CSE-571 Robotics

SLAM: Simultaneous Localization and Mapping

Many slides courtesy of Ryan Eustice, Cyrill Stachniss, John Leonard

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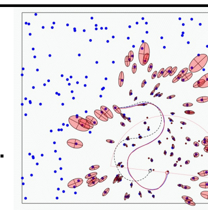
The SLAM Problem



A robot is exploring an unknown, static environment.

Given:

- The robot's controls
- Observations of nearby features

Estimate:

- Map of features
- Path of the robot

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SLAM Applications

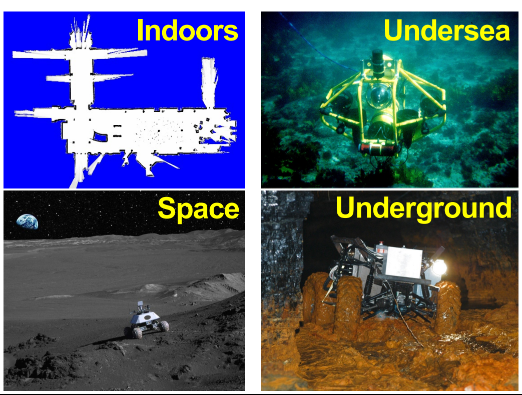

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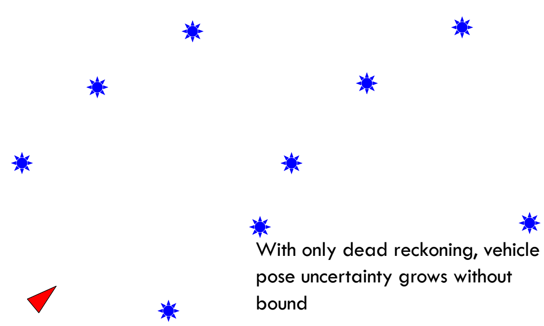
Illustration of SLAM without Landmarks



With only dead reckoning, vehicle pose uncertainty grows without bound

Courtesy J. Leonard

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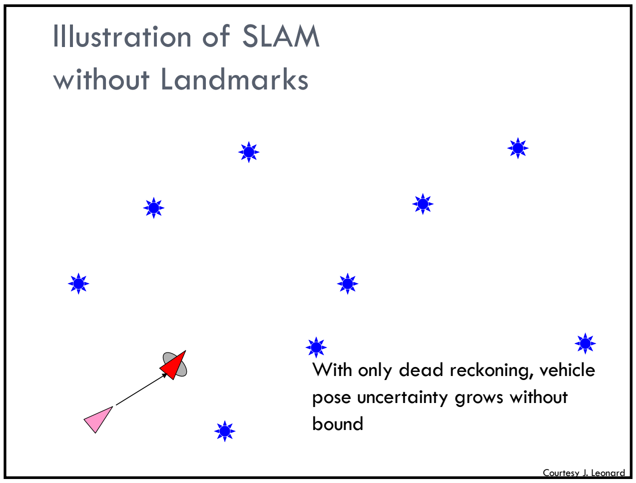
Illustration of SLAM
without Landmarks
With only dead reckoning, vehicle pose uncertainty grows without bound
Courtesy J. Leonard

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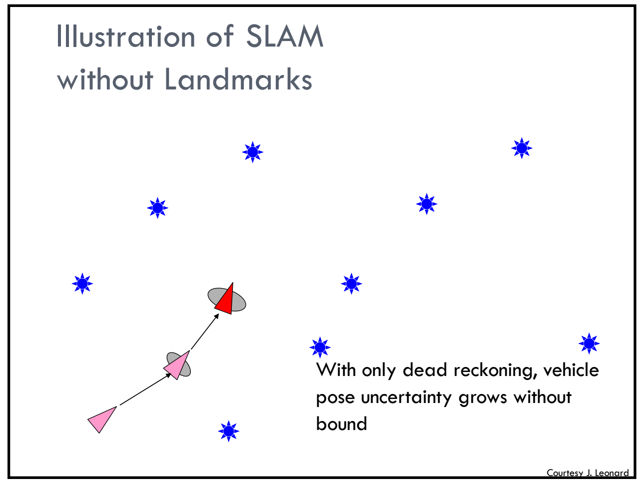
Illustration of SLAM
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With only dead reckoning, vehicle pose uncertainty grows without bound
Courtesy J. Leonard

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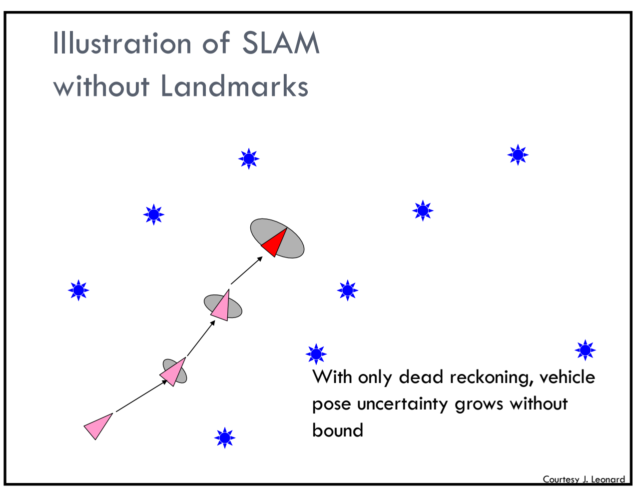
Illustration of SLAM
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With only dead reckoning, vehicle pose uncertainty grows without bound
Courtesy J. Leonard

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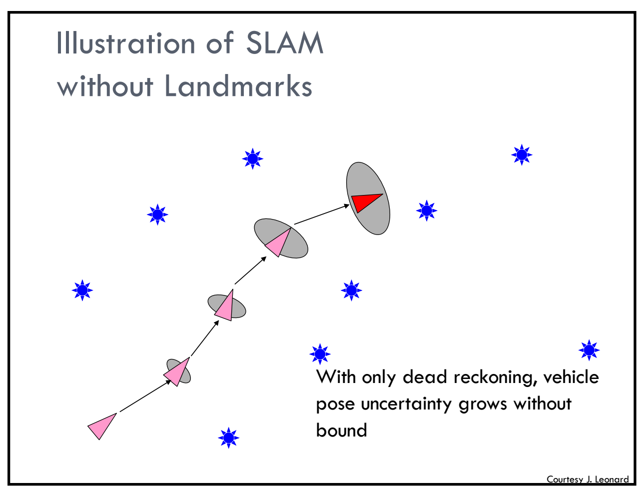
Illustration of SLAM
without Landmarks
With only dead reckoning, vehicle pose uncertainty grows without bound
Courtesy J. Leonard

