CSE-571
Sampling-Based Motion Planning

Built on Dieter's Spring 2020 slides
Slides based on Pieter Abbeel, Zoe McCarthy
Many images from Lavalle, Planning Algorithms

Motion Planning: Outline

- Configuration Space
- Probabilistic Roadmap
- Rapidly-exploring Random Trees (RRTs)
- Extensions
- Smoothing

Configuration Space (C-Space)

Any configuration of a robot can be described as a unique point in its configuration space (C-space)

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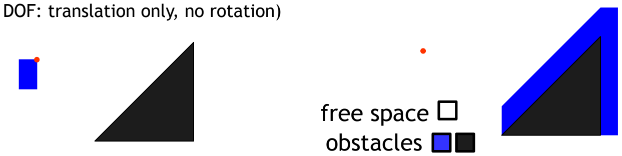

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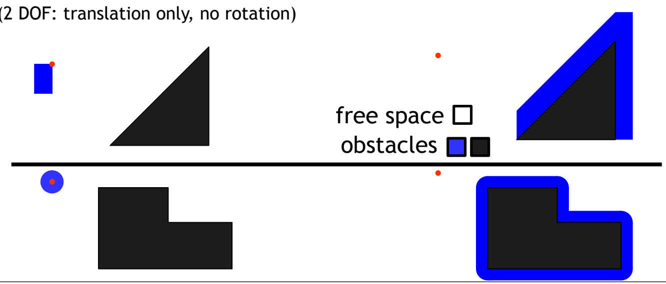

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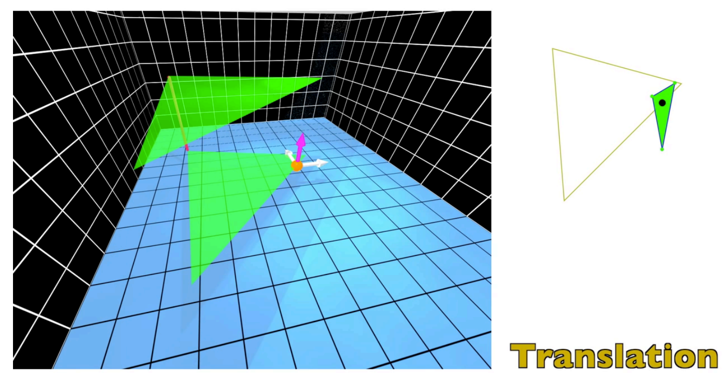
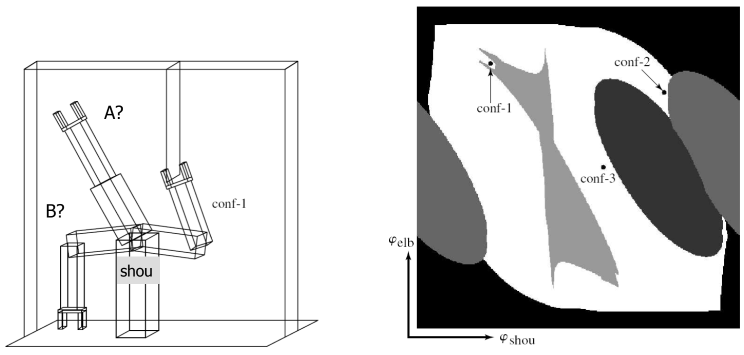
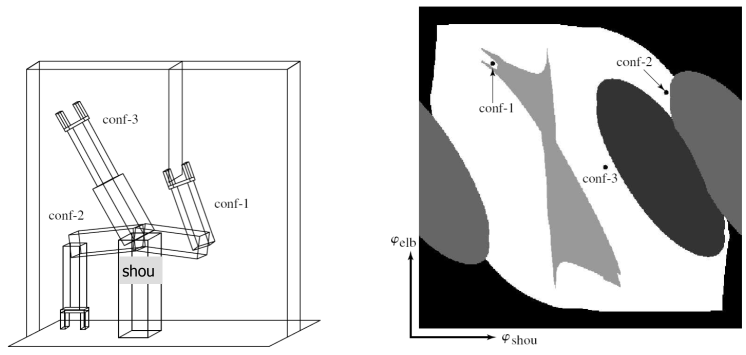


