 0, \quad i = 1, 2, \dots, n_{ineq}$$

$$h_i(\mathbf{x}) = 0, \quad i = 1, 2, \dots, n_{eq}$$

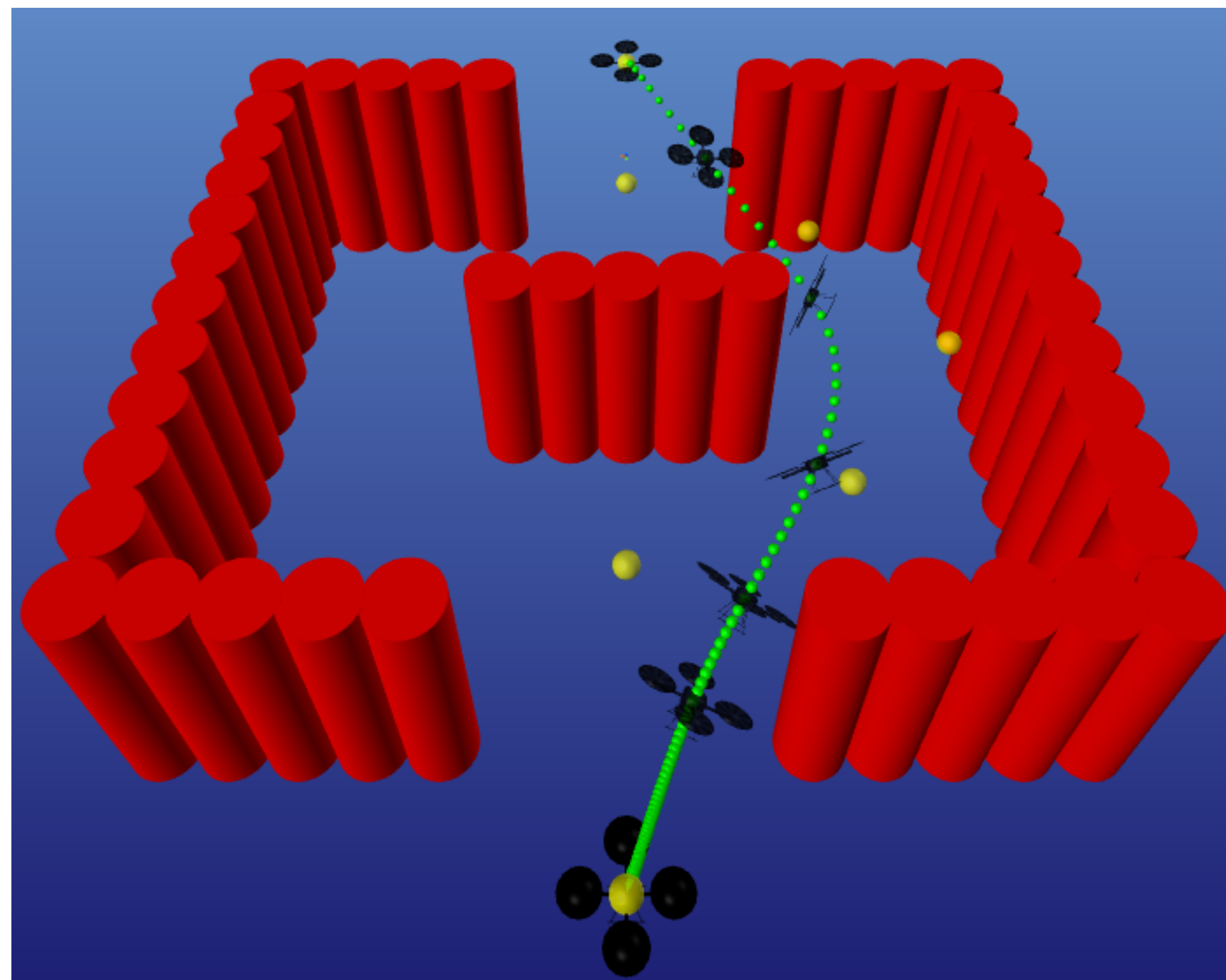
- Collision constraints enforced via **signed distance (sd)**



$sd > 0$



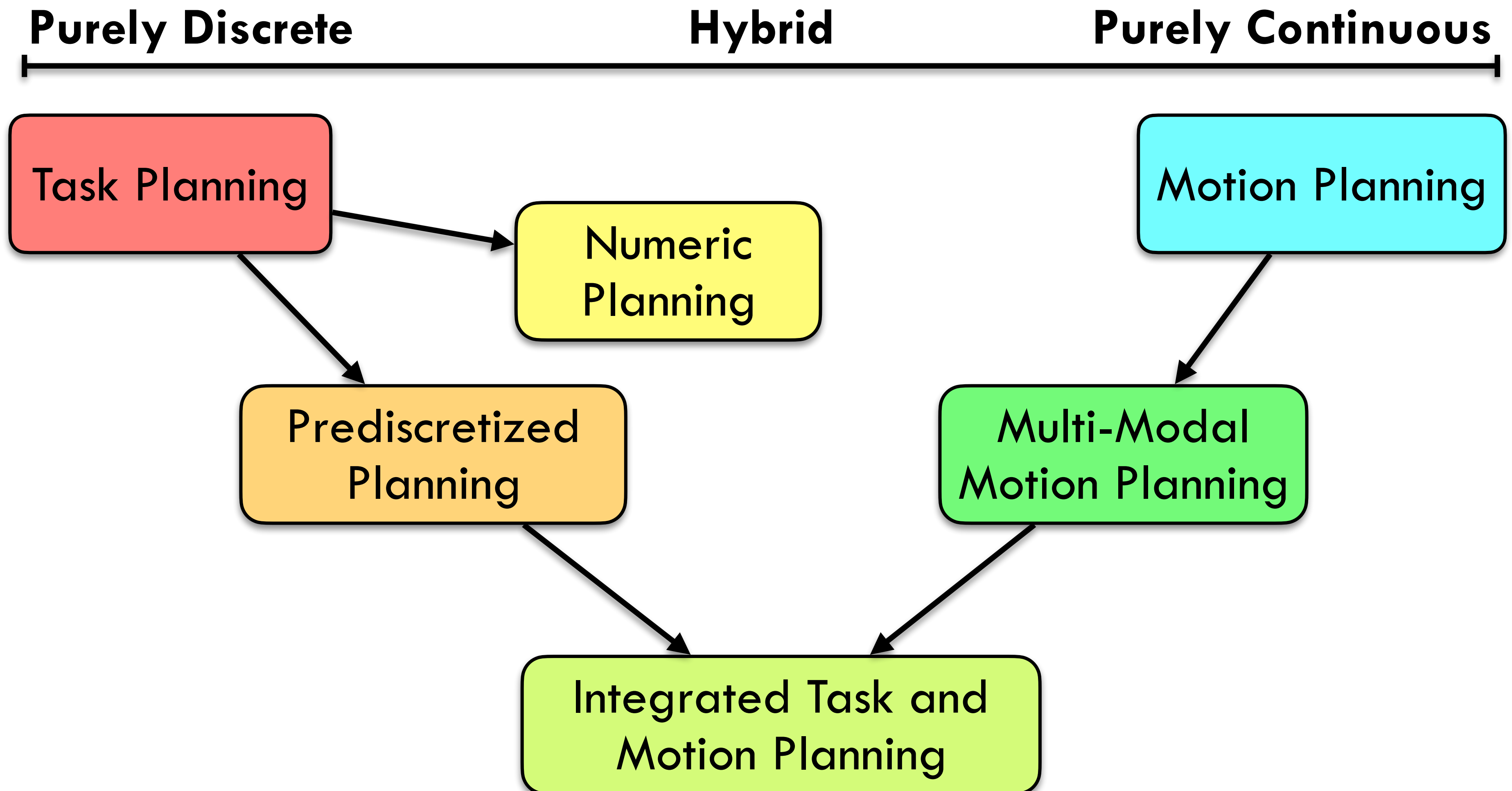
$sd < 0$



[Ratliff 2009][Schulman 2013]

# Hybrid Planning Spectrum

51

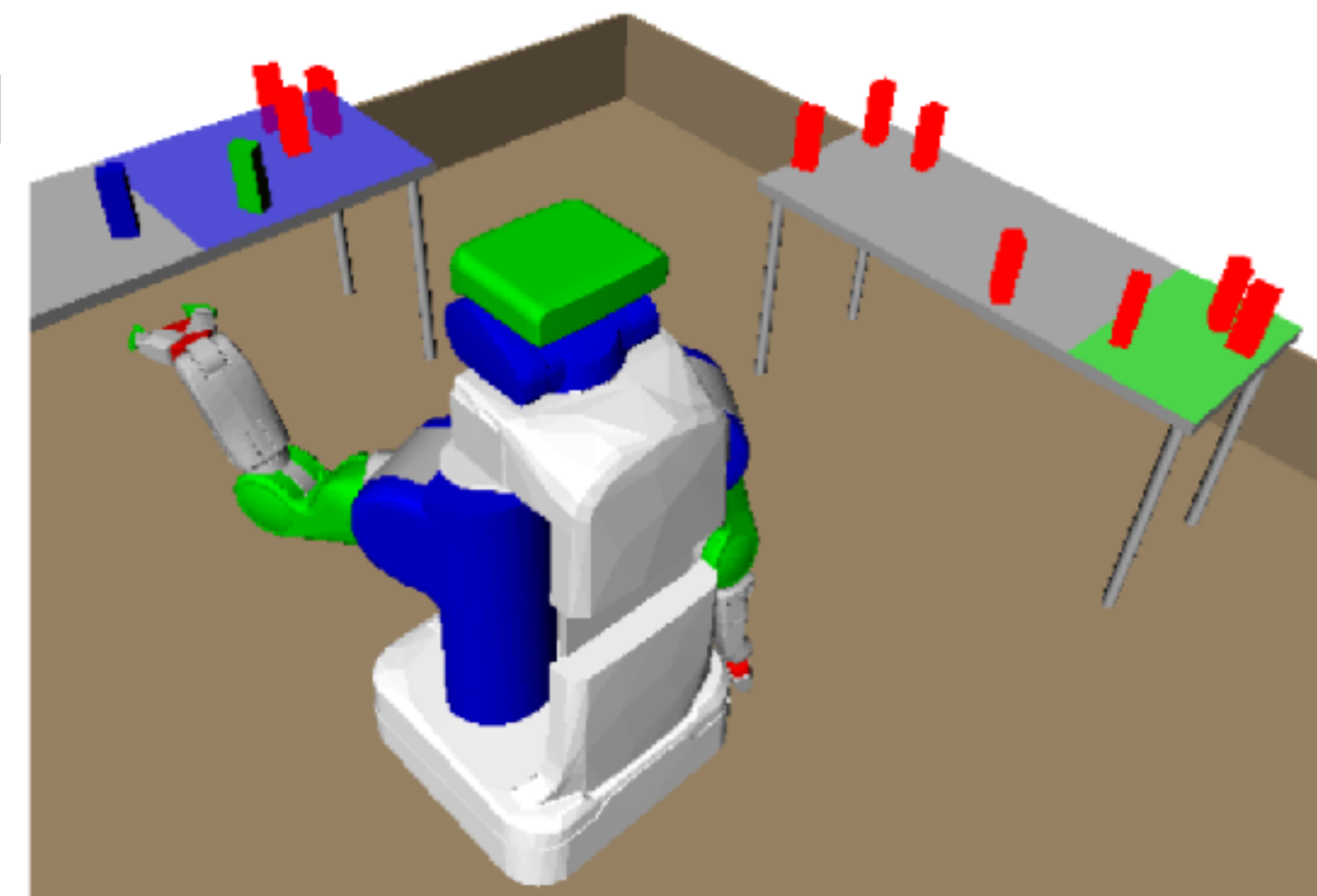
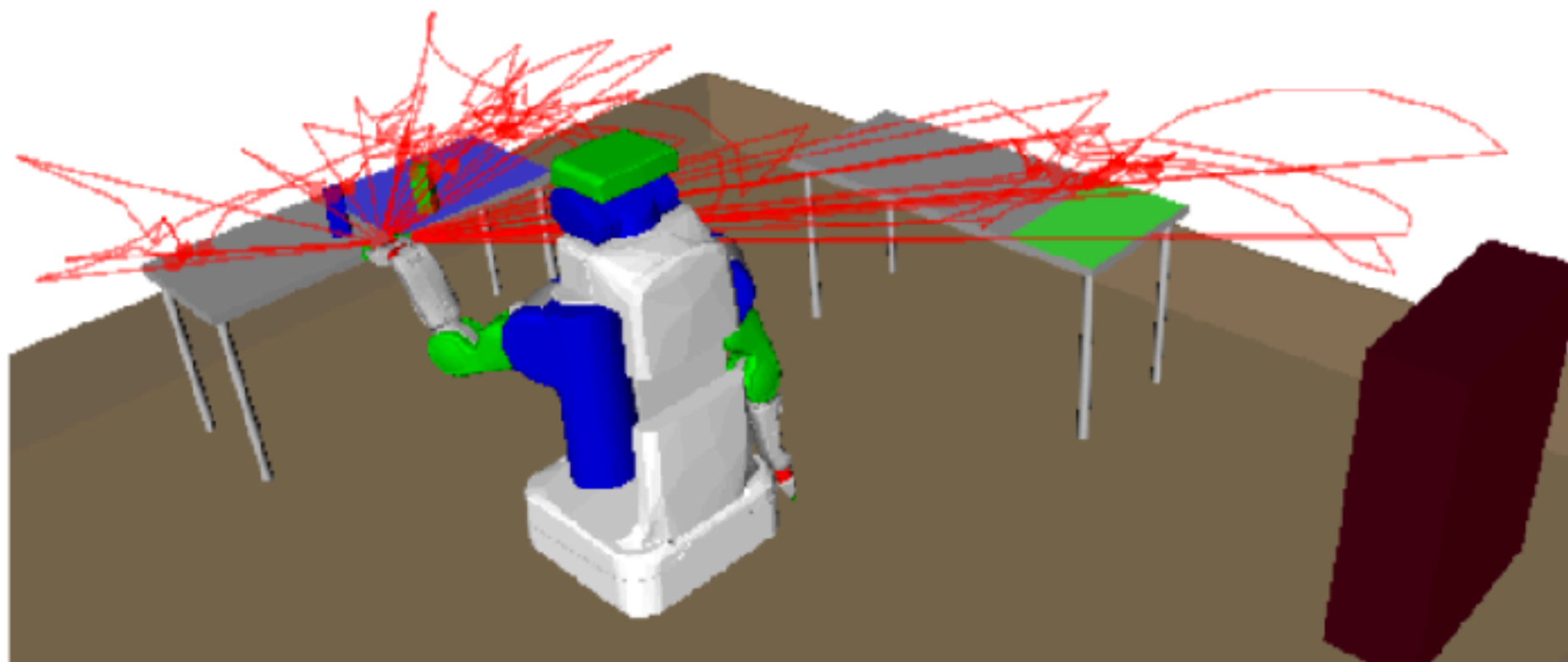


# Prediscretized & Numeric Planning

# Prediscretized Planning

53

- Assumes that a **finite set** of object placements, object grasps, and (sometimes) robot configurations are **given**
- Can **directly** perform discrete task planning
- Still need to evaluate **reachability**
  - **Eagerly in batch** [Lozano-Pérez 2014][Garrett 2017][Ferrer-Mestres 2017]
  - **Eagerly during search** [Dornhege 2009]
  - **Lazily** [Erdem 2011][Dantam 2018][Lo 2018]



# Discrete-Control Numeric Planning

54

- Classical planning with **real-valued variables** and **durative actions**
- **Examples:** time and energy
- Most planners only support **linear/polynomial dynamics**
- **Non-linear** dynamics addressed by **discretizing time**
- **Example:** battery domain

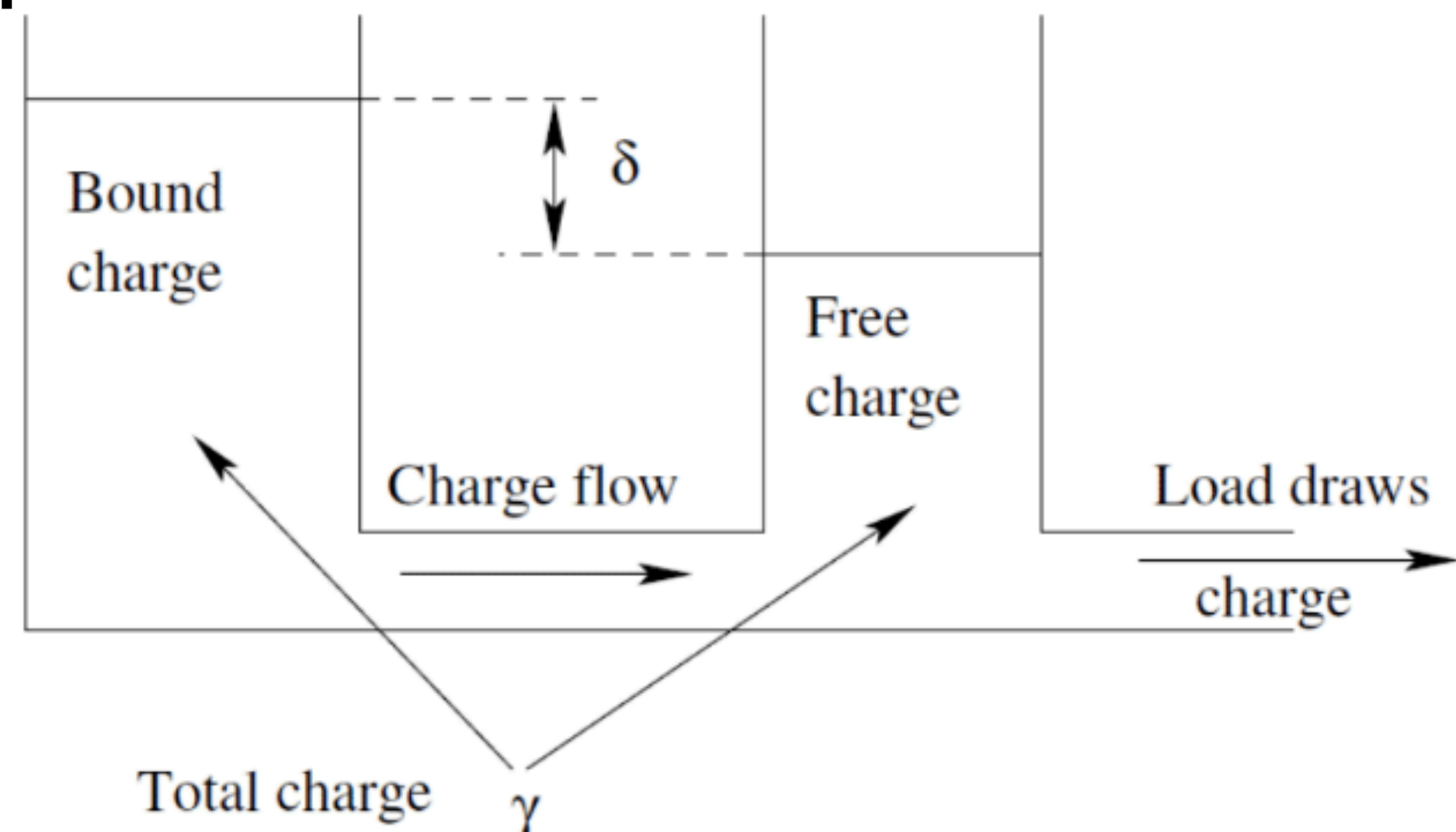
$$\frac{d\delta}{dt} = \frac{i(t)}{c} - k'\delta$$

$\xrightarrow{\text{load}}$   
 $\xrightarrow{\text{Fixed conductan}}$

$$\frac{d\gamma}{dt} = -i(t)$$

$\xrightarrow{\text{battery capacity}}$

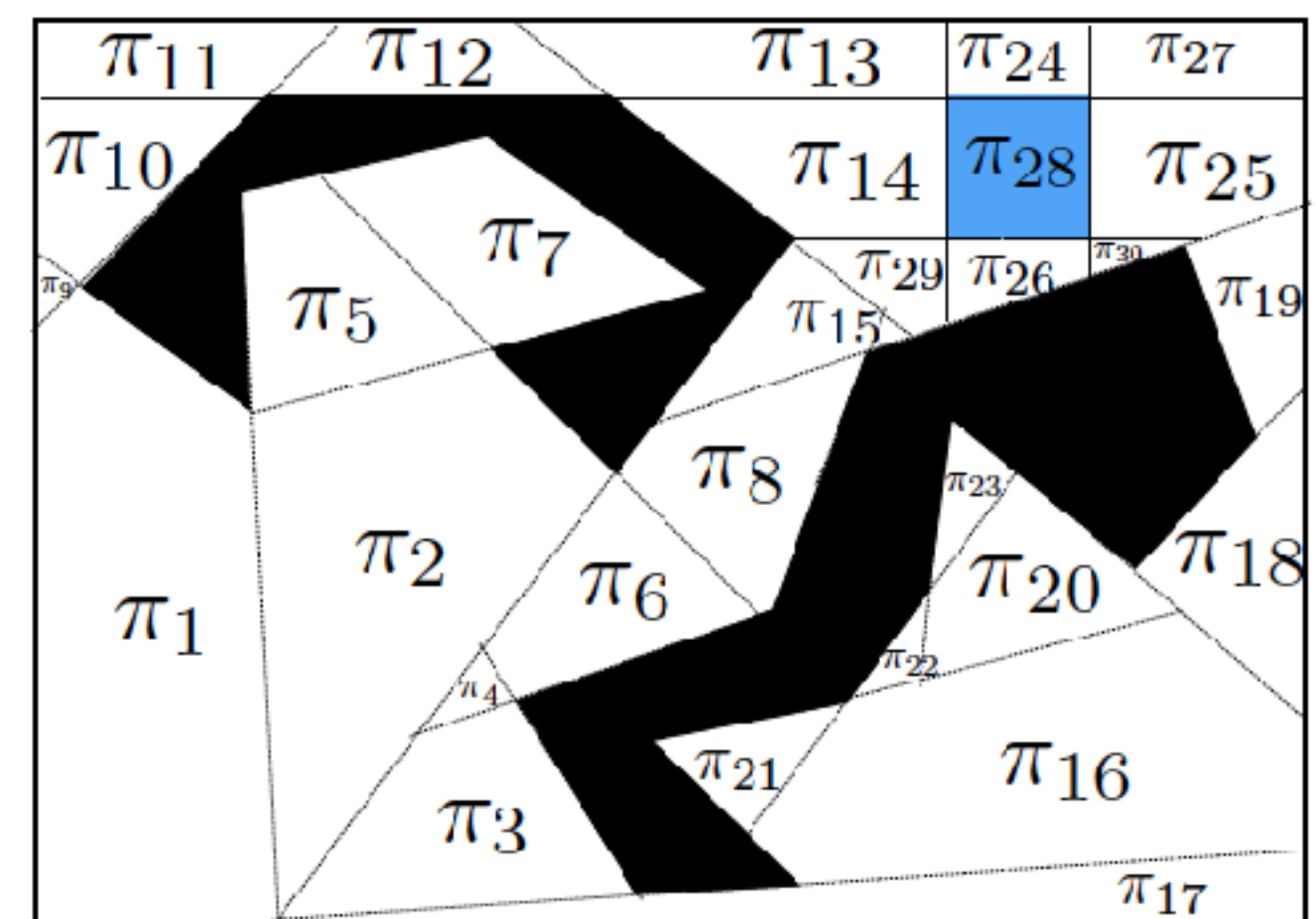
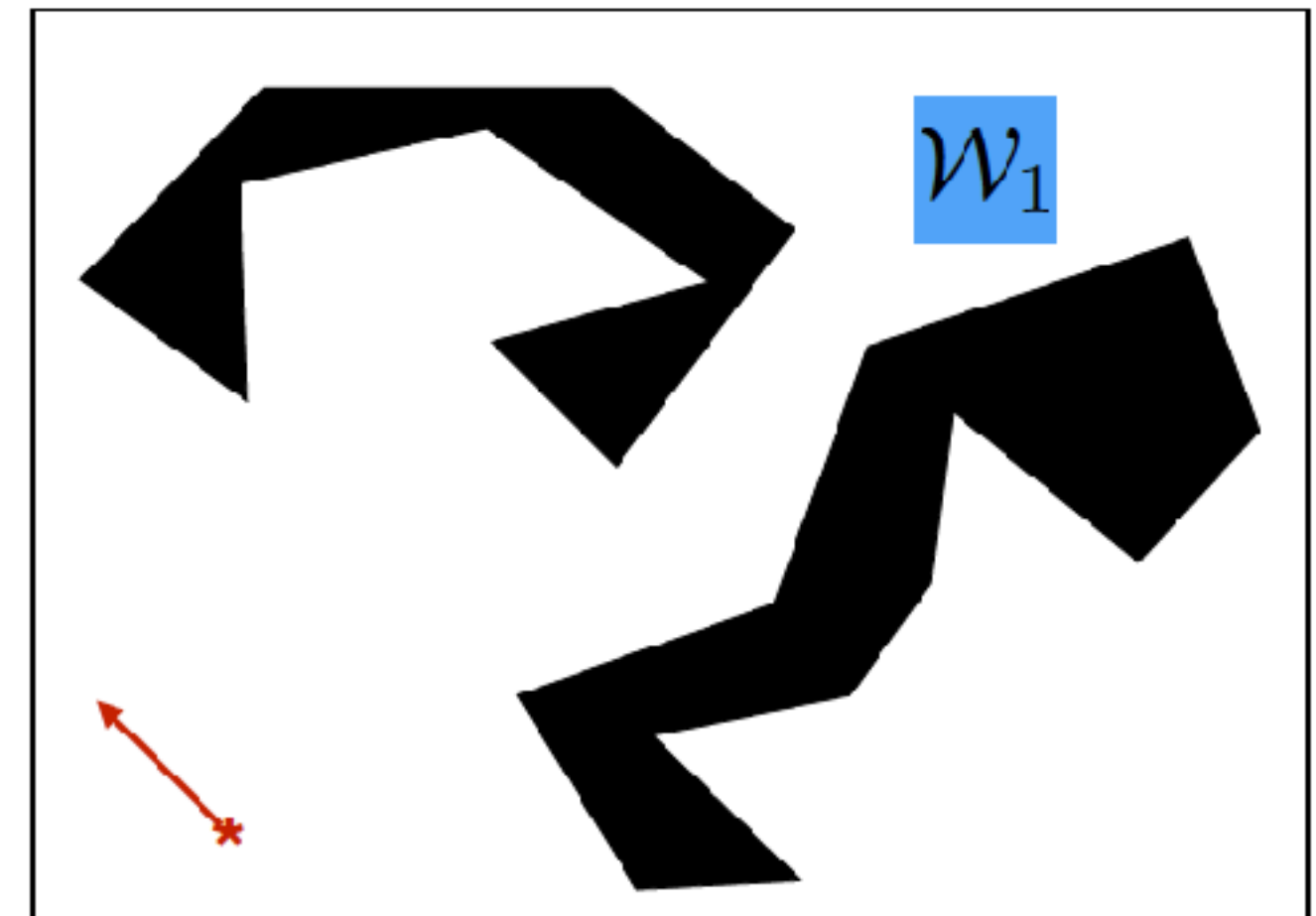
$$\delta(t) = \frac{I}{c} \cdot \frac{1 - e^{-k't}}{k'}$$
$$\gamma(t) = C - It$$



# Continuous-Control Numeric Planning

55

- **Continuous control** parameters
- Tackle **convex dynamics** using **cone programming**
- Non-convexity handled by **partitioning** the state-space
- **In contrast, TAMP is often:**
  - **High-dimensional**
  - **Non-convex**
    - 3D collision constraints
  - **Less sophisticated dynamically**



[Deits 2015][Shoukry 2016]  
[Fernandez-Gonzalez 2018]

