CSE-571
Robotics

Gaussian Distributions

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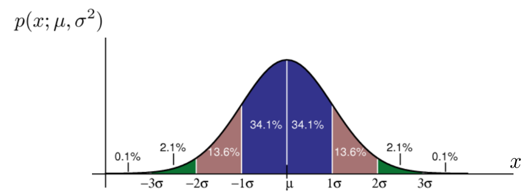
Gaussians (1D)

- Gaussian with mean (μ) and standard deviation (σ)

$$X \sim \mathcal{N}(\mu, \sigma^2)$$

$$p(x; \mu, \sigma^2) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$$



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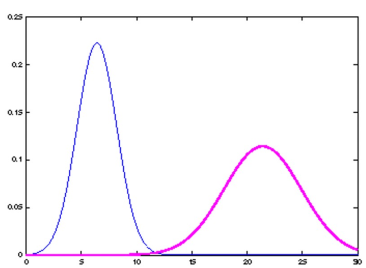
Properties of Gaussians

$$\left.\begin{matrix} X \sim N(\mu, \sigma^2) \\ Y = aX + b \end{matrix}\right\} \Rightarrow \quad Y \sim N(a\mu + b, a^2\sigma^2)$$



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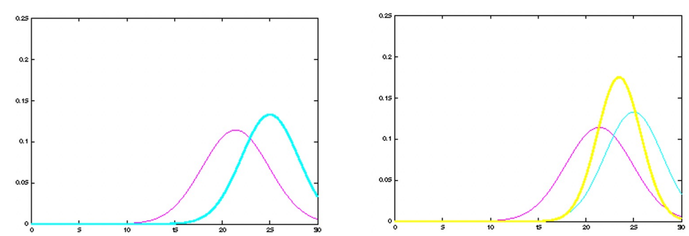
Properties of Gaussians

$$\left.\begin{matrix} X_1 \sim N(\mu_1, \sigma_1^2) \\ X_2 \sim N(\mu_2, \sigma_2^2) \end{matrix}\right\} \Rightarrow p(X_1) \cdot p(X_2) \sim N\left(\frac{\sigma_2^2}{\sigma_1^2 + \sigma_2^2}\mu_1 + \frac{\sigma_1^2}{\sigma_1^2 + \sigma_2^2}\mu_2, \quad \frac{1}{\sigma_1^{-2} + \sigma_2^{-2}}\right)$$



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Picture from [Bishop: Pattern Recognition and Machine Learning, 2006]

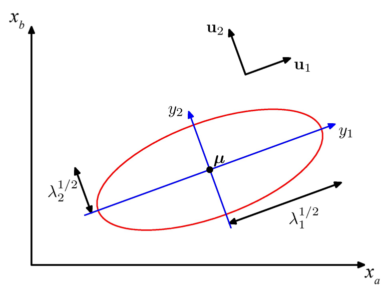
Gaussians (2D)

$$p(\mathbf{x}) = \mathrm{N}(\mu, \Sigma)$$

$$\mathbf{x} = \begin{pmatrix} x_a \\ x_b \end{pmatrix}, \quad \mu = \begin{pmatrix} \mu_a \\ \mu_b \end{pmatrix}$$

$$\Sigma = \begin{pmatrix} \Sigma_{aa} & \Sigma_{ab} \\ \Sigma_{ba} & \Sigma_{bb} \end{pmatrix}$$

$$p(\mathbf{x}) = \frac{1}{(2\pi)^{d/2}|\Sigma|^{1/2}} e^{-\frac{1}{2}(\mathbf{x}-\mu)^T \Sigma^{-1}(\mathbf{x}-\mu)}$$



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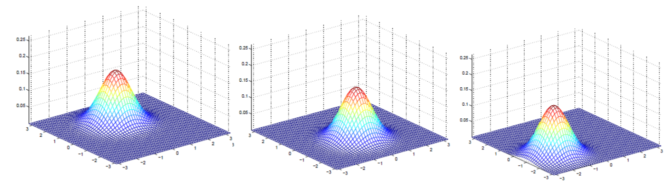
2D examples



