CSE-571
Sampling-Based Motion Planning

Slides based on those from Pieter Abbeel, Zoe McCarthy
Many images from Lavalle, Planning Algorithms

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Solve by Nonlinear Optimization for Control?

- Could try by, for example, following formulation:

$$\begin{aligned} \min_{u,x} \quad & (x_T - x_G)^\top (x_T - x_G) \\ \text{s.t.} \quad & x_{t+1} = f(x_t, u_t) \quad \forall t \\ & u_t \in \mathcal{U}_t \\ & x_t \in \mathcal{X}_t \\ & x_0 = x_S \end{aligned}$$

X_t can encode obstacles

- Or, with constraints, (which would require using an infeasible method):

$$\begin{aligned} \min_{u,x} \quad & \|u\| \\ \text{s.t.} \quad & x_{t+1} = f(x_t, u_t) \quad \forall t \\ & u_t \in \mathcal{U}_t \\ & x_t \in \mathcal{X}_t \\ & x_0 = x_S \\ & X_T = x_G \end{aligned}$$

- *Can work surprisingly well, but for more complicated problems can get stuck in infeasible local minima*

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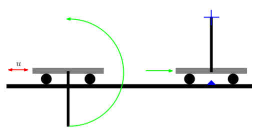
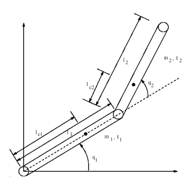
Examples


Helicopter path planning



Cartpole swing-up


Acrobot

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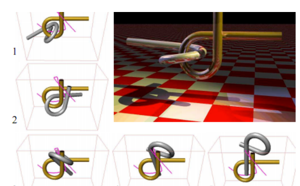
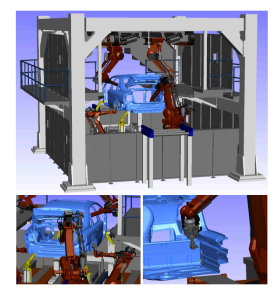
Examples





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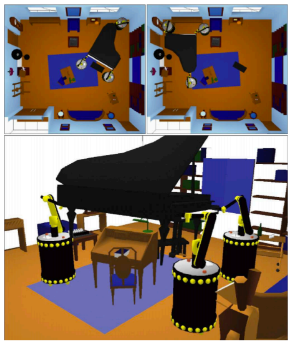
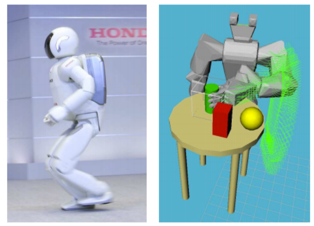
Examples





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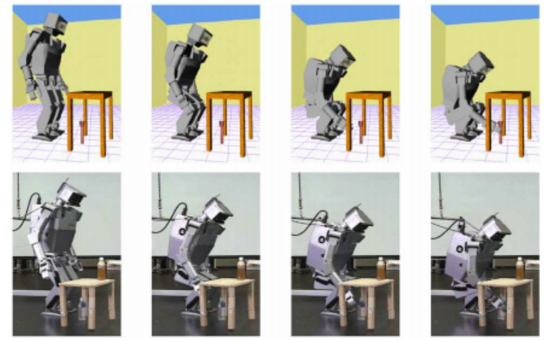
Examples



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Motion Planning: Outline

- Configuration Space
- Probabilistic Roadmap
- Rapidly-exploring Random Trees (RRTs)
- Extensions
- Smoothing

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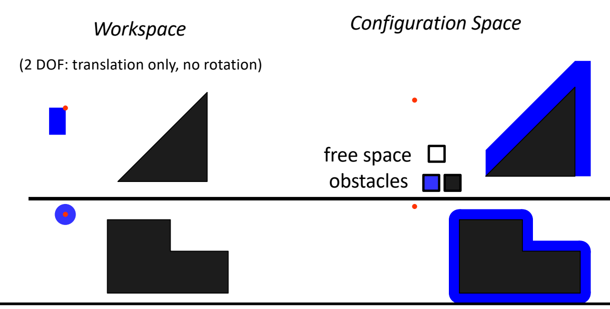
Configuration Space (C-Space)