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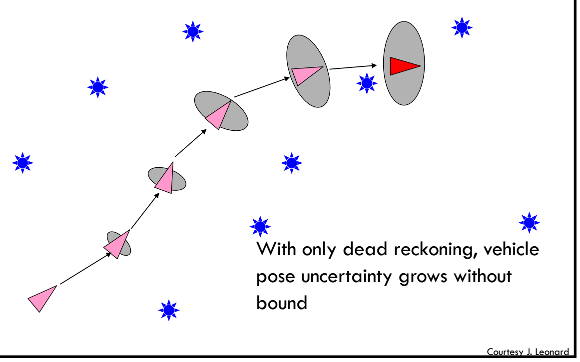
Illustration of SLAM without Landmarks



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Mapping with Raw Odometry

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Repeat, with Measurements of Landmarks

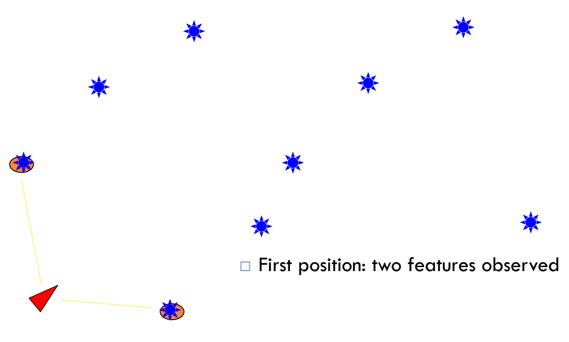

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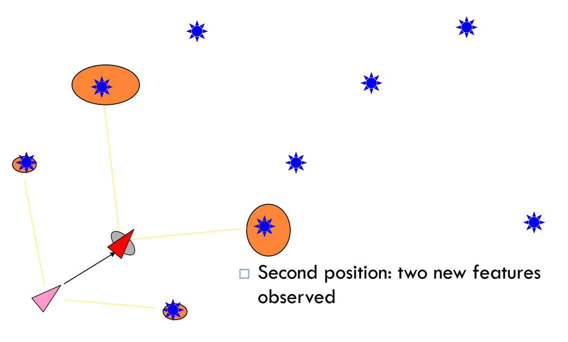
Illustration of SLAM with Landmarks



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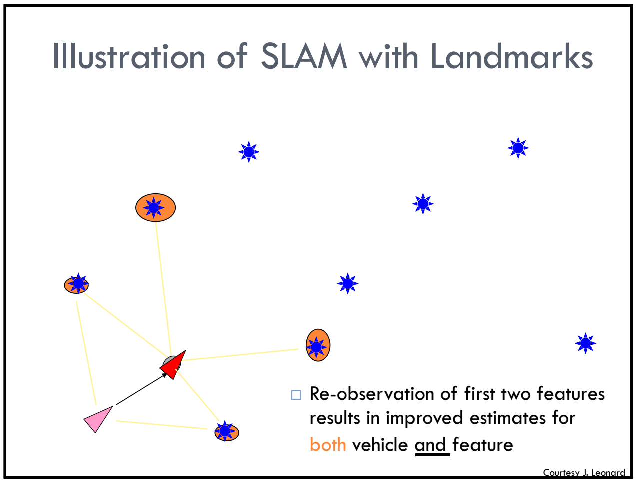
Illustration of SLAM with Landmarks
Re-observation of first two features results in improved estimates for both vehicle and feature
Courtesy J. Leonard

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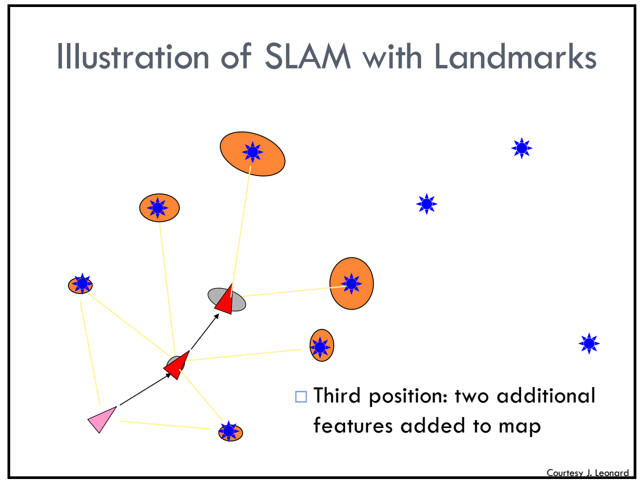
Illustration of SLAM with Landmarks
Third position: two additional features added to map
Courtesy J. Leonard

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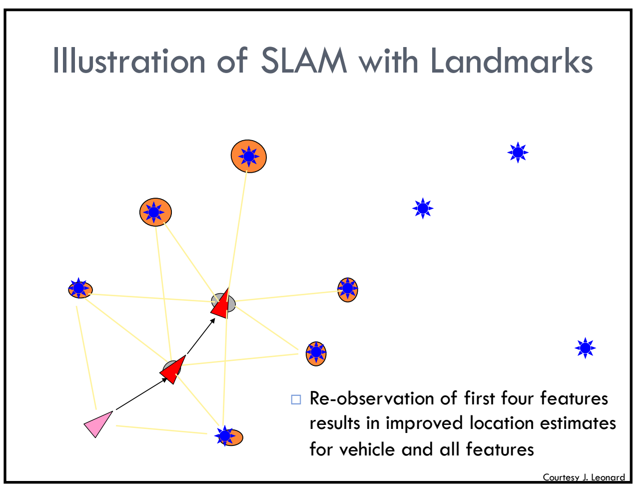
Illustration of SLAM with Landmarks
Re-observation of first four features results in improved location estimates for vehicle and all features
Courtesy J. Leonard

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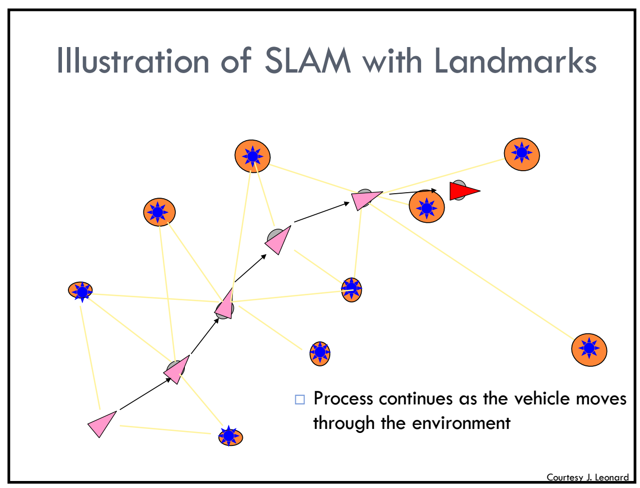
Illustration of SLAM with Landmarks
Process continues as the vehicle moves through the environment
Courtesy J. Leonard

