CSE 571 - Robotics
Homework 2 - EKF and Particle Filter for Localization

Due Tuesday May 11th @ 11:59pm

The key goal of this homework is to get an understanding of the properties of Kalman filters and Particle filters for state estimation. The homework consists of four questions for which you'll provide a written answer, as well as one extra credit question.

Useful reading material: Lecture notes, Chapters 3,4,5,7 & 8 of Probabilistic Robotics, Thrun, Burgard and Fox (pdf shared with class).

Collaboration: Students can discuss questions, but each student MUST write up their own solution, and code their own solution. We will be checking code/PDFs for plagiarism.

1 Kalman Gain [10 points]

Recall that if we have two Gaussian probability density functions:

$$f(x) = \mathcal{N}(x; \mu_1, \sigma_1^2)$$
$$g(x) = \mathcal{N}(x; \mu_2, \sigma_2^2)$$

then, their product is still a Gaussian (up to a scaling factor):

$$f(x)g(x) \; \propto \; \mathcal{N}\left(x; \frac{\sigma_2^2}{\sigma_1^2+\sigma_2^2}\mu_1 + \frac{\sigma_1^2}{\sigma_1^2+\sigma_2^2}\mu_2, \frac{1}{\sigma_1^{-2}+\sigma_2^{-2}}\right)$$

Show that in 1D, the Kalman filter correction step shown below is equivalent (up to a scaling factor) to a multiplication of the predicted state ($\mathcal{N}(x; \bar{\mu}, \bar{\sigma}^2)$) and observation ($\mathcal{N}(z; x, \sigma_{obs}^2)$) Gaussians with mean and variance given by:

$$\mu = \bar{\mu} + K(z - \bar{\mu})$$
$$\sigma^2 = (1 - K)\bar{\sigma}^2$$

where $K = \frac{\bar{\sigma}^2}{\bar{\sigma}^2 + \sigma_{obs}^2}$.

2 Simple Kalman Filter [10 points]

Imagine that we want to estimate the translational velocity, x_t, of a car by using the gas pedal, which provides information regarding the acceleration, u_t, plus some noise, η_t. The propagation (motion) model is $x_t = x_{t-1} + \Delta t \cdot (u_t + \eta_t)$, where $\eta_t \sim \mathcal{N}(0, M)$ where $M = 4$. The observation (sensor) model is $z_t = x_t + \delta_t$, where $\delta_t \sim \mathcal{N}(0, Q)$, and $Q = 10$. Given $\Delta t = 0.5$, $x_0 = 0$, $\Sigma_0 = 10$, calculate

1. $\bar{bel}(x_1)$ (belief after propagation), given that $u_1 = 10$
2. $bel(x_1)$ (belief after correction), given the observation $z_1 = 8$

3 Motion Model [10 points]

Given the state transition function $\boldsymbol{x}_t = g(\boldsymbol{x}_{t-1}, u_t, \epsilon_t) = \begin{bmatrix} e^{(x_{t-1}+u_t)^2} + \epsilon_t \\ e^{(y_{t-1}+u_t)^2} + \epsilon_t \end{bmatrix}$, where $\boldsymbol{x}_t = \begin{bmatrix} x_t \\ y_t \end{bmatrix} \in \mathbb{R}^2$, $u_t \in \mathbb{R}$, and $\epsilon \sim \mathcal{N}(0, M)$.

1. Calculate the Jacobians analytically, one in terms of state and one in terms of control.
2. Give a probabilistic model of the state after a single iteration, given the prior $x \sim \mathcal{N}(\mu, \Sigma)$. Numerically evaluate this model given $x \sim \mathcal{N}\left(\begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}\right)$, $u_t = 1$, and $M = 0.1$.

4 Particle Filter [10 points]

Explain particle deprivation. Explain one possible solution to this problem.

5 Extra Credit [10 points]

1. Localization is an essential part of both Augmented Reality (AR) and Virtual Reality (VR). Describe some potential applications of AR/VR that would be socially beneficial. What are some potential misuses (i.e. those that will have what you perceive as negative consequences)? Do you see any potential dangers to the widespread adoption of these technologies?
2. We often think about localization within a known map. In the case of a self-driving car, this could be a map of your neighborhood, city, or country. These maps capture high fidelity information on the environments they represent. To an interested party (for example, a company or government), localization within these maps can provide implicit information on your activities, behaviors, and preferences. How would you go about creating safeguards to ensure private data stay private? Or if engineered safeguards are unnecessary, why?

6 Submission

You will be using Gradescope `https://www.gradescope.com/` to submit the homework. Please write your solutions in LaTeX and submit the assignment as a PDF.