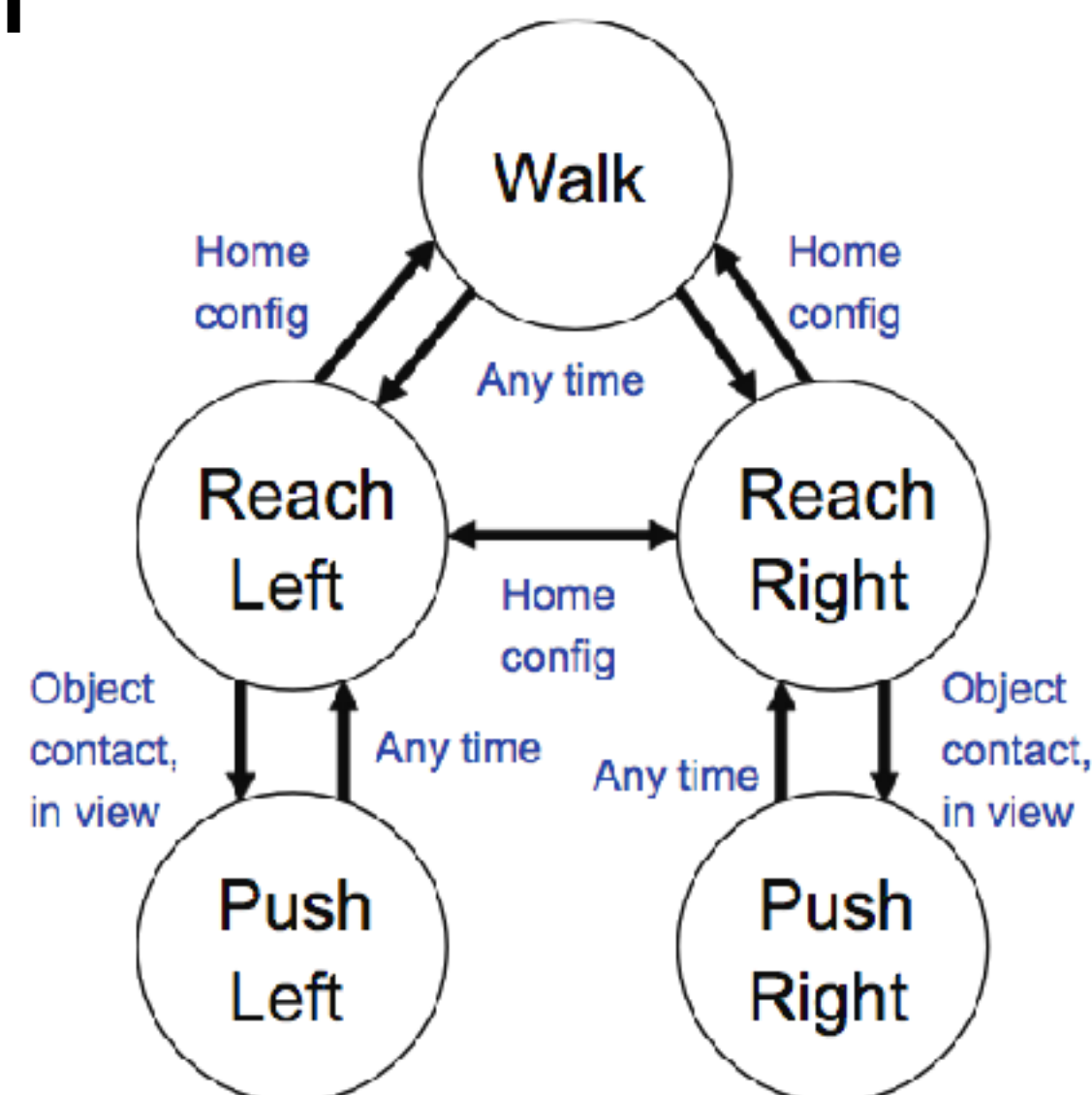
# Multi-Modal Motion Planning



# Multi-Modal Motion Planning

57

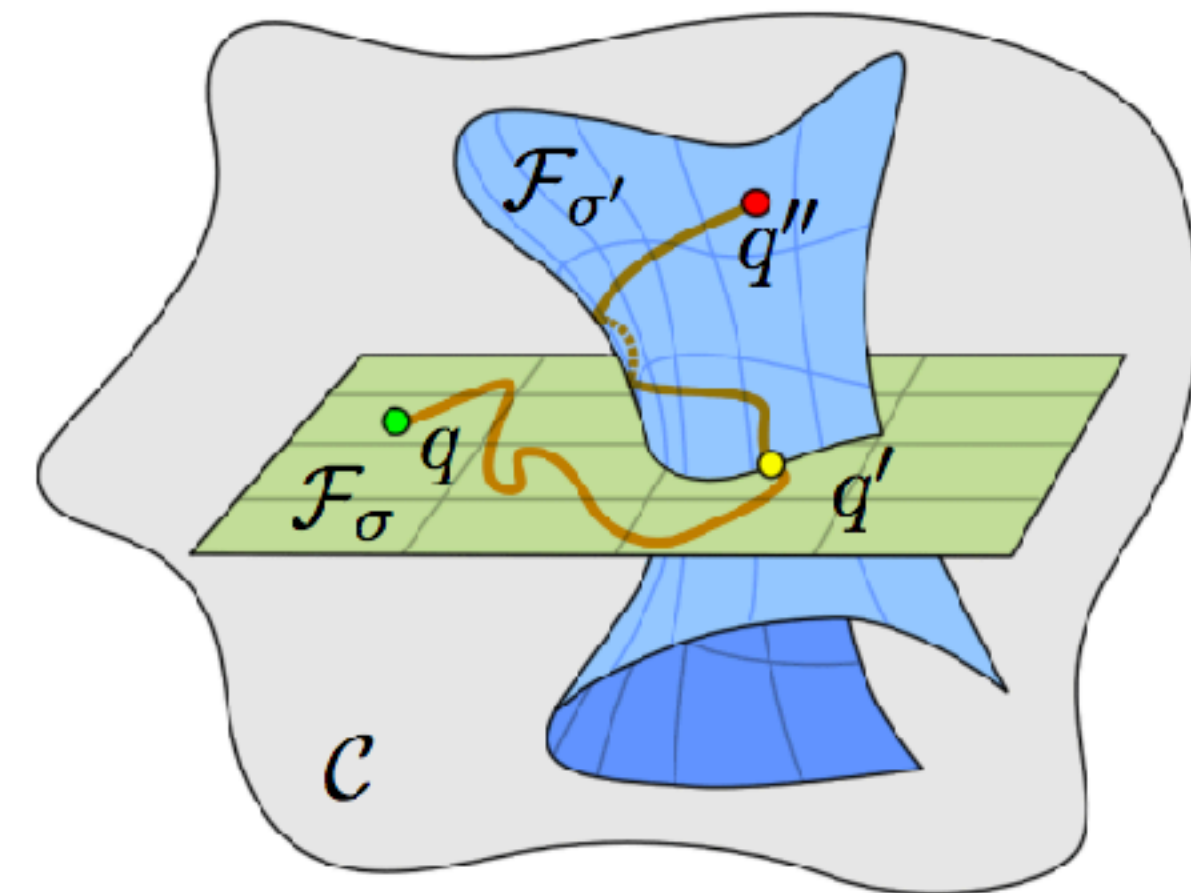
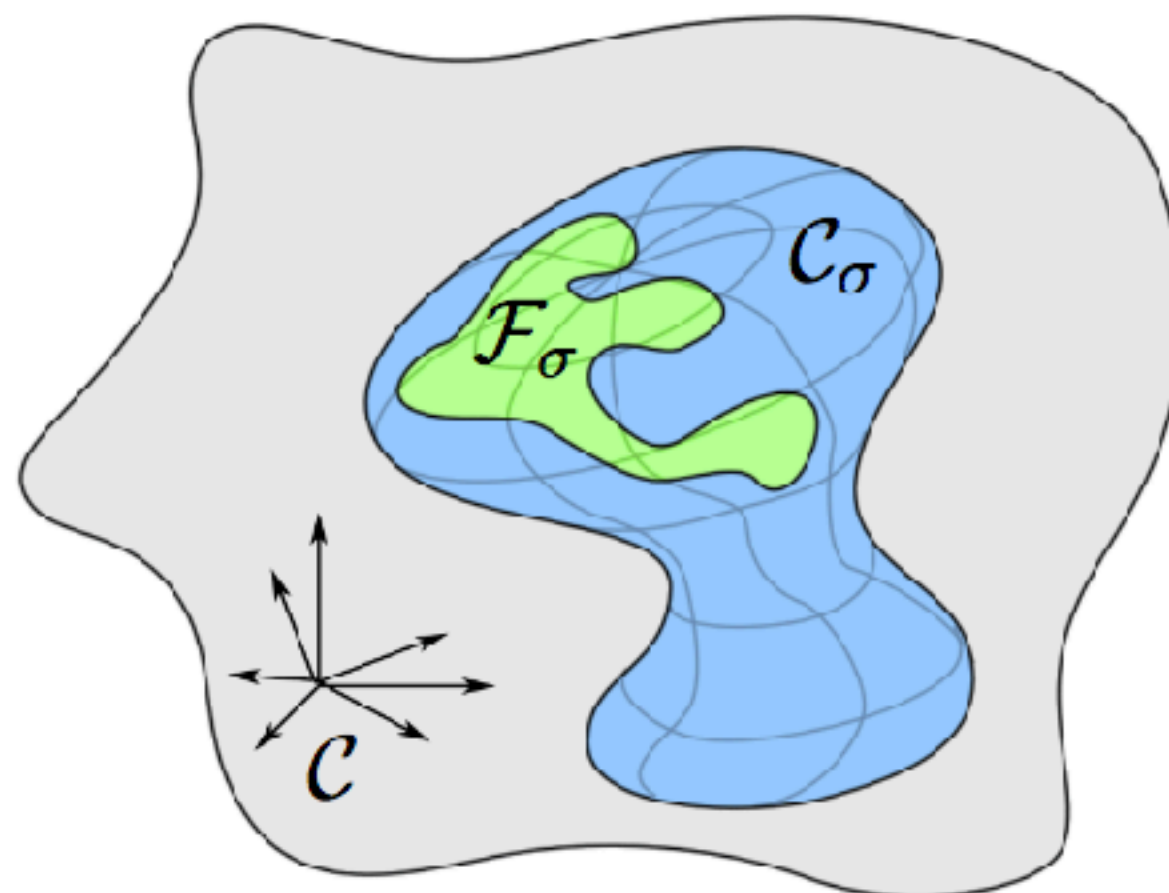
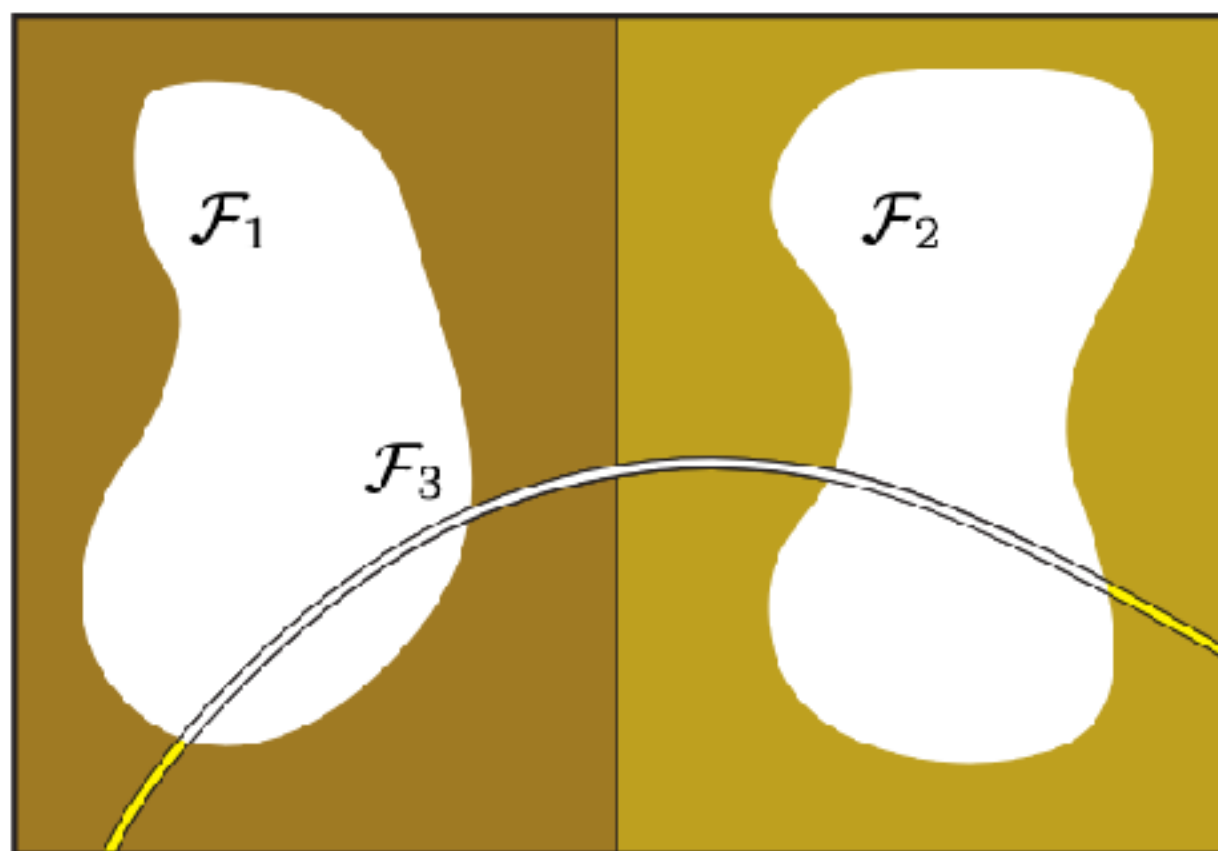
- Collision-free configuration space **changes** when objects are manipulated
- Use a **sequence** of motion planning problems each defined by a **mode**
- **Mode**: a set of motion constraints
  - Gripper is empty
  - Relative object pose remains **constant**



# Low-dimensional Intersections

58

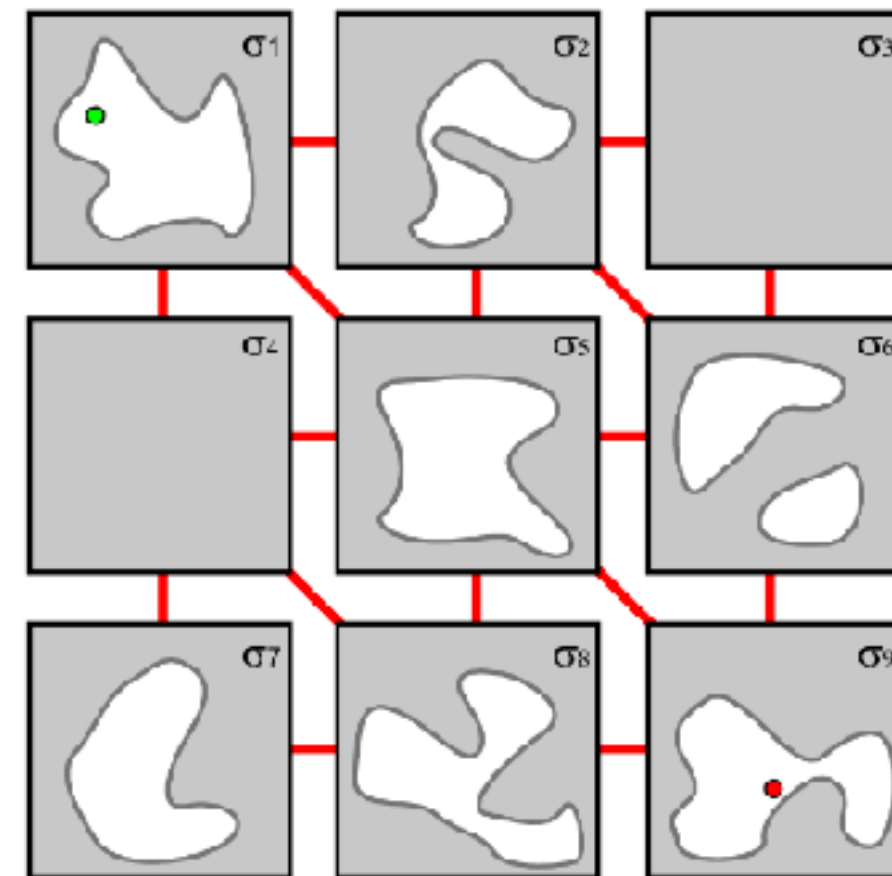
- Need samples that **connect** adjacent modes
- Intersection of two modes is often **low-dimensional**
  - **Special-purpose** samplers are needed
- **Example:** transition from gripper **empty** to **holding**
- Configurations at the **intersection** obtained using **inverse kinematics (IK)**



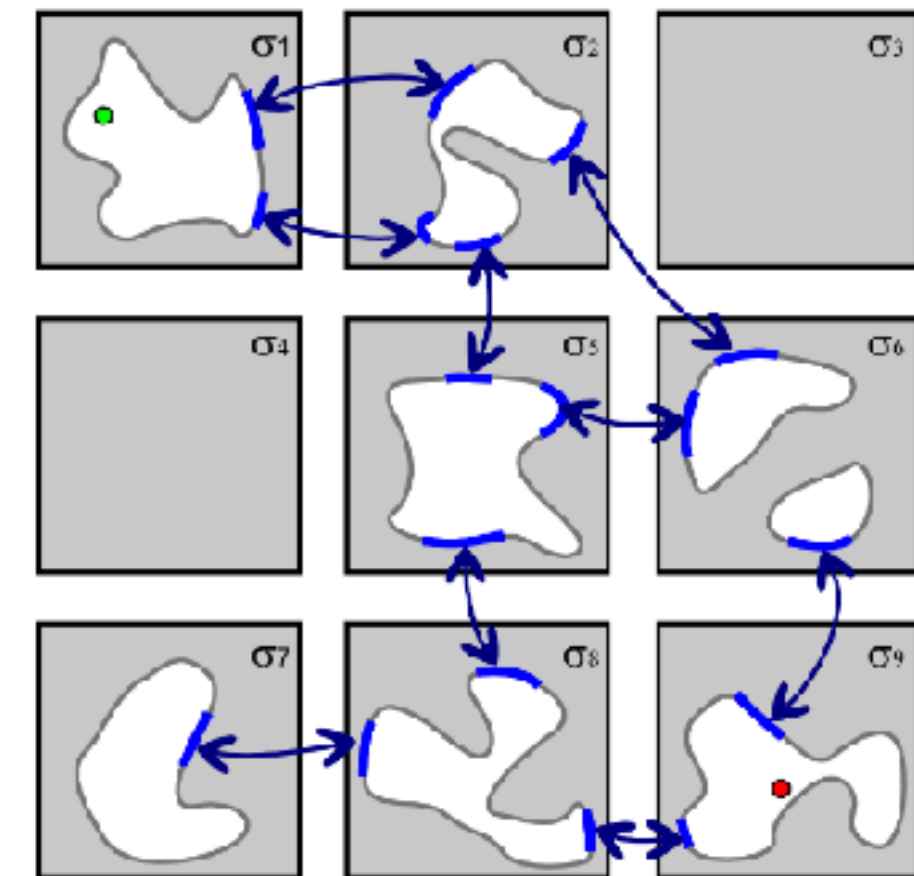
# Sampling-Based Multi-Modal Planning

59

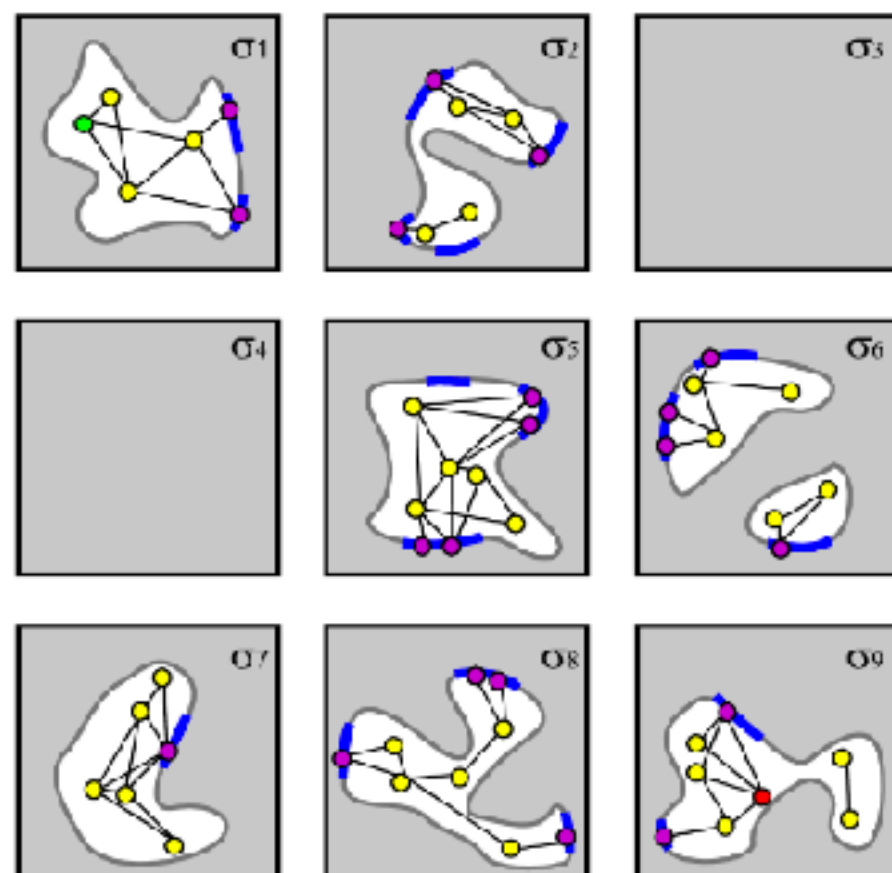
1. Sample from the set of **modes**
2. Sample at the **low-dimensional intersection** of adjacent modes
3. Sample a roadmap **within each mode**
4. Discrete search on the **multi-modal roadmap**



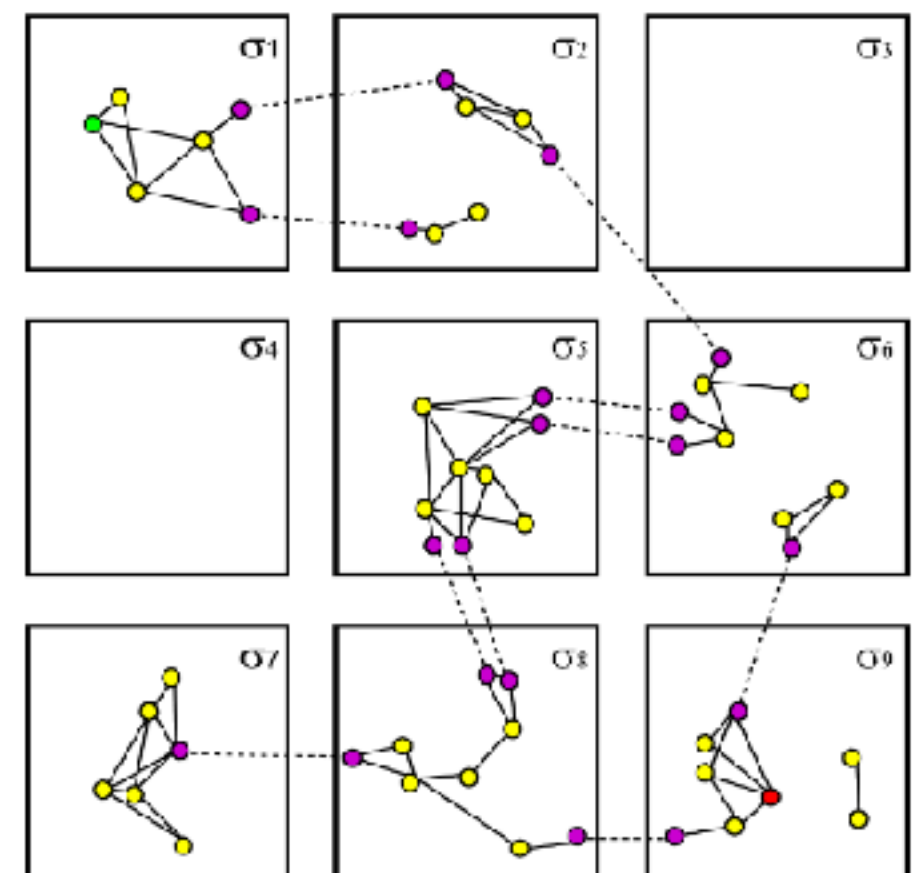
Adjacent modes



Intersections



Individual mode roadmaps

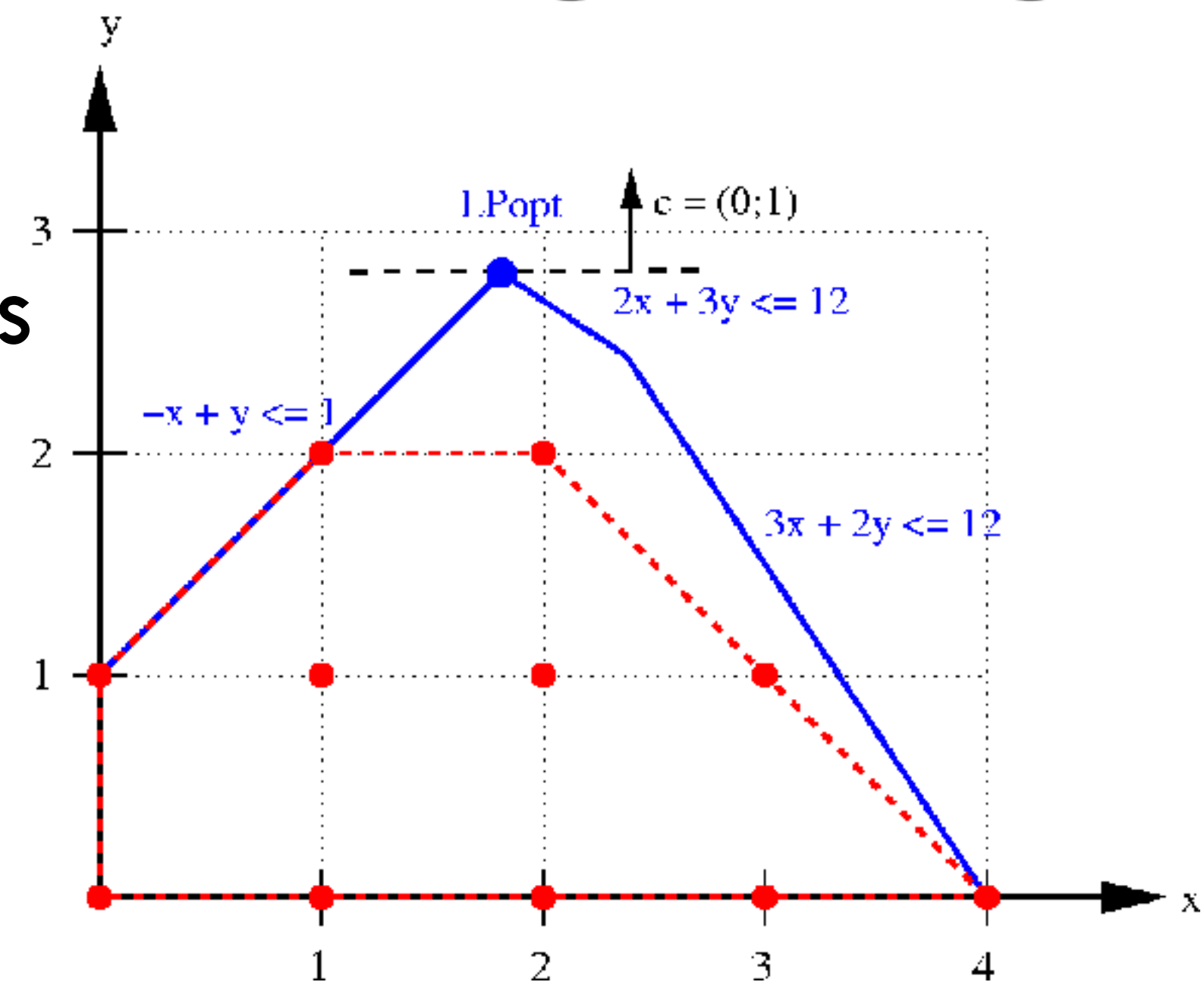
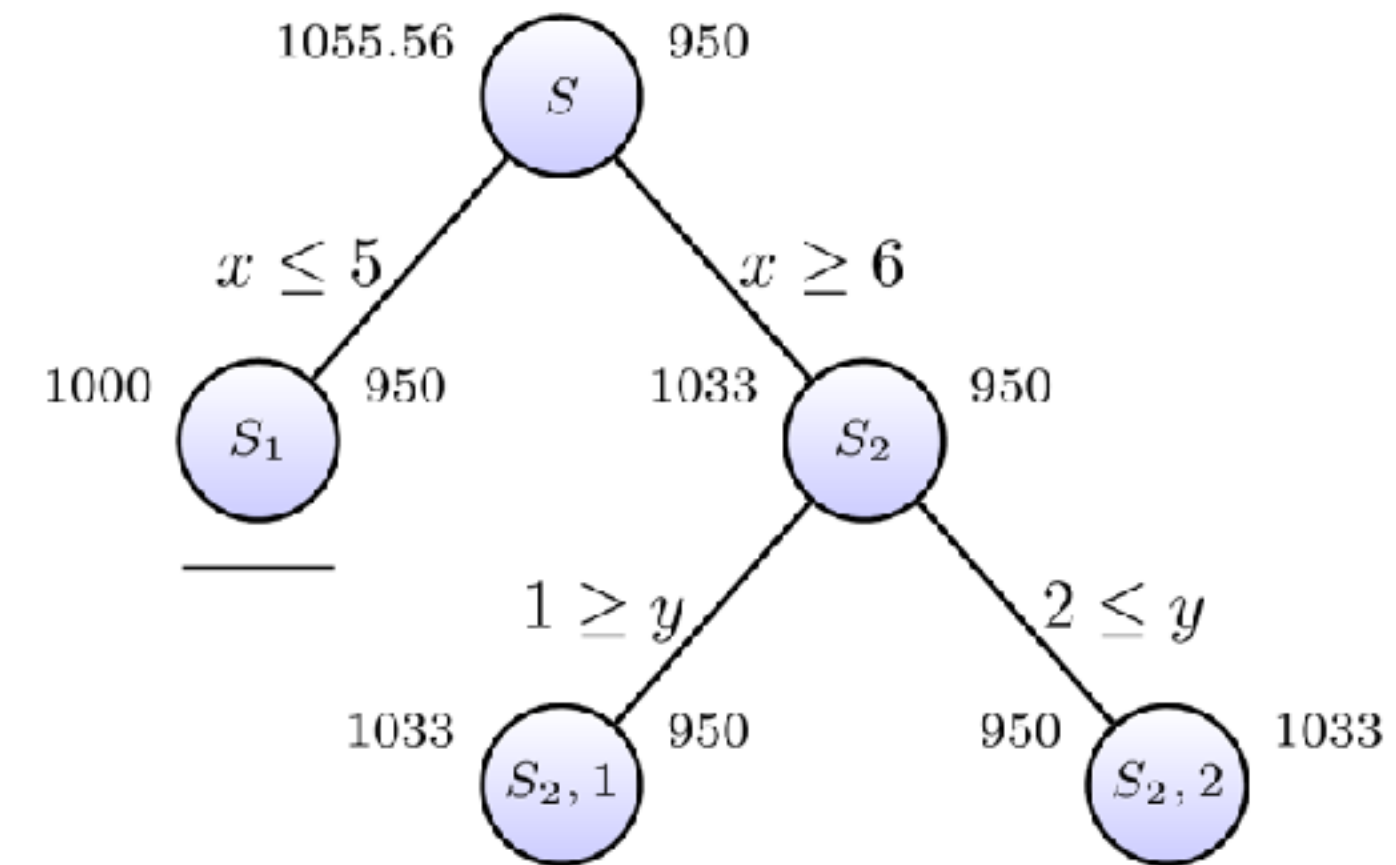


Combined Roadmap

# Mixed Integer Programming (MIP)

60

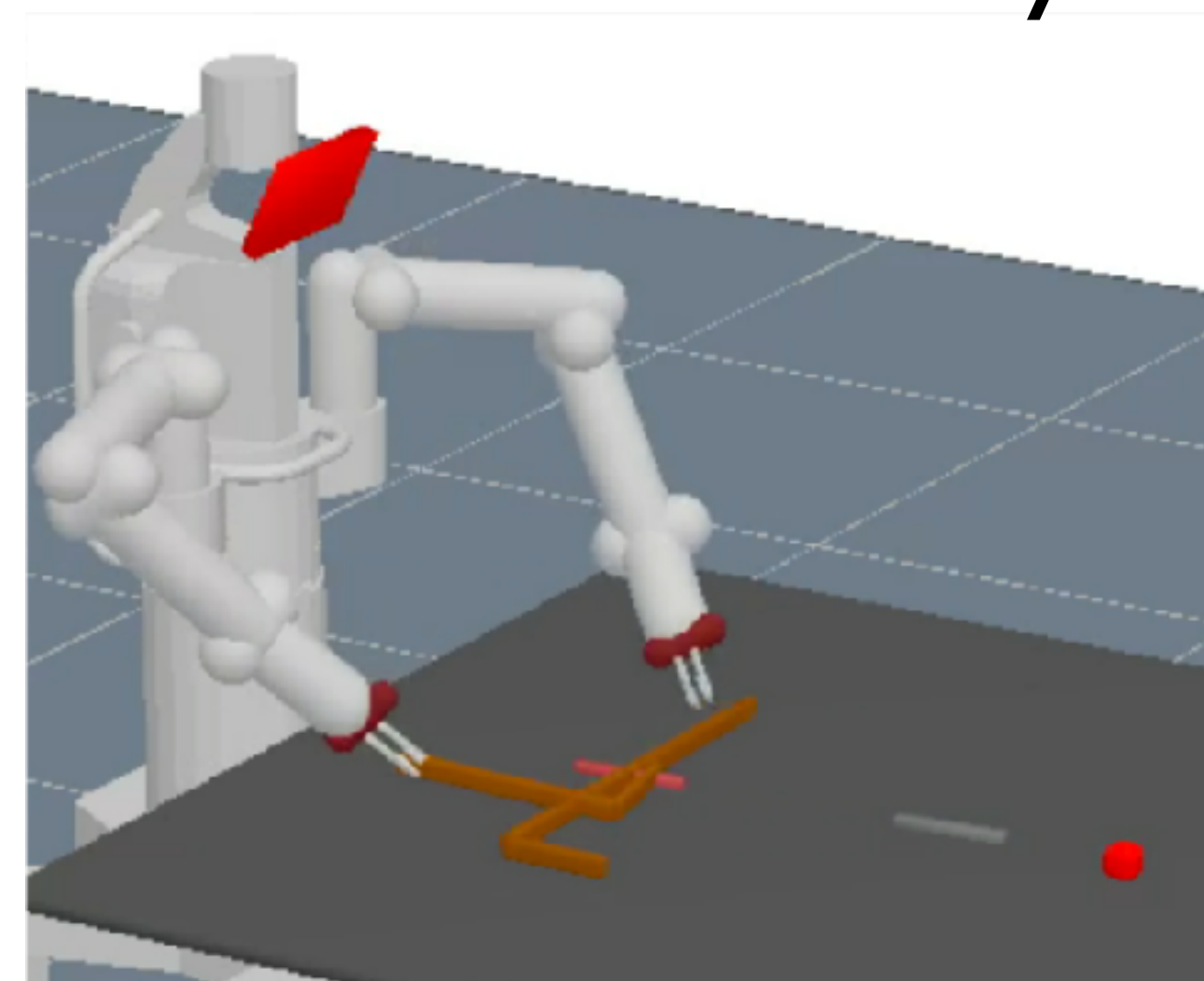
- Continuous and integer variables
- Convex constraints and costs
- **Branch-and-bound**
  - Split on integer variables
- **Integrality relaxation**
  - Lower bound on cost
  - Loose when **logical** operations
- Planning limitation
  - # of variables may be **exponential** in problem size



# Optimization-Based Multi-Modal Motion Planning

61

- Discrete search over sequences of **mode switches**
- Sequences have **varying length**
- Each sequence induces a **non-convex constrained optimization problem**
- Sequences can be pruned using **lower bounds** [Lagriffoul 2014] obtained by **relaxing** some constraints



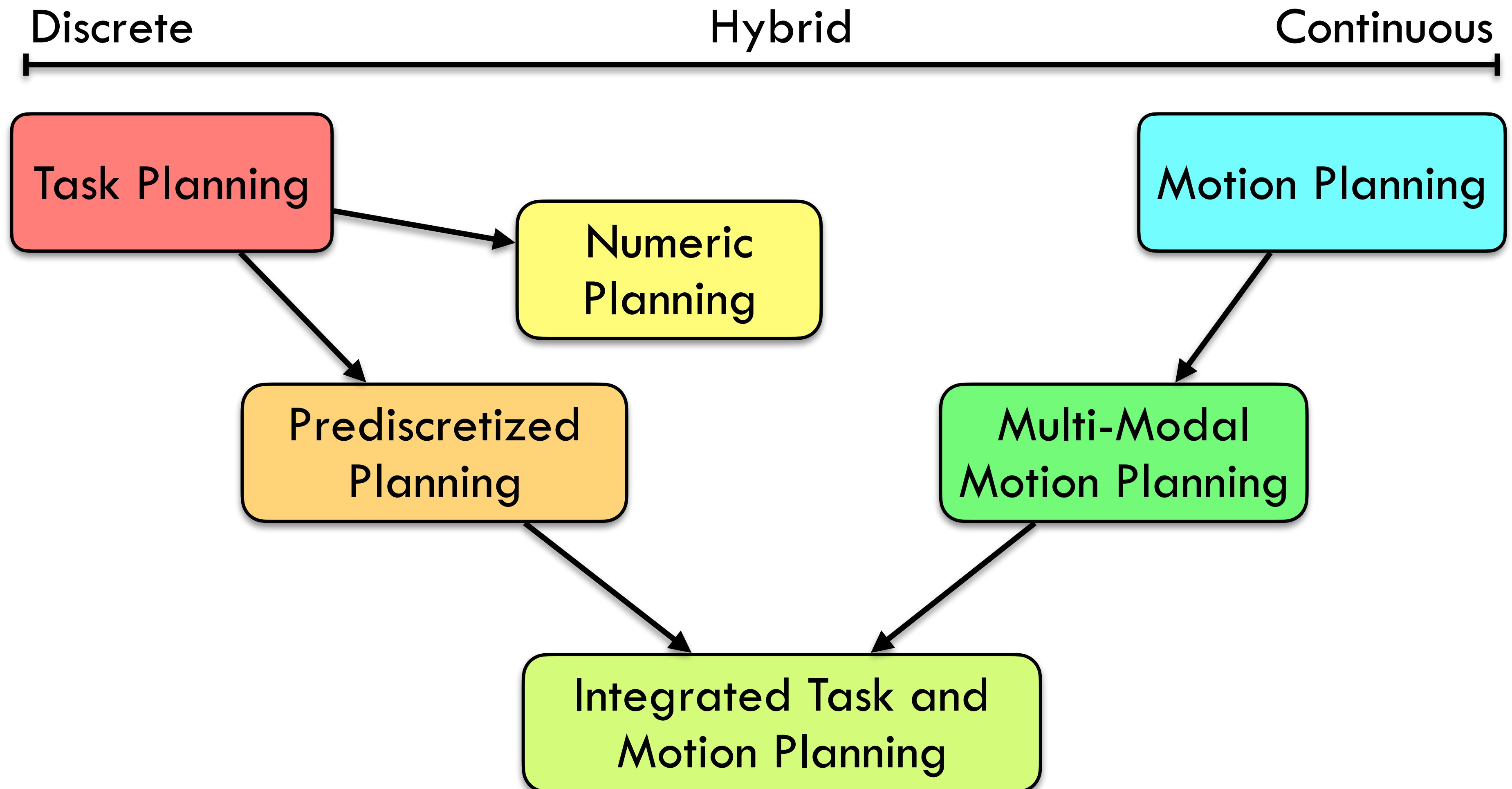
[Toussaint 2015]

[Toussaint 2018]

$$\begin{aligned} \min_{x, a_{1:K}, s_{1:K}} \quad & \int_0^T f_{\text{path}}(\bar{x}(t)) dt + f_{\text{goal}}(x(T)) \\ \text{s.t.} \quad & x(0) = x_0, \quad h_{\text{goal}}(x(T)) = 0, \quad g_{\text{goal}}(x(T)) \leq 0, \\ & \forall t \in [0, T] : \quad h_{\text{path}}(\bar{x}(t), s_{k(t)}) = 0, \\ & \quad \quad \quad g_{\text{path}}(\bar{x}(t), s_{k(t)}) \leq 0 \\ & \forall k \in \{1, \dots, K\} : \quad h_{\text{switch}}(\hat{x}(t_k), a_k) = 0, \\ & \quad \quad \quad g_{\text{switch}}(\hat{x}(t_k), a_k) \leq 0, \\ & \quad \quad \quad s_k \in \text{succ}(s_{k-1}, a_k) . \end{aligned}$$

# Hybrid Planning Spectrum Revisited

62



# Task and Motion Planning (TAMP)

# Shakey the Robot (1969)

64

- **First autonomous mobile manipulator (via pushing)**
  - Visibility graph, A\* search, and STRIPS!
- **Decoupled task and motion planning**
  - Task planning **then** motion planning

[Fikes 1971]

[Nilsson 1984]