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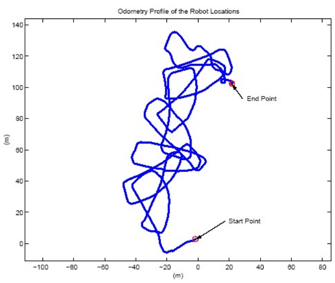
SLAM Using Landmarks



Courtesy J. Leonard

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Test Environment (Point Landmarks)

Courtesy J. Leonard

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View from Vehicle

Courtesy J. Leonard

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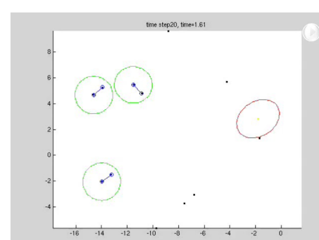
SLAM Using Landmarks

1. Move
2. Sense
3. Associate measurements with known features
4. Update state estimates for robot and previously mapped features
5. Find new features from unassociated measurements
6. Initialize new features
7. Repeat

MIT Indoor Track



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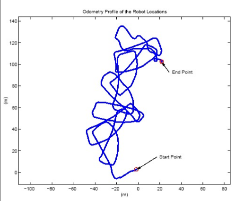
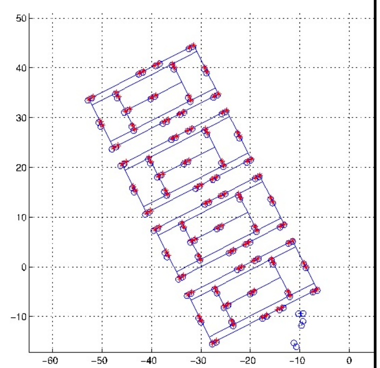
Comparison with Ground Truth



odometry



SLAM result

Courtesy J. Leonard

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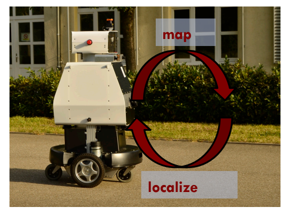
Simultaneous Localization and Mapping (SLAM)

- Building a map and locating the robot in the map at the same time
- Chicken-and-egg problem



Courtesy: Cyrill Stachniss

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Definition of the SLAM Problem

Given

- The robot's controls
  $u_{1:T} = \{u_1, u_2, u_3, \dots, u_T\}$
- Observations
  $z_{1:T} = \{z_1, z_2, z_3, \dots, z_T\}$

Wanted

- Map of the environment
  m
- Path of the robot
  $x_{0:T} = \{x_0, x_1, x_2, \dots, x_T\}$

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Three Main Paradigms

Kalman filter	Graph-based	Particle filter

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Bayes Filter

- Recursive filter with prediction and correction step
- Prediction

$$\overline{bel}(x_t) = \int p(x_t \mid u_t, x_{t-1})\ bel(x_{t-1})\, dx_{t-1}$$

- Correction

$$bel(x_t) = \eta\ p(z_t \mid x_t)\ \overline{bel}(x_t)$$

Courtesy: Cyrill Stachniss

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EKF SLAM

- Application of the EKF to SLAM
- Estimate robot's pose and locations of landmarks in the environment
- Assumption: known correspondences
- State space (for the 2D plane) is

$$x_t = (\underbrace{x, y, \theta}_{\text{robot's pose}}, \underbrace{m_{1,x}, m_{1,y}}_{\text{landmark 1}}, \ldots, \underbrace{m_{n,x}, m_{n,y}}_{\text{landmark n}})^T$$

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EKF SLAM: State Representation

- Map with n landmarks: (3+2n)-dimensional Gaussian
- Belief is represented by

$$\underbrace{\begin{pmatrix} x \\ y \\ \theta \\ m_{1,x} \\ m_{1,y} \\ \vdots \\ m_{n,x} \\ m_{n,y} \end{pmatrix}}_{\mu} \quad \underbrace{\begin{pmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{x\theta} & \sigma_{xm_{1,x}} & \sigma_{xm_{1,y}} & \cdots & \sigma_{xm_{n,x}} & \sigma_{xm_{n,y}} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{y\theta} & \sigma_{ym_{1,x}} & \sigma_{ym_{1,y}} & \cdots & \sigma_{m_{n,x}} & \sigma_{m_{n,y}} \\ \sigma_{\theta x} & \sigma_{\theta y} & \sigma_{\theta\theta} & \sigma_{\theta m_{1,x}} & \sigma_{\theta m_{1,y}} & \cdots & \sigma_{\theta m_{n,x}} & \sigma_{\theta m_{n,y}} \\ \sigma_{m_{1,x}x} & \sigma_{m_{1,x}y} & \sigma_{\theta} & \sigma_{m_{1,x}m_{1,x}} & \sigma_{m_{1,x}m_{1,y}} & \cdots & \sigma_{m_{1,x}m_{n,x}} & \sigma_{m_{1,x}m_{n,y}} \\ \sigma_{m_{1,y}x} & \sigma_{m_{1,y}y} & \sigma_{\theta} & \sigma_{m_{1,y}m_{1,x}} & \sigma_{m_{1,y}m_{1,y}} & \cdots & \sigma_{m_{1,y}m_{n,x}} & \sigma_{m_{1,y}m_{n,y}} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \sigma_{m_{n,x}x} & \sigma_{m_{n,x}y} & \sigma_{\theta} & \sigma_{m_{n,x}m_{1,x}} & \sigma_{m_{n,x}m_{1,y}} & \cdots & \sigma_{m_{n,x}m_{n,x}} & \sigma_{m_{n,x}m_{n,y}} \\ \sigma_{m_{n,y}x} & \sigma_{m_{n,y}y} & \sigma_{\theta} & \sigma_{m_{n,y}m_{1,x}} & \sigma_{m_{n,y}m_{1,y}} & \cdots & \sigma_{m_{n,y}m_{n,x}} & \sigma_{m_{n,y}m_{n,y}} \end{pmatrix}}_{\Sigma}$$

Courtesy: Cyrill Stachniss

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EKF SLAM: State Representation

- More compactly

$$\underbrace{\begin{pmatrix} x_R \\ m_1 \\ \vdots \\ m_n \end{pmatrix}}_{\mu} \quad \underbrace{\begin{pmatrix} \Sigma_{x_R x_R} & \Sigma_{x_R m_1} & \cdots & \Sigma_{x_R m_n} \\ \Sigma_{m_1 x_R} & \Sigma_{m_1 m_1} & \cdots & \Sigma_{m_1 m_n} \\ \vdots & \vdots & \ddots & \vdots \\ \Sigma_{m_n x_R} & \Sigma_{m_n m_1} & \cdots & \Sigma_{m_n m_n} \end{pmatrix}}_{\Sigma}$$