Visualization developed by Dror Atariah and Günter Rote - https://www.youtube.com/watch?v=SBFwgR4K1Gk

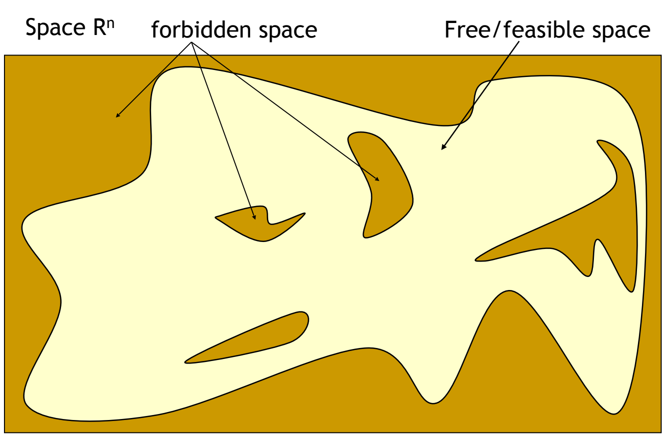
Motion planning



Motion planning

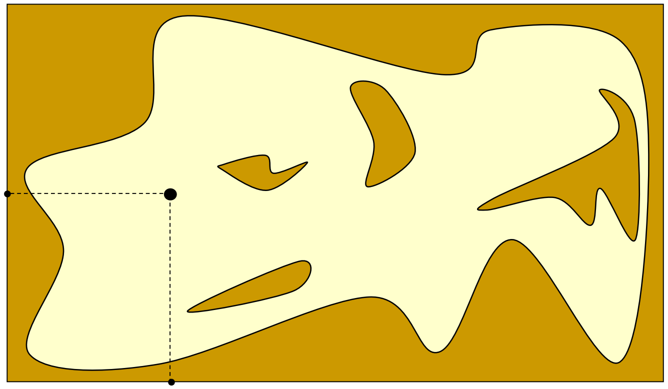


Probabilistic Roadmap (PRM)



Probabilistic Roadmap (PRM)

Configurations are sampled by picking coordinates at random



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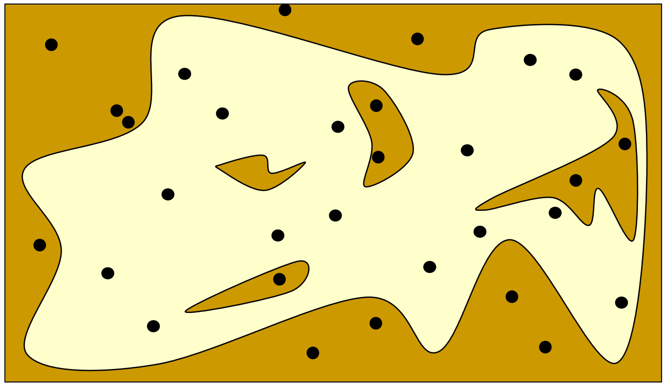
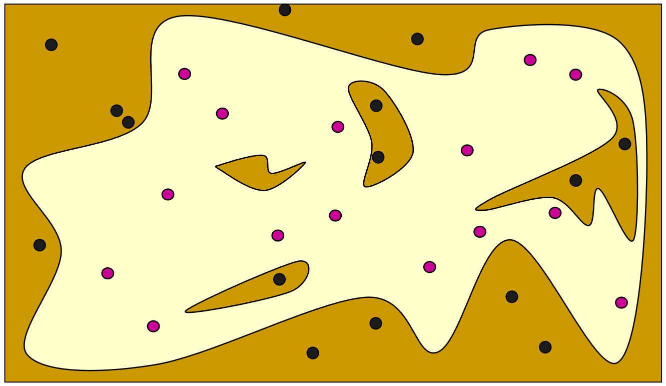