= { x | x is a pose of the robot}

- obstacles → configuration space obstacles



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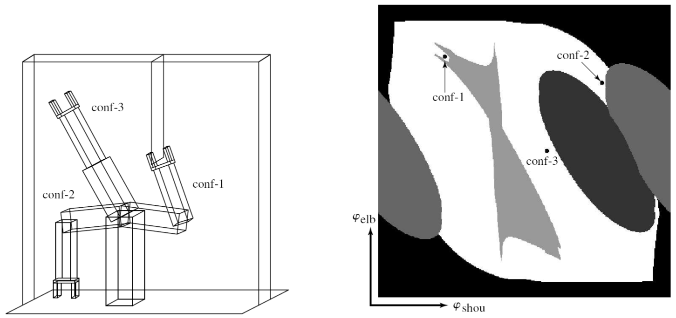
Motion planning



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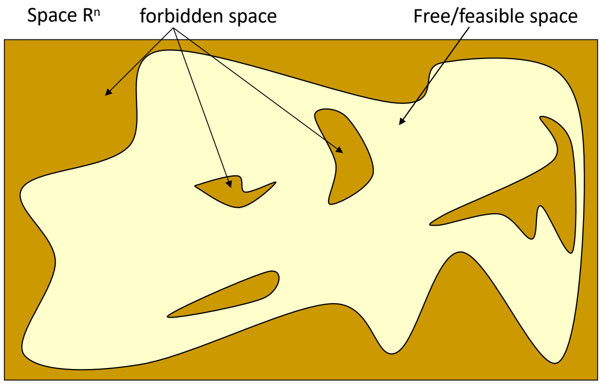
Probabilistic Roadmap (PRM)



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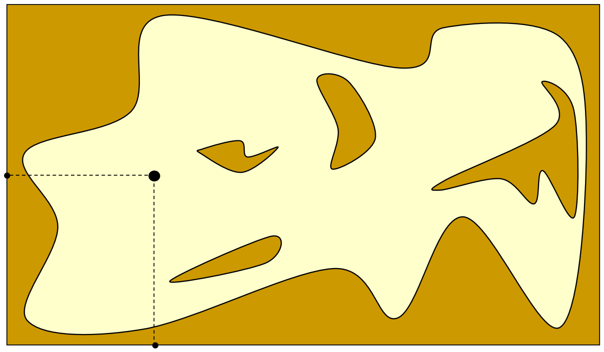
Probabilistic Roadmap (PRM)

Configurations are sampled by picking coordinates at random



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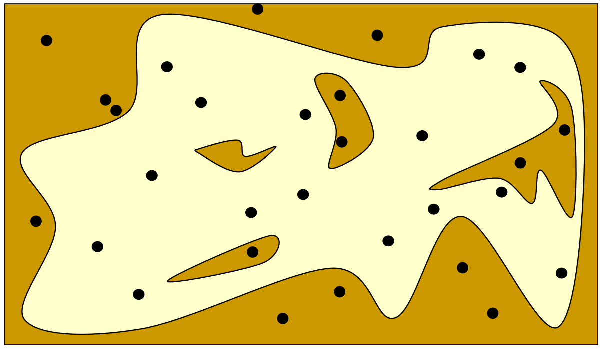
Probabilistic Roadmap (PRM)

Configurations are sampled by picking coordinates at random



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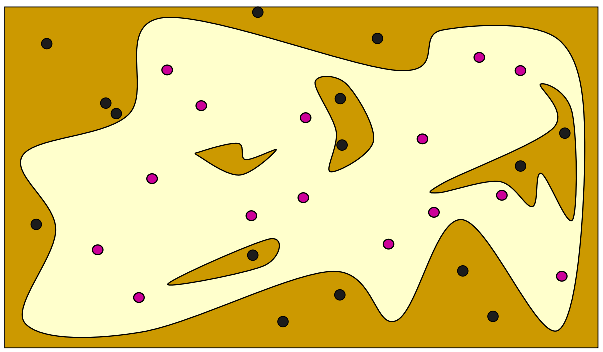
Probabilistic Roadmap (PRM)