Courtesy: Cyrill Stachniss

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EKF SLAM: State Representation

- Even more compactly (note: $x_R \rightarrow x$)

$$\underbrace{\begin{pmatrix} x \\ m \end{pmatrix}}_{\mu} \quad \underbrace{\begin{pmatrix} \Sigma_{xx} & \Sigma_{xm} \\ \Sigma_{mx} & \Sigma_{mm} \end{pmatrix}}_{\Sigma}$$

Courtesy: Cyrill Stachniss

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EKF SLAM: Filter Cycle

1. State prediction
2. Measurement prediction
3. Measurement
4. Data association
5. Update

Courtesy: Cyrill Stachniss

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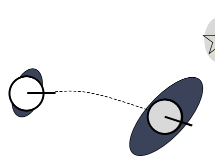
EKF SLAM: State Prediction



$$\underbrace{\begin{pmatrix} x_R \\ m_1 \\ \vdots \\ m_n \end{pmatrix}}_{\mu} \quad \underbrace{\begin{pmatrix} \Sigma_{x_R x_R} & \Sigma_{x_R m_1} & \dots & \Sigma_{x_R m_n} \\ \Sigma_{m_1 x_R} & \Sigma_{m_1 m_1} & \dots & \Sigma_{m_1 m_n} \\ \vdots & \vdots & \ddots & \vdots \\ \Sigma_{m_n x_R} & \Sigma_{m_n m_1} & \dots & \Sigma_{m_n m_n} \end{pmatrix}}_{\Sigma}$$

Courtesy: Cyrill Stachniss

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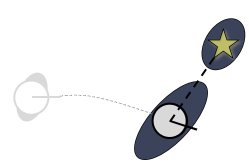
EKF SLAM: Measurement Prediction



$$\underbrace{\begin{pmatrix} x_R \\ m_1 \\ \vdots \\ m_n \end{pmatrix}}_{\mu} \quad \underbrace{\begin{pmatrix} \Sigma_{x_R x_R} & \Sigma_{x_R m_1} & \dots & \Sigma_{x_R m_n} \\ \Sigma_{m_1 x_R} & \Sigma_{m_1 m_1} & \dots & \Sigma_{m_1 m_n} \\ \vdots & \vdots & \ddots & \vdots \\ \Sigma_{m_n x_R} & \Sigma_{m_n m_1} & \dots & \Sigma_{m_n m_n} \end{pmatrix}}_{\Sigma}$$

Courtesy: Cyrill Stachniss

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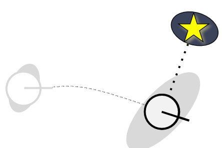
EKF SLAM: Obtained Measurement



$$\underbrace{\begin{pmatrix} x_R \\ m_1 \\ \vdots \\ m_n \end{pmatrix}}_{\mu} \quad \underbrace{\begin{pmatrix} \Sigma_{x_R x_R} & \Sigma_{x_R m_1} & \cdots & \Sigma_{x_R m_n} \\ \Sigma_{m_1 x_R} & \Sigma_{m_1 m_1} & \cdots & \Sigma_{m_1 m_n} \\ \vdots & \vdots & \ddots & \vdots \\ \Sigma_{m_n x_R} & \Sigma_{m_n m_1} & \cdots & \Sigma_{m_n m_n} \end{pmatrix}}_{\Sigma}$$

Courtesy: Cyrill Stachniss

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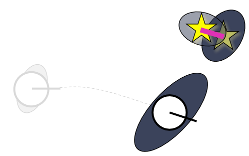
EKF SLAM: Data Association and Difference Between h(x) and z



$$\underbrace{\begin{pmatrix} x_R \\ m_1 \\ \vdots \\ m_n \end{pmatrix}}_{\mu} \quad \underbrace{\begin{pmatrix} \Sigma_{x_R x_R} & \Sigma_{x_R m_1} & \cdots & \Sigma_{x_R m_n} \\ \Sigma_{m_1 x_R} & \Sigma_{m_1 m_1} & \cdots & \Sigma_{m_1 m_n} \\ \vdots & \vdots & \ddots & \vdots \\ \Sigma_{m_n x_R} & \Sigma_{m_n m_1} & \cdots & \Sigma_{m_n m_n} \end{pmatrix}}_{\Sigma}$$

Courtesy: Cyrill Stachniss

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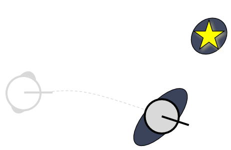
EKF SLAM: Update Step



$$\underbrace{\begin{pmatrix} x_R \\ m_1 \\ \vdots \\ m_n \end{pmatrix}}_{\mu} \quad \underbrace{\begin{pmatrix} \Sigma_{x_R x_R} & \Sigma_{x_R m_1} & \cdots & \Sigma_{x_R m_n} \\ \Sigma_{m_1 x_R} & \Sigma_{m_1 m_1} & \cdots & \Sigma_{m_1 m_n} \\ \vdots & \vdots & \ddots & \vdots \\ \Sigma_{m_n x_R} & \Sigma_{m_n m_1} & \cdots & \Sigma_{m_n m_n} \end{pmatrix}}_{\Sigma}$$

Courtesy: Cyrill Stachniss

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EKF SLAM: Concrete Example

Setup

- Robot moves in the 2D plane
- Velocity-based motion model
- Robot observes point landmarks
- Range-bearing sensor
- Known data association
- Known number of landmarks

Courtesy: Cyrill Stachniss

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Initialization

- Robot starts in its own reference frame (all landmarks unknown)
- 2N+3 dimensions

$$\mu_0 = \begin{pmatrix} 0 & 0 & 0 & \dots & 0 \end{pmatrix}^T$$

$$\Sigma_0 = \begin{pmatrix} 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \infty & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & \infty \end{pmatrix}$$

Courtesy: Cyrill Stachniss

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Extended Kalman Filter Algorithm

1: **Extended_Kalman_filter**$(\mu_{t-1}, \Sigma_{t-1}, u_t, z_t)$**:**

2: $\bar{\mu}_t = g(u_t, \mu_{t-1})$

3: $\bar{\Sigma}_t = G_t \, \Sigma_{t-1} \, G_t^T + R_t$

4: $K_t = \bar{\Sigma}_t \, H_t^T (H_t \, \bar{\Sigma}_t \, H_t^T + Q_t)^{-1}$

5: $\mu_t = \bar{\mu}_t + K_t(z_t - h(\bar{\mu}_t))$

6: $\Sigma_t = (I - K_t \, H_t) \, \bar{\Sigma}_t$

7: *return* μ_t, Σ_t

Courtesy: Cyrill Stachniss

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Prediction Step (Motion)

- Goal: Update state space based on the robot's motion
- Robot motion in the plane

$$\begin{pmatrix} x' \\ y' \\ \theta' \end{pmatrix} = \underbrace{\begin{pmatrix} x \\ y \\ \theta \end{pmatrix} + \begin{pmatrix} -\frac{v_t}{\omega_t}\sin\theta + \frac{v_t}{\omega_t}\sin(\theta + \omega_t \Delta t) \\ \frac{v_t}{\omega_t}\cos\theta - \frac{v_t}{\omega_t}\cos(\theta + \omega_t \Delta t) \\ \omega_t \, \Delta t \end{pmatrix}}_{g_{x,y,\theta}(u_t,(x,y,\theta)^T)}$$

- How to map that to the 2N+3 dim space?