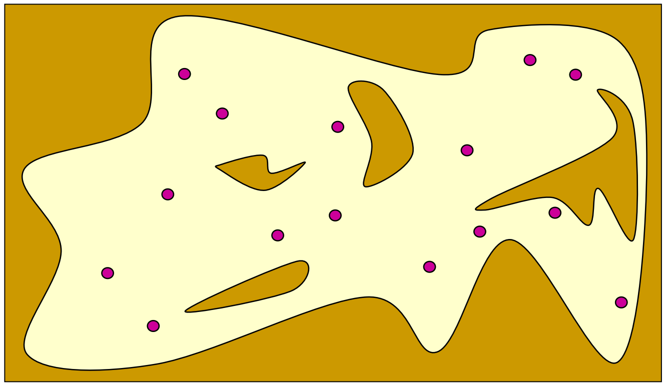
Probabilistic Roadmap (PRM)

Sampled configurations are tested for collision



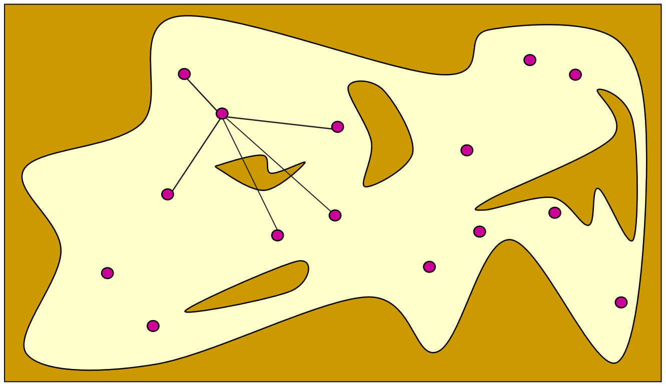
Probabilistic Roadmap (PRM)

The collision-free configurations are retained as milestones



Probabilistic Roadmap (PRM)

Each milestone is linked by straight paths to its nearest neighbors



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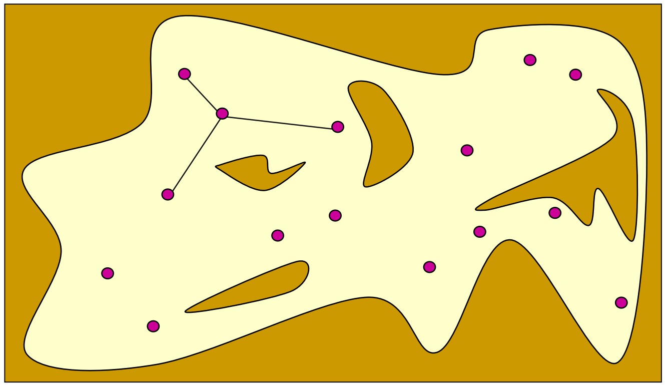
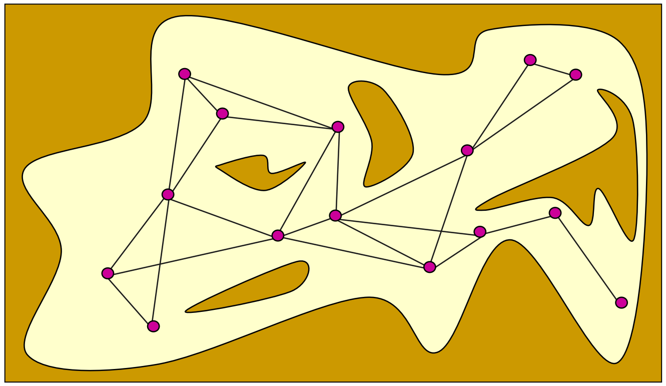


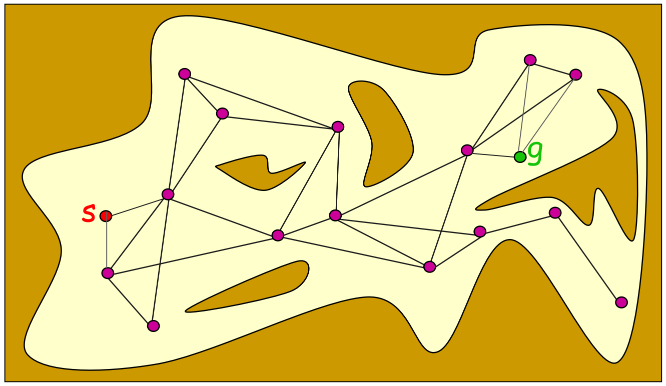
Probabilistic Roadmap (PRM)

