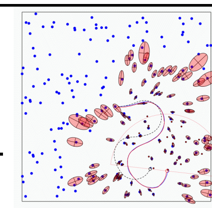
CSE-571 Robotics

SLAM: Simultaneous Localization and Mapping

Many slides courtesy of Ryan Eustice, Cyrill Stachniss, John Leonard

1

The SLAM Problem



A robot is exploring an unknown, static environment.

Given:

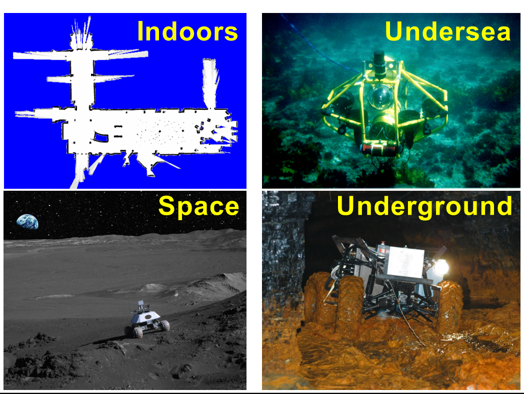
- The robot's controls
- Observations of nearby features

Estimate:

- Map of features
- Path of the robot

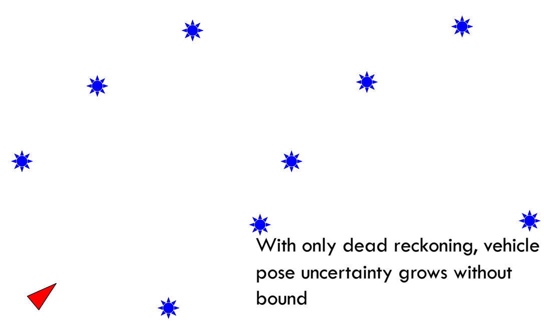
2

SLAM Applications



3

Illustration of SLAM without Landmarks



With only dead reckoning, vehicle pose uncertainty grows without bound

Courtesy J. Leonard

4

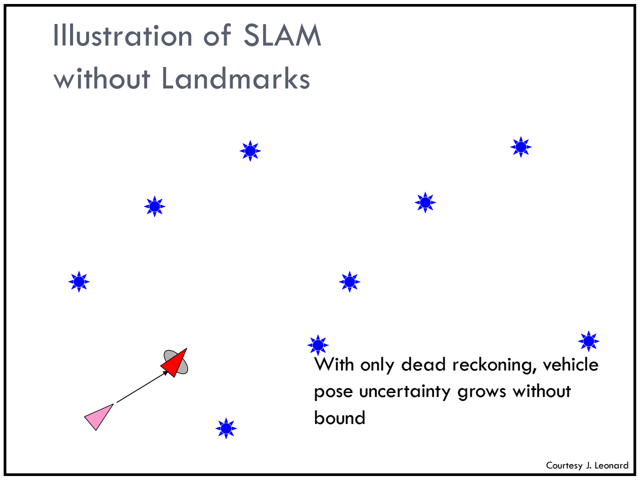
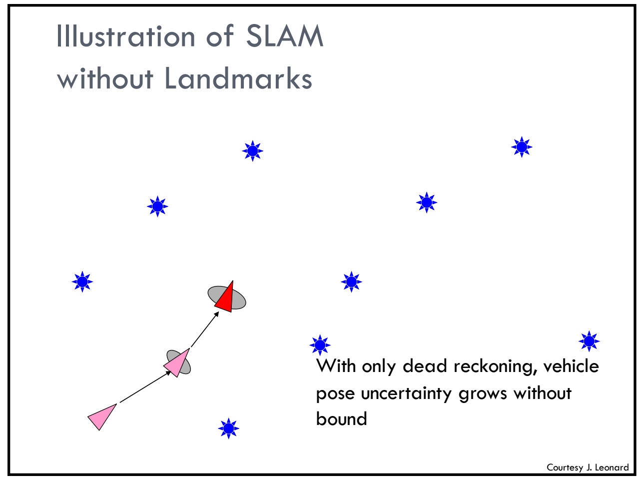


Illustration of SLAM without Landmarks
With only dead reckoning, vehicle pose uncertainty grows without bound
Courtesy J. Leonard