Courtesy: Cyrill Stachniss

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Update the State Space

- From the motion in the plane

$$\begin{pmatrix} x' \\ y' \\ \theta' \end{pmatrix} = \begin{pmatrix} x \\ y \\ \theta \end{pmatrix} + \begin{pmatrix} -\frac{v_t}{\omega_t}\sin\theta + \frac{v_t}{\omega_t}\sin(\theta + \omega_t \Delta t) \\ \frac{v_t}{\omega_t}\cos\theta - \frac{v_t}{\omega_t}\cos(\theta + \omega_t \Delta t) \\ \omega_t \, \Delta t \end{pmatrix}$$

- to the 2N+3 dimensional space

$$\begin{pmatrix} x' \\ y' \\ \theta' \\ \vdots \end{pmatrix} = \underbrace{\begin{pmatrix} x \\ y \\ \theta \\ \vdots \end{pmatrix} + \underbrace{\begin{pmatrix} 1 & 0 & 0 & 0\dots0 \\ 0 & 1 & 0 & 0\dots0 \\ 0 & 0 & 1 & \underbrace{0\dots0}_{2N cols} \end{pmatrix}^T}_{F_x^T} \begin{pmatrix} -\frac{v_t}{\omega_t}\sin\theta + \frac{v_t}{\omega_t}\sin(\theta + \omega_t \Delta t) \\ \frac{v_t}{\omega_t}\cos\theta - \frac{v_t}{\omega_t}\cos(\theta + \omega_t \Delta t) \\ \omega_t \, \Delta t \end{pmatrix}}_{g(u_t, x_t)}$$

Courtesy: Cyrill Stachniss

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Extended Kalman Filter Algorithm

1: **Extended_Kalman_filter**$(\mu_{t-1}, \Sigma_{t-1}, u_t, z_t)$**:**

2: ~~$\bar{\mu}_t = g(u_t, \mu_{t-1})$~~ **DONE**
3: $\bar{\Sigma}_t = G_t\ \Sigma_{t-1}\ G_t^T + R_t$

4: $K_t = \bar{\Sigma}_t\ H_t^T (H_t\ \bar{\Sigma}_t\ H_t^T + Q_t)^{-1}$
5: $\mu_t = \bar{\mu}_t + K_t(z_t - h(\bar{\mu}_t))$
6: $\Sigma_t = (I - K_t\ H_t)\ \bar{\Sigma}_t$
7: return μ_t, Σ_t

Courtesy: Cyrill Stachniss

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Update Covariance

- The function g only affects the robot's motion and not the landmarks

Jacobian of the motion (3x3)

$$G_t = \begin{pmatrix} G_t^x & 0 \\ 0 & I \end{pmatrix}$$

Identity (2N x 2N)

Courtesy: Cyrill Stachniss

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This Leads to the Time Propagation

1: **Extended_Kalman_filter**$(\mu_{t-1}, \Sigma_{t-1}, u_t, z_t)$**:**

2: ~~$\bar{\mu}_t = g(u_t, \mu_{t-1})$~~ **Apply & DONE**
3: $\bar{\Sigma}_t = G_t\ \Sigma_{t-1}\ G_t^T + R_t$

$$\begin{aligned} \bar{\Sigma}_t &= G_t \Sigma_{t-1} G_t^T + R_t \\ &= \begin{pmatrix} G_t^x & 0 \\ 0 & I \end{pmatrix} \begin{pmatrix} \Sigma_{xx} & \Sigma_{xm} \\ \Sigma_{mx} & \Sigma_{mm} \end{pmatrix} \begin{pmatrix} (G_t^x)^T & 0 \\ 0 & I \end{pmatrix} + R_t \\ &= \begin{pmatrix} G_t^x \Sigma_{xx} (G_t^x)^T & G_t^x \Sigma_{xm} \\ (G_t^x \Sigma_{xm})^T & \Sigma_{mm} \end{pmatrix} + R_t \end{aligned}$$

Courtesy: Cyrill Stachniss

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Extended Kalman Filter Algorithm

1: **Extended_Kalman_filter**$(\mu_{t-1}, \Sigma_{t-1}, u_t, z_t)$**:**

2: ~~$\bar{\mu}_t = g(u_t, \mu_{t-1})$~~ **DONE**
3: ~~$\bar{\Sigma}_t = G_t\ \Sigma_{t-1}\ G_t^T + R_t$~~ **DONE**

4: $K_t = \bar{\Sigma}_t\ H_t^T (H_t\ \bar{\Sigma}_t\ H_t^T + Q_t)^{-1}$
5: $\mu_t = \bar{\mu}_t + K_t(z_t - h(\bar{\mu}_t))$
6: $\Sigma_t = (I - K_t\ H_t)\ \bar{\Sigma}_t$
7: return μ_t, Σ_t

Courtesy: Cyrill Stachniss

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EKF SLAM: Correction Step

- Known data association
- $c_t^i = j$: i-th measurement at time t observes the landmark with index j
- Initialize landmark if unobserved
- Compute the expected observation
- Compute the Jacobian of h
- Proceed with computing the Kalman gain

Courtesy: Cyrill Stachniss

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Range-Bearing Observation

- Range-Bearing observation $z_t^i = (r_t^i, \phi_t^i)^T$
- If landmark has not been observed

$$\begin{pmatrix} \bar{\mu}_{j,x} \\ \bar{\mu}_{j,y} \end{pmatrix} = \begin{pmatrix} \bar{\mu}_{t,x} \\ \bar{\mu}_{t,y} \end{pmatrix} + \begin{pmatrix} r_t^i \cos(\phi_t^i + \bar{\mu}_{t,\theta}) \\ r_t^i \sin(\phi_t^i + \bar{\mu}_{t,\theta}) \end{pmatrix}$$

observed location of landmark j — estimated robot's location — relative measurement

Courtesy: Cyrill Stachniss

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Jacobian for the Observation