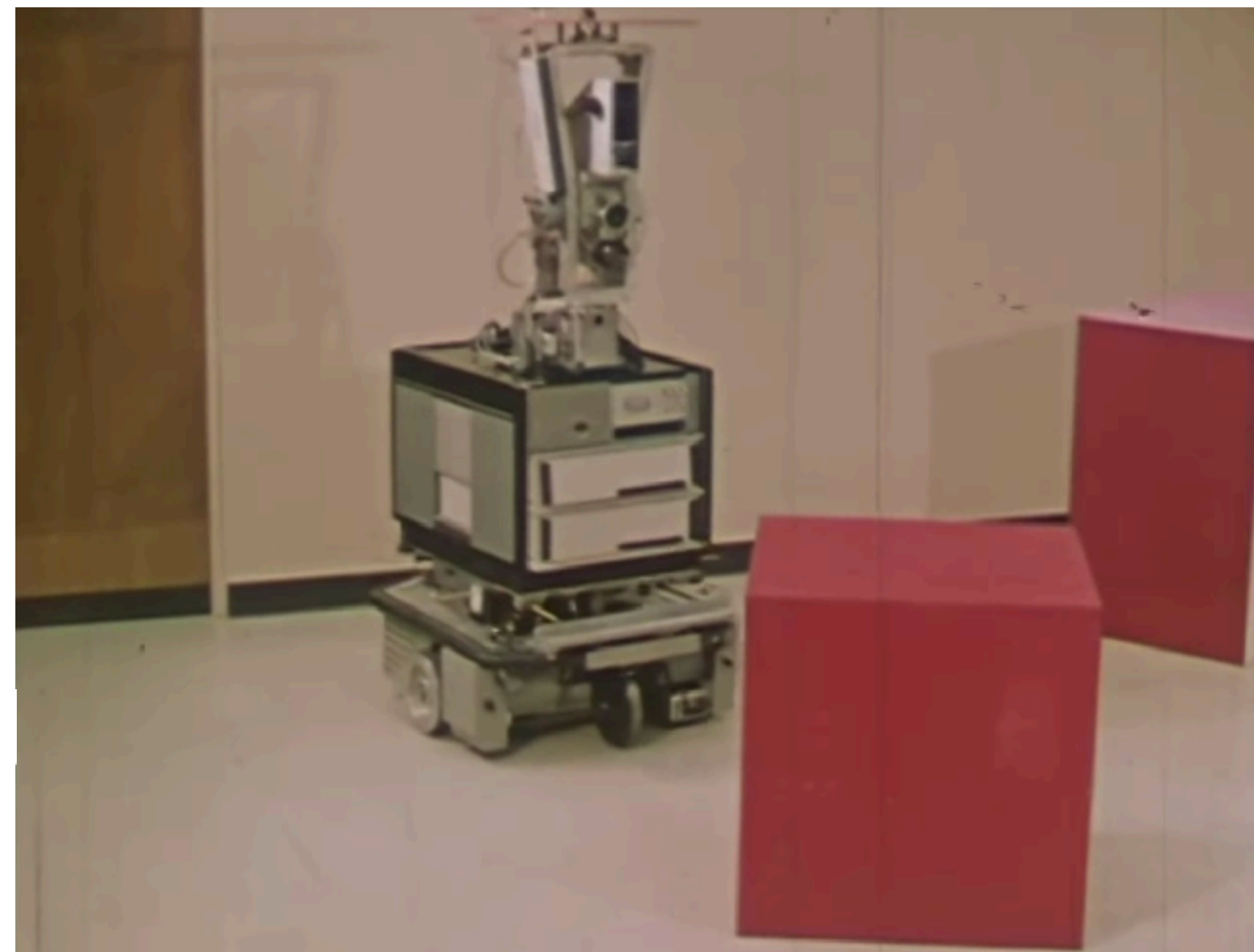
```
type(robot robot)    type(ol object)
name(robot shakey)    name(ol box1)
at(robot 4.1 7.2)    at(ol 3.1 5.2)
theta(robot 90.1)    inroom(ol r1)
                    shape(ol wedge)
                    radius(ol 3.1)
```

GOTHRU(d,r1,r2)

Precondition INROOM(ROBOT,r1)  $\wedge$  CONNECTS(d,r1,r2)

Delete List INROOM(ROBOT,\$)

Add List INROOM(ROBOT,r2)

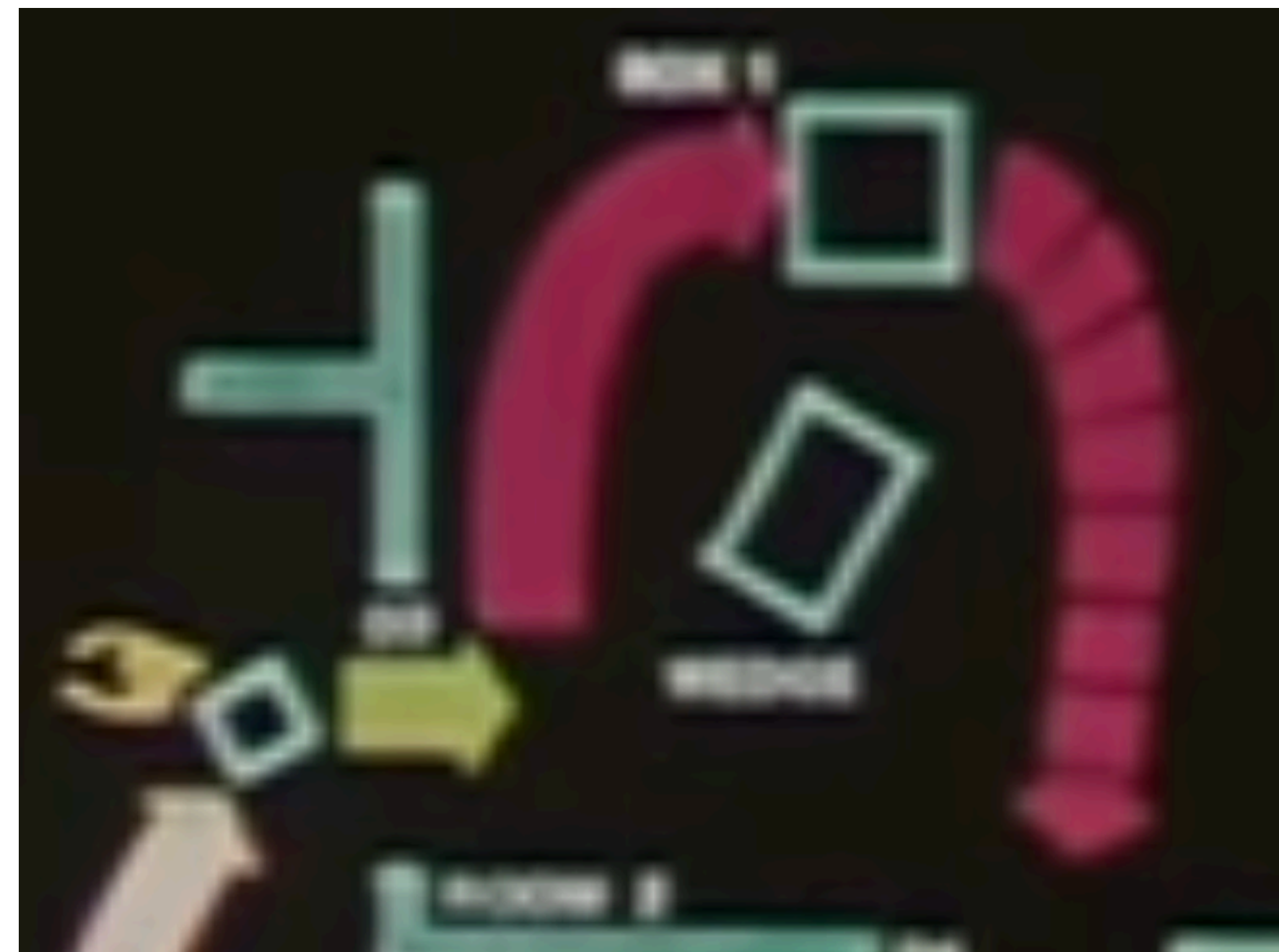
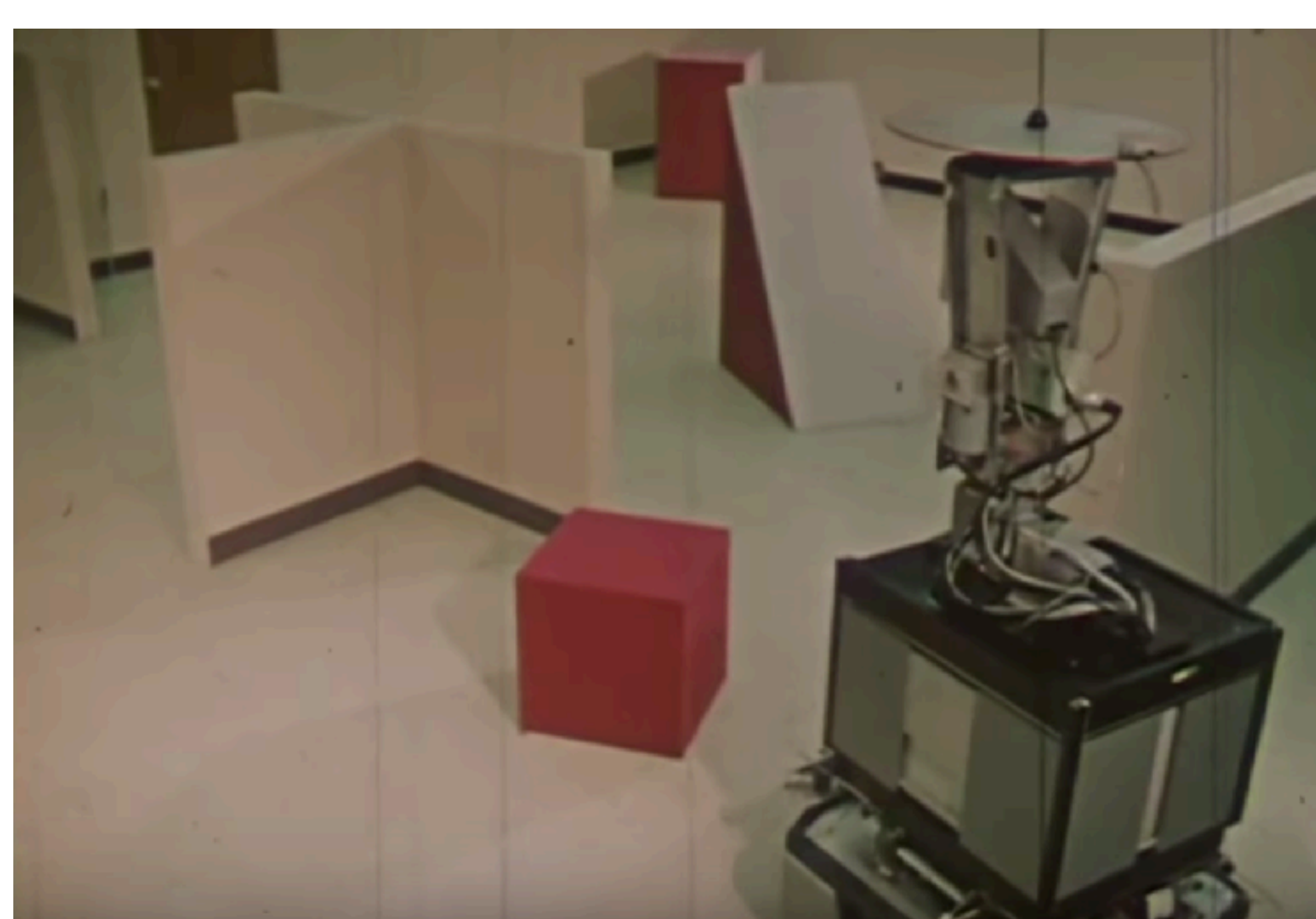




# Obstacle Blocks Shakey's Path

65

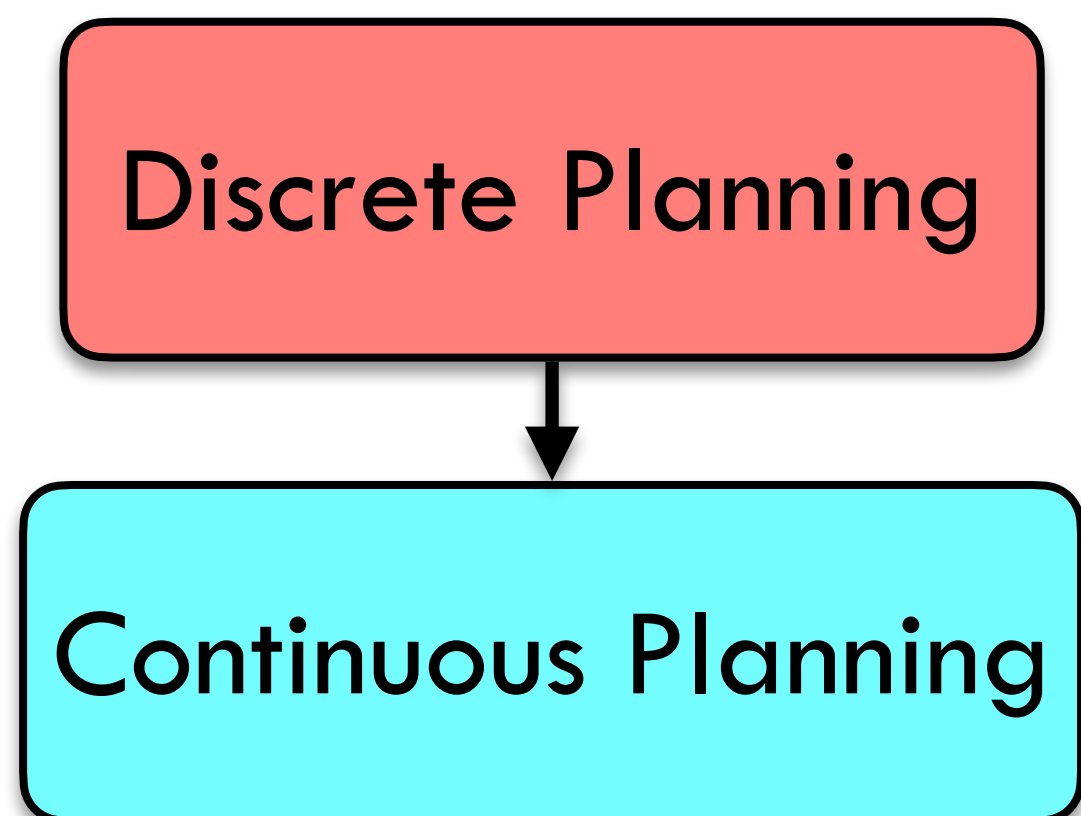
- What if a movable block **prevented** Shakey from safely moving into the adjacent room?
- Shakey could **push** it out of the way or **go around** it
  - What's more efficient? How to push it? ...



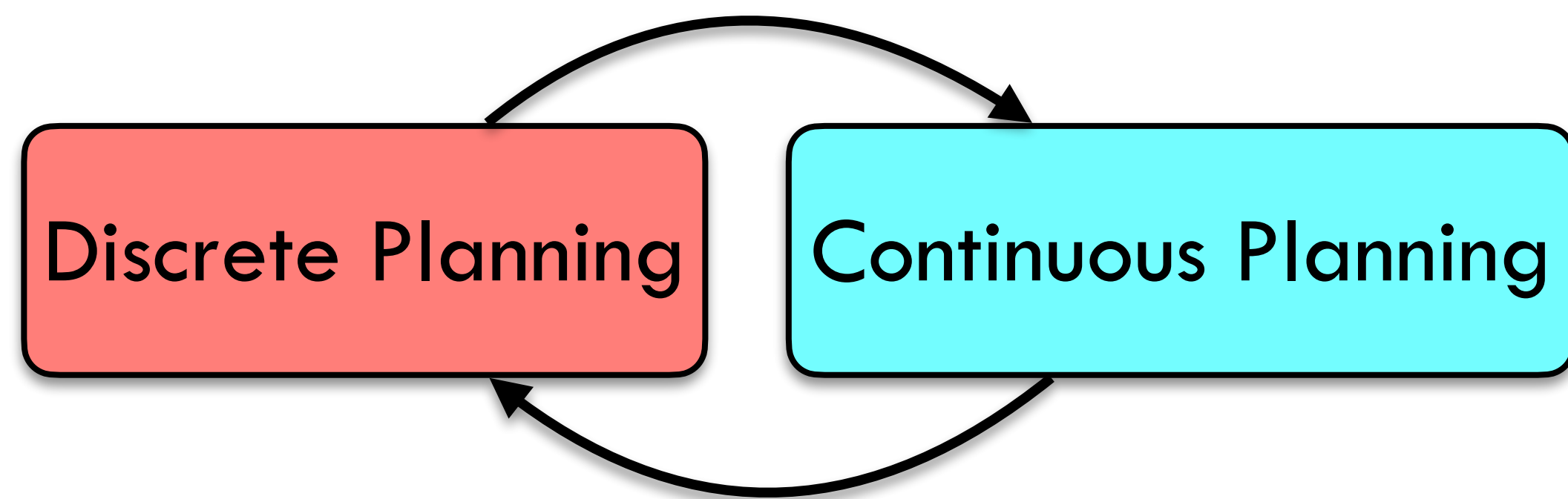
# Decoupled vs Integrated TAMP

66

- **Decoupled:** discrete (task) planning **then** continuous (motion) planning
- Requires a strong **downward refinement** assumption
- **Every** correct discrete plan can be **refined** into a correct continuous plan (from hierarchal planning)
- **Integrated:** **simultaneous** discrete & continuous planning



**Decoupled**

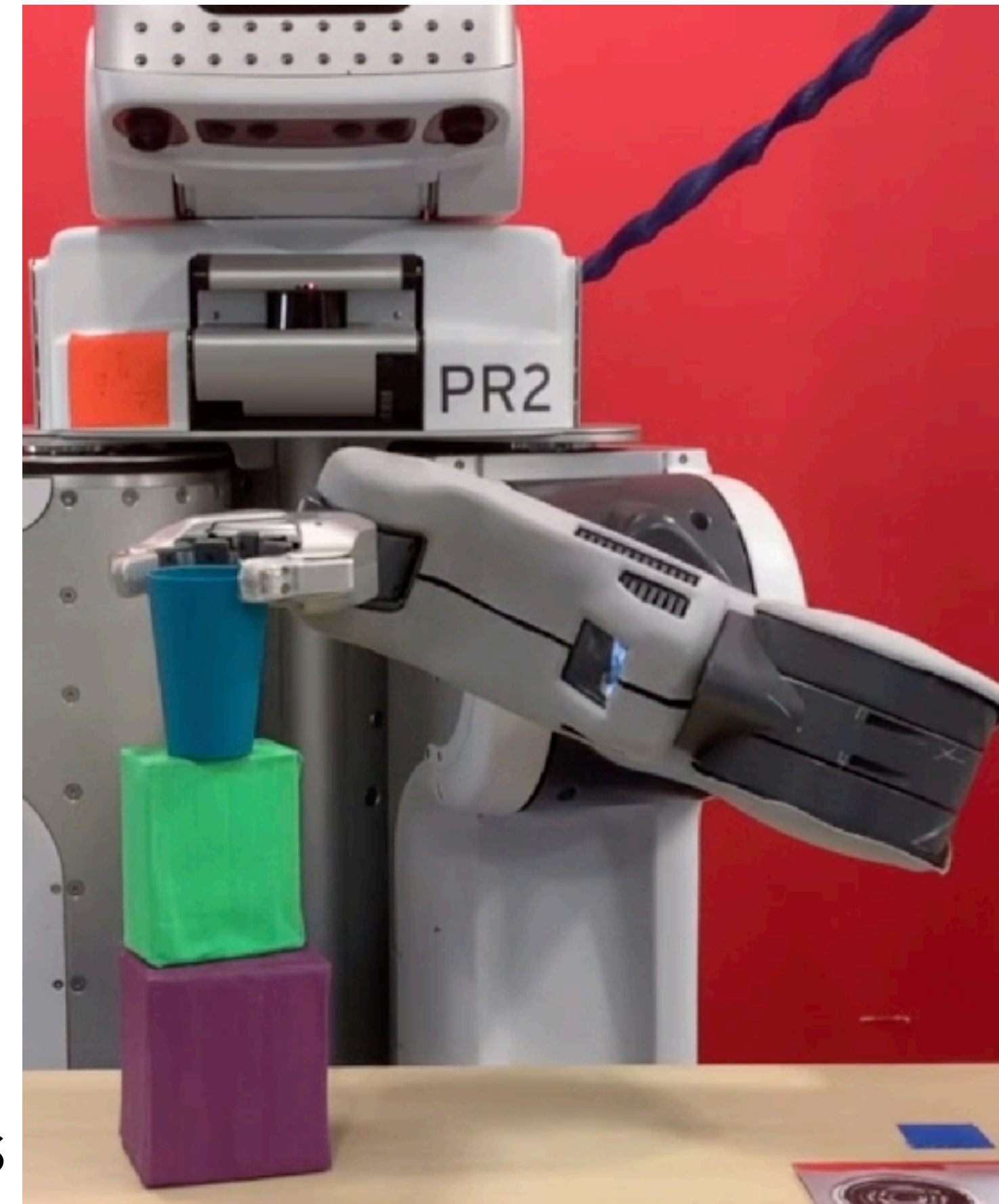


**Integrated**

# Task and Motion Planning (TAMP)

67

- **Continuous** robot motion with **discrete-time** actions
- Mixed discrete/continuous (**hybrid**) states and actions:
- State variables include:
  - **Continuous:** robot config, object poses, door joint angles
  - **Discrete:** is-on, is-cooked
- Solution components:
  - Plan **structure:** action sequence
  - Action **parameter** values: placements, grasps, ...
  - **Control** values: continuous motions



# TAMP Example: Cook Object A

68

$s_0 = \{ \text{atRob} = q_0, \text{at}[A] = p_0, \text{holding} = \text{None}, \text{cooked}[A] = \text{False} \}$

Goal conditions:  $\text{cooked}[A] = \text{True}$

Initial state

atRob  $q_0$

holding None

at[A]  $p_0$

cooked[A] False

$s_0$



Goal state(s)

$q_*$

None

$p_*$

True

$s_*$

# Plan Skeleton & Action Parameters

69

## Plan skeleton (structure)



atRob **q<sub>0</sub>**

holding **None**

at[A] **p<sub>0</sub>**

cooked[A] **False**

$s_0$

$s_1$

$s_2$

$s_3$

$s_4$

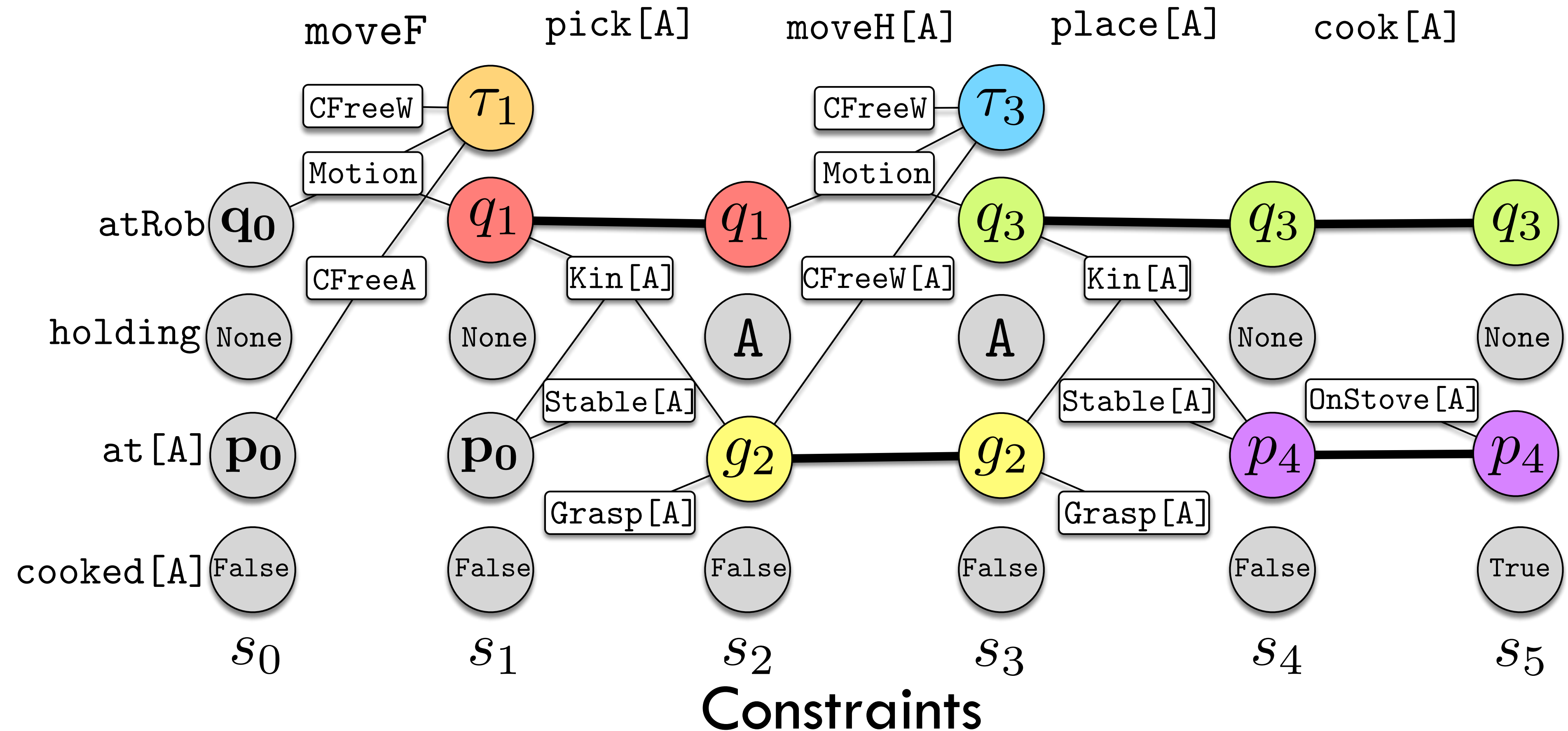
$s_5$

State variable values

# Plan Constraints & Parameter Values

70

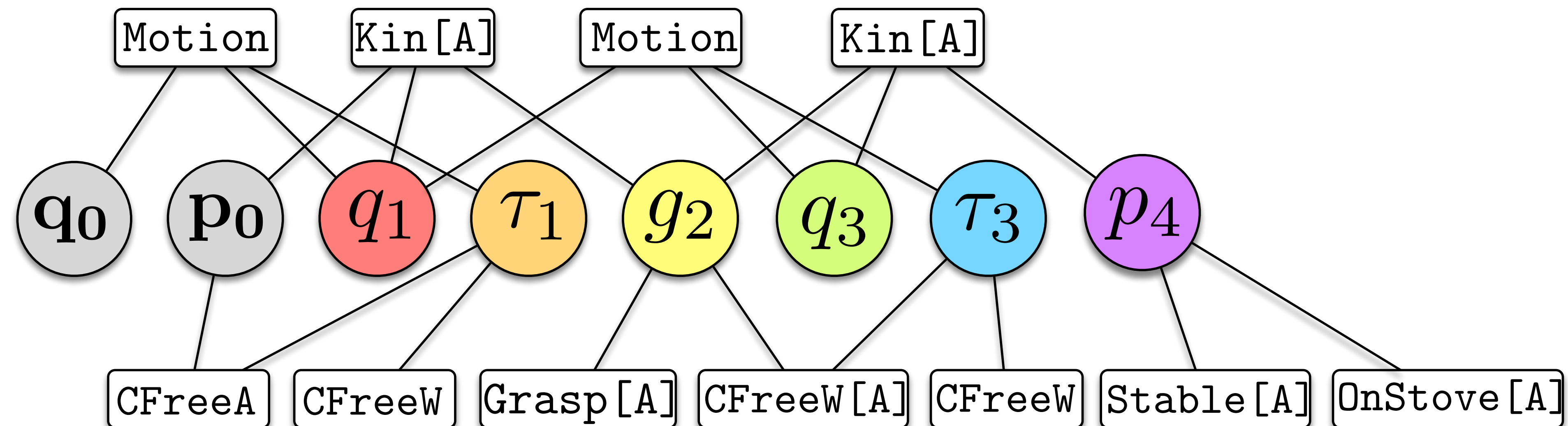
Example plan:  $\pi = [\text{moveF}(q_0, \tau_1, q_1), \text{pick}[A](q_1, p_0, g_2),$   
 $\text{moveH}[A](q_1, \tau_3, q_3), \text{place}[A](q_3, p_4, g_2), \text{cook}[A](p_4)]$



# Constraint Network (Factor Graph)

71

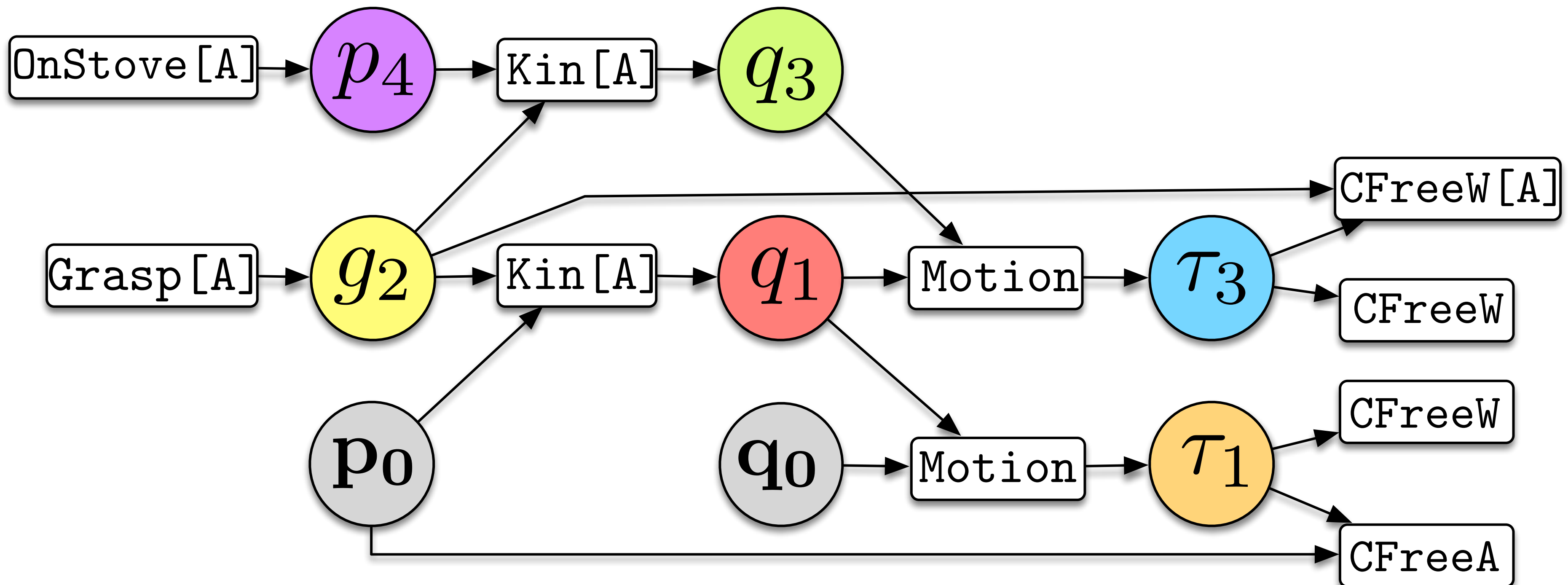
- Compress plan skeleton into a constraint network
- Undirected bipartite graph of **variables & constraints**
- Can address with **optimization** and/or **sampling**



# Sampling Network

72

- Satisfy constraint network **compositionally**
- Directed acyclic graph (DAG)
- **Conditional** samplers consume **inputs** and generate completing **outputs**





# The Need for Integrated Planning

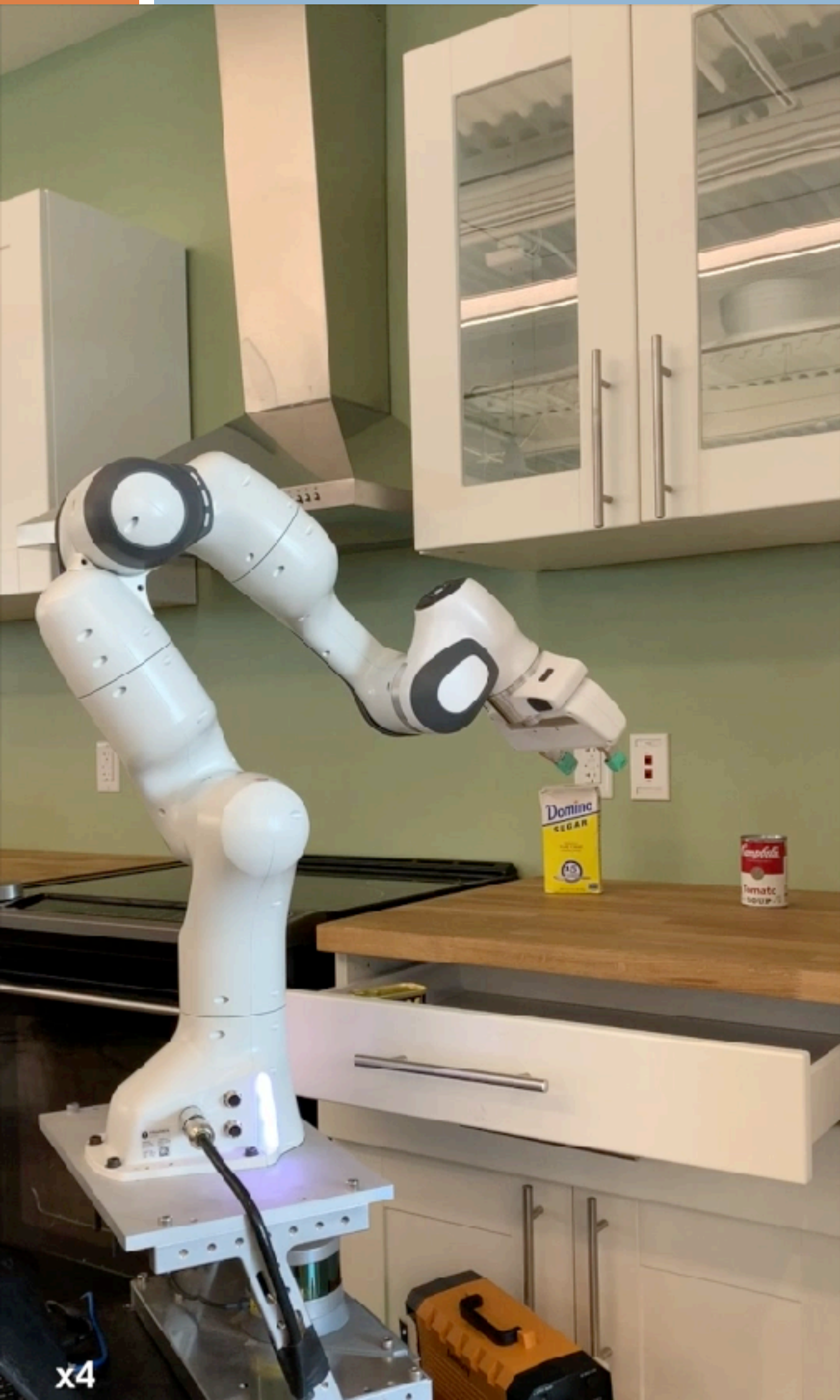
73

- Continuous constraints **limit** feasible **plan structures**
- Kinematics, joint limits, collisions, stable grasps, visibility, stability, stiffness, dynamics
- Strict **hierarchy** (task planning *then* motion planning) **fails**
- Reachability, **obstruction**, occupancy, occlusion
- Need to plan **jointly**