5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Courtesy J. Leonard

6

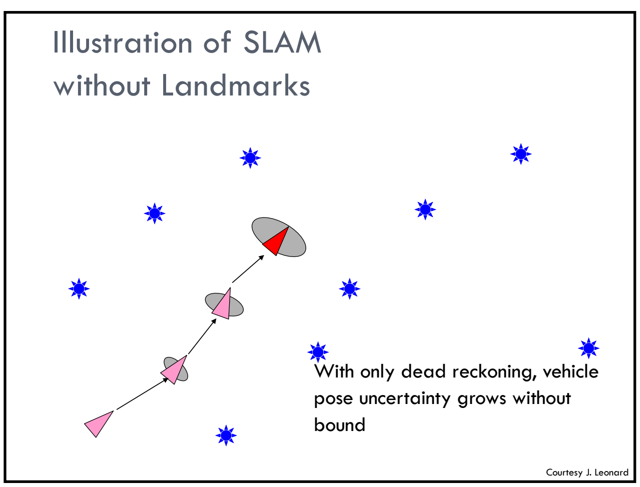


Illustration of SLAM without Landmarks
With only dead reckoning, vehicle pose uncertainty grows without bound
Courtesy J. Leonard

7

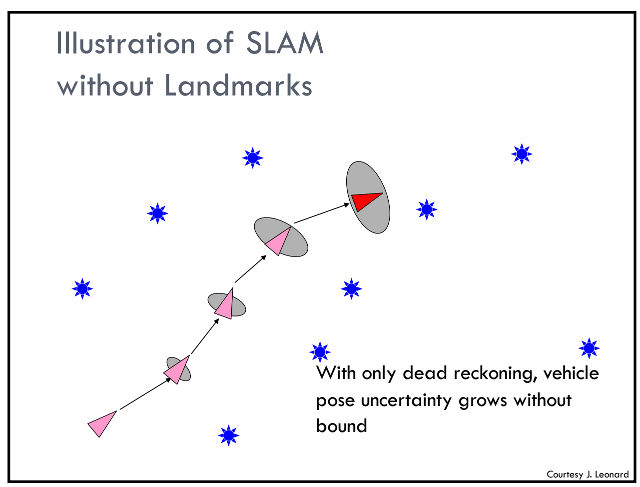


Illustration of SLAM without Landmarks
With only dead reckoning, vehicle pose uncertainty grows without bound
Courtesy J. Leonard

8

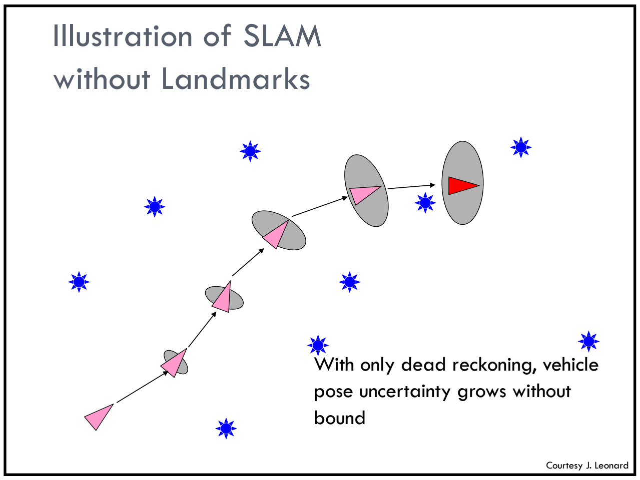
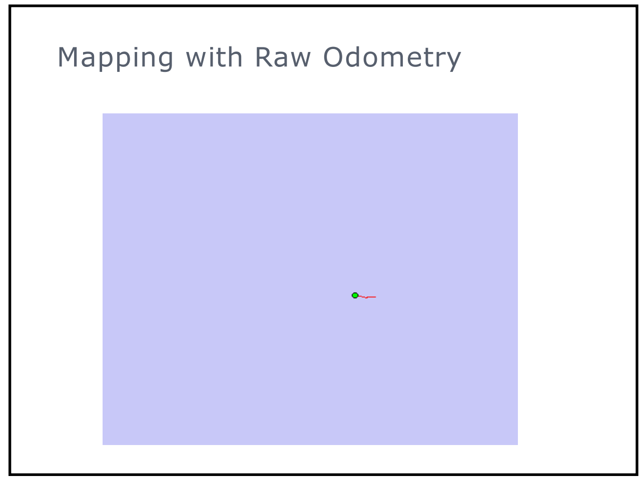
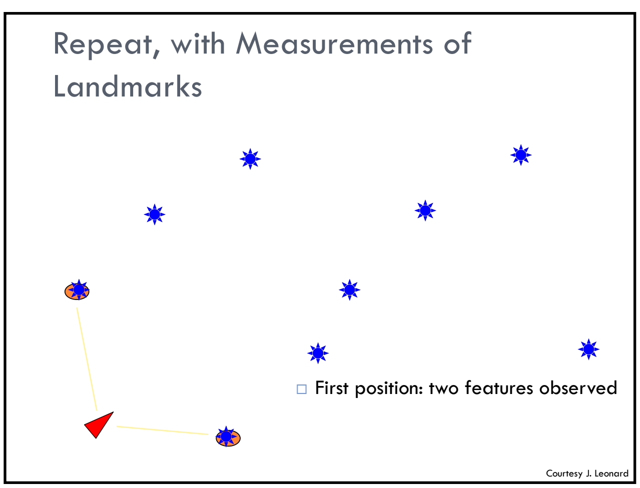