- Based on

$$\begin{aligned} \delta &= \begin{pmatrix} \delta_x \\ \delta_y \end{pmatrix} = \begin{pmatrix} \bar{\mu}_{j,x} - \bar{\mu}_{t,x} \\ \bar{\mu}_{j,y} - \bar{\mu}_{t,y} \end{pmatrix} \\ q &= \delta^T \delta \\ \hat{z}_t^i &= \begin{pmatrix} \sqrt{q} \\ \operatorname{atan2}(\delta_y, \delta_x) - \bar{\mu}_{t,\theta} \end{pmatrix} \end{aligned}$$

- Compute the Jacobian

$$\begin{aligned} {}^{\text{low}}H_t^i &= \frac{\partial h(\bar{\mu}_t)}{\partial \bar{\mu}_t} \\ &= \frac{1}{q} \begin{pmatrix} -\sqrt{q}\delta_x & -\sqrt{q}\delta_y & 0 & +\sqrt{q}\delta_x & \sqrt{q}\delta_y \\ \delta_y & -\delta_x & -q & -\delta_y & \delta_x \end{pmatrix} \end{aligned}$$

Courtesy: Cyrill Stachniss

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Jacobian for the Observation

- Use the computed Jacobian

$${}^{\text{low}}H_t^i = \frac{1}{q} \begin{pmatrix} -\sqrt{q}\delta_x & -\sqrt{q}\delta_y & 0 & +\sqrt{q}\delta_x & \sqrt{q}\delta_y \\ \delta_y & -\delta_x & -q & -\delta_y & \delta_x \end{pmatrix}$$

- map it to the high dimensional space

$$H_t^i = {}^{\text{low}}H_t^i \, F_{x,j}$$

$$F_{x,j} = \begin{pmatrix} 1 & 0 & 0 & 0\cdots0 & 0 & 0 & 0\cdots0 \\ 0 & 1 & 0 & 0\cdots0 & 0 & 0 & 0\cdots0 \\ 0 & 0 & 1 & 0\cdots0 & 0 & 0 & 0\cdots0 \\ 0 & 0 & 0 & 0\cdots0 & 1 & 0 & 0\cdots0 \\ 0 & 0 & 0 & \underbrace{0\cdots0}_{2j-2} & 0 & 1 & \underbrace{0\cdots0}_{2N-2j} \end{pmatrix}$$

Courtesy: Cyrill Stachniss

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Next Steps as Specified…

1: **Extended_Kalman_filter**$(\mu_{t-1}, \Sigma_{t-1}, u_t, z_t)$:

2: ~~$\bar{\mu}_t = g(u_t, \mu_{t-1})$~~ DONE
3: ~~$\bar{\Sigma}_t = G_t\, \Sigma_{t-1}\, G_t^T + R_t$~~ DONE

4: ➡ $K_t = \bar{\Sigma}_t\, H_t^T (H_t\, \bar{\Sigma}_t\, H_t^T + Q_t)^{-1}$
5: $\mu_t = \bar{\mu}_t + K_t(z_t - h(\bar{\mu}_t))$
6: $\Sigma_t = (I - K_t\, H_t)\, \bar{\Sigma}_t$
7: *return* μ_t, Σ_t

Courtesy: Cyrill Stachniss

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Extended Kalman Filter Algorithm

1: **Extended_Kalman_filter**$(\mu_{t-1}, \Sigma_{t-1}, u_t, z_t)$:

2: ~~$\bar{\mu}_t = g(u_t, \mu_{t-1})$~~ DONE
3: ~~$\bar{\Sigma}_t = G_t\, \Sigma_{t-1}\, G_t^T + R_t$~~ DONE

4: ~~$K_t = \bar{\Sigma}_t\, H_t^T (H_t\, \bar{\Sigma}_t\, H_t^T + Q_t)^{-1}$~~ Apply & DONE
5: ~~$\mu_t = \bar{\mu}_t + K_t(z_t - h(\bar{\mu}_t))$~~ Apply & DONE
6: ~~$\Sigma_t = (I - K_t\, H_t)\, \bar{\Sigma}_t$~~ Apply & DONE
7: ➡ *return* μ_t, Σ_t

Courtesy: Cyrill Stachniss

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EKF SLAM Complexity

- Cubic complexity depends only on the measurement dimensionality
- Cost per step: dominated by the number of landmarks: $O(n^2)$
- Memory consumption: $O(n^2)$
- The EKF becomes computationally intractable for large maps!

Courtesy: Cyrill Stachniss

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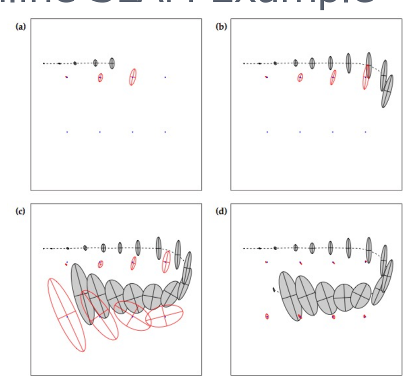
Online SLAM Example



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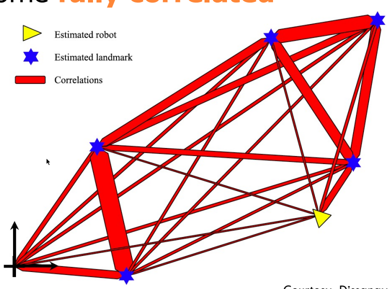
EKF SLAM Correlations

- In the limit, the landmark estimates become **fully correlated**



Courtesy: Dissanayake

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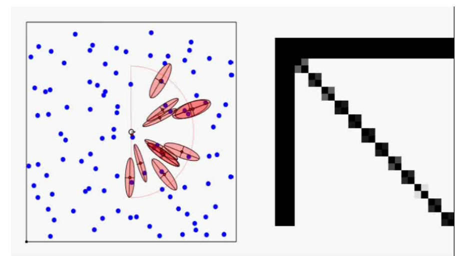
EKF SLAM Correlations



Blue path = true path **Red path** = estimated path **Black path** = odometry

- Approximate the SLAM posterior with a high-dimensional Gaussian [Smith & Cheesman, 1986] …
- Single hypothesis data association

Courtesy: M. Montemerlo

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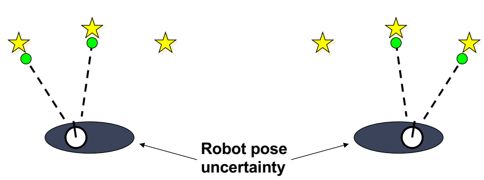
Data Association in SLAM



- In the real world, the mapping between observations and landmarks is unknown
- Picking wrong data associations can have catastrophic consequences
  - EKF SLAM is brittle in this regard
- Pose error correlates data associations

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Loop-Closing

- Loop-closing means recognizing an already mapped area
- Data association under
  - high ambiguity
  - possible environment symmetries
- Uncertainties **collapse** after a loop-closure (whether the closure was correct or not)