Sampled configurations are tested for collision



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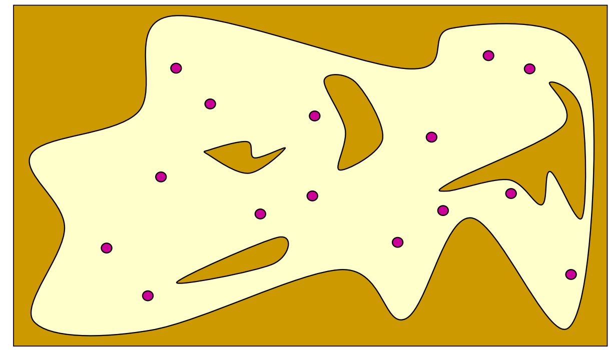
Probabilistic Roadmap (PRM)

The collision-free configurations are retained as milestones



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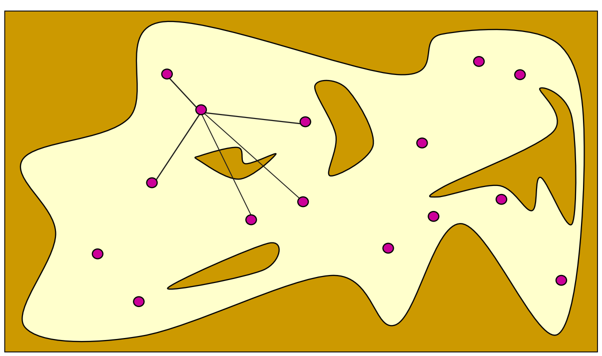
Probabilistic Roadmap (PRM)

Each milestone is linked by straight paths to its nearest neighbors



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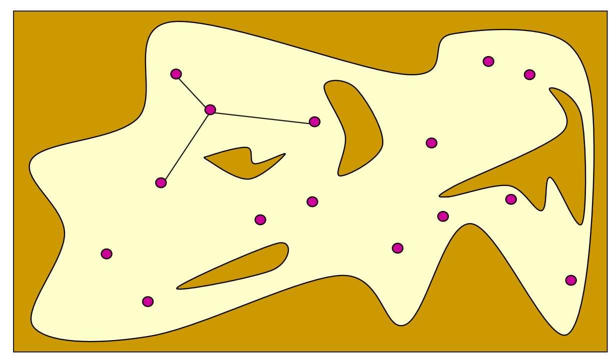
Probabilistic Roadmap (PRM)

Each milestone is linked by straight paths to its nearest neighbors



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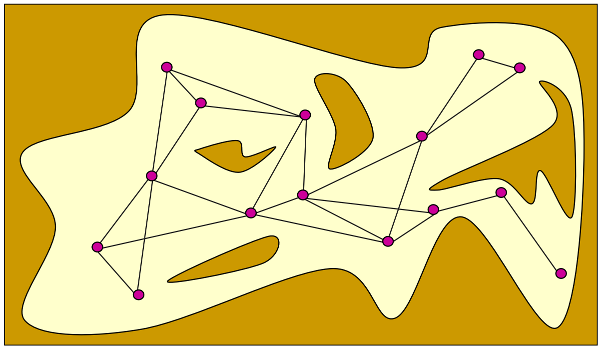
Probabilistic Roadmap (PRM)

The collision-free links are retained as local paths to form the PRM



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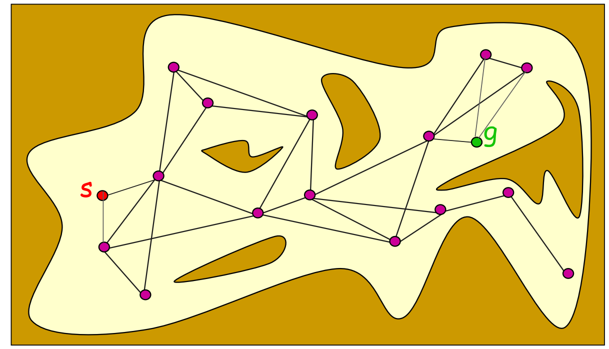
Probabilistic Roadmap (PRM)