The collision-free links are retained as local paths to form the PRM



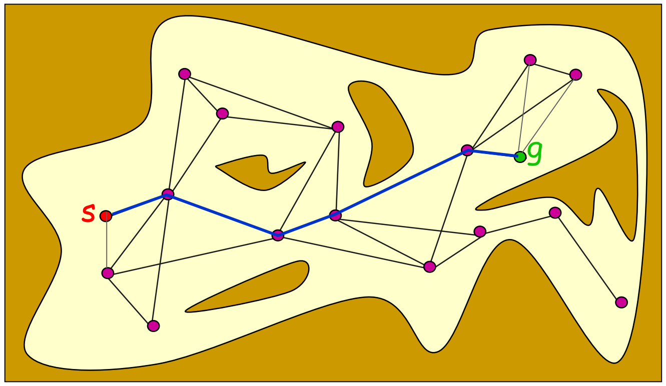
Probabilistic Roadmap (PRM)

The start and goal configurations are included as milestones



Probabilistic Roadmap (PRM)

The PRM is searched for a path from s to g



Probabilistic Roadmap (PRM)

```
for i = 1, ..., N do
    q_i ← sample from C_free
    add q_i to Roadmap R
end for
for i = 1, ..., N do
    N(q_i) ← k closest neighbors of q_i
    for each q ∈ N(q_i) do
        if there is a collision free local path from q to q_i and there is not already
           an edge from q to q_i then
            add an edge from q to q_i to the Roadmap R
        end if
    end for
end for
return R
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The resulting R depends on:

- N - number of samples
- k - number of neighbors
- Sampler
- Local path planner

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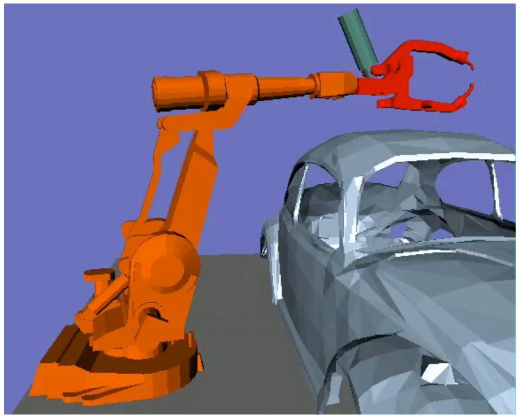
PRM is a multiple-query planner: invest time in generating a good representation of the free C-space, that can be used to solve several motion planning problems.

Probabilistic Roadmap (PRM)

Demonstration - https://demonstrations.wolfram.com/ProbabilisticRoadmapMethodForRobotArm/

PRM Example 1



PRM Example 2

PRM's Pros and Cons

- Pro:
  - Probabilistically complete: i.e., with probability one, if run for long enough the graph will contain a solution path if one exists.
- Cons:
  - Required to solve 2-point boundary value problem
  - Build graph over state space but no focus on generating a path

Rapidly exploring Random Tree (RRT)

Steve LaValle (98)

- Basic idea:
  - Build up a tree through generating "next states" in the tree by executing random controls
  - However: not exactly above to ensure good coverage

Demonstration - https://demonstrations.wolfram.com/RapidlyExploringRandomTreeRRTAndRRT/

Rapidly exploring Random Tree (RRT)

- Select random point, and expand nearest vertex towards it
  - Biases samples towards largest Voronoi region