# Spam in Left Cabinet & Door Closed

74

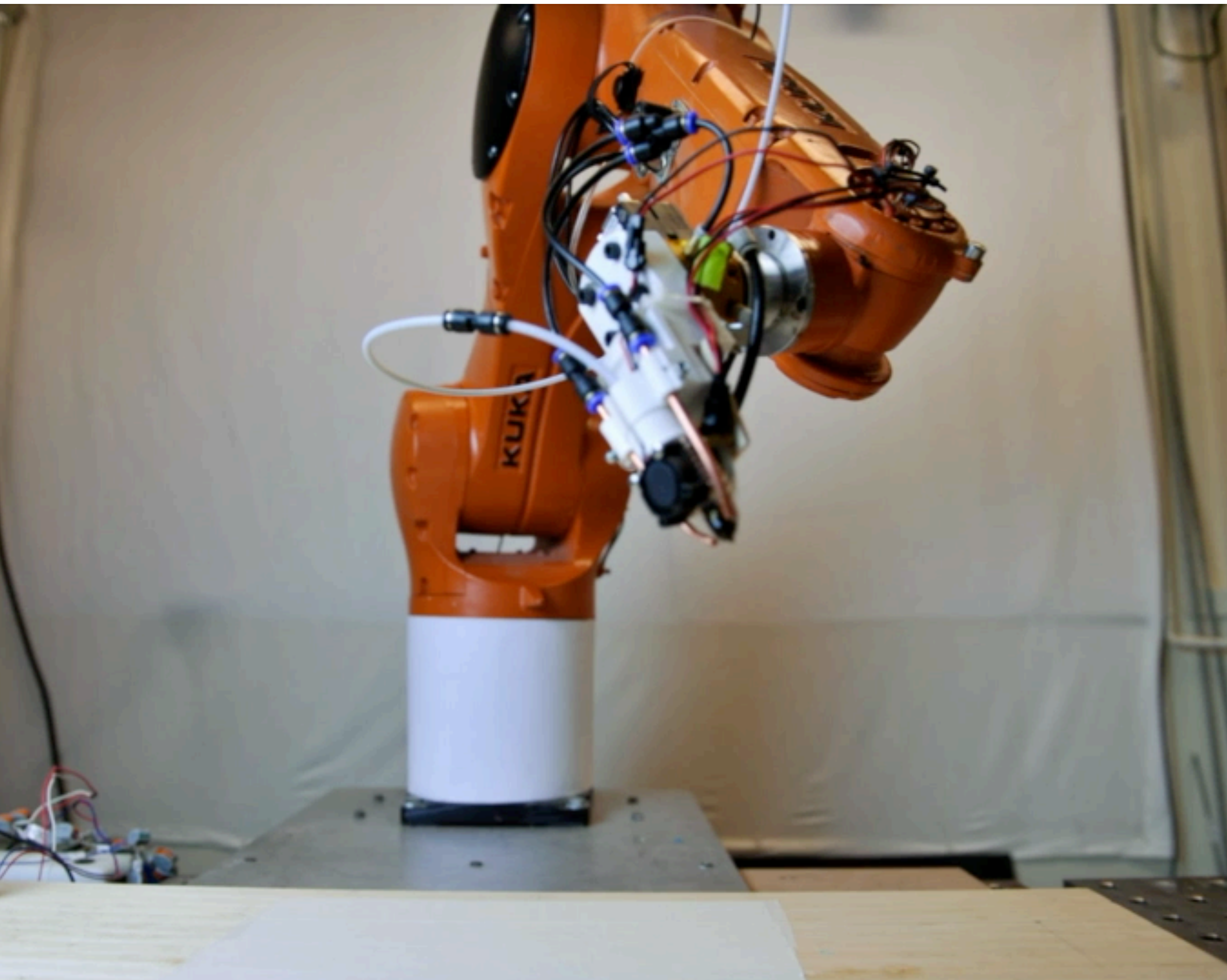


- Robot forced to **regrasp** the spam
- Change from a **top** to a **side** grasp
- **Non-monotonic** problem
  - Plan must temporarily **undo goals**
  - **Open** then later **close** the door
- Planning automatically discovers through **propagating constraints**

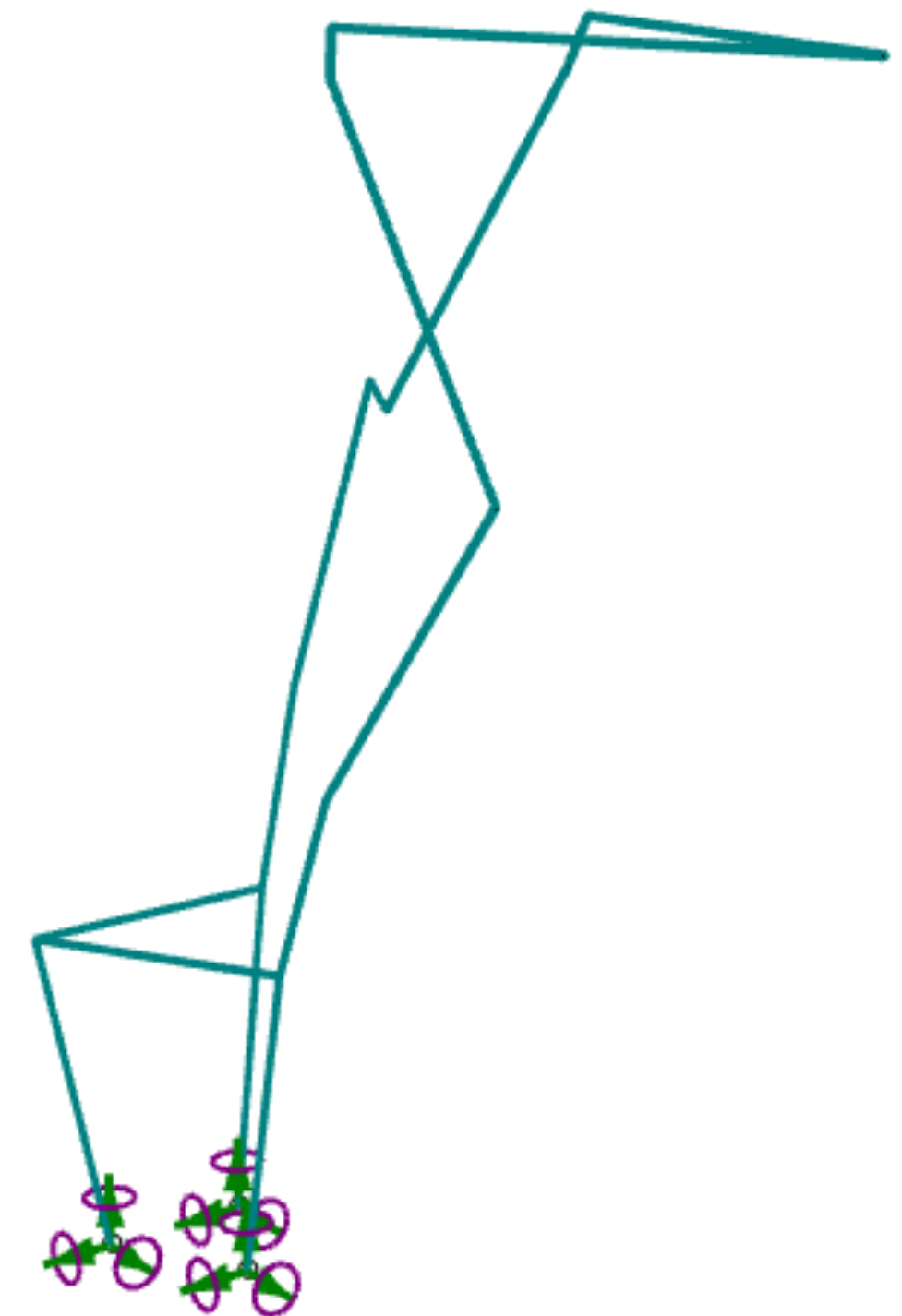
# 3D Print (Extrude) Klein Bottle Design

75

- Plan sequence & motions for **246** extrusions



**Stiffness constraint**



[*Garrett, Huang, Lozano-Pérez, & Mueller 2020*]

# Taxonomy of TAMP Approaches

76

	Pre-discretized	Sampling	Optimization
Satisfaction first	Ferrer-Mestres et al. (84, 85) <sup>b</sup>	Siméon et al. (22) <sup>a</sup> Hauser et al. (13, 14, 29) <sup>a</sup> <b>Garrett et al. (21, 86)<sup>b</sup></b> Krontiris & Bekris (87, 88) <sup>a</sup> Akbari & Rosell (89) <sup>b</sup> Vega-Brown & Roy (90) <sup>a</sup>	
Interleaved	Dornhege et al. (62, 63, 91) <sup>b</sup> Gaschler et al. (92–94) <sup>b</sup> Colledanchise et al. (95) <sup>b</sup>	Gravot et al. (96, 97) <sup>b</sup> Stilman et al. (23, 98, 99) <sup>a</sup> Plaku & Hager (100) <sup>a</sup> Kaelbling & Lozano-Pérez (101, 102) <sup>b</sup> Barry et al. (30, 103, 104) <sup>a</sup> Garrett et al. (70, 71) <sup>b</sup> Thomason & Knepper (105) <sup>b</sup> Kim et al. (106, 107) <sup>b</sup> Kingston et al. (108) <sup>a</sup>	Fernández-González et al. (109) <sup>b</sup>
Sequencing first	Nilsson (3) <sup>b</sup> Erdem et al. (74, 75) <sup>b</sup> Lagriffoul et al. (65–67) <sup>b</sup> Pandey et al. (110, 111) <sup>b</sup> Lozano-Pérez & Kaelbling (112) <sup>b</sup> Dantam et al. (77–79) <sup>b</sup> Lo et al. (113) <sup>b</sup>	Wolfe et al. (114) <sup>b</sup> Srivastava et al. (60, 76) <sup>b</sup> <b>Garrett et al. (55, 73)<sup>b</sup></b>	Toussaint et al. (61, 68, 69) <sup>b</sup> Shoukry et al. (81–83) <sup>b</sup> Hadfield-Menell et al. (115) <sup>b</sup>

<sup>a</sup>Approaches for MMMP.

<sup>b</sup>Approaches for TAMP.

# My Approach: PDDLStream

77

- Extends Planning Domain Definition Language (**PDDL**)
  - States and actions described using **predicate logic**
  - Standardized, **factored**, lifted, domain-independent
- Specification of **sampling** procedures as **streams**
  - Can model domains with **infinitely-many** actions
- Algorithms plan while treating streams as **blackboxes**
- **Reduce** planning to a sequence of **finite** problems
  - **PDDL heuristic search** algorithms as subroutines

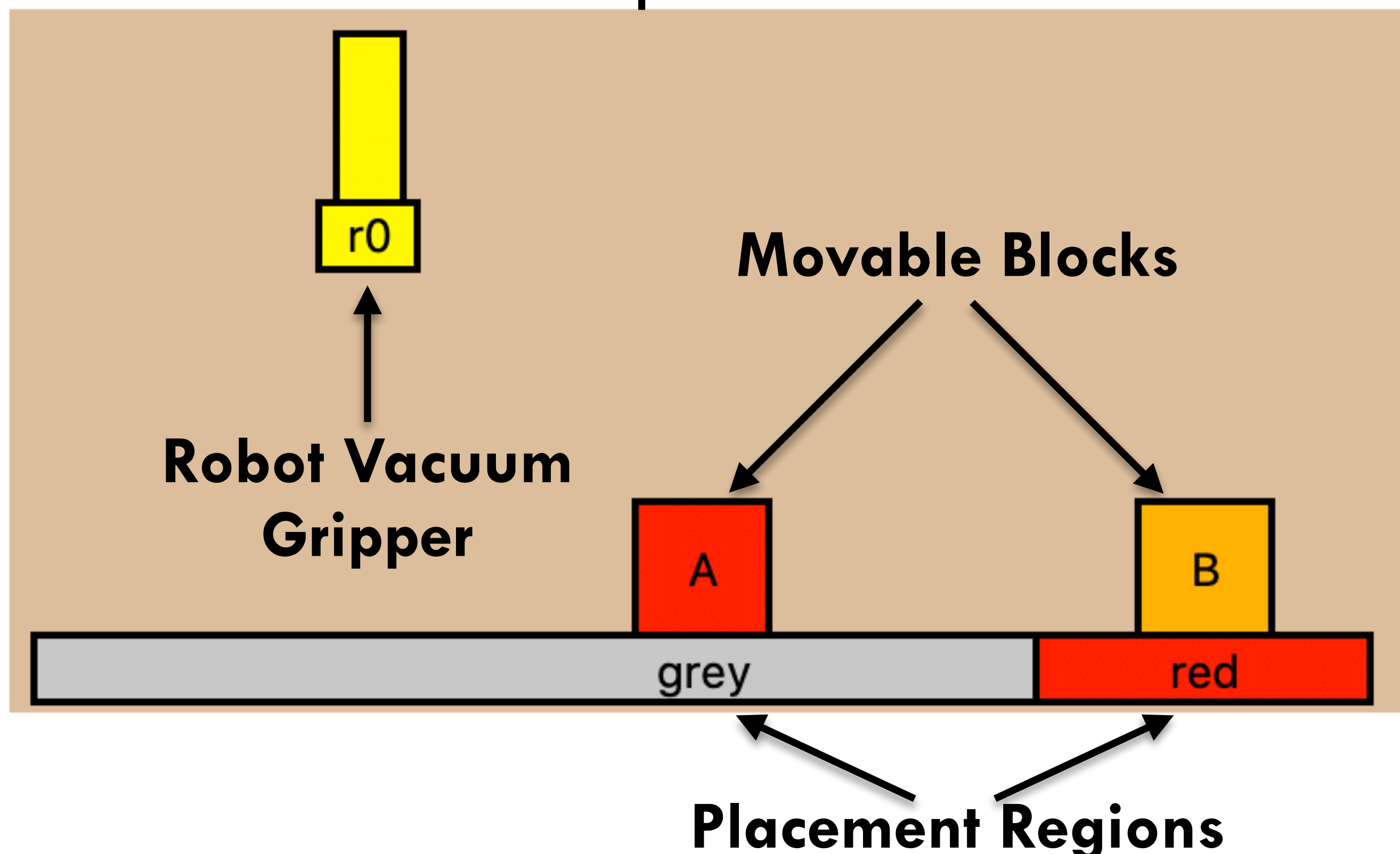
# PDDLStream Language

[Garrett, Lozano-Pérez, Kaelbling 2020a]

# 2D Pick-and-Place Domain

79

- Robot and block poses are continuous  $[x \ y]$  pairs
- **Goal:** block **A** within the **red** region
- Block **B** obstructs the placement of **A**

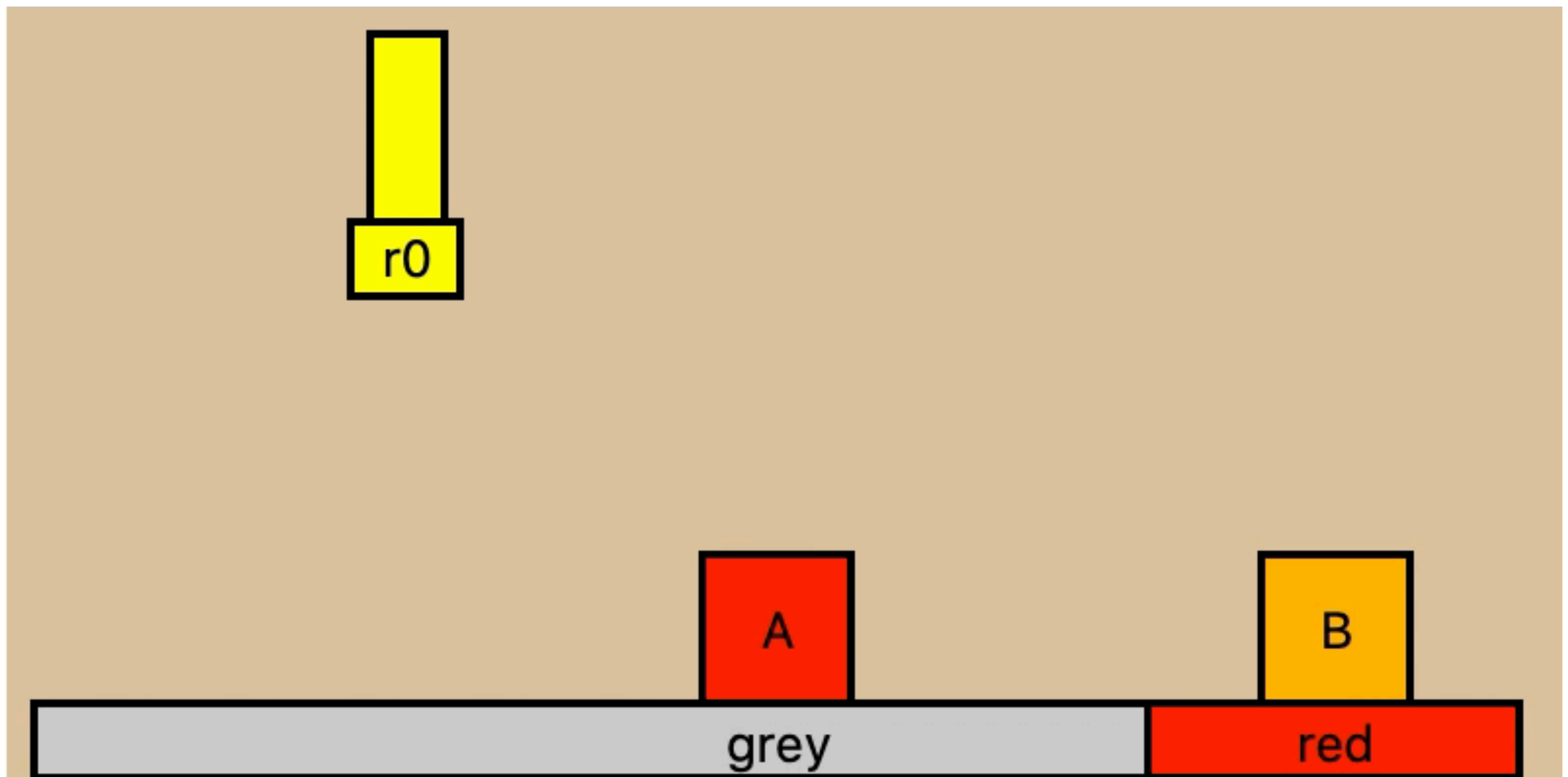


# 2D Pick-and-Place Solution

80

- One (of infinitely many) possible solutions

[move (...), pick (**B**, ...), move (...), place (**B**, ...),  
move (...), pick (**A**, ...), move (...), place (**A**, ...)]





# 2D Pick-and-Place Initial & Goal

81

- Not all values are discrete, some are **continuous**
- **Static** (constant) initial facts - **satisfied constraints**

$$F = \{ \text{Block}(\mathbf{A}), \text{Block}(\mathbf{B}), \text{Region}(\mathbf{red}), \\ \text{Region}(\mathbf{grey}), \text{Conf}(\underline{[-7.5, 5]}), \\ \text{Pose}(\mathbf{A}, \underline{[0. 0.]}), \text{Pose}(\mathbf{B}, \underline{[7.5 0.]}) \}$$

- **Fluent** (changing) initial facts - **state variables**

$$S_0 = \{ \text{AtConf}(\underline{[-7.5 5.]}), \text{HandEmpty}(), \\ \text{AtPose}(\mathbf{A}, \underline{[0. 0.]}), \text{AtPose}(\mathbf{B}, \underline{[7.5 0.]}) \}$$

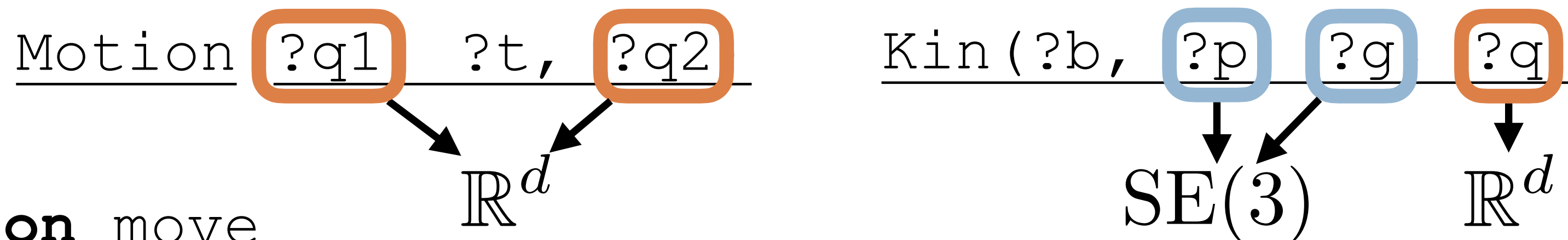
- **Goal** logical formula - set of **goal states**

$$S_* = \mathbf{exists} (?p) \{ \text{Contained}(\mathbf{A}, ?p, \mathbf{red}), \text{AtPose}(\mathbf{A}, ?p) \}$$

# Pick-and-Place Actions

82

- Typical PDDL action description except that arguments are **high-dimensional & continuous!**
- To use, must **satisfy static facts (constraints)**



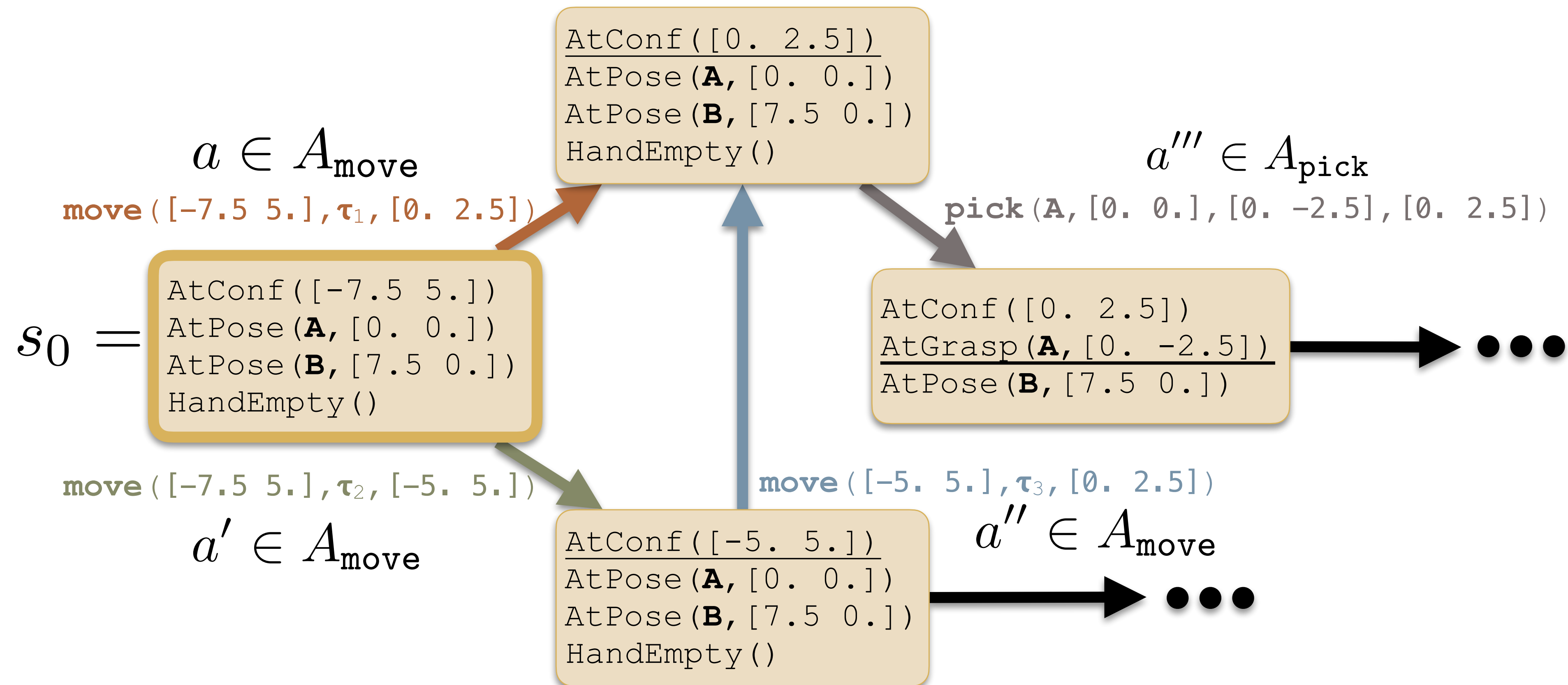
```
(:action move
:parameters (?q1, ?t, ?q2)
:precondition {Motion(?q1, ?t, ?q2), AtConf(?q1)}
:effect {AtConf(?q2), ¬AtConf(?q1)})
Amove

(:action pick
:parameters (?b, ?p, ?g, ?q)
:precondition {Kin(?b, ?p, ?g, ?q), AtConf(?q),
              AtPose(?b, ?p), HandEmpty()}
:effect {AtGrasp(?b, ?g), ¬AtPose(?b, ?p), ¬HandEmpty()})
Apick
```

# Search in Discretized State-Space

Suppose an **oracle** gave us the following values and facts:

$$F = \{ \text{Motion}([-7.5 \ 5.], \tau_1, [0. \ 2.5]), \text{Motion}([-7.5 \ 5.], \tau_2, [-5. \ 5.]), \\ \text{Motion}([-5. \ 5.], \tau_3, [0. \ 2.5]), \text{Kin}(\mathbf{A}, [0. \ 0.], [0. \ -2.5], [0. \ 2.5]), \dots \}$$



# No a Priori Discretization

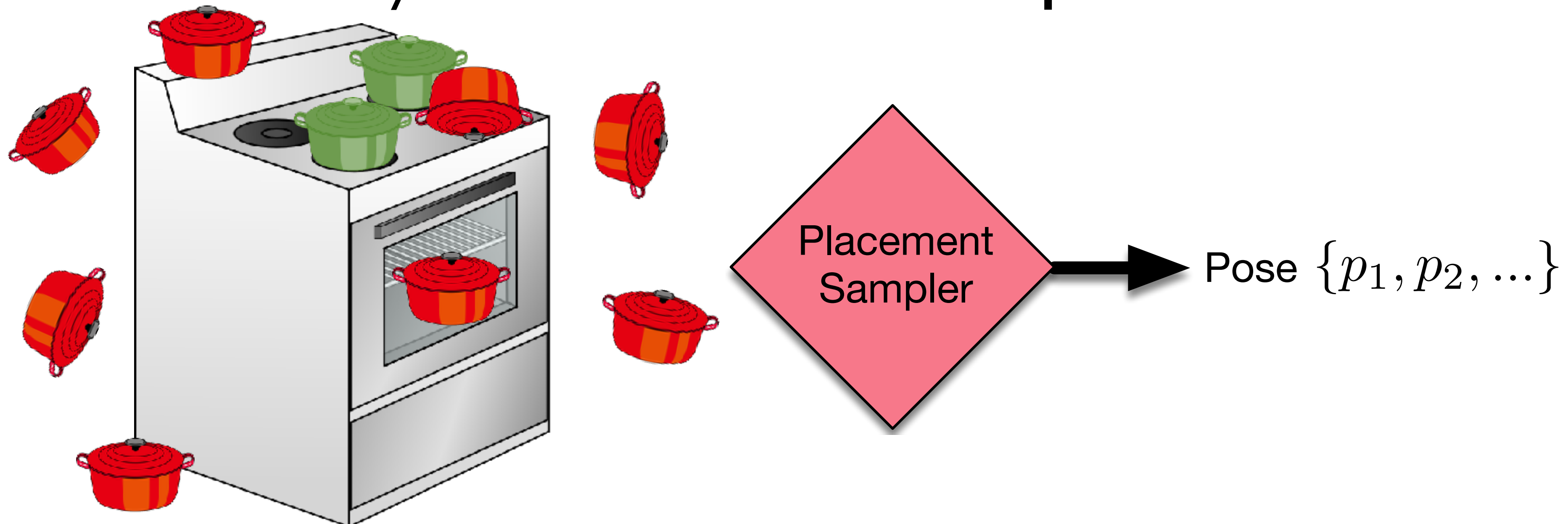
84

- Values **given** at start:
  - 1 initial configuration: `Conf ([-7.5 5.])`
  - 2 initial poses: `Pose (A, [0. 0.])`  
`Pose (B, [7.5 0.])`
- **Planner needs to find:**
  - 1 pose for **A** within **red**: `Contain (A, ?p, red)`
  - 1 collision-free pose for **B**: `CFree (A, ?p, B, ?p2)`
  - 1 grasp for **A** and **B**: `Grasp (A, ?g) , Grasp (B, ?g)`
  - 4 grasping configurations: `Kin (?b, ?p, ?g, ?q)`
  - 4 robot trajectories: `Motion (?q1, ?t, ?q2)`

# What Samplers Do We Need?

85

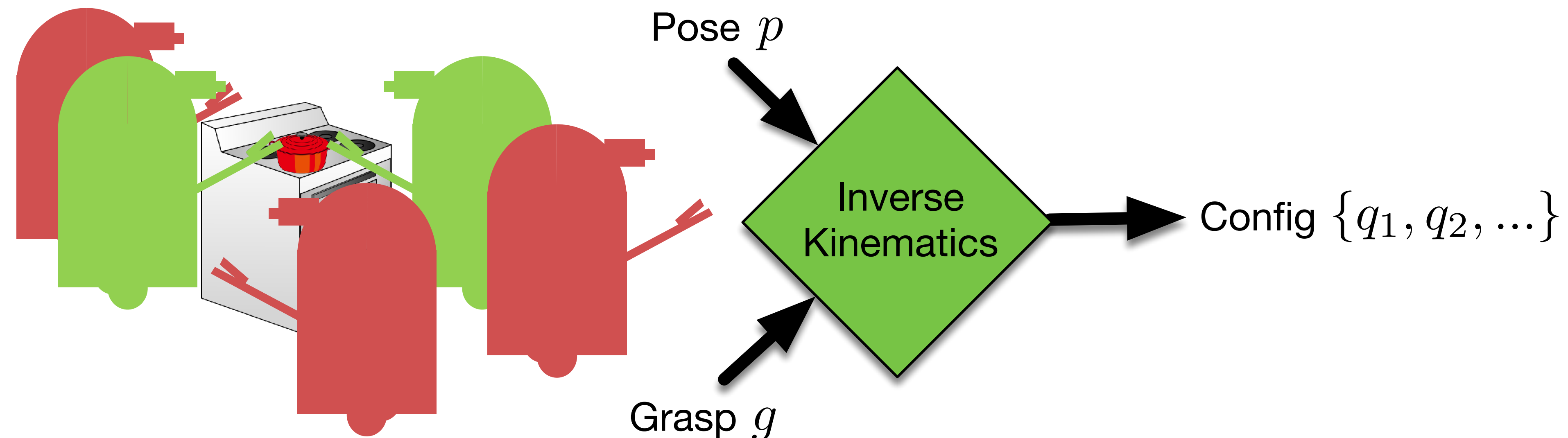
- **Low-dimensional** placement stability constraint (`Contain`)
  - e.g. 1D line embedded in 2D placement space
- **Directly sample values that satisfy the constraint**
- May need **arbitrarily many** samples
- Gradually enumerate an **infinite sequence**



# Intersection of Constraints

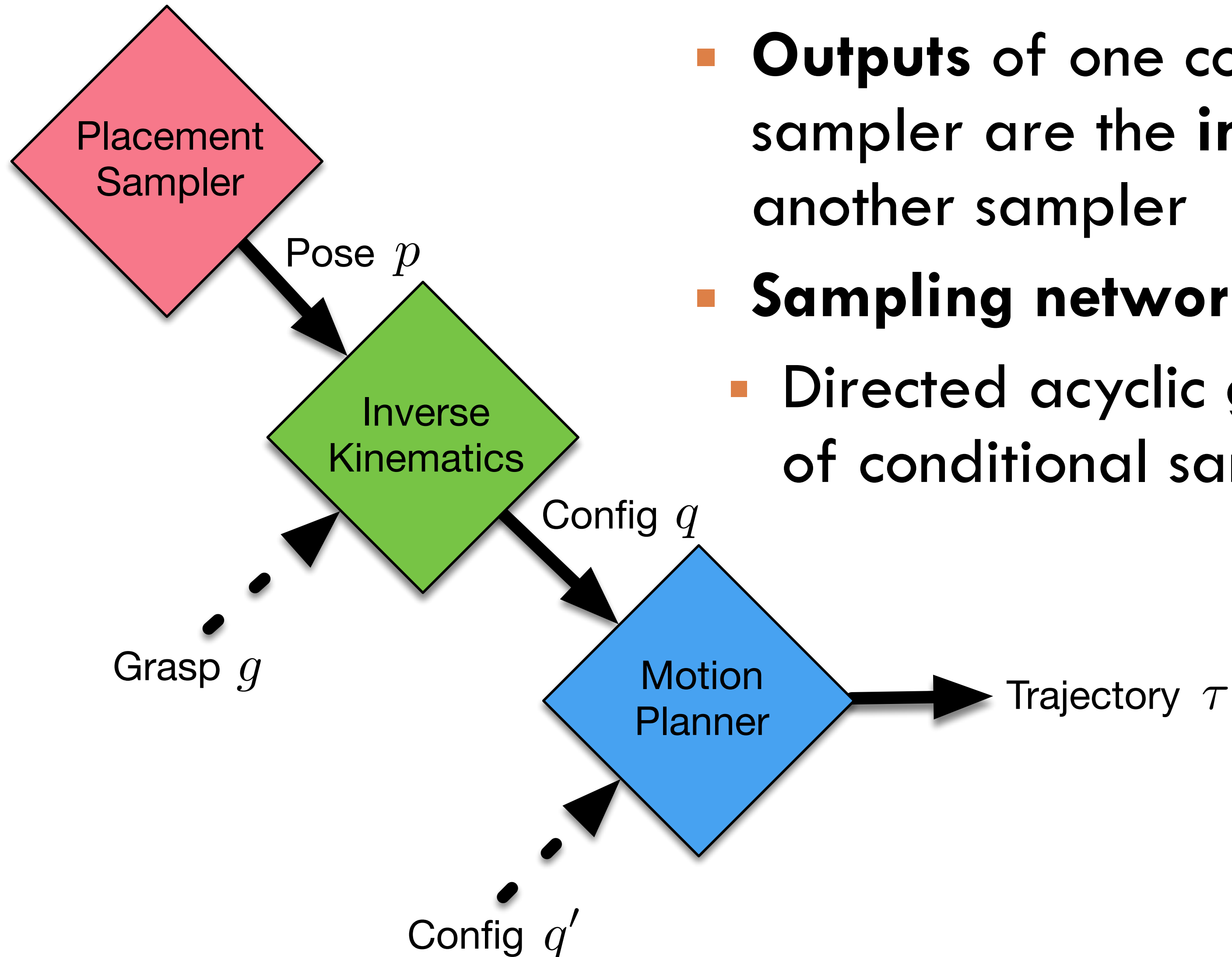
86

- **Kinematic constraint** ( $K_{in}$ ) involves poses, grasps, and configurations
- **Conditional samplers** - function from **input values** to a sampler that generates **output values**