The start and goal configurations are included as milestones



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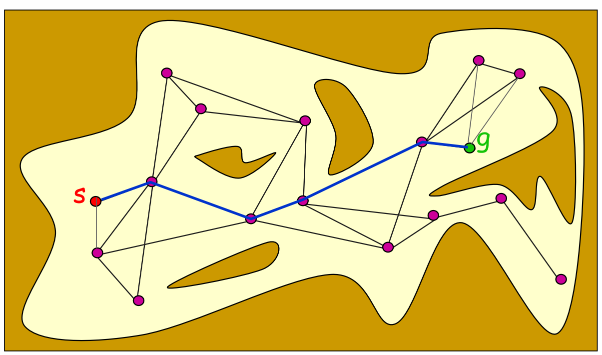
Probabilistic Roadmap (PRM)

The PRM is searched for a path from s to g



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Probabilistic Roadmap

- Initialize set of points with x_S and x_G
- Randomly sample points in configuration space
- Connect nearby points if they can be reached from each other
- Find path from x_S to x_G in the graph
  - Alternatively: keep track of connected components incrementally, and declare success when x_S and x_G are in same connected component

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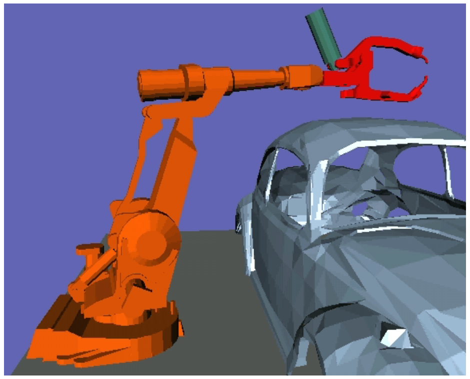
PRM Example 1



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PRM Example 2

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PRM's Pros and Cons

- Pro:
  - Probabilistically complete: i.e., with probability one, if run for long enough the graph will contain a solution path if one exists.
- Cons:
  - Required to solve 2-point boundary value problem
  - Build graph over state space but no focus on generating a path

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Rapidly exploring Random Tree (RRT)

Steve LaValle (98)

- Basic idea:
  - Build up a tree through generating "next states" in the tree by executing random controls
  - However: not exactly above to ensure good coverage

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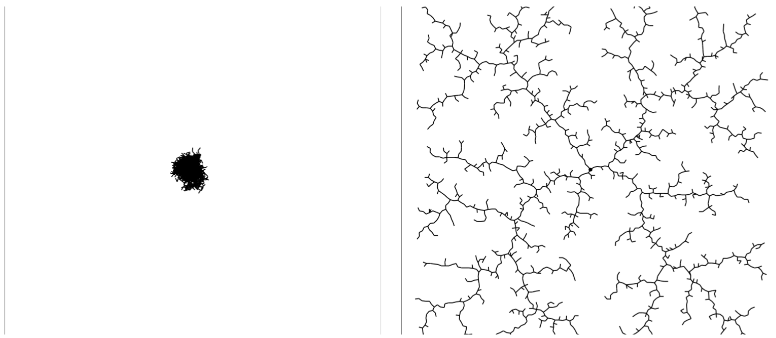
How to Sample



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Rapidly exploring Random Tree (RRT)

- Select random point, and expand nearest vertex towards it
  - Biases samples towards largest Voronoi region

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Rapidly exploring Random Tree (RRT)

GENERATE_RRT($x_{init}, K, \Delta t$)
$\mathcal{T}$.init(x_{init});
for $k = 1$ **to** K **do**
    $x_{rand} \leftarrow$ RANDOM_STATE();
    $x_{near} \leftarrow$ NEAREST_NEIGHBOR($x_{rand}, \mathcal{T}$);
    $u \leftarrow$ SELECT_INPUT(x_{rand}, x_{near});
    $x_{new} \leftarrow$ NEW_STATE($x_{near}, u, \Delta t$);
    $\mathcal{T}$.add_vertex(x_{new});
    $\mathcal{T}$.add_edge(x_{near}, x_{new}, u);
Return $\mathcal{T}$

RANDOM_STATE(): often uniformly at random over space with probability 99%, and the goal state with probability 1%, this ensures it attempts to connect to goal semi-regularly