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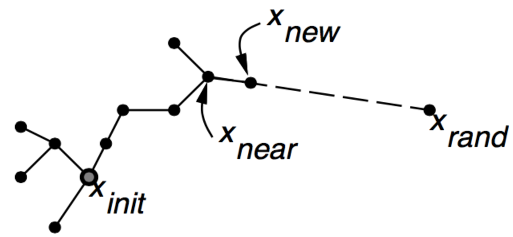

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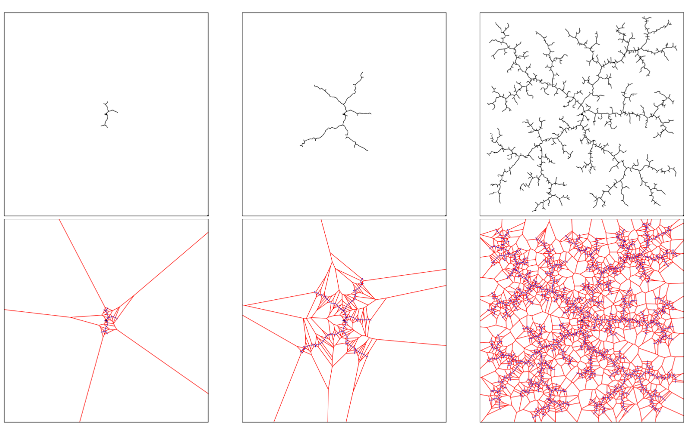
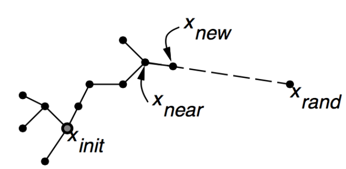


Source: LaValle and Kuffner 01

Rapidly exploring Random Tree (RRT)

GENERATE_RRT($x_{init}, K, \Delta t$)

$\mathcal{T}$.init(x_{init});
for $k = 1$ **to** K **do**
  $x_{rand} \leftarrow$ RANDOM_STATE();
  $x_{near} \leftarrow$ NEAREST_NEIGHBOR($x_{rand}, \mathcal{T}$);
  $u \leftarrow$ SELECT_INPUT(x_{rand}, x_{near});
  $x_{new} \leftarrow$ NEW_STATE($x_{near}, u, \Delta t$);
  $\mathcal{T}$.add_vertex(x_{new});
  $\mathcal{T}$.add_edge(x_{near}, x_{new}, u);
Return $\mathcal{T}$



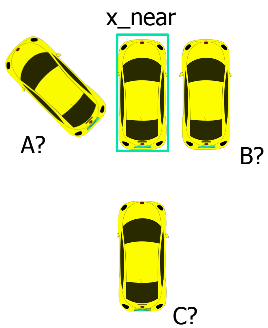
RANDOM_STATE(): often uniformly at random over space with probability 99%, and the goal state with probability 1%, this ensures it attempts to connect to goal semi-regularly

RRT Practicalities

- NEAREST_NEIGHBOR(x_{rand}, T): need to find (approximate) nearest neighbor efficiently
  - KD Trees data structure (upto 20-D) [e.g., FLANN]
  - Locality Sensitive Hashing
- SELECT_INPUT(x_{rand}, x_{near})
  - Two point boundary value problem
    - If too hard to solve, often just select best out of a set of control sequences. This set could be random, or some well chosen set of primitives.

RRT Extension

- Non-holonomic: approximately (sometimes as approximate as picking best of a few random control sequences) solve two-point boundary value problem



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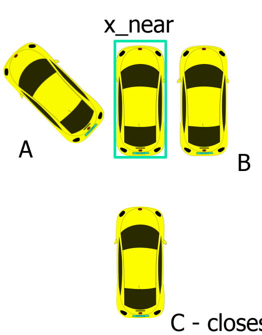

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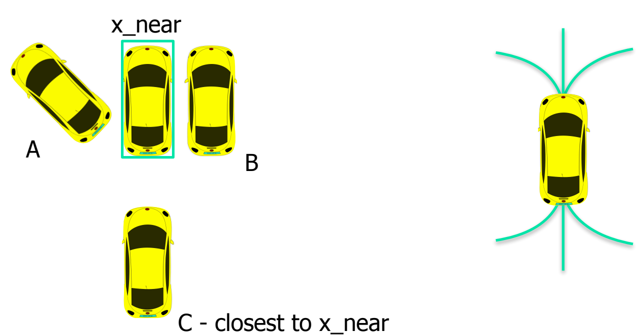
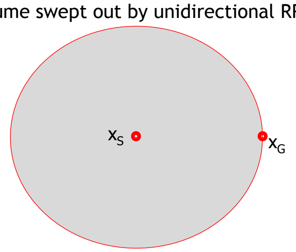
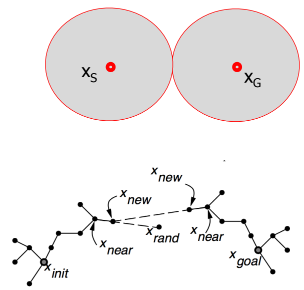


Bi-directional RRT

- Volume swept out by unidirectional RRT:



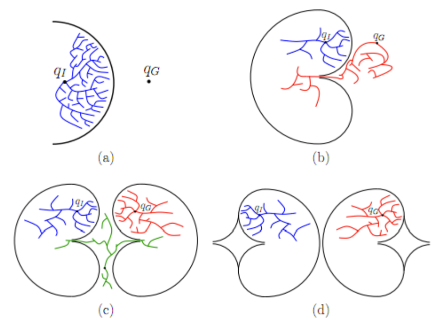
- Volume swept out by bi-directional RRT:



- Difference more and more pronounced as dimensionality increases

Multi-directional RRT