Courtesy: Cyrill Stachniss

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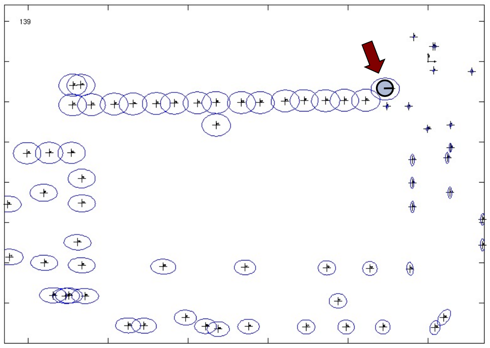
Before the Loop-Closure



Courtesy: K. Arras

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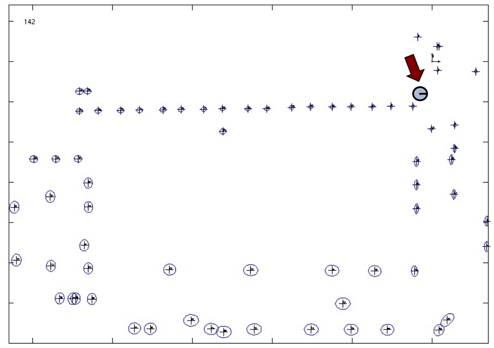
After the Loop-Closure



Courtesy: K. Arras

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Example: Victoria Park Dataset

Courtesy: E. Nebot

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Victoria Park: Data Acquisition

Courtesy: E. Nebot

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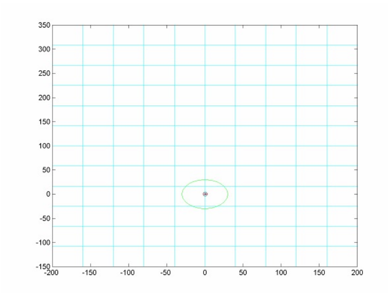
Victoria Park: EKF Estimate



Courtesy: E. Nebot

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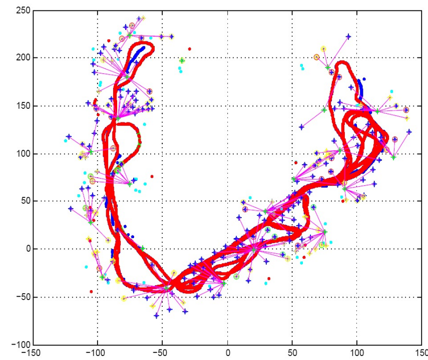
Victoria Park: EKF Estimate



Courtesy: E. Nebot

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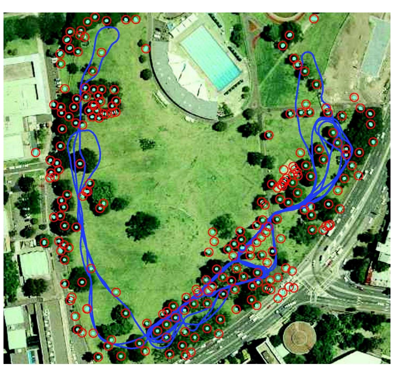
Victoria Park: Landmarks



Courtesy: E. Nebot

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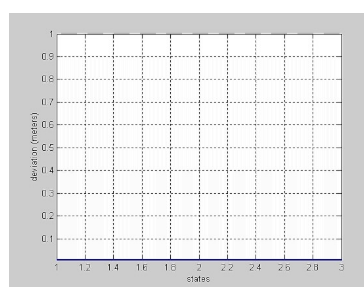
Victoria Park: Landmark Covariance



Courtesy: E. Nebot

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Andrew Davison: MonoSLAM

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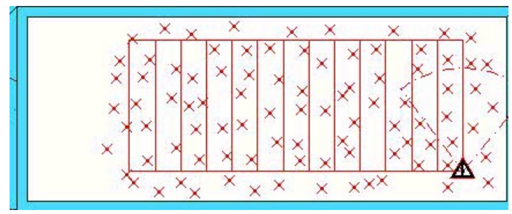
Sub-maps for EKF SLAM



[Leonard et al 1998]

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EKF SLAM Summary

- Quadratic in the number of landmarks: $O(n^2)$
- Convergence results for the linear case.
- Can diverge if nonlinearities are large!
- Have been applied successfully in large-scale environments.
- Approximations reduce the computational complexity.

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Literature

EKF SLAM

- "Probabilistic Robotics", Chapter 10
- Smith, Self, & Cheeseman: "Estimating Uncertain Spatial Relationships in Robotics"
- Dissanayake et al.: "A Solution to the Simultaneous Localization and Map Building (SLAM) Problem"
- Durrant-Whyte & Bailey: "SLAM Part 1" and "SLAM Part 2" tutorials

Courtesy: Cyrill Stachniss

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Graph-SLAM

- Full SLAM technique
- Generates probabilistic links
- Computes map only occasionally
- Based on Information Filter form

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Information Form

- Represent posterior in canonical form

$$\Omega = \Sigma^{-1} \quad \text{Information matrix}$$

$$\xi = \Sigma^{-1}\mu \quad \text{Information vector}$$

- One-to-one transform between canonical and moment representation

$$\Sigma = \Omega^{-1}$$

$$\mu = \Omega^{-1}\xi$$

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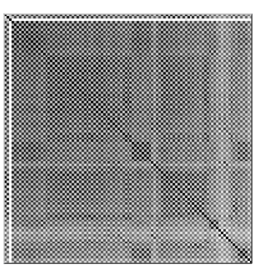
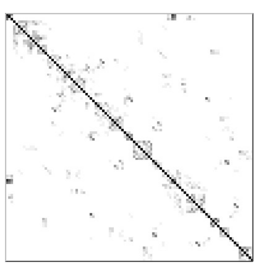
Information vs. Moment Form



Correlation matrix



Information matrix

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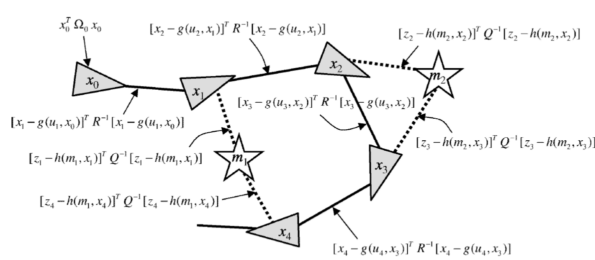
Graph-SLAM Idea



Sum of all constraints:

$$J_{\text{GraphSLAM}} = x_0^T \Omega_0 x_0 + \sum_t [x_t - g(u_t, x_{t-1})]^T R^{-1} [x_t - g(u_t, x_{t-1})] + \sum_t [z_t - h(m_{c_t}, x_t)]^T Q^{-1} [z_t - h(m_{c_t}, x_t)]$$