Illustration of SLAM
without Landmarks
With only dead reckoning, vehicle pose uncertainty grows without bound
Courtesy J. Leonard

9


Mapping with Raw Odometry

10


Repeat, with Measurements of
Landmarks
First position: two features observed
Courtesy J. Leonard

11

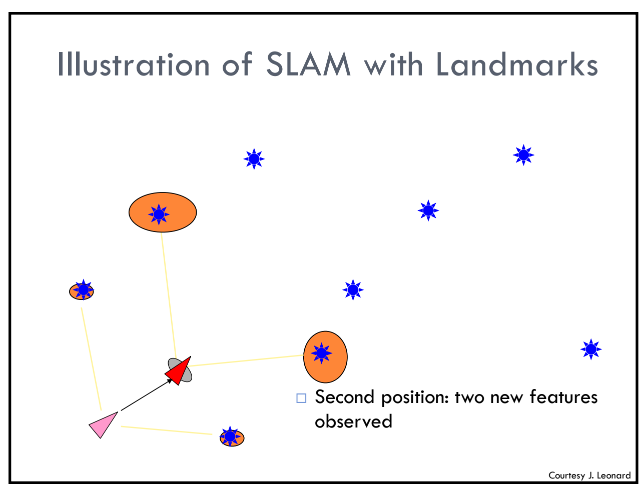


Illustration of SLAM with Landmarks
Second position: two new features observed
Courtesy J. Leonard

12

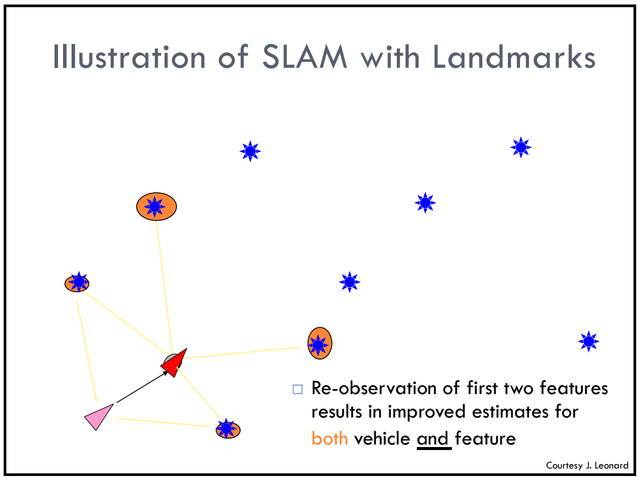


Illustration of SLAM with Landmarks
Re-observation of first two features results in improved estimates for both vehicle and feature
Courtesy J. Leonard

13

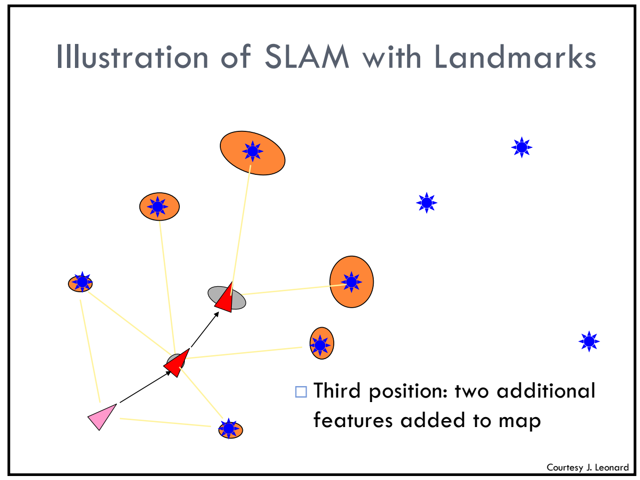


Illustration of SLAM with Landmarks
Third position: two additional features added to map
Courtesy J. Leonard