- Planning around obstacles or through narrow passages can often be easier in one direction than the other



RRT*

Algorithm 6: RRT*

$V \leftarrow \{x_{\text{init}}\}$; $E \leftarrow \emptyset$;
for $i = 1, \ldots, n$ **do**
  $x_{\text{rand}} \leftarrow \texttt{SampleFree}_i$;
  $x_{\text{nearest}} \leftarrow \texttt{Nearest}(G = (V, E), x_{\text{rand}})$;
  $x_{\text{new}} \leftarrow \texttt{Steer}(x_{\text{nearest}}, x_{\text{rand}})$;
  if $\texttt{ObtacleFree}(x_{\text{nearest}}, x_{\text{new}})$ **then**
    $X_{\text{near}} \leftarrow \texttt{Near}(G = (V, E), x_{\text{new}}, \min\{\gamma_{\text{RRT}^*}(\log(\text{card}\,(V))/\,\text{card}\,(V))^{1/d}, \eta\})$;
    $V \leftarrow V \cup \{x_{\text{new}}\}$;
    $x_{\text{min}} \leftarrow x_{\text{nearest}}$; $c_{\text{min}} \leftarrow \texttt{Cost}(x_{\text{nearest}}) + c(\texttt{Line}(x_{\text{nearest}}, x_{\text{new}}))$;
    foreach $x_{\text{near}} \in X_{\text{near}}$ **do** // Connect along a minimum-cost path
      if $\texttt{CollisionFree}(x_{\text{near}}, x_{\text{new}}) \wedge \texttt{Cost}(x_{\text{near}}) + c(\texttt{Line}(x_{\text{near}}, x_{\text{new}})) < c_{\text{min}}$ **then**
        $x_{\text{min}} \leftarrow x_{\text{near}}$; $c_{\text{min}} \leftarrow \texttt{Cost}(x_{\text{near}}) + c(\texttt{Line}(x_{\text{near}}, x_{\text{new}}))$
    $E \leftarrow E \cup \{(x_{\text{min}}, x_{\text{new}})\}$;
    foreach $x_{\text{near}} \in X_{\text{near}}$ **do** // Rewire the tree
      if $\texttt{CollisionFree}(x_{\text{new}}, x_{\text{near}}) \wedge \texttt{Cost}(x_{\text{new}}) + c(\texttt{Line}(x_{\text{new}}, x_{\text{near}})) < \texttt{Cost}(x_{\text{near}})$
      then $x_{\text{parent}} \leftarrow \texttt{Parent}(x_{\text{near}})$;
      $E \leftarrow (E \setminus \{(x_{\text{parent}}, x_{\text{near}})\}) \cup \{(x_{\text{new}}, x_{\text{near}})\}$
return $G = (V, E)$;

Same as RRT

Find the set of neighbors to x_new in the G
Add x_new to G

Find a neighbor from the set that will lead to x_new at less cost.
Add an edge to that neighbor and x_new