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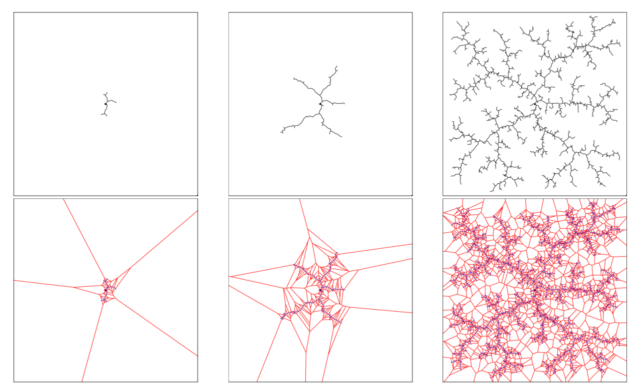
Rapidly exploring Random Tree (RRT)



Source: LaValle and Kuffner 01

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RRT Practicalities

- NEAREST_NEIGHBOR(x_{rand}, T): need to find (approximate) nearest neighbor efficiently
  - KD Trees data structure (upto 20-D) [e.g., FLANN]
  - Locality Sensitive Hashing
- SELECT_INPUT(x_{rand}, x_{near})
  - Two point boundary value problem
    - If too hard to solve, often just select best out of a set of control sequences. This set could be random, or some well chosen set of primitives.

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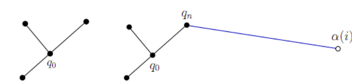
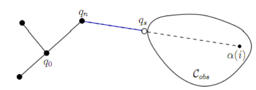
RRT Extension

- No obstacles, holonomic:



- With obstacles, holonomic:



- Non-holonomic: approximately (sometimes as approximate as picking best of a few random control sequences) solve two-point boundary value problem

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Growing RRT

Demo: http://en.wikipedia.org/wiki/File:Rapidly-exploring_Random_Tree_(RRT)_500x373.gif

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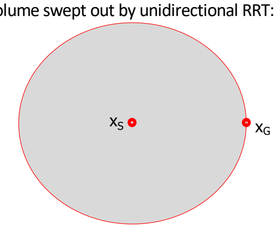
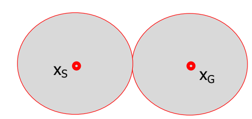
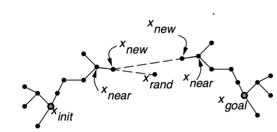
Bi-directional RRT

- Volume swept out by unidirectional RRT:



- Volume swept out by bi-directional RRT:





- Difference more and more pronounced as dimensionality increases

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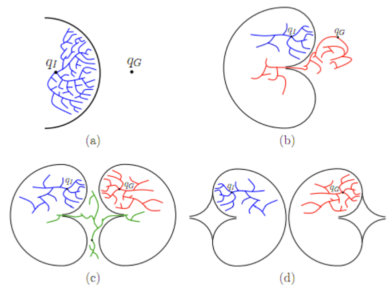
Multi-directional RRT

- Planning around obstacles or through narrow passages can often be easier in one direction than the other



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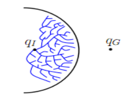
Resolution-Complete RRT (RC-RRT)

- Issue: nearest points chosen for expansion are (too) often the ones stuck behind an obstacle



RC-RRT solution:

- Choose a maximum number of times, m, you are willing to try to expand each node
- For each node in the tree, keep track of its Constraint Violation Frequency (CVF)
- Initialize CVF to zero when node is added to tree
- Whenever an expansion from the node is unsuccessful (e.g., per hitting an obstacle):
  - Increase CVF of that node by 1
  - Increase CVF of its parent node by 1/m, its grandparent $1/m^2$, …
- When a node is selected for expansion, skip over it with probability CVF/m

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RRT*

Algorithm 6: RRT*

```
V ← {x_init}; E ← ∅;
for i = 1, ..., n do
    x_rand ← SampleFree_i;
    x_nearest ← Nearest(G = (V, E), x_rand);
    x_new ← Steer(x_nearest, x_rand) ;
    if ObtacleFree(x_nearest, x_new) then
        X_near ← Near(G = (V, E), x_new, min{γ_RRT*(log(card (V))/ card (V))^(1/d), η}) ;
        V ← V ∪ {x_new};
        x_min ← x_nearest; c_min ← Cost(x_nearest) + c(Line(x_nearest, x_new));
        foreach x_near ∈ X_near do                    // Connect along a minimum-cost path
            if CollisionFree(x_near, x_new) ∧ Cost(x_near) + c(Line(x_near, x_new)) < c_min then
                x_min ← x_near; c_min ← Cost(x_near) + c(Line(x_near, x_new))
        E ← E ∪ {(x_min, x_new)};
        foreach x_near ∈ X_near do                                    // Rewire the tree
            if CollisionFree(x_new, x_near) ∧ Cost(x_new) + c(Line(x_new, x_near)) < Cost(x_near)
            then x_parent ← Parent(x_near);
            E ← (E \ {(x_parent, x_near)}) ∪ {(x_new, x_near)}
return G = (V, E);
```