14

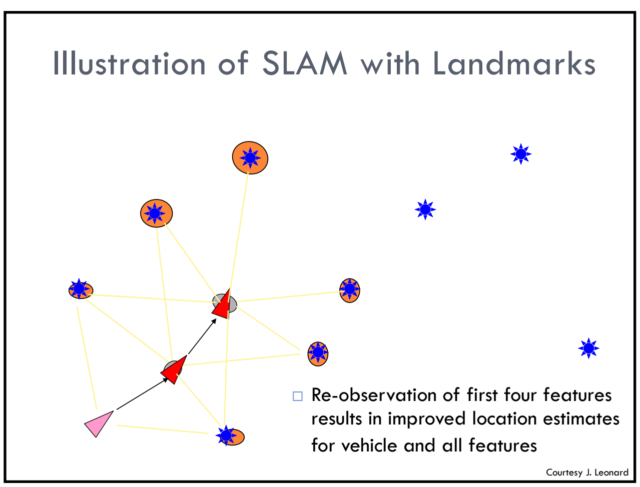


Illustration of SLAM with Landmarks
Re-observation of first four features results in improved location estimates for vehicle and all features
Courtesy J. Leonard

15

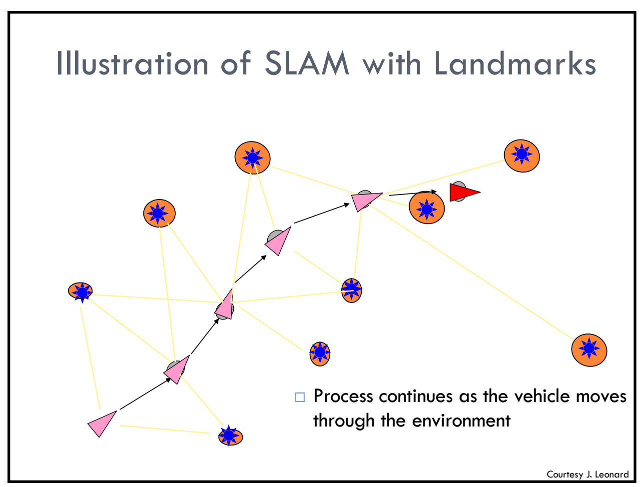
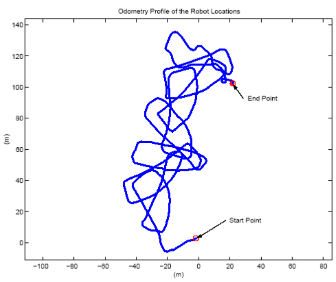


Illustration of SLAM with Landmarks
Process continues as the vehicle moves through the environment
Courtesy J. Leonard

16

SLAM Using Landmarks



Courtesy J. Leonard

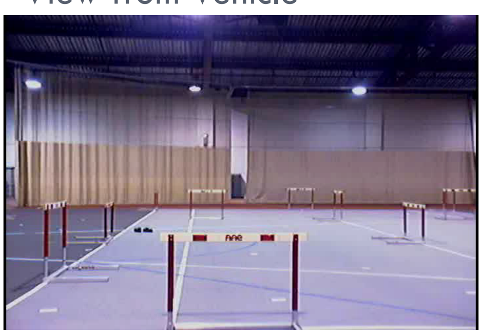
17

Test Environment (Point Landmarks)

Courtesy J. Leonard

18

View from Vehicle



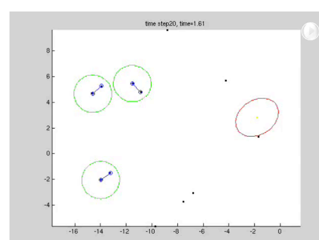
Courtesy J. Leonard

19

SLAM Using Landmarks

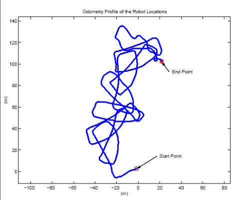
1. Move
2. Sense
3. Associate measurements with known features
4. Update state estimates for robot and previously mapped features
5. Find new features from unassociated measurements
6. Initialize new features
7. Repeat