- μ = [1; 0]
- Σ = [1 0; 0 1]
- μ = [-.5; 0]
- Σ = [1 0; 0 1]
- μ = [-1; -1.5]
- Σ = [1 0; 0 1]

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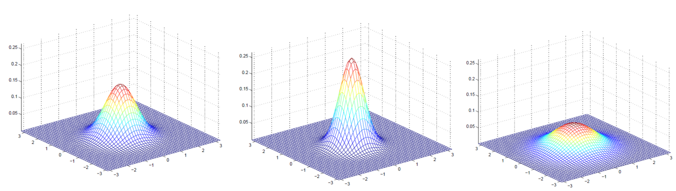
2D examples



- μ = [0; 0]
- Σ = [1 0 ; 0 1]
- μ = [0; 0]
- Σ = [.6 0 ; 0 .6]
- μ = [0; 0]
- Σ = [2 0 ; 0 2]

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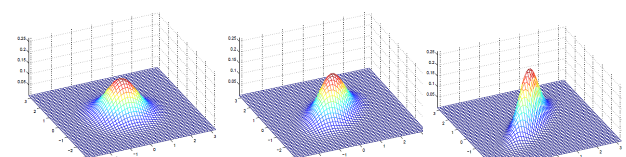
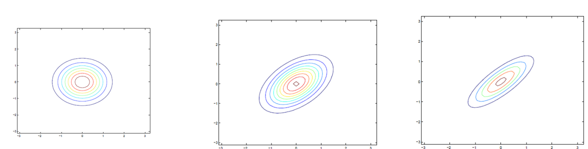
2D examples



- μ = [0; 0]
- Σ = [1 0; 0 1]
- μ = [0; 0]
- Σ = [1 0.5; 0.5 1]
- μ = [0; 0]
- Σ = [1 0.8; 0.8 1]



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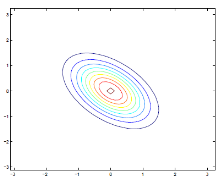
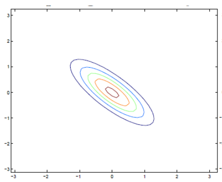
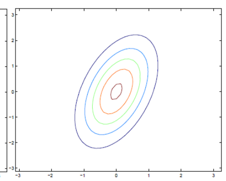
2D examples



- μ = [0; 0]
- Σ = [1 -0.5 ; -0.5 1]



- μ = [0; 0]
- Σ = [1 -0.8 ; -0.8 1]



- μ = [0; 0]
- Σ = [1 0.8 ; 0.8 3]

Pictures from [Bishop: PRML, 2006]

Marginalization / Conditioning

- Marginalizing joint distribution results in a Gaussian

$$p\left(\begin{bmatrix} x_a \\ x_b \end{bmatrix}\right) = N\left(\begin{bmatrix} \mu_a \\ \mu_b \end{bmatrix}, \begin{bmatrix} \Sigma_{aa} & \Sigma_{ab} \\ \Sigma_{ba} & \Sigma_{bb} \end{bmatrix}\right)$$

$$p(x_a) = \int p(x_a, x_b) dx_b$$

$$p(x_a) = N(\mu_a, \Sigma_{aa})$$

- Conditioning also leads to a Gaussian

$$p(x_a \mid x_b) = N(\mu_{a|b}, \Sigma_{a|b})$$

$$\mu_{a|b} = \mu_a + \Sigma_{ab} \Sigma_{bb}^{-1} (x_b - \mu_b)$$

Cross co-variance, Prior Variance (b), Observed value, Prior mean (b)

$$\Sigma_{a|b} = \Sigma_{aa} - \Sigma_{ab} \Sigma_{bb}^{-1} \Sigma_{ba}$$

Prior Variance (a), Shrink term (>= 0)