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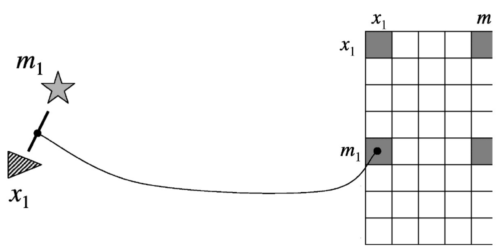
Graph-SLAM Idea (1)



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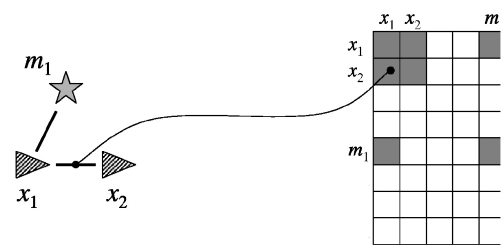
Graph-SLAM Idea (2)



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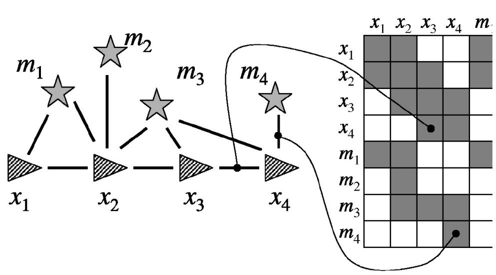
Graph-SLAM Idea (3)



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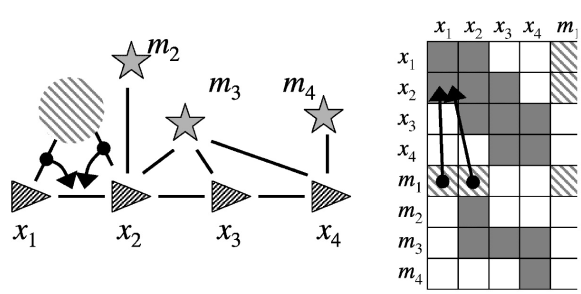
Graph-SLAM Inference (1)



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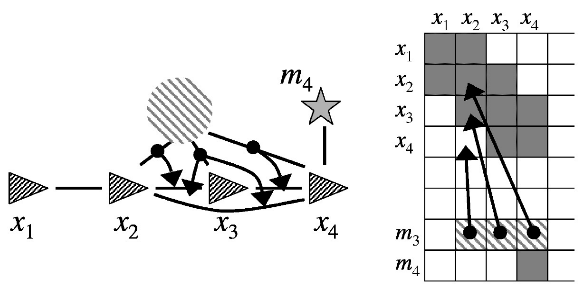
Graph-SLAM Inference (2)



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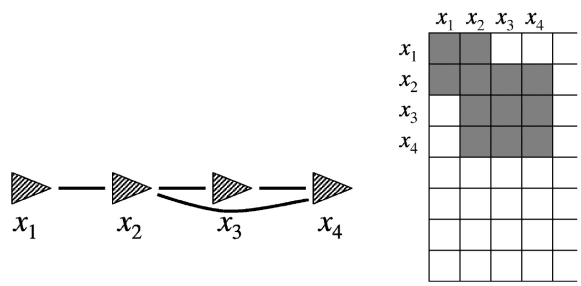
Graph-SLAM Inference (3)



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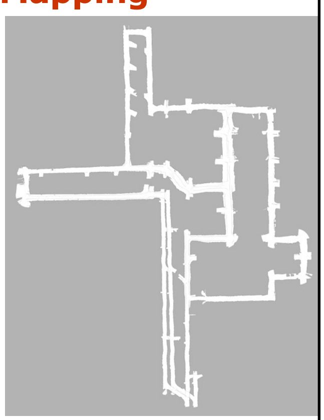
Mine Mapping



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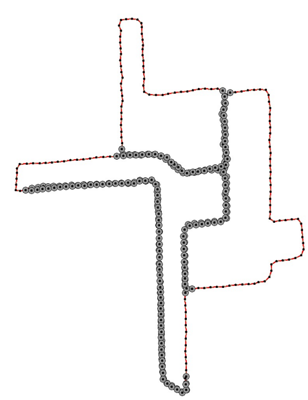
Mine Mapping: Data Associations



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Efficient Map Recovery

- Information matrix inversion can be avoided if only best map estimate is required
- Minimize constraint function $J_{GraphSLAM}$ using standard optimization techniques (gradient descent, Levenberg Marquardt, conjugate gradient)

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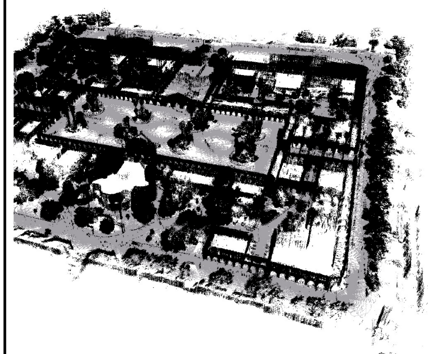
3D Outdoor Mapping



10^8 features, 10^5 poses, only few secs using cg.

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Map Before Optimization

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Map After Optimization

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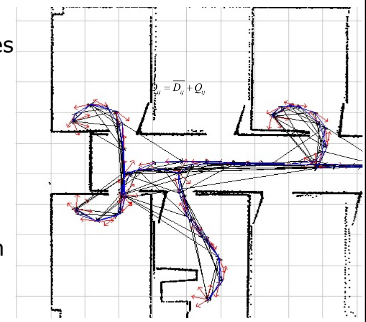
Robot Poses and Scans [Lu and Milios 1997]

- Successive robot poses connected by odometry
- Laser scan matching yields constraints between poses
- Loop closure based on map patches created from multiple scans



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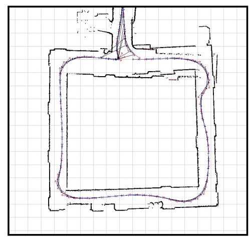
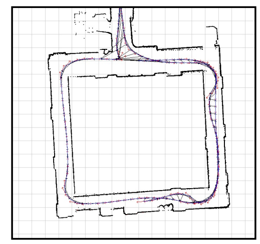
Loop Closure

- Use scan patches to detect loop closure
- Add new position constraints
- Deform the network based on covariances of matches



Before loop closure



After loop closure

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Mapping the Allen Center

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Graph-SLAM Summary

- Adresses full SLAM problem
- Constructs link graph between poses and poses/landmarks
- Graph is sparse: number of edges linear in number of nodes
- Inference performed by building information matrix and vector (linearized form)
- Map recovered by reduction to robot poses, followed by conversion to moment representation, followed by estimation of landmark positions
- ML estimate by minimization of $J_{GraphSLAM}$
- Data association by iterative greedy search

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