# Composing Conditional Samplers

87



- **Outputs** of one conditional sampler are the **inputs** to another sampler
- **Sampling network**
- Directed acyclic graph (DAG) of conditional samplers

# Stream: Specification for a Sampler

88

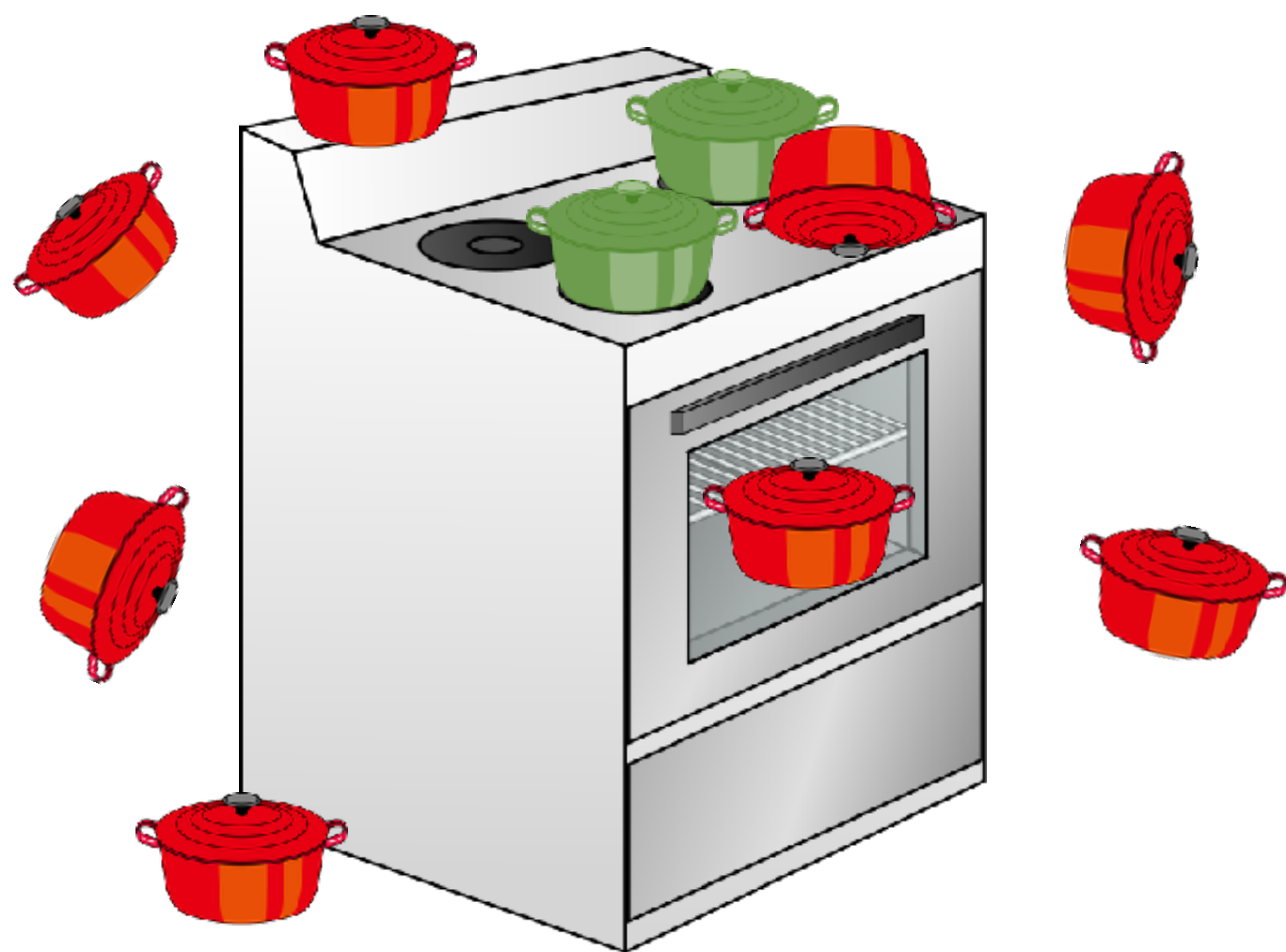
- What do **inputs & outputs** represent?
  - Communicate semantics using **predicates** (constraints)
- Declarative stream specification:
  - **Domain facts** - static facts declaring legal **inputs**
    - e.g. only configurations can be motion planner inputs
  - **Certified facts** - static facts that all **outputs** are **asserted** to satisfy with their corresponding **inputs**
    - e.g. poses sampled from a region are within it



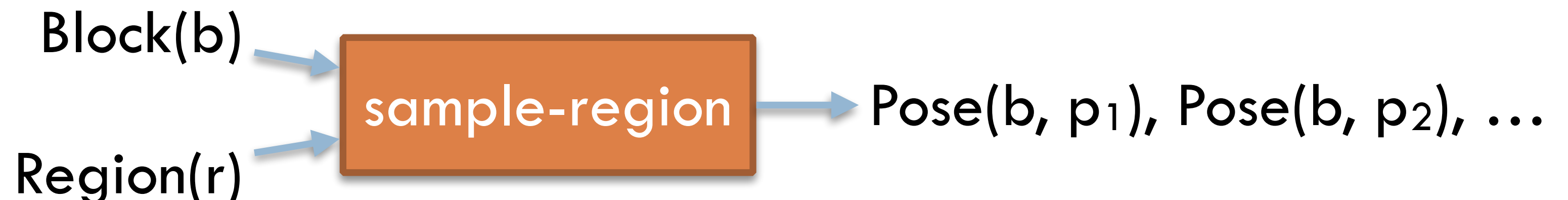
# Sampling Placements in a Region

89

```
(:stream sample-region
:inputs (?b, ?r)
:domain {Block(?b), Region(?r)}
:outputs (?p)
:certified {Pose(?b, ?p), Contain(?b, ?p, ?r)})
```



```
def sample_region(b, r):
    x_min, x_max = REGIONS[r]
    w = BLOCKS[b].width
    while True:
        x = random.uniform(x_min + w/2,
                           x_max - w/2)
        p = np.array([x, 0.])
        yield (p,)
```

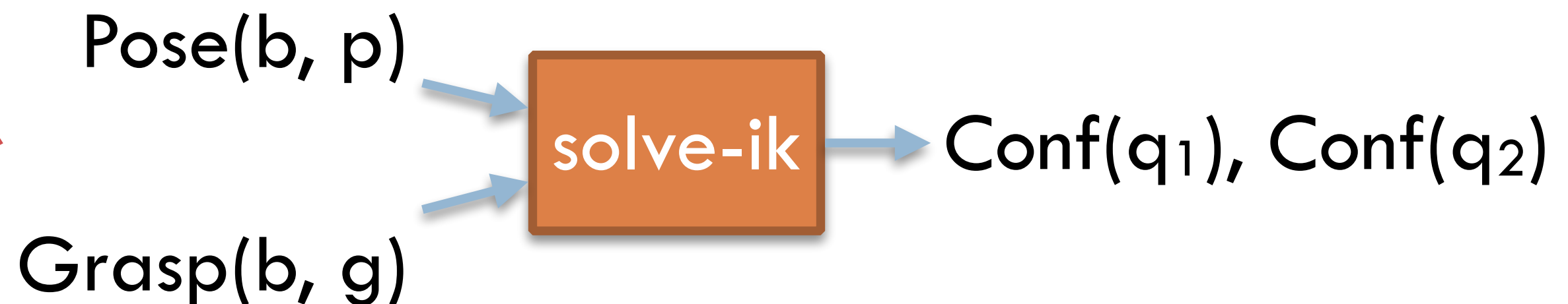
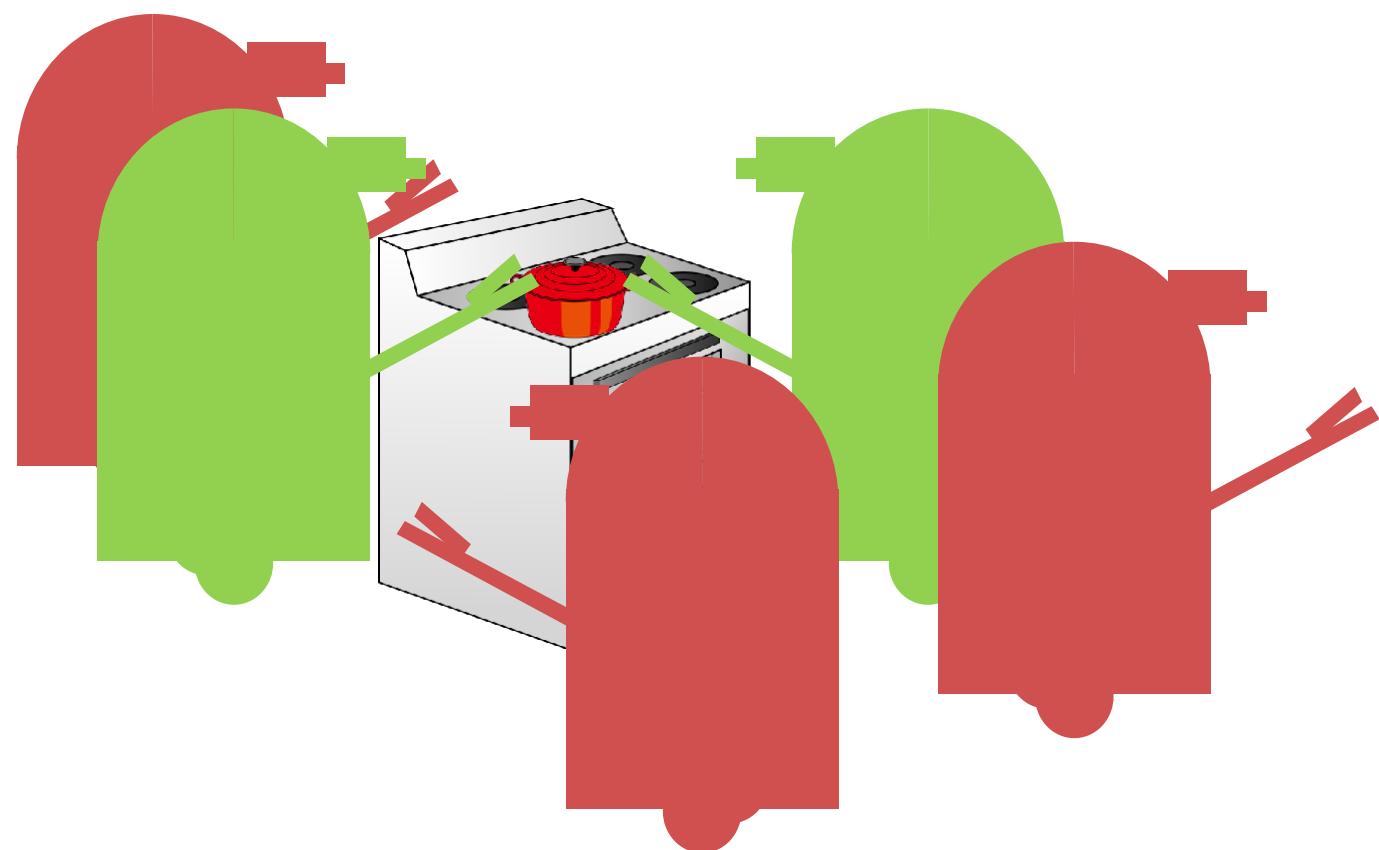


# Sampling IK Solutions

90

- **Inverse kinematics (IK)** to produce robot grasping configurations
- Trivial in 2D, non-trivial in general (e.g. 7-DOF arm)

```
(:stream solve-ik  
:inputs (?b, ?p, ?g)  
:domain {Pose(?b, ?p), Grasp(?b, ?g)}  
:outputs (?q)  
:certified {Conf(?q), Kin(?b, ?p, ?g, ?q)})
```

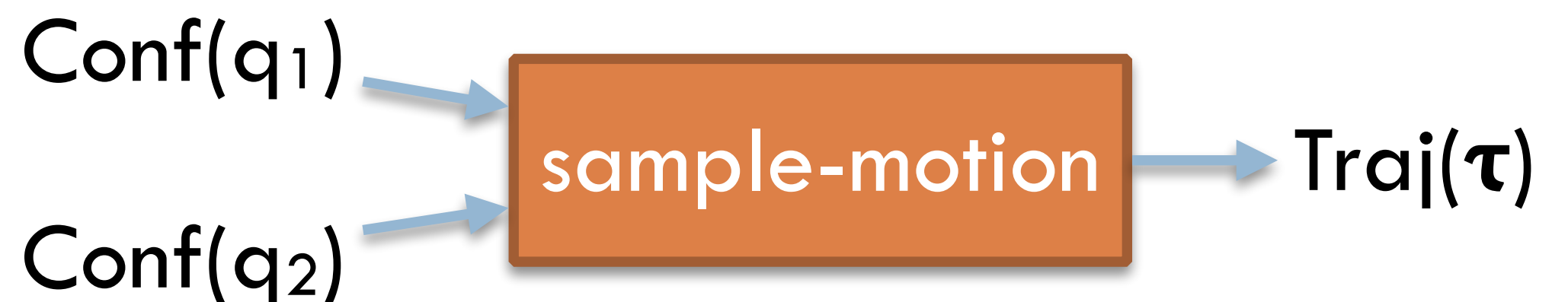
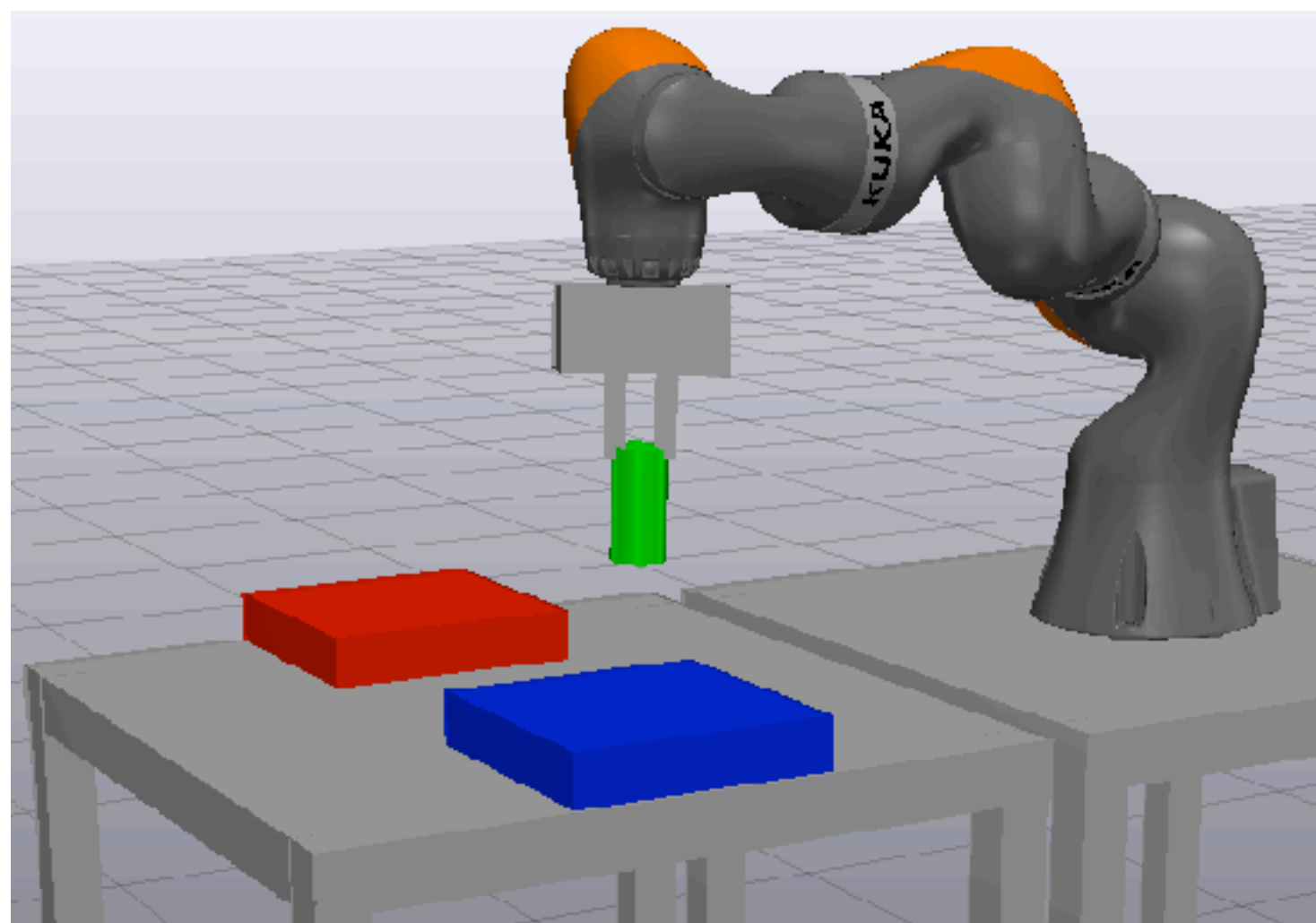


# Invoking a Motion Planner

91

- “Sample” multi-waypoint robot trajectories
- Use off-the-shelf motion planner (e.g. RRT)

```
(:stream sample-motion  
:inputs (?q1, ?q2)  
:domain {Conf(?q1), Conf(?q2)}  
:outputs (?t)  
:certified {Traj(?t), Motion(?q1, ?t, ?q2)})
```



# PDDLStream Algorithms

[Garrett, Lozano-Pérez, Kaelbling 2020a]

# Two PDDLStream Algorithms

93

- PDDLStream **algorithms decide** which streams to use
- **Reduce** planning to a sequence of PDDL problems
  1. **Search** a finite PDDL problem for plan
  2. **Modify** the PDDL problem (depending on the plan)



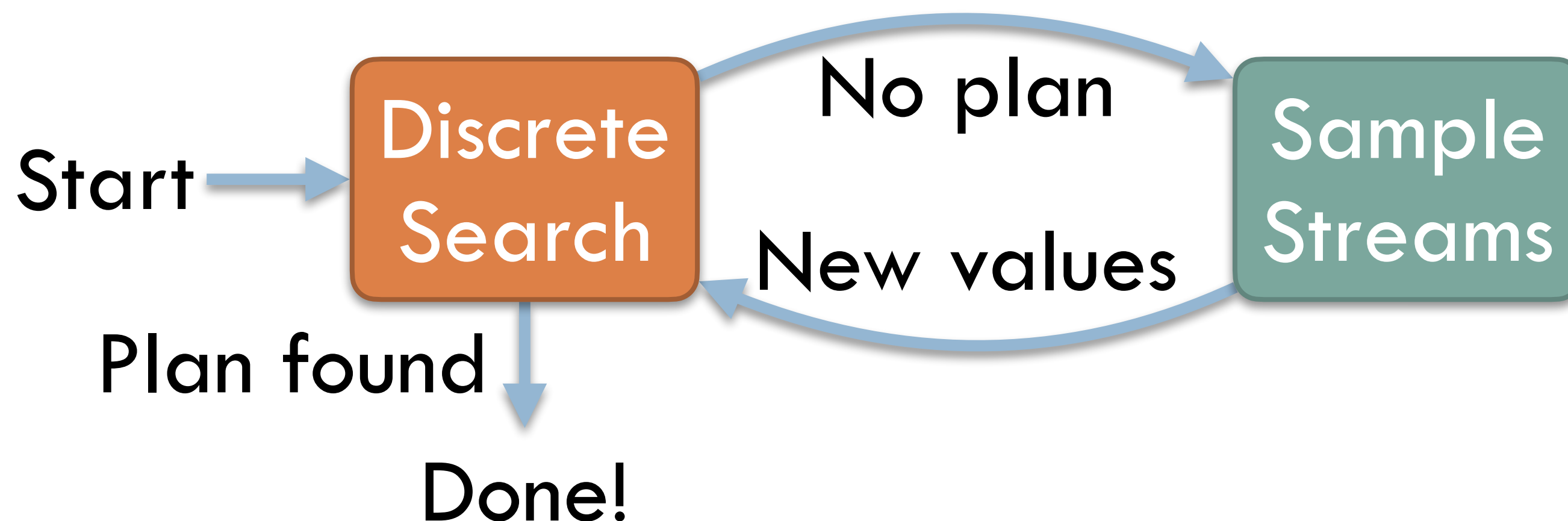
*[Garrett 2018]*  
*[Garrett 2020a]*

- Implement search using off-the-shelf domain-independent **PDDL planners** (e.g. FastDownward)
  - **Greedy** best-first heuristic search
  - Exploit **factoring** in PDDL for heuristics (e.g.  $h_{FF}$ )

# Incremental Algorithm





94

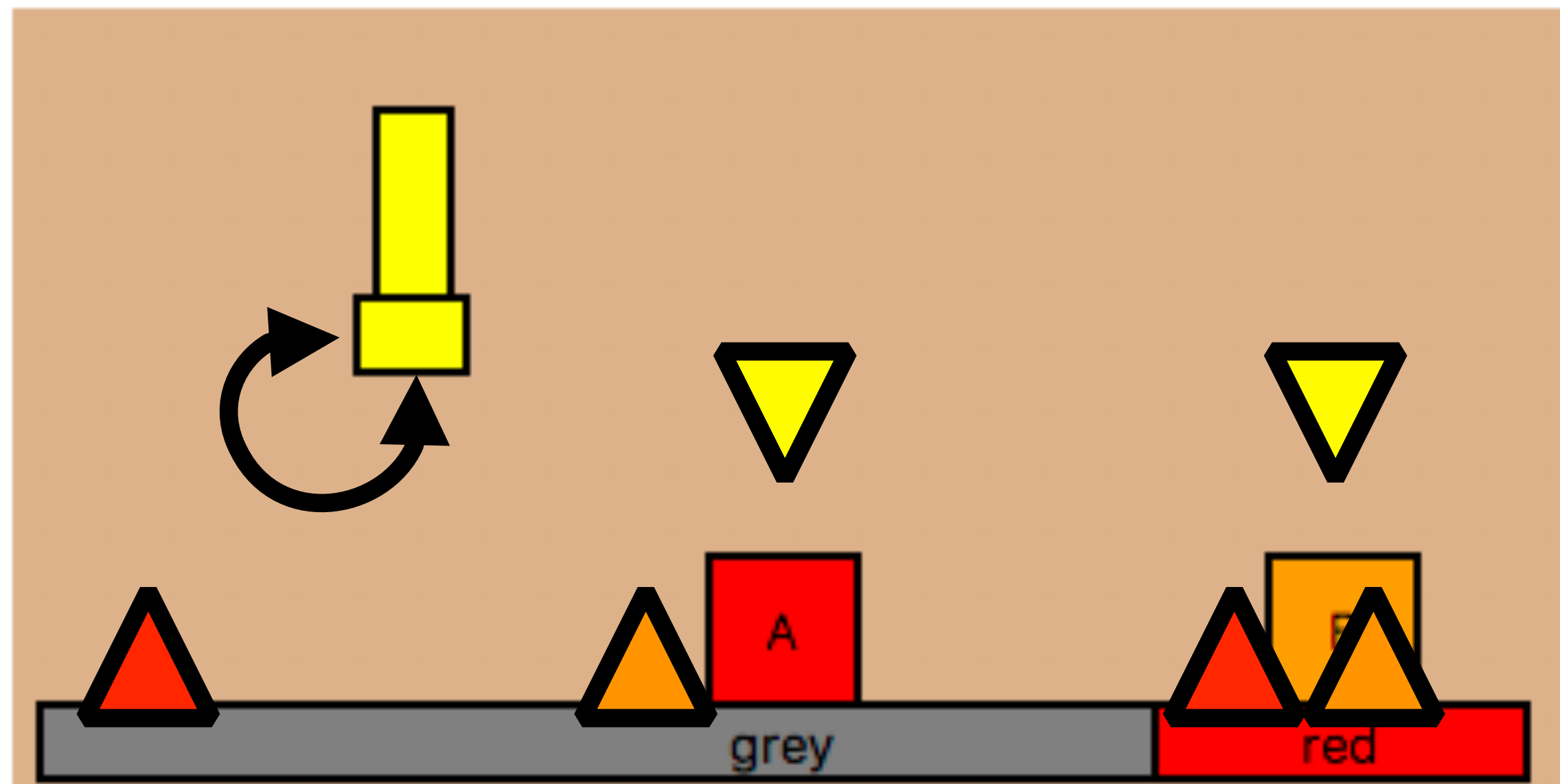
- **Incrementally** grow the set of values and facts
- Repeat:
  1. **Instantiate** and **sample** streams to generate new values and prove new facts
  2. **Search** for a plan using the current values
  3. **Return** when a plan is found



# Incremental: Iteration 1 - Sampling

95

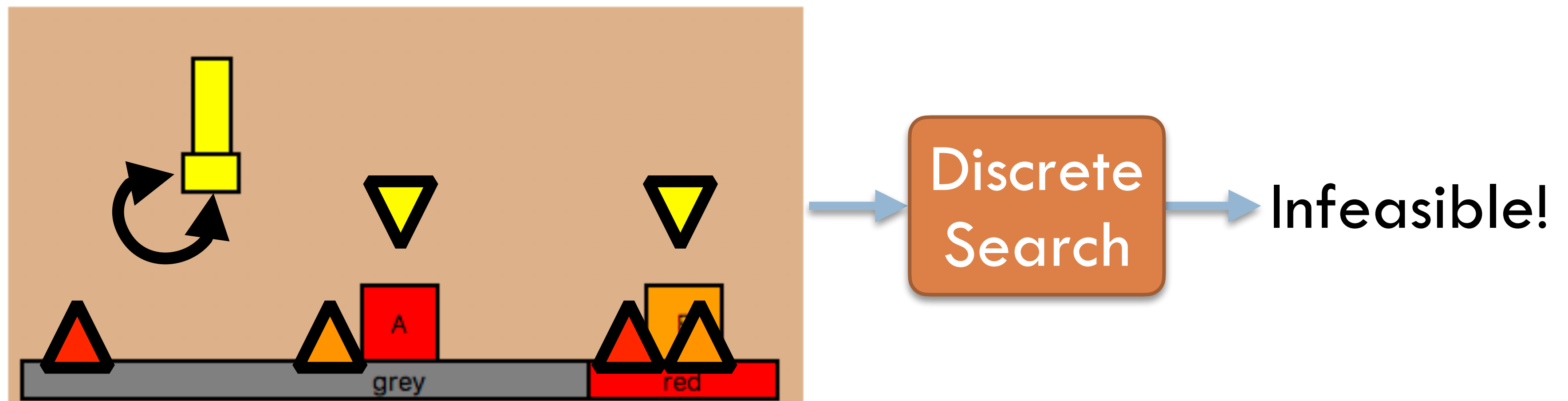
- **Iteration 1** - evaluated 14 streams
- **Sampled:**
  - 4 new block poses:  
  - 2 new robot configurations: 
  - 2 new trajectories: 



# Incremental: Iteration 1 - Search

96





- Pass current discretization to FastDownward
- If **infeasible**, the current set of samples is insufficient

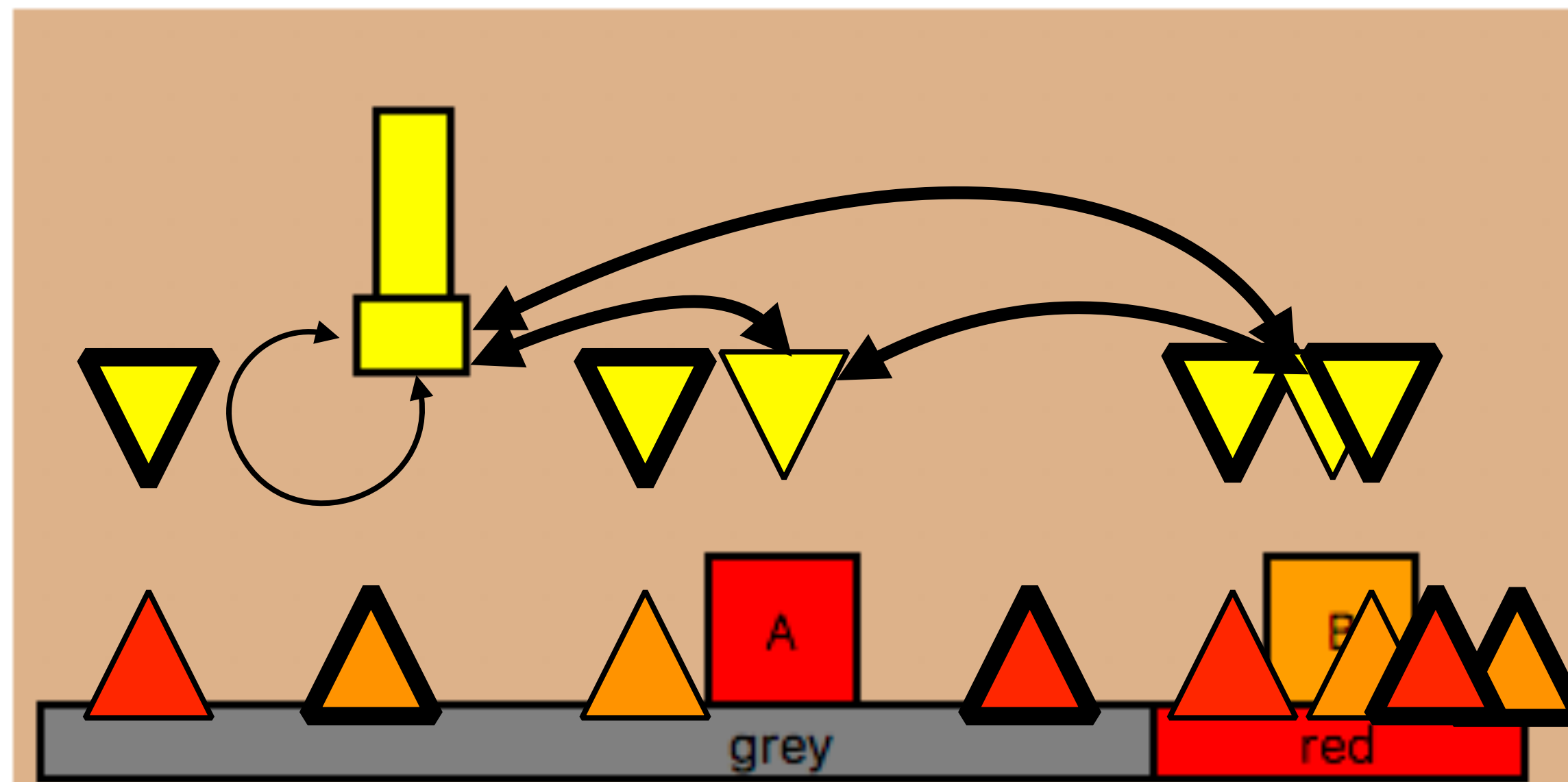




# Incremental: Iteration 2 - Sampling

97

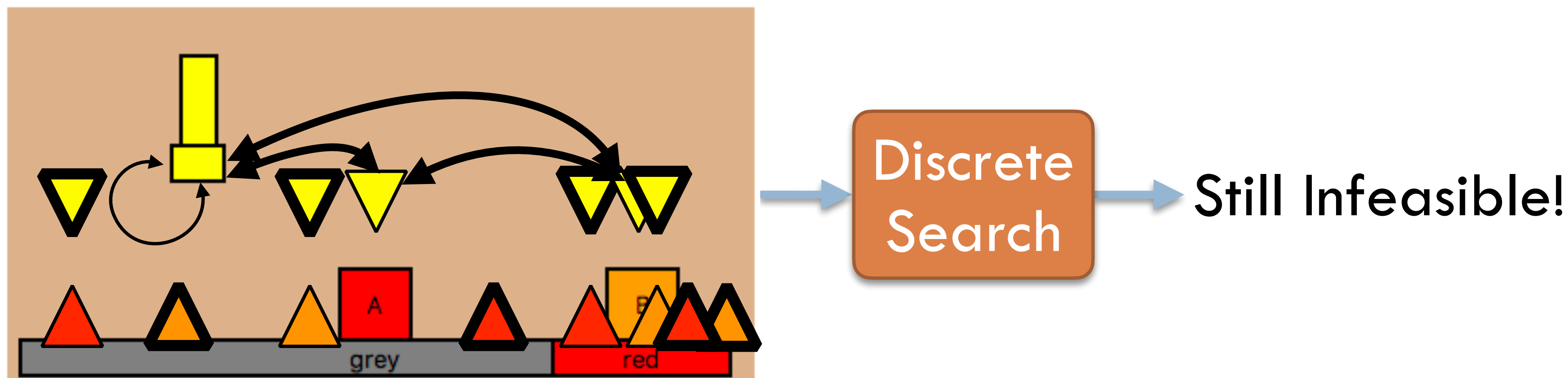
- **Iteration 2** - evaluated 54 streams
- **Sampled:**
  - 4 new block poses:  
  - 4 new robot configurations: 
  - 10 new trajectories: 



# Incremental: Iteration 2 - Search

98

- Pass current discretization to FastDownward
- If **infeasible**, the current set of samples is insufficient



# Incremental Example: Iterations 3-4

99

**Iteration 3** - 118 queried streams - infeasible

**Iteration 4** - 182 queried streams - **solved!**

## **Solution:**

```
1.move ([-7.5 5.],  $\tau_1$ , [7.5 2.5])
2.pick (B, [7.5 0.], [0. -2.5], [7.5 2.5])
3.move ([7.5 2.5],  $\tau_2$ , [10.97 2.5])
4.place (B, [10.97 0.], [0. -2.5], [10.97 2.5])
5.move ([10.97 2.5],  $\tau_3$ , [0. 2.5])
6.pick (A, [0. 0.], [0. -2.5], [0. 2.5])
7.move ([0. 2.5],  $\tau_4$ , [7.65 2.5])
8.place (A, [7.65 0.], [0. -2.5], [7.65 2.5])
```

- **Planner generated** all but the underlined values
- **Drawback** - many unnecessary samples produced

# Optimistic Stream Evaluation

100

- Many TAMP streams are computationally **expensive**
  - Inverse kinematics, collision checking, motion planning
- Only query streams after they are **identified** as useful
  - Plan with **optimistic hypothetical** outputs

- Inductively create unique **optimistic placeholder values** for each stream output (denoted by prefix #)

1. `s-region(A, red) → #p0`

2. `s-ik(A, [0. 0.], [0. -2.5]) → #q0,`

3. `s-ik(A, #p0, [0. -2.5]) → #q2,`

4. `s-motion(A, #q0, #q2) → #t0, ...`

[Garrett 2018]  
[Garrett 2020a]

