

CSE-571 Robotics

Probabilistic Motion Models

1

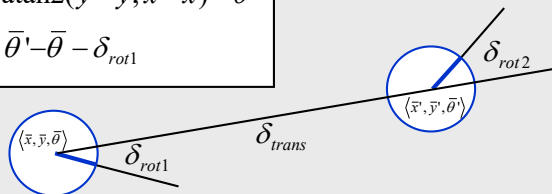
$$Bel(x_t) = \eta P(z_t | x_t) \int P(x_t | u_t, x_{t-1}) Bel(x_{t-1}) dx_{t-1}$$

2

Probabilistic Kinematics

- Robot moves from $\langle \bar{x}, \bar{y}, \bar{\theta} \rangle$ to $\langle \bar{x}', \bar{y}', \bar{\theta}' \rangle$.
- Odometry information $u = \langle \delta_{rot1}, \delta_{rot2}, \delta_{trans} \rangle$.

$$\begin{aligned} \delta_{trans} &= \sqrt{(\bar{x}' - \bar{x})^2 + (\bar{y}' - \bar{y})^2} \\ \delta_{rot1} &= \text{atan2}(\bar{y}' - \bar{y}, \bar{x}' - \bar{x}) - \bar{\theta} \\ \delta_{rot2} &= \bar{\theta}' - \bar{\theta} - \delta_{rot1} \end{aligned}$$



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3

Noise Model for Motion

- The measured motion is given by the true motion corrupted with noise.

$$\begin{aligned} \hat{\delta}_{rot1} &= \delta_{rot1} + \epsilon_{\alpha_1 |\delta_{rot1}| + \alpha_2 |\delta_{trans}|} \\ \hat{\delta}_{trans} &= \delta_{trans} + \epsilon_{\alpha_3 |\delta_{trans}| + \alpha_4 |\delta_{rot1} + \delta_{rot2}|} \\ \hat{\delta}_{rot2} &= \delta_{rot2} + \epsilon_{\alpha_1 |\delta_{rot2}| + \alpha_2 |\delta_{trans}|} \end{aligned}$$

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4

Sample Odometry Motion Model

1. Algorithm **sample_motion_model**(u, x):

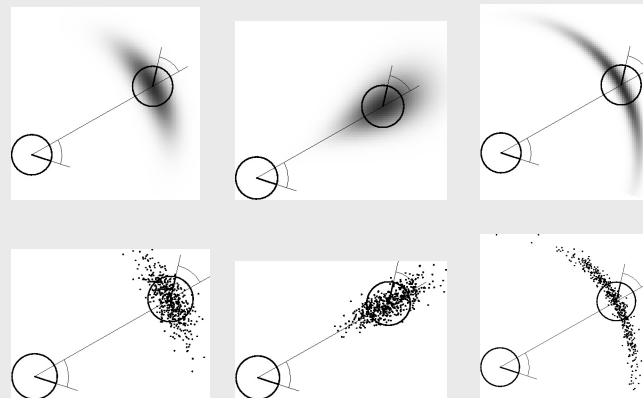
$$u = \langle \delta_{rot1}, \delta_{rot2}, \delta_{trans} \rangle, x = \langle x, y, \theta \rangle$$

1. $\hat{\delta}_{rot1} = \delta_{rot1} + \text{sample}(\alpha_1 | \delta_{rot1}| + \alpha_2 \delta_{trans})$
2. $\hat{\delta}_{trans} = \delta_{trans} + \text{sample}(\alpha_3 \delta_{trans} + \alpha_4 (|\delta_{rot1}| + |\delta_{rot2}|))$
3. $\hat{\delta}_{rot2} = \delta_{rot2} + \text{sample}(\alpha_1 | \delta_{rot2}| + \alpha_2 \delta_{trans})$
4. $x' = x + \hat{\delta}_{trans} \cos(\theta + \hat{\delta}_{rot1})$
5. $y' = y + \hat{\delta}_{trans} \sin(\theta + \hat{\delta}_{rot1})$
6. $\theta' = \theta + \hat{\delta}_{rot1} + \hat{\delta}_{rot2}$
7. Return $\langle x', y', \theta' \rangle$

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5

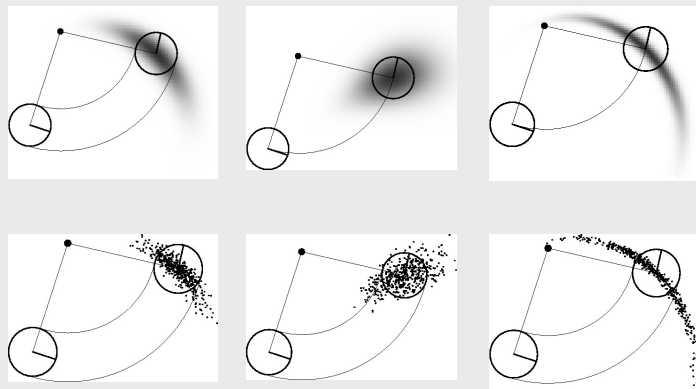
Examples (odometry based)



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6

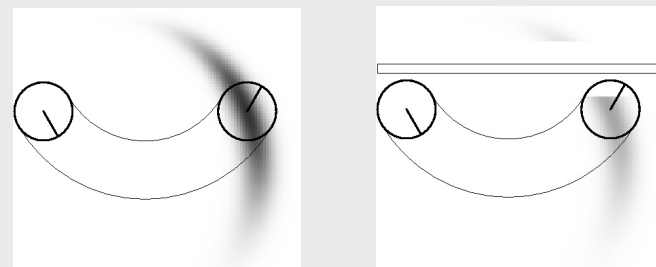
Examples (velocity based)



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7

Motion Model with Map



$$P(x|u, x')$$

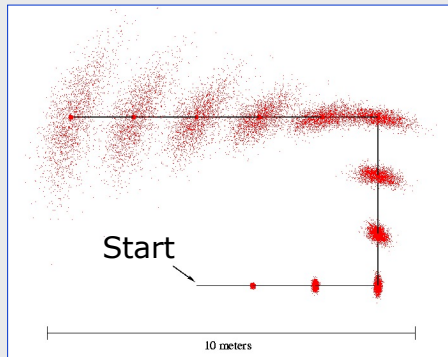
$$P(x|u, x', m) \approx P(x|m) P(x|u, x')$$

- When does this approximation fail?

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8

Sample-based Motion



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