Source: Karaman and Frazzoli

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RRT*

- Asymptotically optimal
- Main idea:
  - Swap new point in as parent for nearby vertices who can be reached along shorter path through new point than through their original (current) parent

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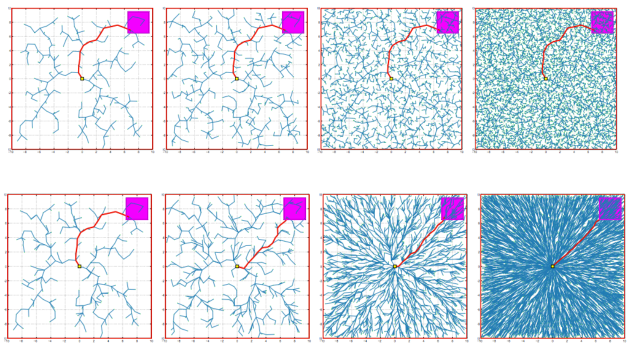
RRT*

RRT

RRT*



Source: Karaman and Frazzoli

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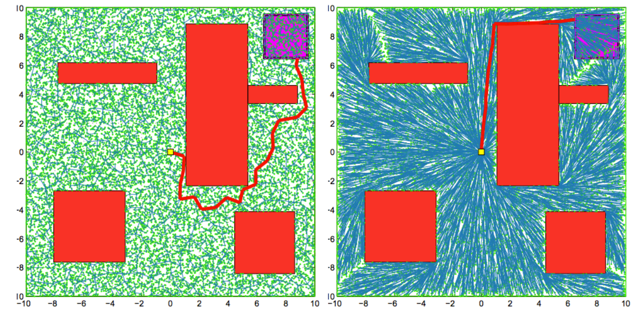
RRT*

RRT RRT*



Source: Karaman and Frazzoli

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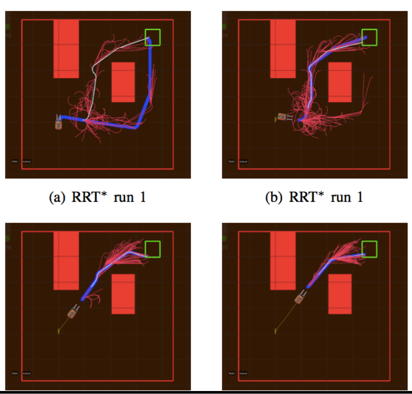
Real Time RRT*



(a) RRT* run 1 (b) RRT* run 1

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Smoothing

Randomized motion planners tend to find not so great paths for execution: very jagged, often much longer than necessary.

→ In practice: do smoothing before using the path

- Shortcutting:
  - along the found path, pick two vertices x_{t1}, x_{t2} and try to connect them directly (skipping over all intermediate vertices)
- Nonlinear optimization for optimal control
  - Allows to specify an objective function that includes smoothness in state, control, small control inputs, etc.

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Additional Resources

- Marco Pavone (http://asl.stanford.edu/):
  - Sampling-based motion planning on GPUs: https://arxiv.org/pdf/1705.02403.pdf
  - Learning sampling distributions: https://arxiv.org/pdf/1709.05448.pdf
- Sidd Srinivasa (https://personalrobotics.cs.washington.edu/)
  - Batch informed trees: https://robotic-esp.com/code/bitstar/
  - Expensive edge evals: https://arxiv.org/pdf/2002.11853.pdf
- Michael Yip (https://www.ucsdarclab.com/)
  - Neural Motion Planners: https://www.ucsdarclab.com/neuralplanning
- Lydia Kavraki (http://www.kavrakilab.org/)
  - Motion in human workspaces: http://www.kavrakilab.org/nsf-nri-1317849.html

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