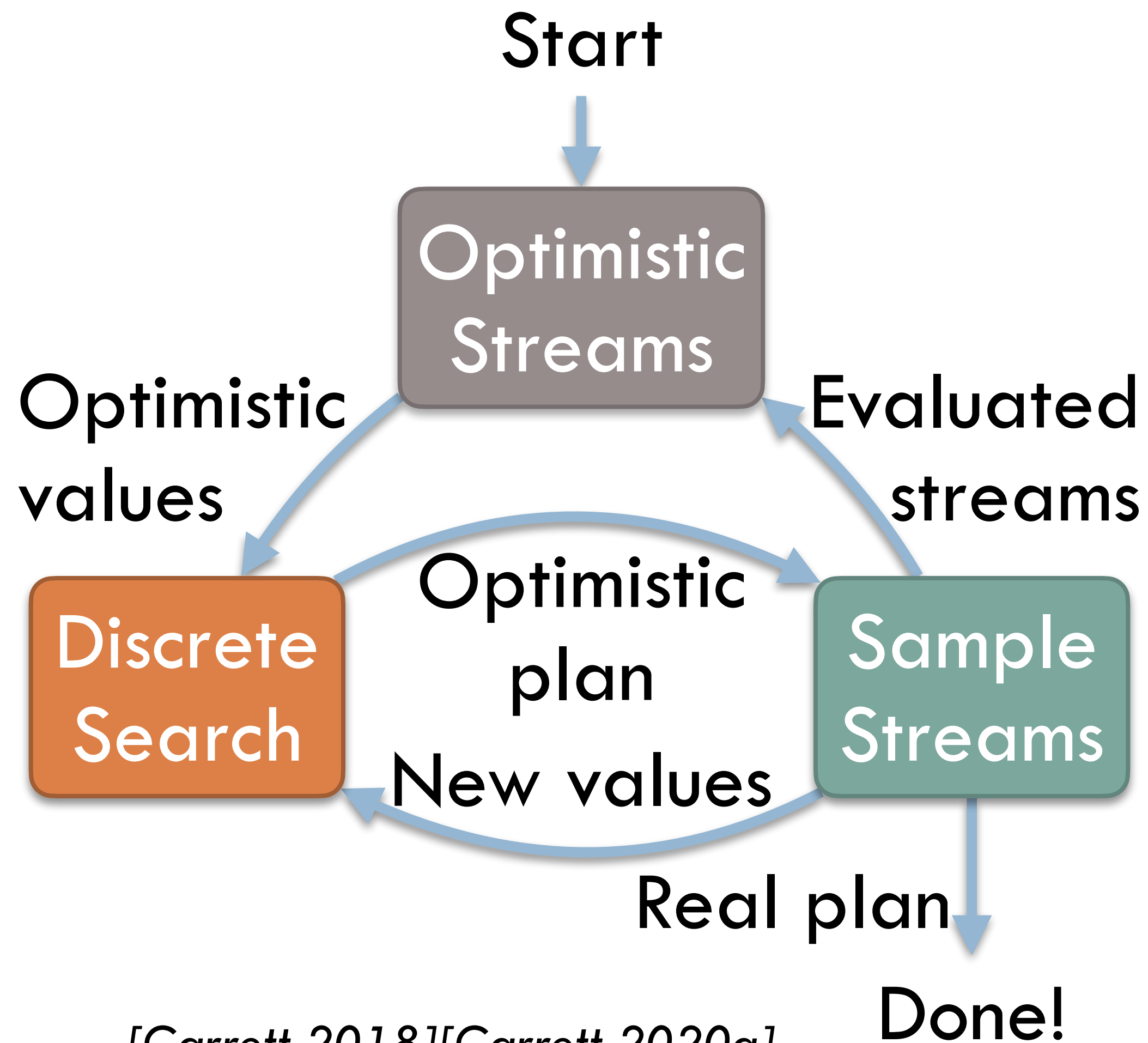
# Focused Algorithm

101

- **Lazily** plan using optimistic values **before** real values

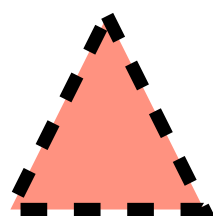
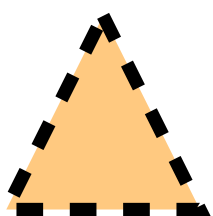
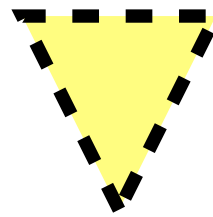
- Repeat:

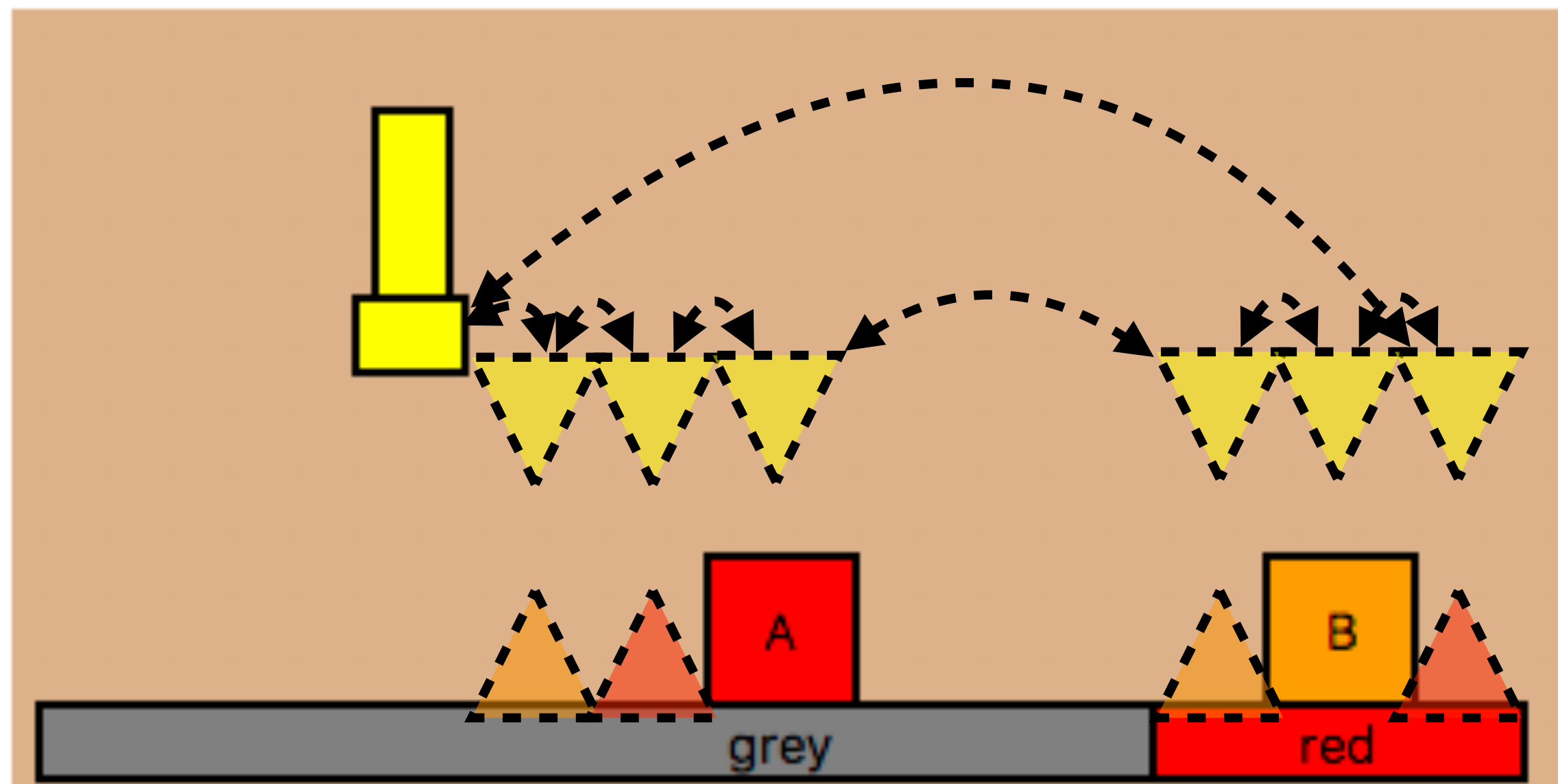
1. **Construct** optimistic stream outputs
2. **Search** with real & optimistic values
3. **Retrace** and **evaluate** streams
4. **Replace** optimistic with real if they exist
5. **Return** if all succeed



# Focused: Iteration 1

102

- **Iteration 1** - optimistically evaluated 46 streams
- **Created:**
  - 4 optimistic block poses:  
  - 6 optimistic robot configurations: 
  - 36 optimistic trajectories: ----->

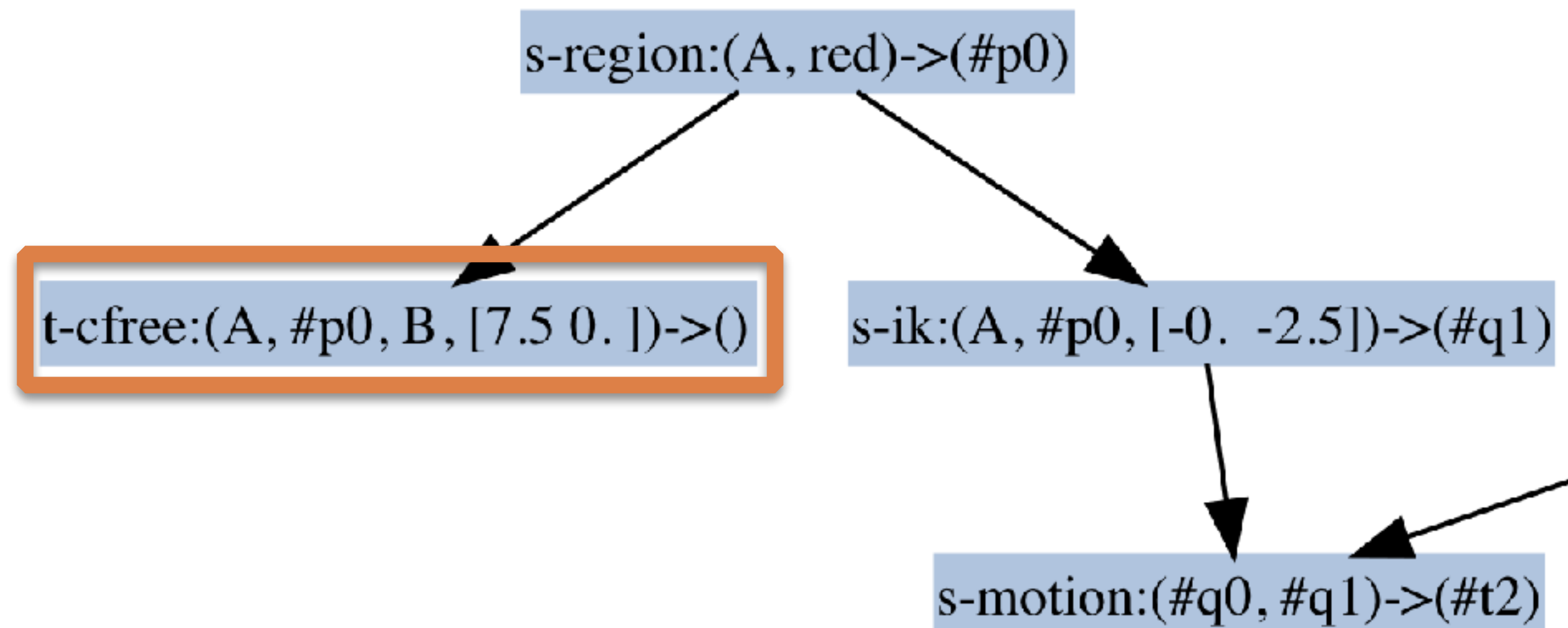
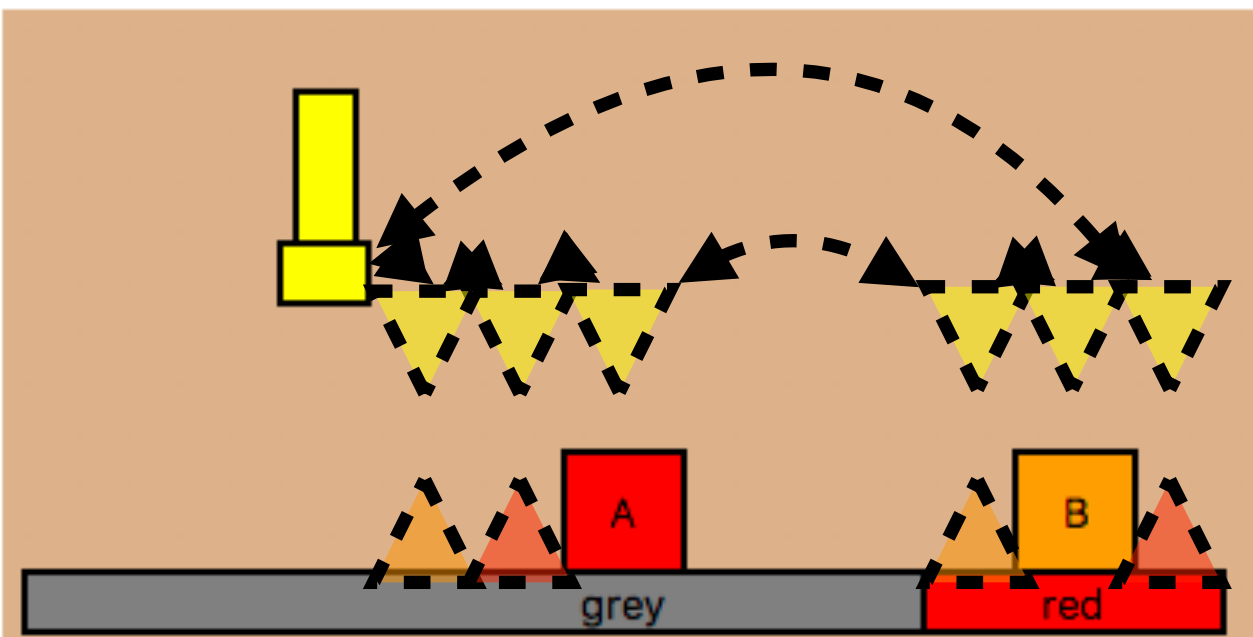


# Focused: Iteration 1 - Sampling

103

## Optimistic plan:

```
[move([-5. 5.], #t0, #q0), pick(A, [0. 0.], [0. -2.5], #q0),  
move(#q0, #t2, #q1), place(A, #p0, [0. -2.5], #q1)]
```



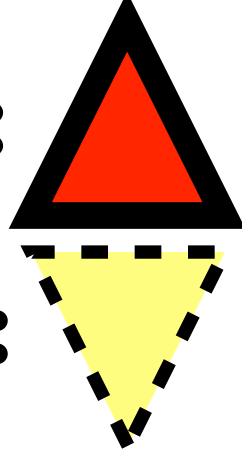
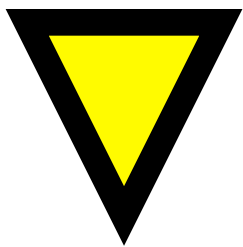
## Queried streams:

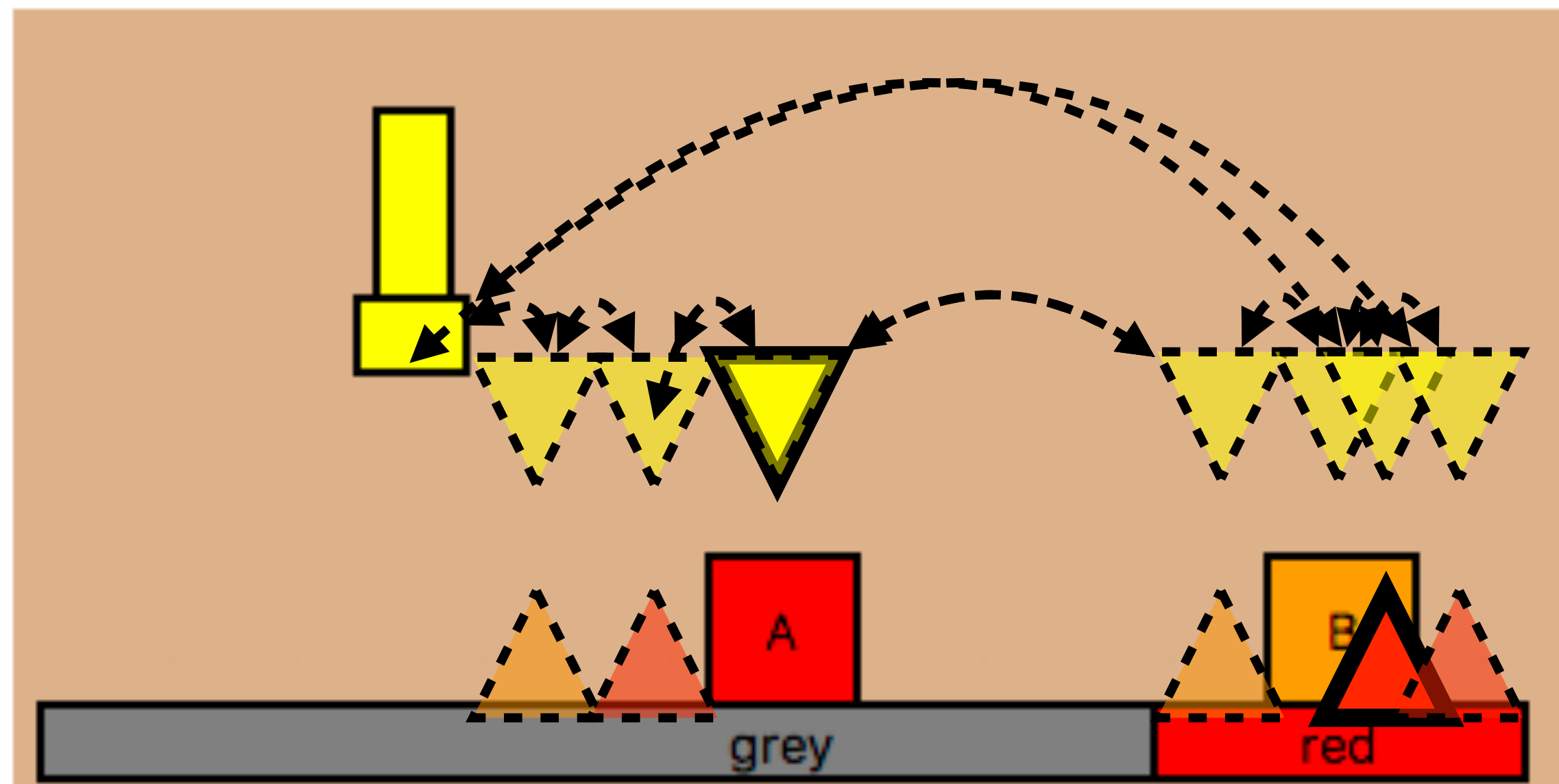
1. `s-region(A, red)` → [8.21 0.]
2. `s-ik(A, [0. 0.], [0. -2.5])` → [0. 2.5]
3. `t-cfree(A, [8.21 0.], B, [7.5 0.])` → False

**Temporarily remove** these streams from the next search

# Focused: Iteration 2

104

- **Iteration 2** - optimistically evaluated 42 streams
- Removed **optimistic** pose and configuration
- Added **sampled** pose and configuration: 
- Added 1 **optimistic** robot configurations: 
- Added 14 **optimistic** trajectories: ----->



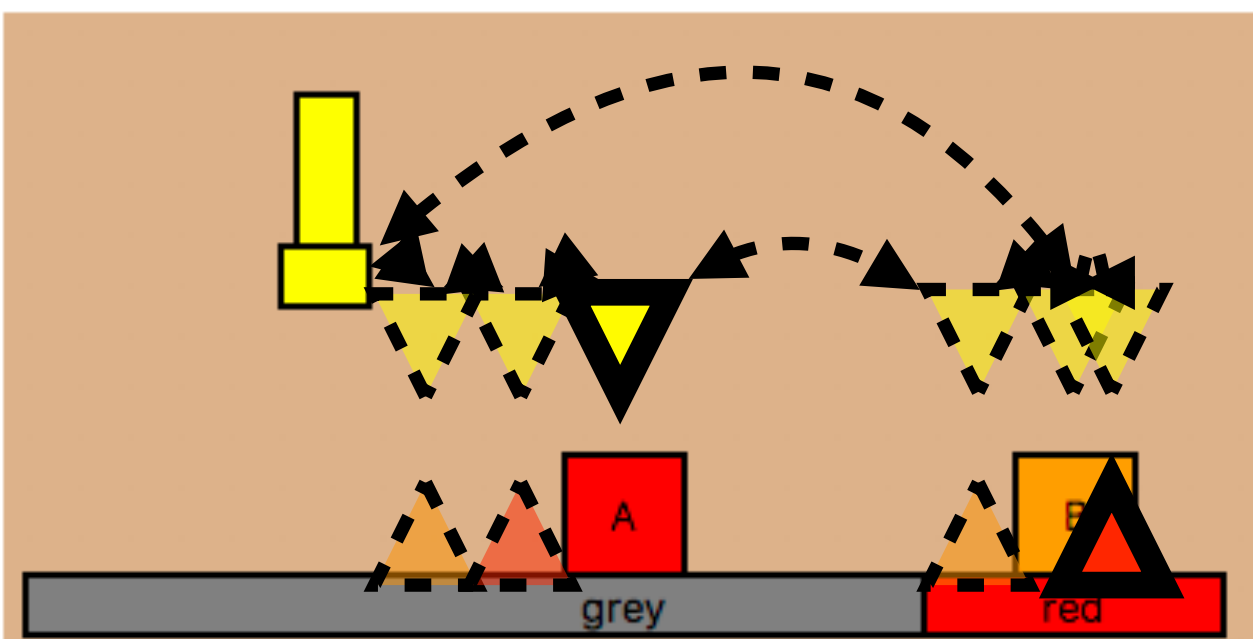


# Focused: Iteration 2 - Sampling

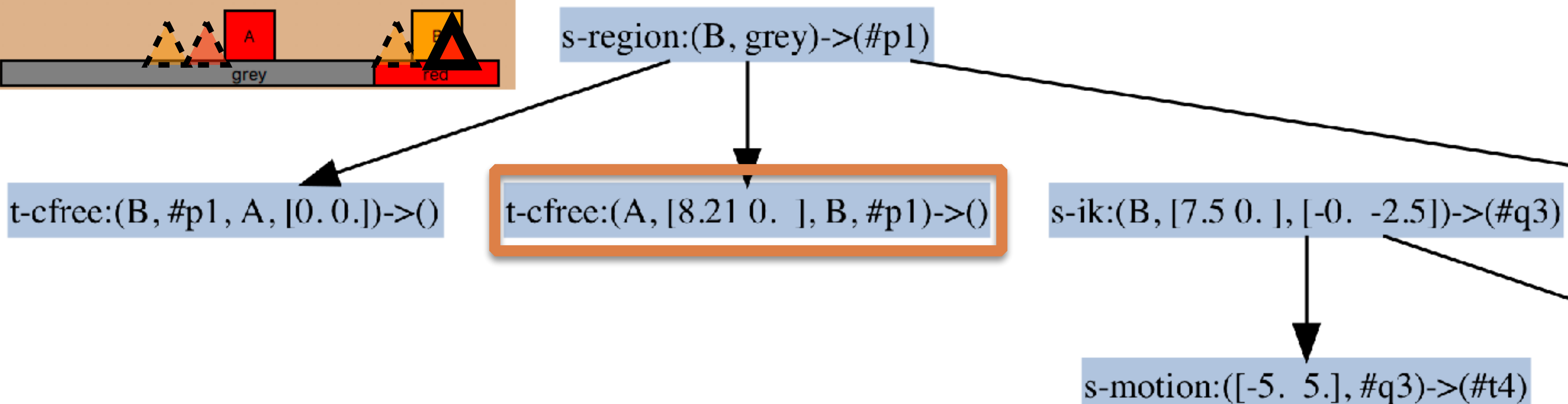
105

## New optimistic plan:

```
[move([-5. 5.], #t4, #q2), pick(B, [7.5 0.], [0. -2.5], #q2),  
move(#q2, #t9, #q3), place(B, #p1, [0. -2.5], #q3),  
move(#q3, #t6, [0. 2.5]), pick(A, [0. 0.], [0. -2.5], [0. 2.5]),  
move([0. 2.5], #t8, #q4), place(A, [8.21 0.], [0. -2.5], #q4)]
```



## Different stream plan might succeed!

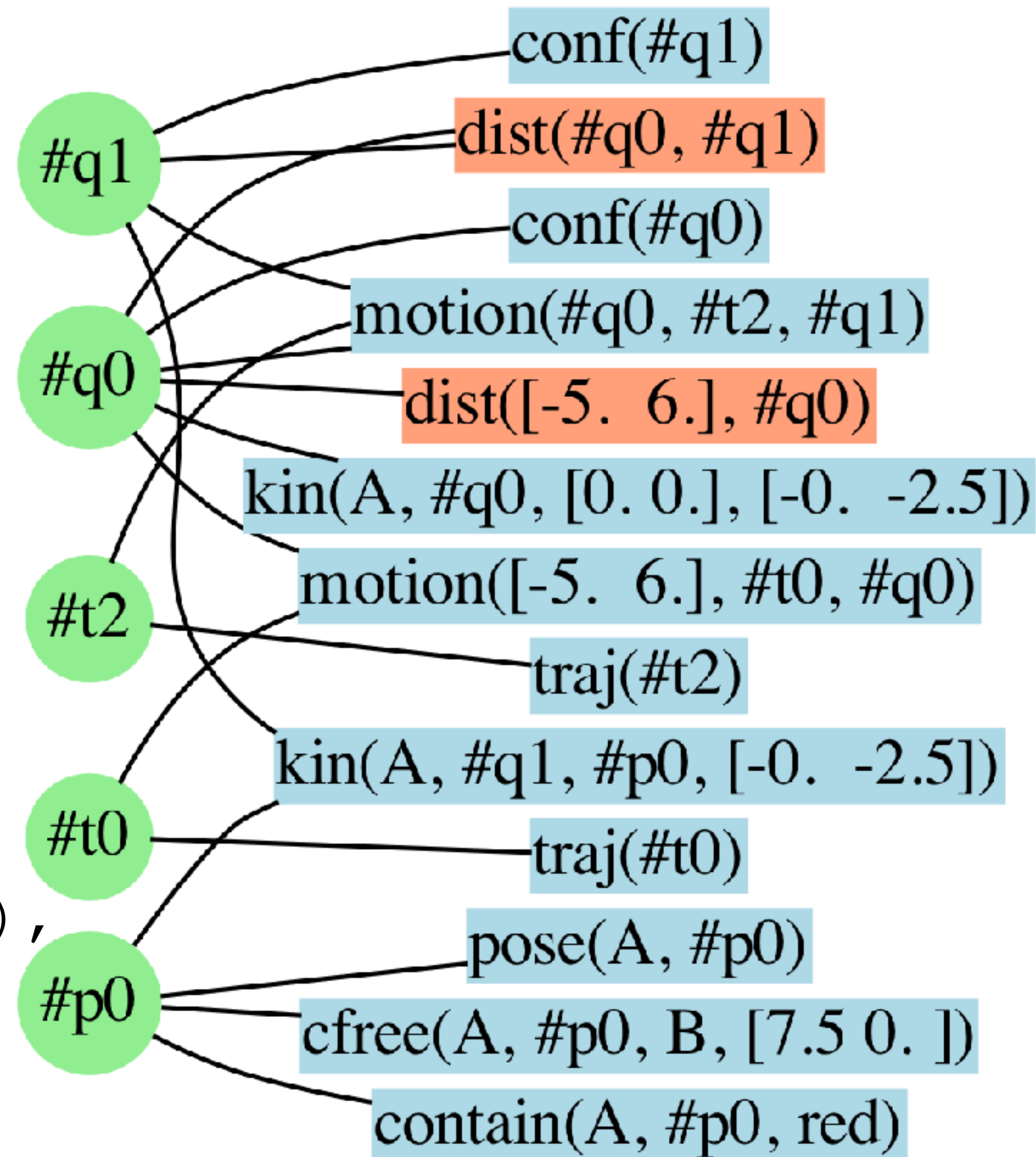


# Optimistic Planning with Optimization

106

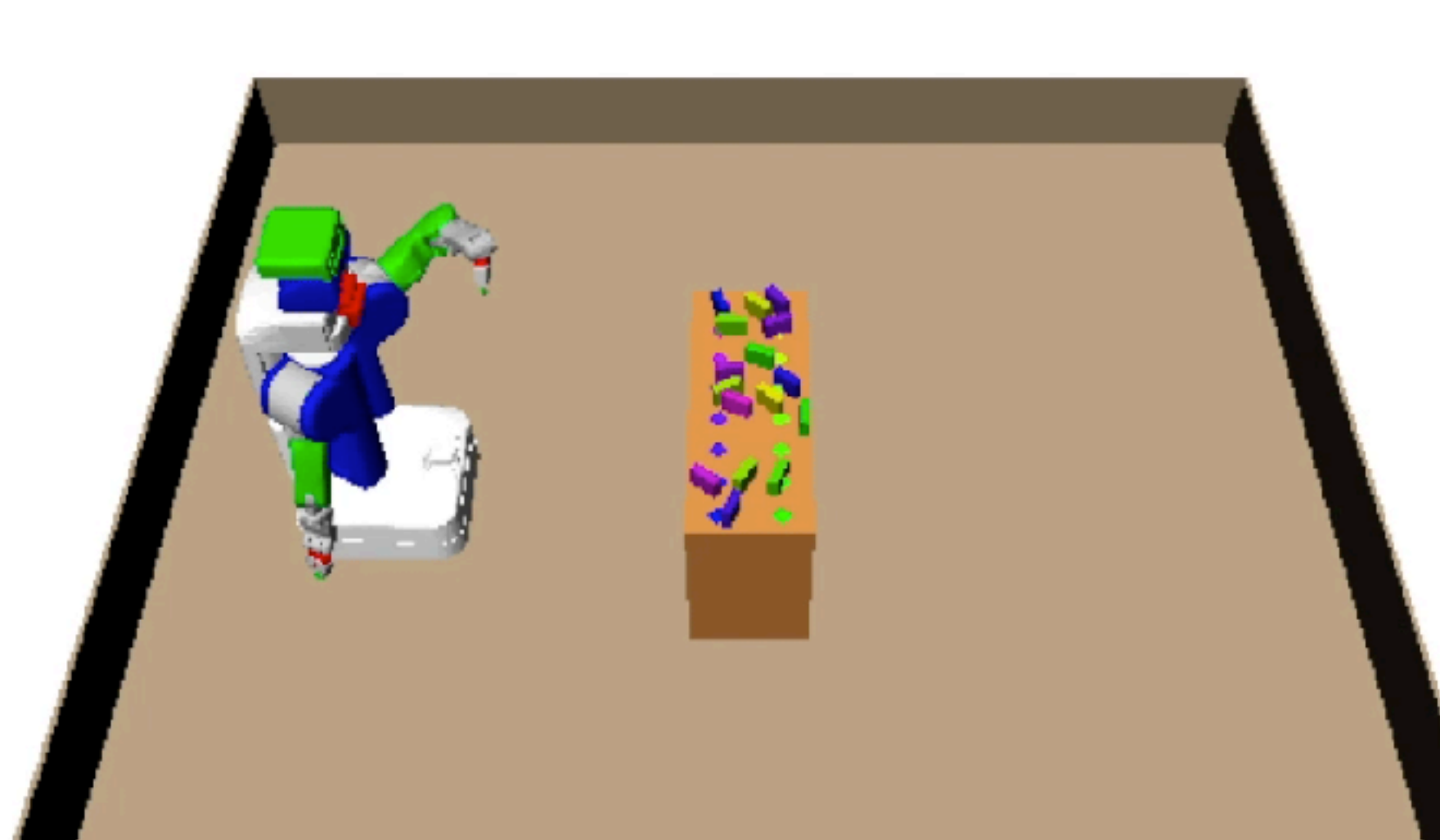
- Instead of sampling, directly **optimize** the constraint network
- Non-convex constrained mathematical program **solver as a stream**
- Additional PDDLStream algorithms...

```
[move([-5. 6.], #t0, #q0),  
pick(A, [0. 0.], [0. -2.5], #q0),  
move(#q0, #t2, #q1),  
place(A, #p0, [0. -2.5], #q1)]
```