Rewire the set of neighbors with edges such that cost to reach any of these nodes from the root is minimum while the new node x_new

Source: Karaman and Frazzoli

RRT*

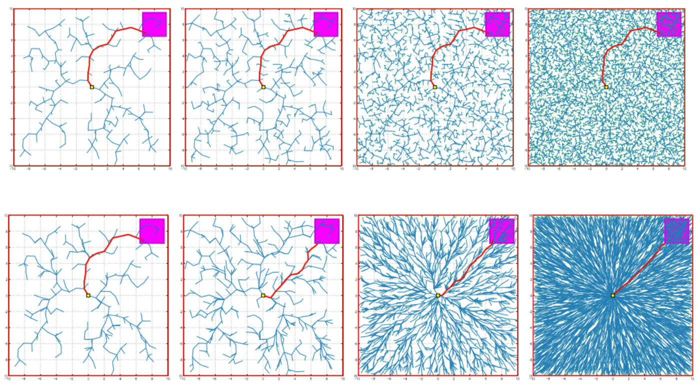
- Asymptotically optimal
- Main idea:
  - Swap new point in as parent for nearby vertices who can be reached along shorter path through new point than through their original (current) parent

Demonstration - https://demonstrations.wolfram.com/RapidlyExploringRandomTreeRRTAndRRT/

RRT*

RRT

RRT*



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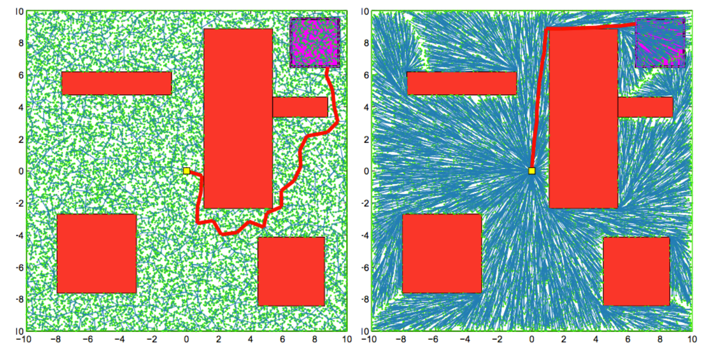
RRT*

RRT RRT*



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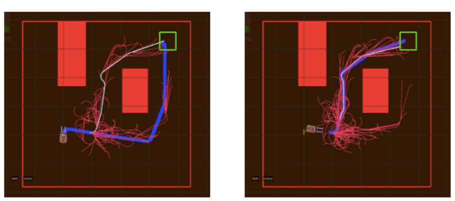
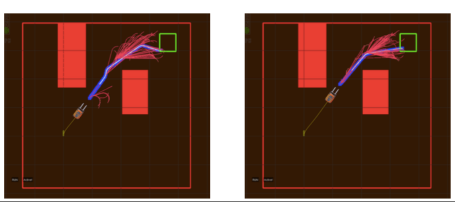
Real Time RRT*



(a) RRT* run 1 (b) RRT* run 1



Smoothing

Randomized motion planners tend to find not so great paths for execution: very jagged, often much longer than necessary.

→ In practice: do smoothing before using the path

- Shortcutting:
  - along the found path, pick two vertices x_{t1}, x_{t2} and try to connect them directly (skipping over all intermediate vertices)
- Nonlinear optimization for optimal control
  - Allows to specify an objective function that includes smoothness in state, control, small control inputs, etc.

Additional Resources

- Marco Pavone (http://asl.stanford.edu/):
  - Sampling-based motion planning on GPUs: https://arxiv.org/pdf/1705.02403.pdf
  - Learning sampling distributions: https://arxiv.org/pdf/1709.05448.pdf
- Sidd Srinivasa (https://personalrobotics.cs.washington.edu/)
  - Batch informed trees: https://robotic-esp.com/code/bitstar/
  - Expensive edge evals: https://arxiv.org/pdf/2002.11853.pdf
  - Lazy search: https://personalrobotics.cs.washington.edu/publications/mandalika2019gls.pdf
- Michael Yip (https://www.ucsdarclab.com/)
  - Neural Motion Planners: https://www.ucsdarclab.com/neuralplanning
- Lydia Kavraki (http://www.kavrakilab.org/)
  - Motion in human workspaces: http://www.kavrakilab.org/nsf-nri-1317849.html