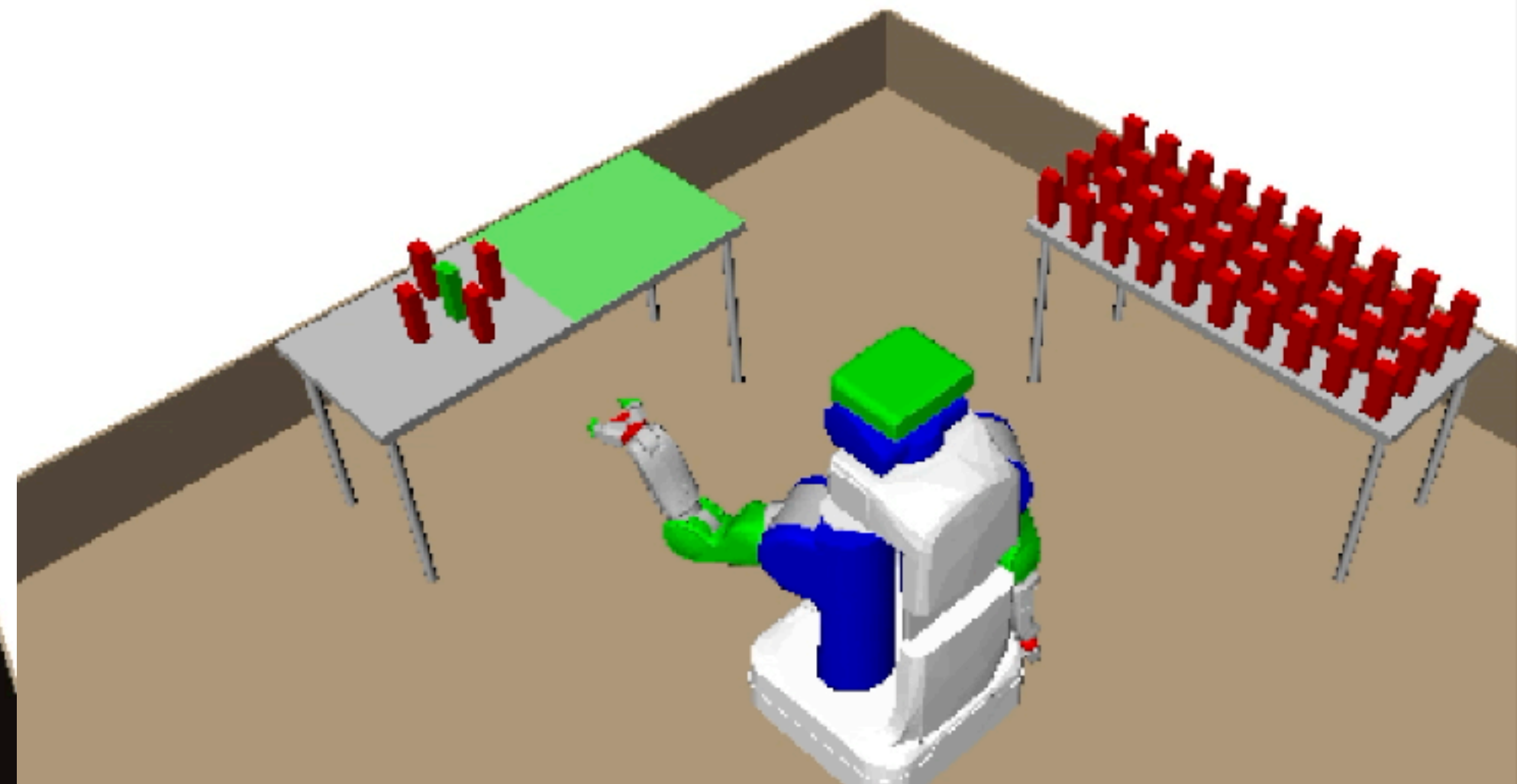


# Scaling Experiments

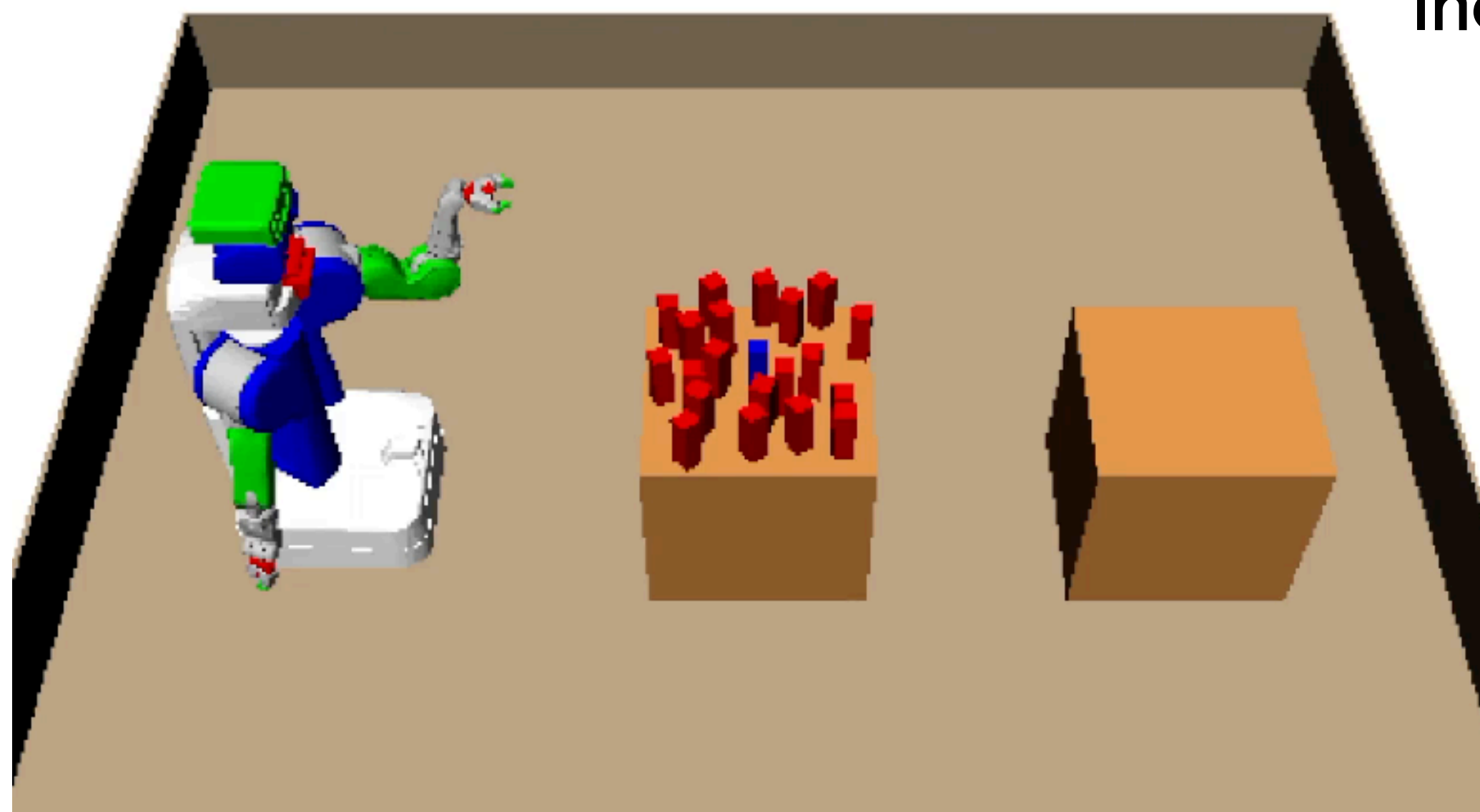
107



Incremental ~20s  
**Focused ~10s**



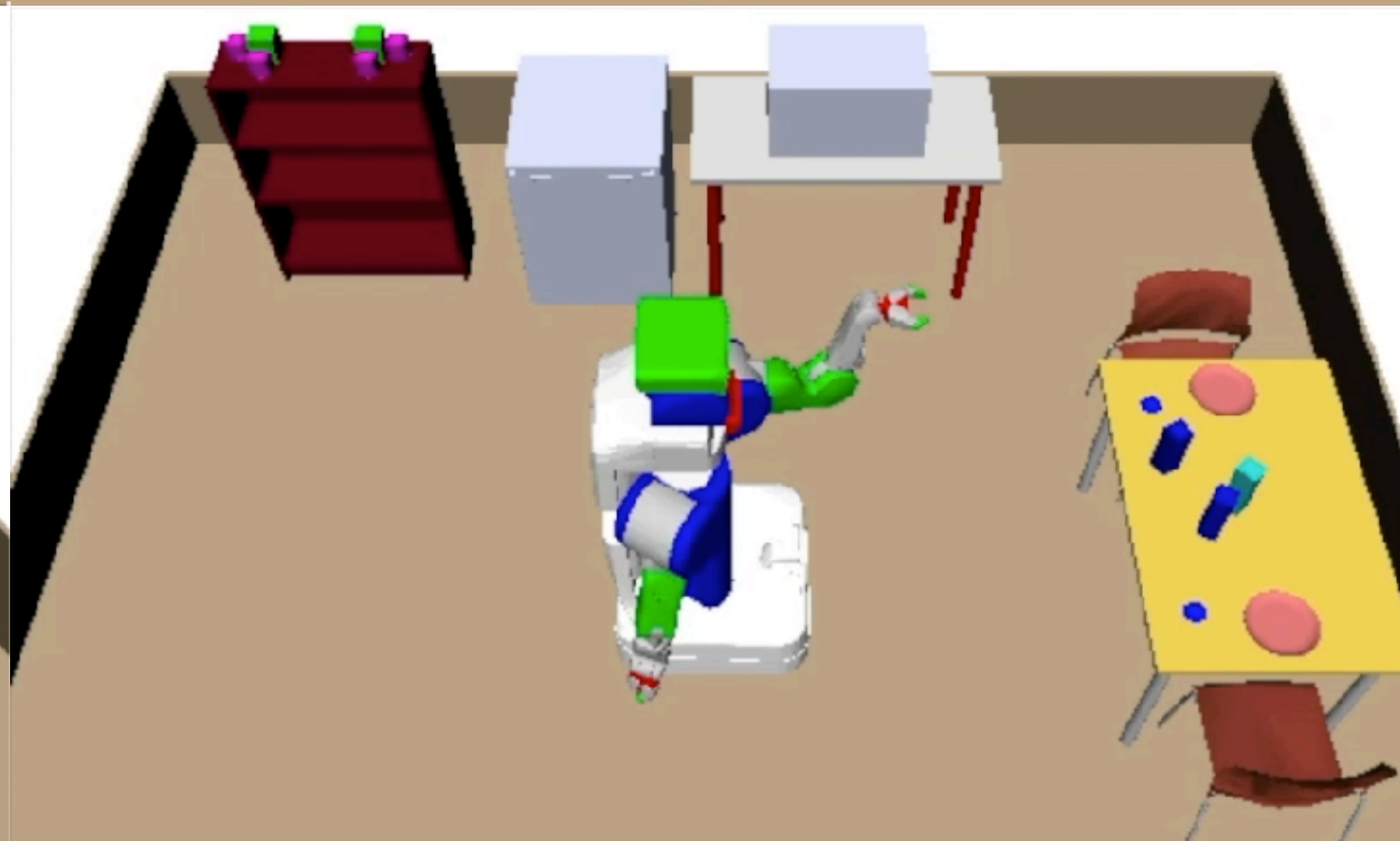
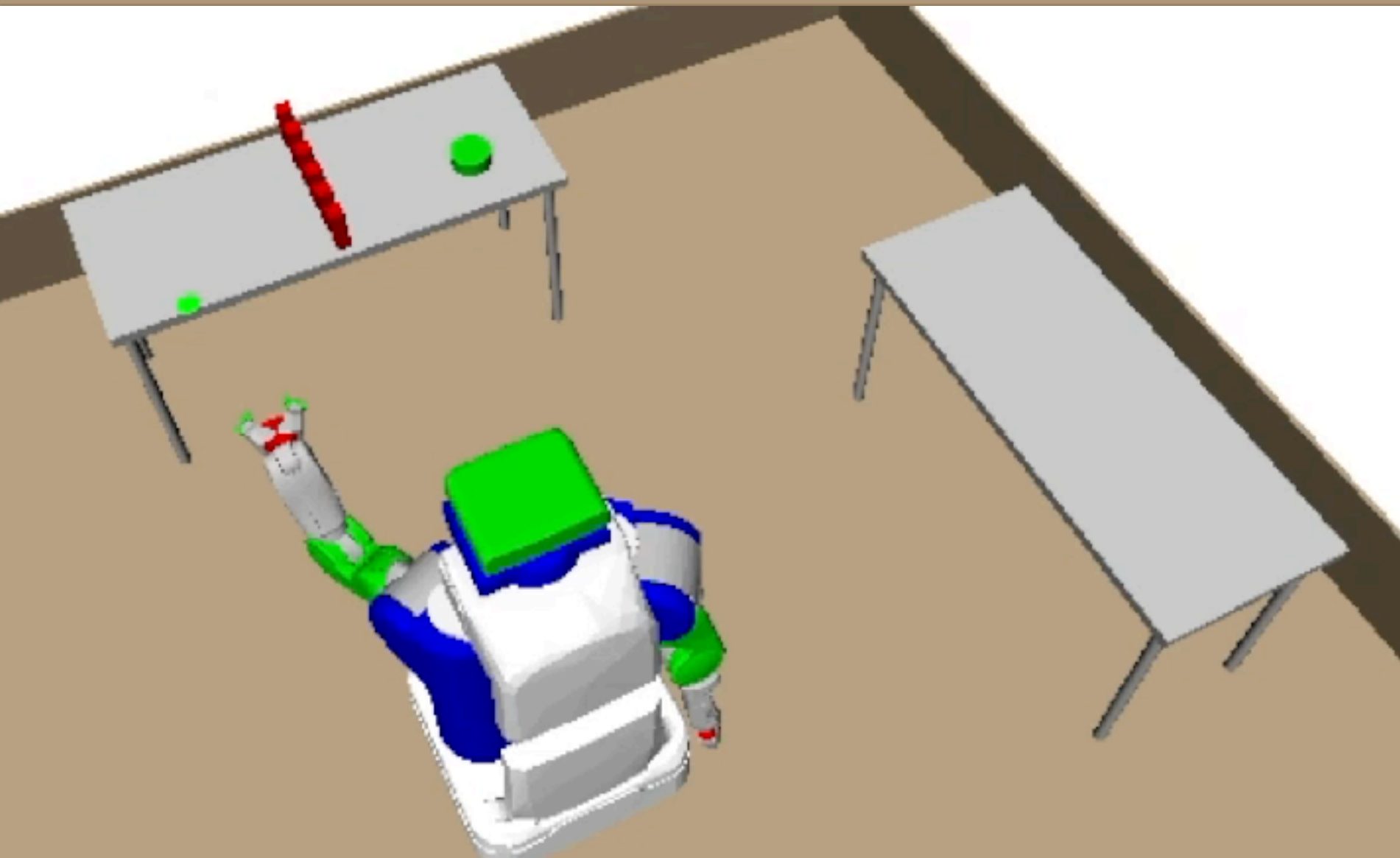
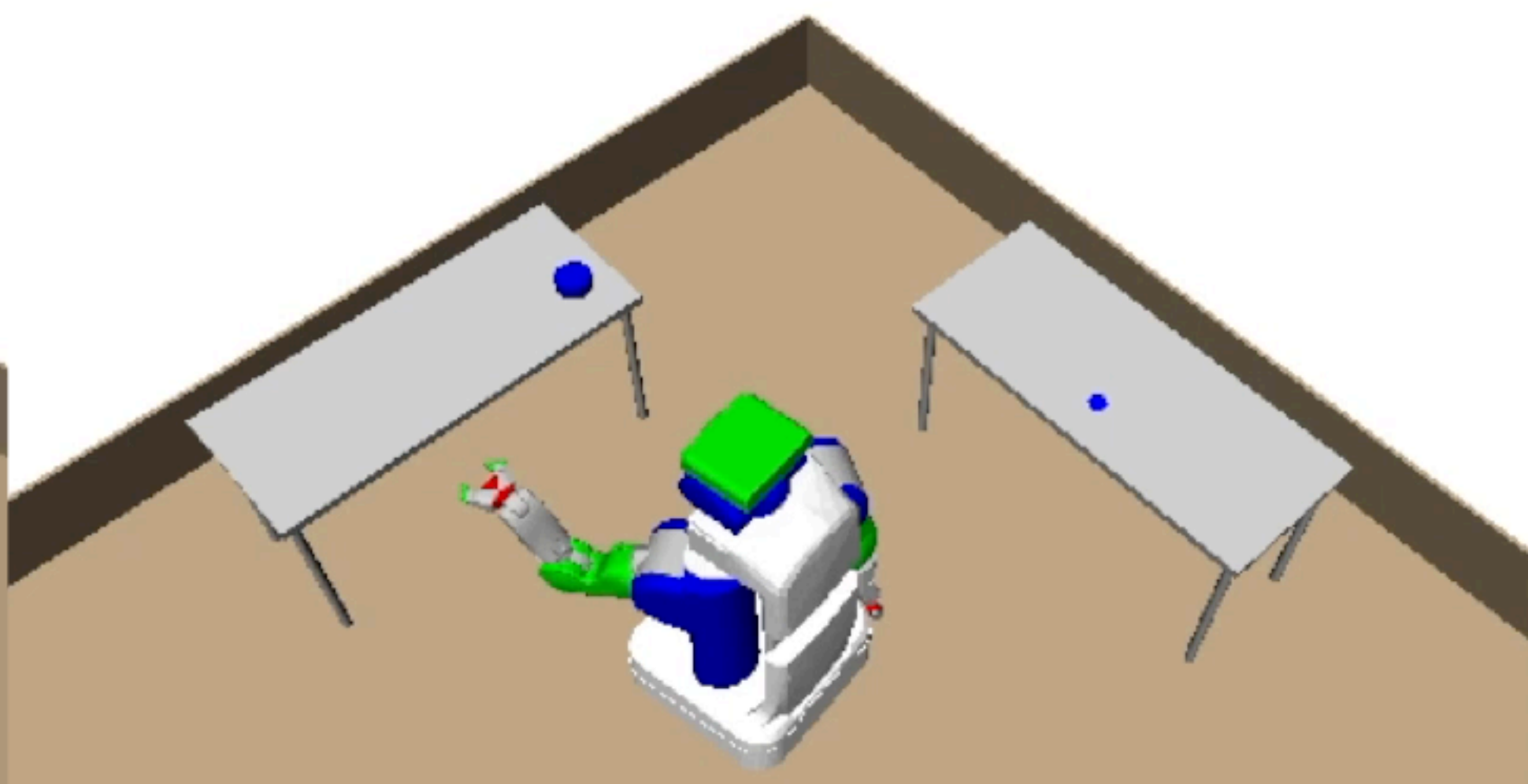
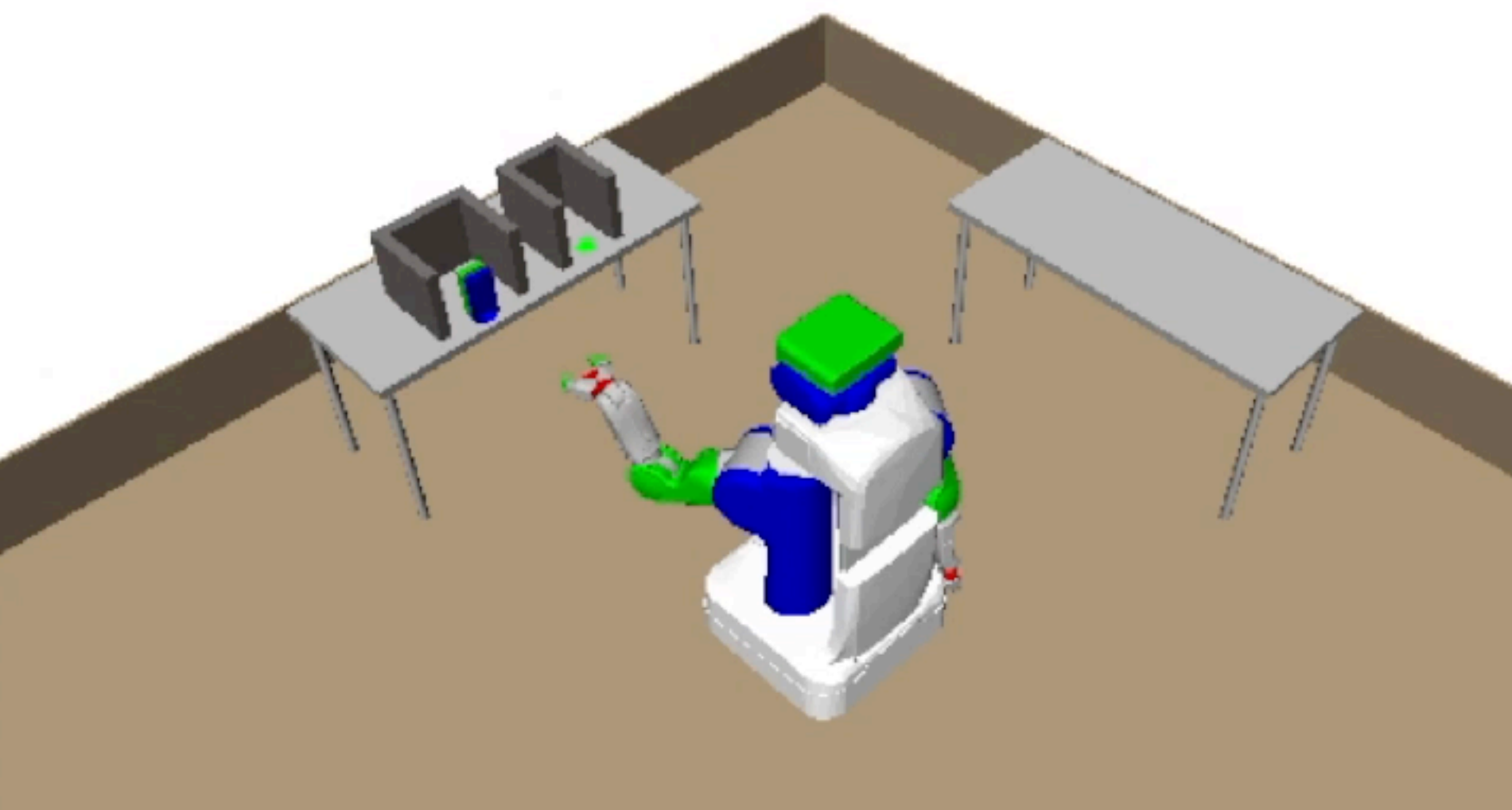
Incremental 120+  
**Focused ~25s**



Incremental 120+  
**Focused ~20s**

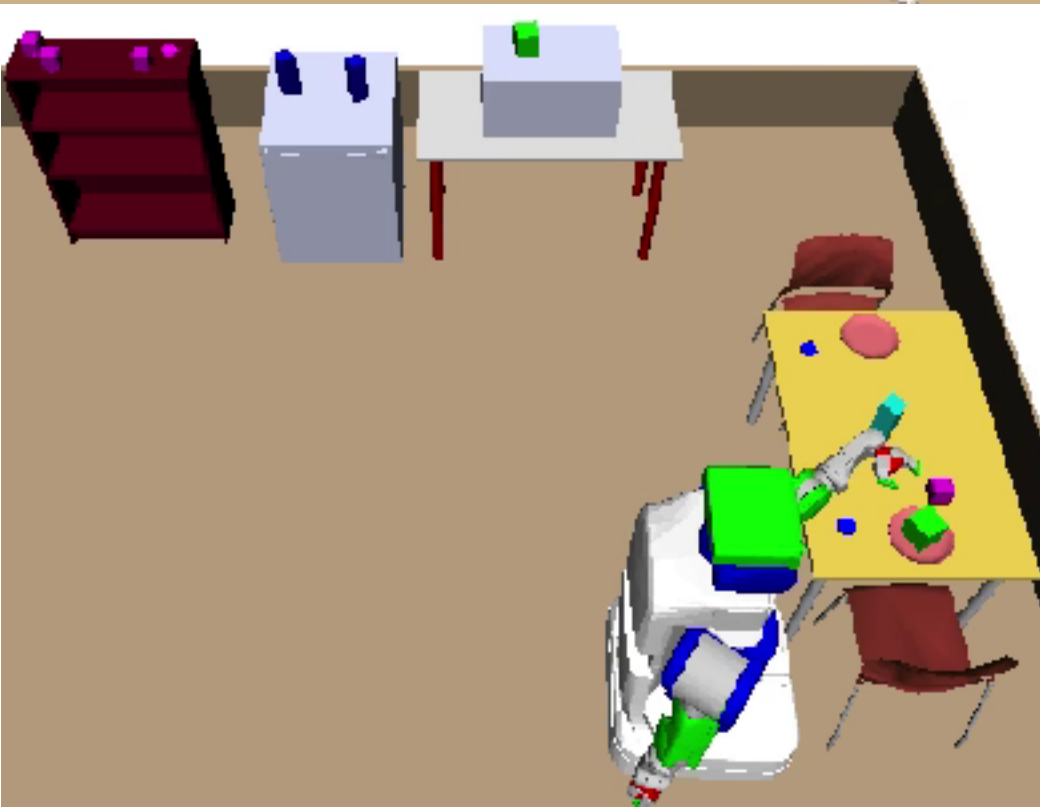
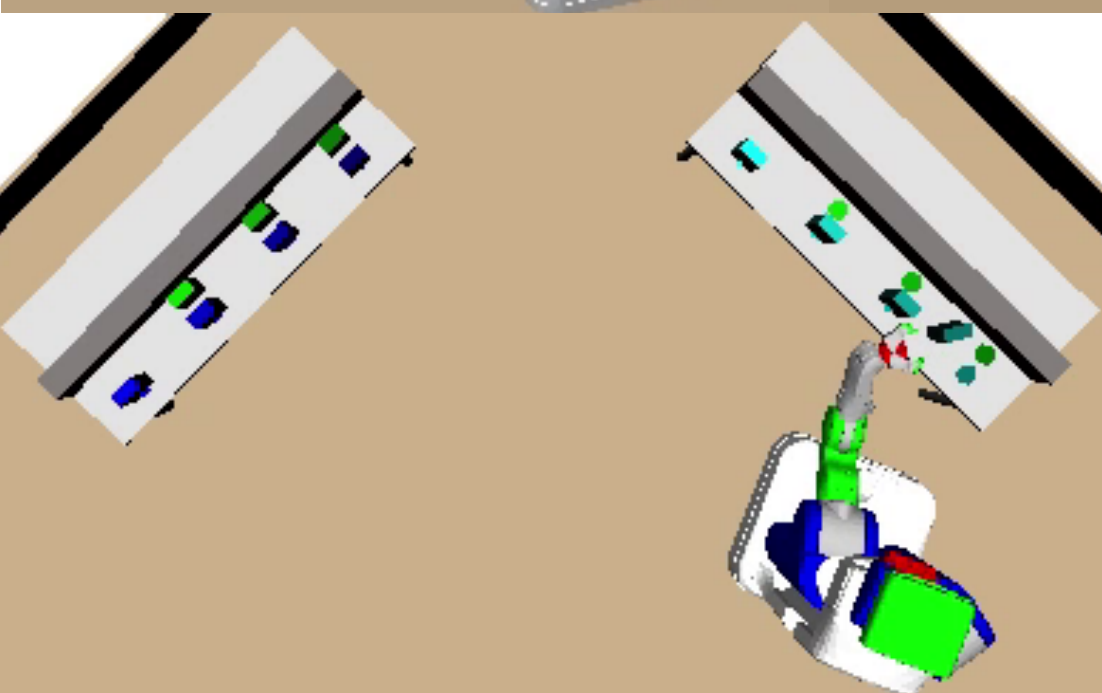
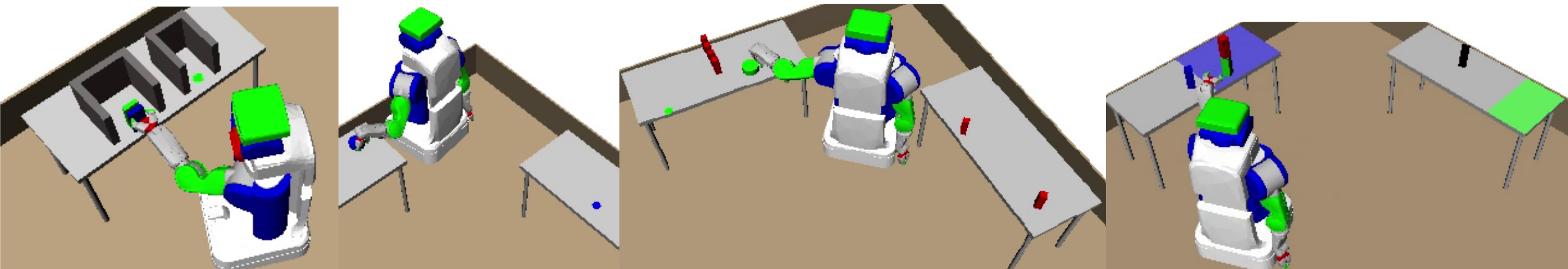
# Diverse Experiments

108



# Diverse Experiments

109



Problem	<i>Incr.</i>		<i>Incr. - H</i>		<i>Focus</i>		<i>Focus - H</i>	
	%	t	%	t	%	t	%	t
<i>Regrasp</i>	98	1	100	2	98	1	95	1
<i>Push</i>	100	11	100	13	100	13	100	9
<i>Wall</i>	95	10	98	13	100	6	100	8
<i>Stacking</i>	100	9	100	9	100	2	100	3
<i>Nonmon.</i>	25	21	98	15	0	-	88	43
<i>Dinner</i>	0	-	100	27	0	-	98	22

Success percentage (%), Average runtime in sec. (t)

# TAMP with Uncertainty

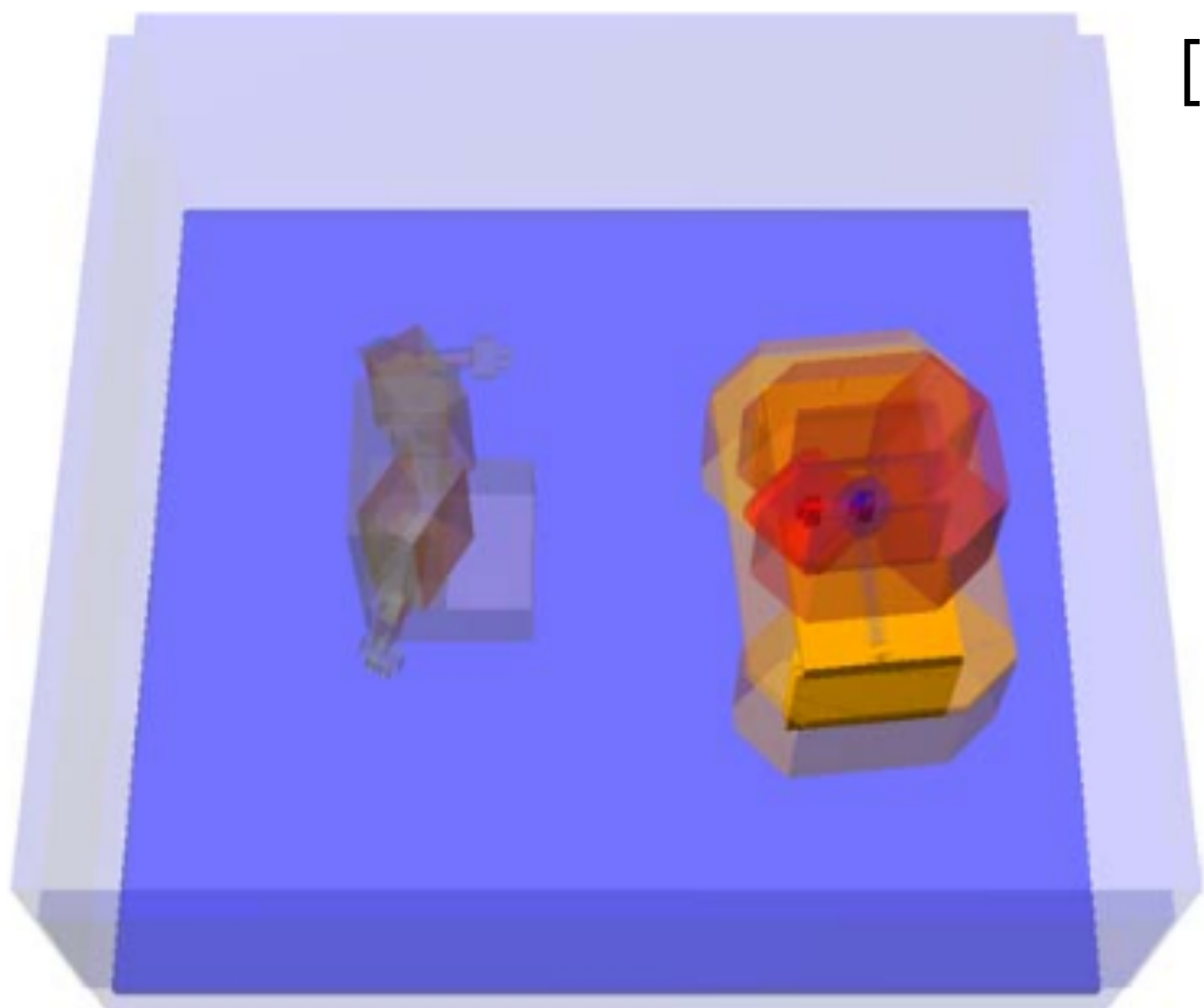
[Garrett, Paxton, Lozano-Pérez, Kaelbling, & Fox 2020b]

# Hybrid MDP/POMDP

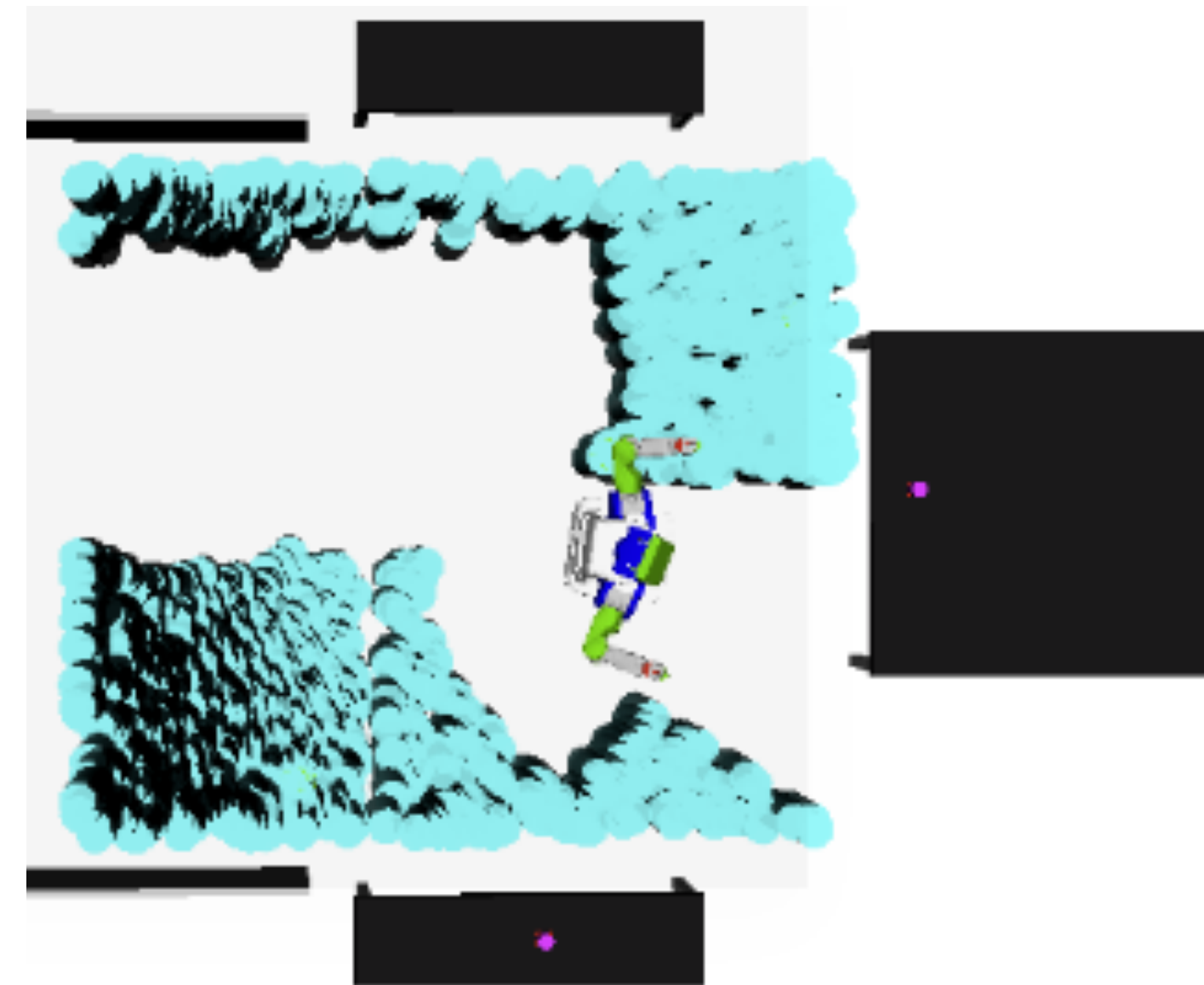
111

- **Nondeterministic outcomes** - stochastic effects
- **Partial observability** - latent state
  - The true state is unknown: **probabilistic inference**
- **Belief space planning**
  - Plan over **beliefs**: probability distributions over states

[Kaelbling 2013]



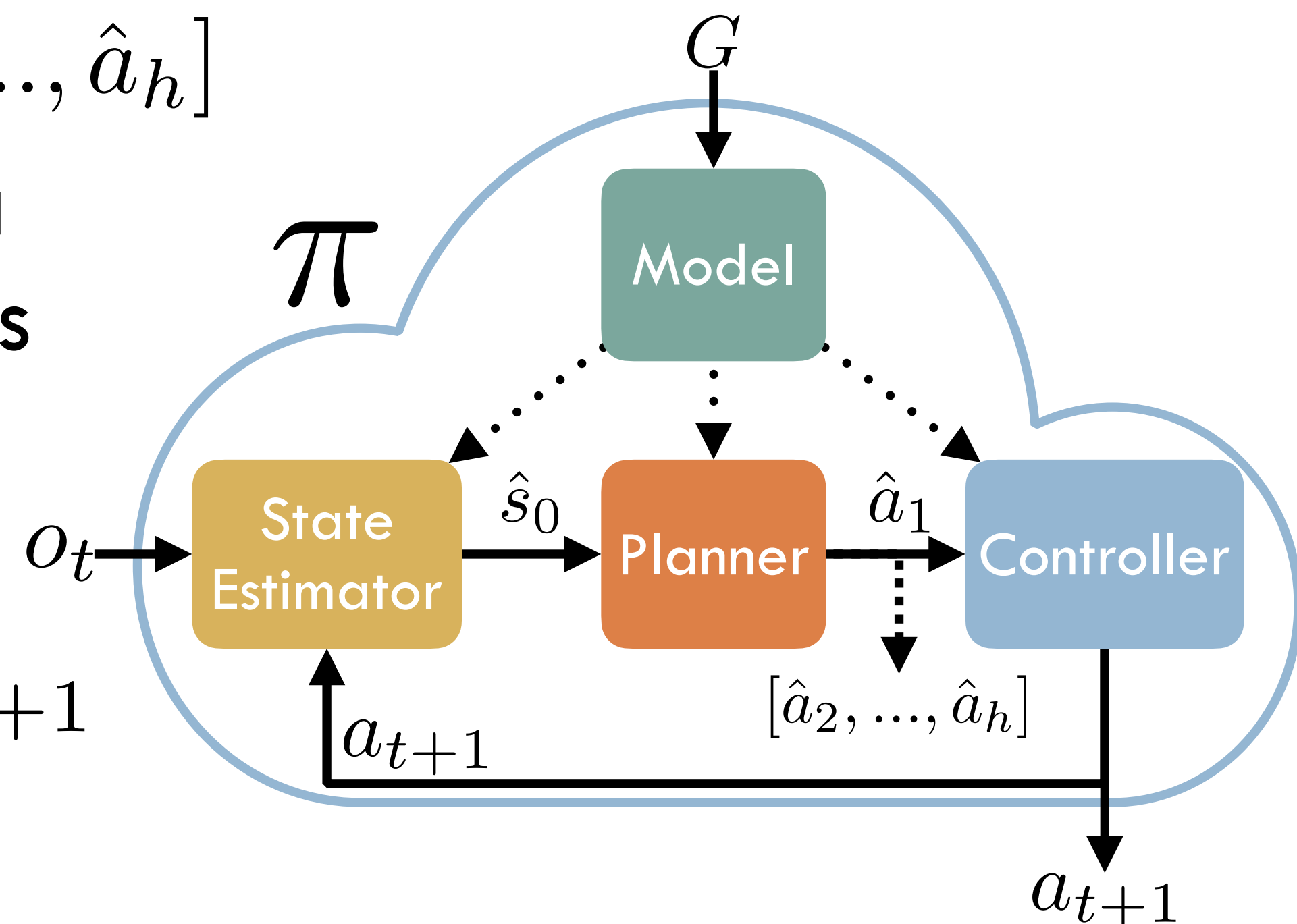
[Hadfield-Menell  
2015]



# Dealing with Action Uncertainty

112

- **Partial observability and stochastic action effects**
- **MPC policy: estimation, replanning, and control**
- State estimator **compresses** history into **belief** statistic  $\hat{s}_t$  that encodes **uncertainty**
- **Planner** finds plan  $[\hat{a}_1, \dots, \hat{a}_h]$ 
  - Tail of plan serves as a **certificate** that plan has low cost-to-go
- **Controller** converts **first** action  $\hat{a}_1$  into torques  $a_{t+1}$





# Localize and Cook Spam (on Stove)

113

[Garrett 2020b]



Goal: Cook Spam  
x4

# Dealing with State Uncertainty

114

- **Occlusions** due to doors, drawers, objects, robot, ...
- State estimator: **particle-filters** over object poses
  - **Multimodal** distributions capture **view-cone** geometry
- Need **active** information gathering to find objects
  - Open doors/drawers
  - Relocate occluding objects
- Plan in **belief space**
  - (Instead of state space)
  - Plan future **observations**



# Belief Space PDDLStream

115

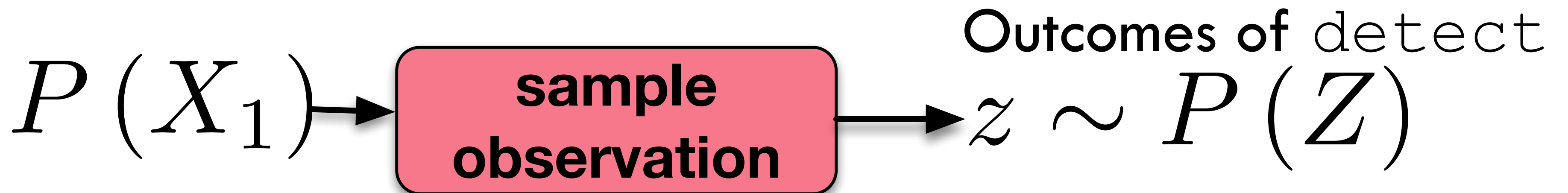
- State variables and action parameters are **probability distributions** (instead of point estimates)
- **Observation** actions model the **belief update** process:
  - **Prior x observation → posterior**

Pose particle  $P(X)$  distribution  $\approx$  View-cone observation

```
(:action detect
:parameters (?o, ?pb1, ?obs, ?pb2)
:precondition {BeliefUpdate(?o, ?pb1, ?obs, ?pb2),
               AtPoseB(?o, ?pb1), BVisible(?o, ?pb1, ?obs) }
:effect {AtPoseB(?o, ?pb2), ¬AtPoseB(?o, ?pb1),
        total-cost+=ObsCost(?o, ?pb1, ?obs) }
```

# Bayesian Inference Streams

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# Prior: Spam in One of the Drawers

117



Goal: Believe Spam in Closed Bottom Drawer  
x4

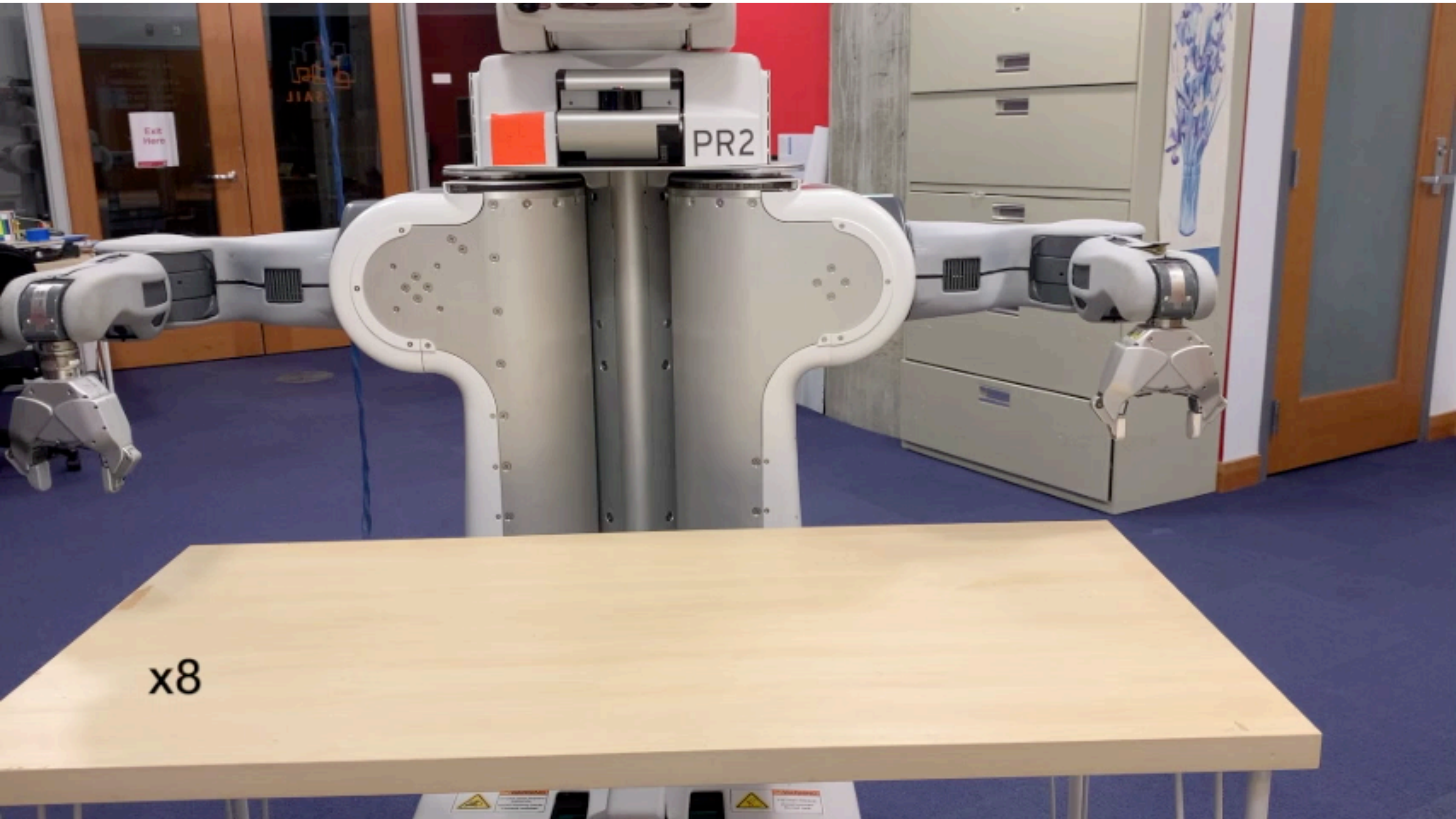
# TAMP with Unknown Objects

[Curtis\*, Fang\*, Lozano-Pérez, Kaelbling, Garrett, 2021]

Goal: all objects are in a bowl of the same color

$\forall obj. \exists bowl. \exists color. \text{In}(obj, bowl) \wedge \text{Color}(obj, color) \wedge \text{Color}(bowl, color)$

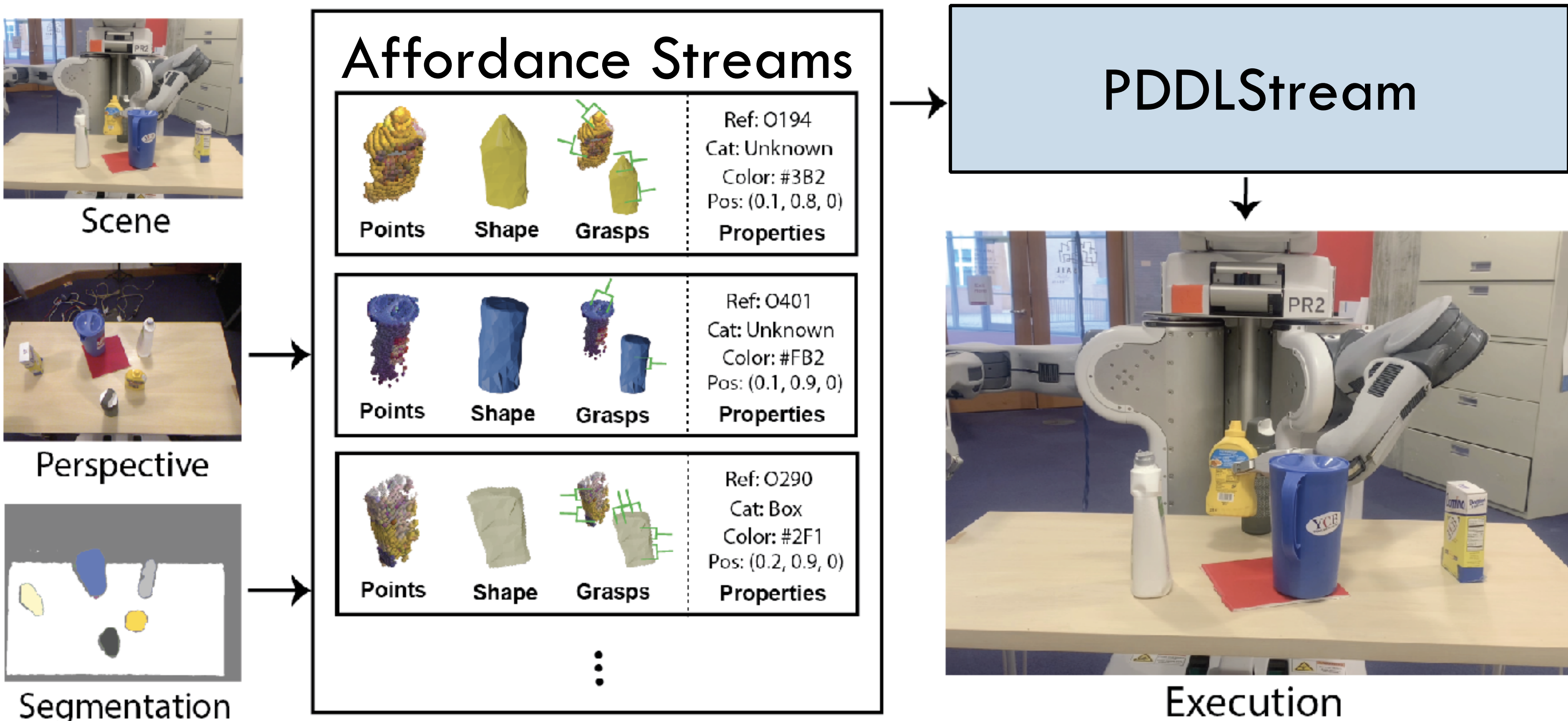
119



# Plan using Estimated Affordances

120

- **Learned** segmentation, shape estimation, grasp prediction
- Streams call perceptual modules using object **point clouds**





Goal: all objects are on a blue target region

$\forall obj. \exists region. On(obj, region) \wedge Color(region, blue)$

121



# Single System Generalizes across Objects, Goals, Initial States

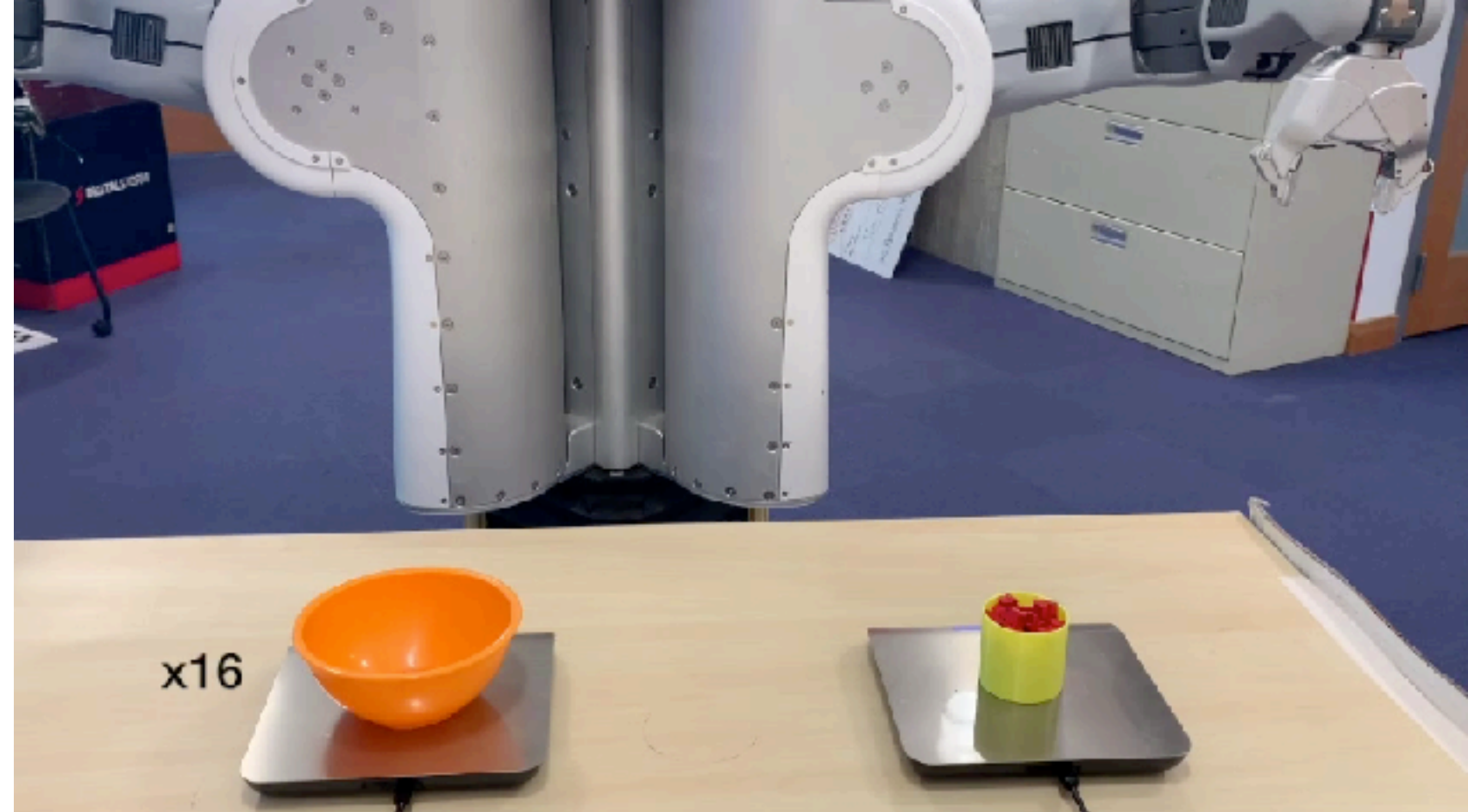
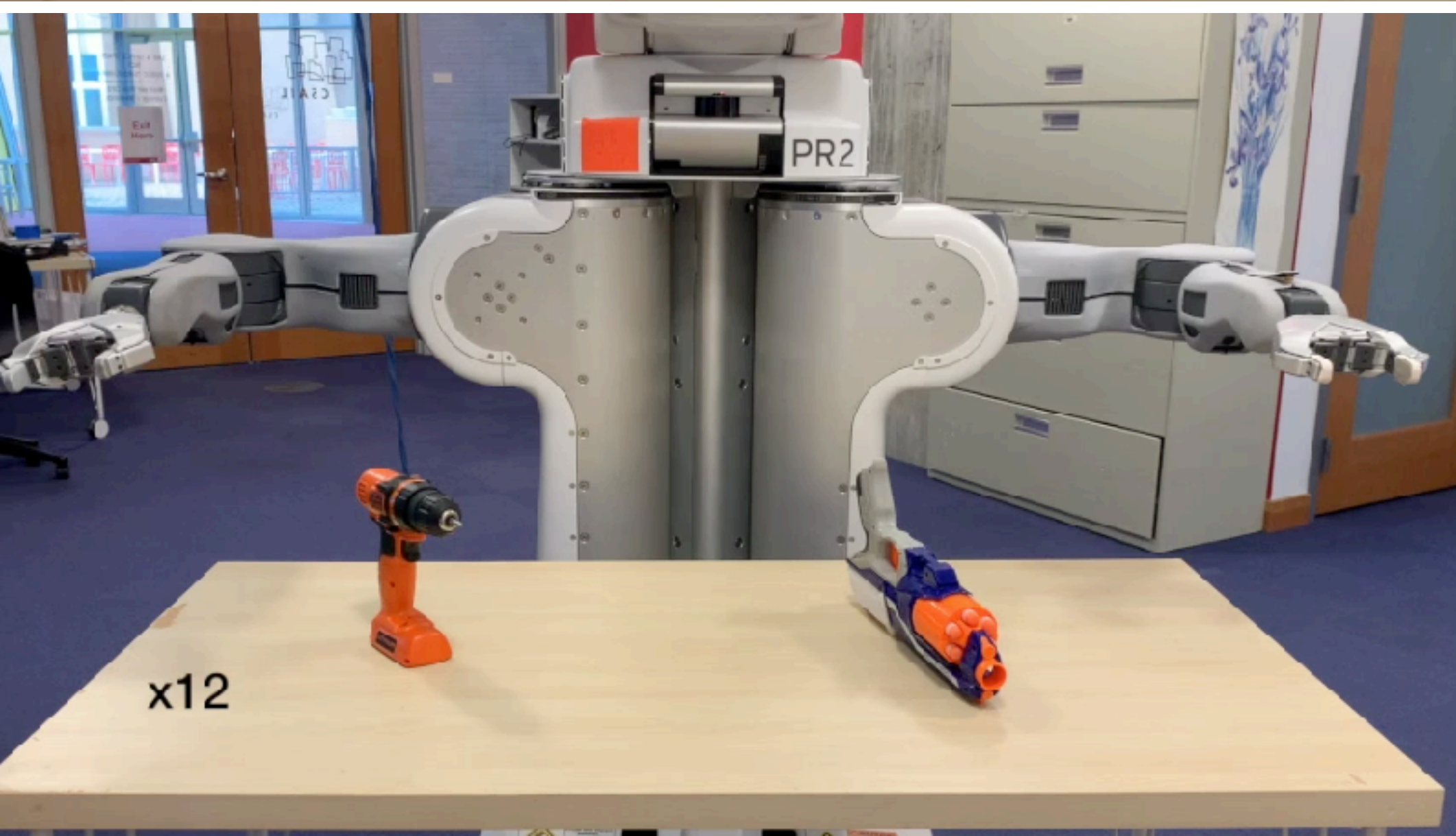
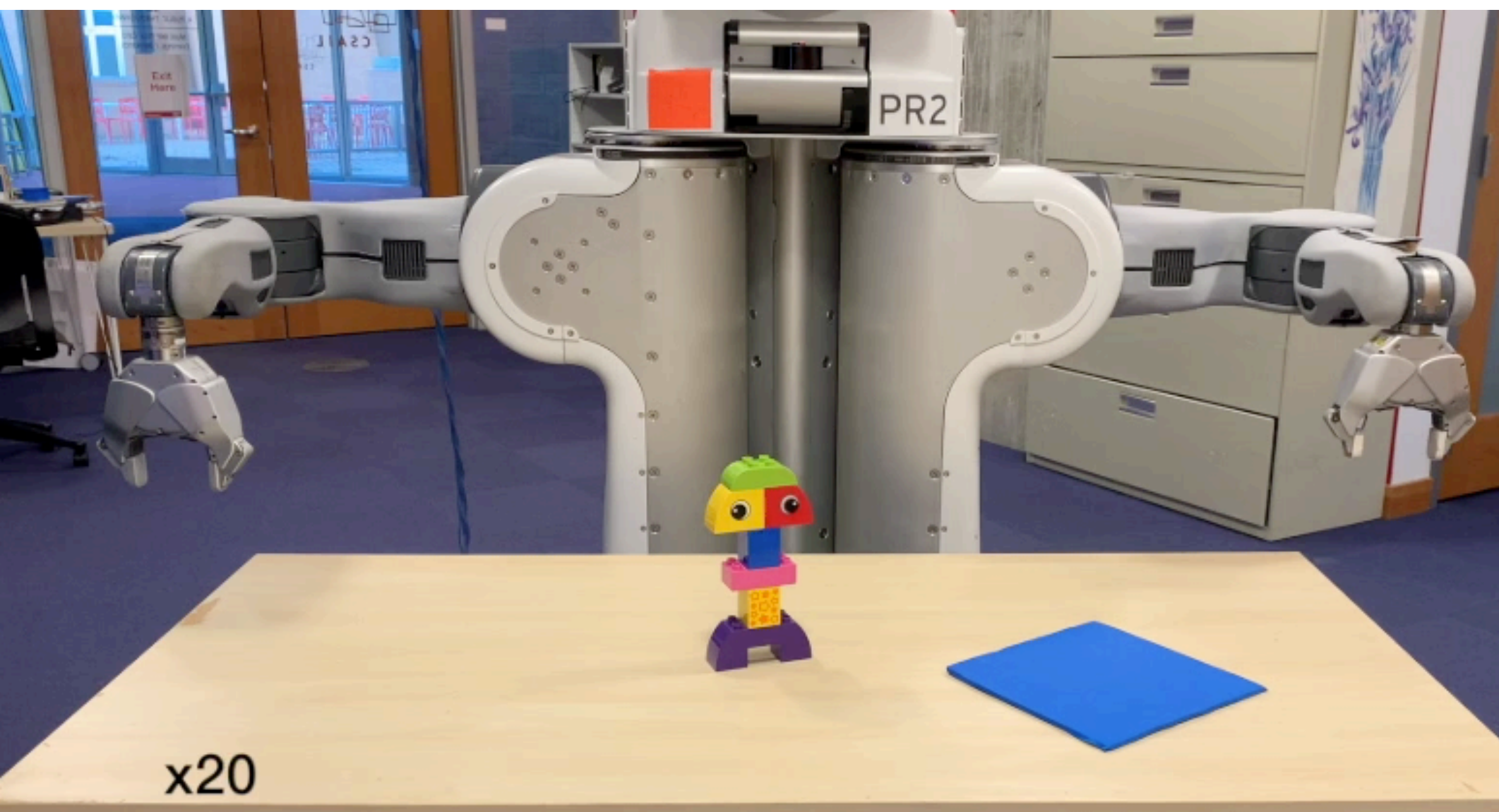
122



# Takeaways

- **Task and Motion Planning (TAMP):** hybrid planning where continuous constraints affect discrete decisions
- **Sampling** is powerful for exploring continuous spaces
- **PDDLStream:** planning language that supports **sampling procedures** as blackbox streams
  - **Domain-independent** algorithms
  - **Lazy/optimistic** planning intelligently queries only a small number of samplers
- Applies to **probabilistic & partially observable TAMP**

# Thanks! Questions?





# References



# Classical Planning

- **[Fikes 1971]** Fikes, R.E. and Nilsson, N.J., 1971. “STRIPS: A new approach to the application of theorem proving to problem solving”. *Artificial intelligence*, 2(3-4), pp.189-208.
- **[Aeronautiques 1998]** Aeronautiques, C., Howe, A., Knoblock, C., McDermott, I.D., Ram, A., Veloso, M., Weld, D., SRI, D.W., Barrett, A., Christianson, D. and Friedman, M., 1998. “PDDL: The Planning Domain Definition Language”.
- **[Hoffman 2001]** Hoffmann, J. and Nebel, B., 2001. “The FF planning system: Fast plan generation through heuristic search”. *Journal of Artificial Intelligence Research*, 14, pp.253-302.
- **[Helmert 2006]** Helmert, M., 2006. “The fast downward planning system”. *Journal of Artificial Intelligence Research*, 26, pp.191-246.
- **[Eyerich 2009]** Eyerich, P., Mattmüller, R. and Röger, G., 2009. “Using the Context-enhanced Additive Heuristic for Temporal and Numeric Planning”. in *Proceedings of the 19th International Conference on Automated Planning and Scheduling (ICAPS)*. AAAI Press, pp. 130–137.

# Motion Planning

- **[Kavraki 1994]** Kavraki, L. E. *et al.*, 1996. “Probabilistic roadmaps for path planning in high-dimensional configuration spaces”. *IEEE Transactions on Robotics and Automation*, 12(4), pp. 566–580.
- **[Bohlin 2000]** Bohlin, R. and Kavraki, L.E., 2000. “Path planning using lazy PRM”. In Proceedings 2000 ICRA. Millennium Conference. *IEEE International Conference on Robotics and Automation*. Symposia Proceedings (Cat. No. 00CH37065) (Vol. 1, pp. 521-528).
- **[Kuffner 2000]** Kuffner Jr, J.J. and LaValle, S.M., 2000. “RRT-connect: An efficient approach to single-query path planning”. In *IEEE International Conference on Robotics and Automation* (Vol. 2).
- **[Kuffner 2001]** LaValle, S.M. and Kuffner Jr, J.J., 2001. “Randomized kinodynamic planning”. *The International Journal of Robotics Research*, 20(5), pp.378-400.
- **[Karaman 2011]** Karaman, S. and Frazzoli, E., 2011. “Sampling-based Algorithms for Optimal Motion Planning”. *International Journal of Robotics Research (IJRR)*, Sage Publications, 30(7), pp. 846–894.

# Pre-discretized Planning

- **[Dornhege 2009]** Dornhege, C., Eyerich, P., Keller, T., Trüg, S., Brenner, M. and Nebel, B., 2009. “Semantic attachments for domain-independent planning systems”. *International Conference on Automated Planning and Scheduling*.
- **[Erdem 2011]** Erdem, E., Haspalamutgil, K., Palaz, C., Patoglu, V. and Uras, T., 2011. “Combining high-level causal reasoning with low-level geometric reasoning and motion planning for robotic manipulation”. *IEEE International Conference on Robotics and Automation* (pp. 4575-4581).
- **[Lagriffoul 2014]** Lagriffoul, F., Dimitrov, D., Bidot, J., Saffiotti, A. and Karlsson, L., 2014. “Efficiently combining task and motion planning using geometric constraints”. *The International Journal of Robotics Research*, 33(14), pp.1726-1747.
- **[Ferrer-Mestres 2017]** Ferrer-Mestres, J., Frances, G. and Geffner, H., 2017. “Combined task and motion planning as classical AI planning”. *arXiv preprint arXiv:1706.06927*.
- **[Dantam 2018]** Dantam, N.T., Kingston, Z.K., Chaudhuri, S. and Kavraki, L.E., 2018. “An incremental constraint-based framework for task and motion planning”. *The International Journal of Robotics Research*, 37(10), pp.1134-1151.
- **[Lo 2018]** Lo, S.Y., Zhang, S. and Stone, P., 2018. “PETLON: Planning Efficiently for Task-Level-Optimal Navigation”. *International Conference on Autonomous Agents and MultiAgent Systems* (pp. 220-228).
- **[Huang 2018]** Huang, Y., Garrett, C.R. and Mueller, C.T., 2018. “Automated sequence and motion planning for robotic spatial extrusion of 3D trusses”. *Construction Robotics*, 2(1-4), pp.15-39.



# Multi-Modal Motion Planning

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- **[Alami 1994]** Alami, R., Laumond, J.P. and Siméon, T., 1994. “Two manipulation planning algorithms”. In *WAFR Proceedings of the workshop on Algorithmic foundations of robotics* (pp. 109-125).
- **[Siméon 2004]** Siméon, T., Laumond, J.P., Cortés, J. and Sahbani, A., 2004. “Manipulation planning with probabilistic roadmaps”. *The International Journal of Robotics Research*, 23(7-8), pp.729-746.
- **[Hauser 2011]** Hauser, K. and Ng-Thow-Hing, V., 2011. “Randomized multi-modal motion planning for a humanoid robot manipulation task”. *The International Journal of Robotics Research*, 30(6), pp.678-698.
- **[Plaku 2010]** Plaku, E. and Hager, G. (2010) “Sampling-based Motion Planning with Symbolic, Geometric, and Differential Constraints”, *IEEE International Conference on Robotics and Automation*.
- **[Barry 2013]** Barry, J., Kaelbling, L.P. and Lozano-Pérez, T., 2013. “A hierarchical approach to manipulation with diverse actions”. *IEEE International Conference on Robotics and Automation* (pp. 1799-1806).
- **[Toussaint 2015]** Toussaint, M., 2015. “Logic-geometric programming: An optimization-based approach to combined task and motion planning”. *International Joint Conference on Artificial Intelligence*.
- **[Vega-Brown 2016]** Vega-Brown, W. and Roy, N., 2016. “Asymptotically optimal planning under piecewise-analytic constraints”. *Workshop on the Algorithmic Foundations of Robotics*.
- **[Toussaint 2018]** Toussaint, M., Allen, K., Smith, K.A. and Tenenbaum, J.B., 2018. “Differentiable Physics and Stable Modes for Tool-Use and Manipulation Planning”. *Robotics: Science and Systems*.

# Task and Motion Planning

- **[Gravot 2005]** Gravot, F., Cambon, S. and Alami, R., 2005. “aSyMov: a planner that deals with intricate symbolic and geometric problems”. In *Robotics Research. The Eleventh International Symposium* (pp. 100-110).
- **[Kaelbling 2011]** Kaelbling, L. P. and Lozano-Pérez, T., 2011. “Hierarchical task and motion planning in the now”. *IEEE International Conference on Robotics and Automation*, Shanghai, 2011, pp. 1470-1477.
- **[Srivastava 2014]** Srivastava, S., Fang, E., Riano, L., Chitnis, R., Russell, S. and Abbeel, P., 2014. “Combined task and motion planning through an extensible planner-independent interface layer”. *IEEE International Conference on Robotics and Automation* (pp. 639-646).
- **[Garrett 2017]** Garrett, C.R., Lozano-Perez, T. and Kaelbling, L.P., 2017. “FFRob: Leveraging symbolic planning for efficient task and motion planning”. *The International Journal of Robotics Research*, 37(1), pp.104-136.
- **[Garrett 2018]** Garrett, C.R., Lozano-Pérez, T. and Kaelbling, L.P., 2018. “Sampling-based methods for factored task and motion planning”. *The International Journal of Robotics Research*, 37(13-14), pp.1796-1825.
- **[Garrett 2020a]** Garrett, C.R., Lozano-Pérez, T. and Kaelbling, L. P., 2020. “PDDLStream: Integrating Symbolic Planners and Blackbox Samplers”. *International Conference on Automated Planning and Scheduling (ICAPS)*.
- **[Garrett 2021]** Garrett, C. R. et al., 2021. “Integrated Task and Motion Planning”. *Annual review of control, robotics, and autonomous systems*, 4.

# Probabilistic & Partially-Observable

- **[Kaelbling 1998]** Kaelbling, L.P., Littman, M.L. and Cassandra, A.R., 1998. “Planning and acting in partially observable stochastic domains”. *Artificial intelligence*, 101(1-2), pp.99-134.
- **[Yoon 2007]** Yoon, S.W., Fern, A. and Givan, R., 2007. “FF-Replan: A Baseline for Probabilistic Planning”. *International Conference on Automated Planning and Scheduling (ICAPS)* (Vol. 7, pp. 352-359).
- **[Keyder 2008]** Keyder, E. and Geffner, H., 2008. “The HMDPP planner for planning with probabilities”. *Sixth International Planning Competition at ICAPS*, 8.
- **[Platt 2010]** Platt, R. *et al.*, 2010. “Belief space planning assuming maximum likelihood observations”. *Robotics: Science and Systems VI*. doi: 10.15607/RSS.2010.VI.037.
- **[Silver 2010]** Silver, D. and Veness, J., 2010. “Monte-Carlo planning in large POMDPs”. *Advances in neural information processing systems*, pp. 2164–2172.
- **[Kaelbling 2013]** Kaelbling, L.P. and Lozano-Pérez, T., 2013. “Integrated task and motion planning in belief space”. *The International Journal of Robotics Research*, 32(9-10), pp.1194-1227.
- **[Garrett 2020b]** Garrett, C. R. *et al.*, 2020. “Online Replanning in Belief Space for Partially Observable Task and Motion Problems”. *International Conference on Robotics and Automation (ICRA)*.