MIT Indoor Track

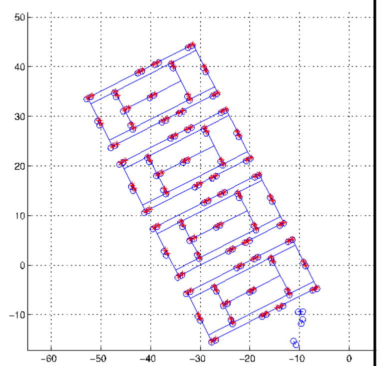


20

Comparison with Ground Truth



odometry



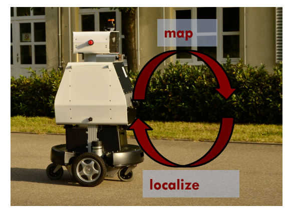
SLAM result

Courtesy J. Leonard

21

Simultaneous Localization and Mapping (SLAM)

- Building a map and locating the robot in the map at the same time
- Chicken-and-egg problem



Courtesy: Cyrill Stachniss

22

Definition of the SLAM Problem

Given

- The robot's controls
  $u_{1:T} = \{u_1, u_2, u_3, \ldots, u_T\}$
- Observations
  $z_{1:T} = \{z_1, z_2, z_3, \ldots, z_T\}$

Wanted

- Map of the environment
  m
- Path of the robot
  $x_{0:T} = \{x_0, x_1, x_2, \ldots, x_T\}$

Courtesy: Cyrill Stachniss

23

Three Main Paradigms

Kalman filter | Particle filter | Graph-based

Courtesy: Cyrill Stachniss

24

Bayes Filter

- Recursive filter with prediction and correction step
- Prediction

$$\overline{bel}(x_t) = \int p(x_t \mid u_t, x_{t-1})\ bel(x_{t-1})\, dx_{t-1}$$

- Correction

$$bel(x_t) = \eta\ p(z_t \mid x_t)\ \overline{bel}(x_t)$$

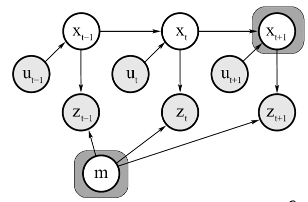
Courtesy: Cyrill Stachniss

25

EKF for Online SLAM

- We consider here the Kalman filter as a solution to the online SLAM problem

$$p(x_t, m \mid z_{1:t}, u_{1:t})$$



Courtesy: Thrun, Burgard, Fox

26

EKF SLAM

- Application of the EKF to SLAM
- Estimate robot's pose and locations of landmarks in the environment
- Assumption: known correspondences
- State space (for the 2D plane) is

$$x_t = (\underbrace{x, y, \theta}_{\text{robot's pose}}, \underbrace{m_{1,x}, m_{1,y}}_{\text{landmark 1}}, \ldots, \underbrace{m_{n,x}, m_{n,y}}_{\text{landmark n}})^T$$

Courtesy: Cyrill Stachniss

27

EKF SLAM: State Representation

- Map with n landmarks: (3+2n)-dimensional Gaussian
- Belief is represented by

$$\underbrace{\begin{pmatrix} x \\ y \\ \theta \\ m_{1,x} \\ m_{1,y} \\ \vdots \\ m_{n,x} \\ m_{n,y} \end{pmatrix}}_{\mu} \quad \underbrace{\begin{pmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{x\theta} & \sigma_{xm_{1,x}} & \sigma_{xm_{1,y}} & \cdots & \sigma_{xm_{n,x}} & \sigma_{xm_{n,y}} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{y\theta} & \sigma_{ym_{1,x}} & \sigma_{ym_{1,y}} & \cdots & \sigma_{m_{n,x}} & \sigma_{m_{n,y}} \\ \sigma_{\theta x} & \sigma_{\theta y} & \sigma_{\theta\theta} & \sigma_{\theta m_{1,x}} & \sigma_{\theta m_{1,y}} & \cdots & \sigma_{\theta m_{n,x}} & \sigma_{\theta m_{n,y}} \\ \sigma_{m_{1,x}x} & \sigma_{m_{1,x}y} & \sigma_{\theta} & \sigma_{m_{1,x}m_{1,x}} & \sigma_{m_{1,x}m_{1,y}} & \cdots & \sigma_{m_{1,x}m_{n,x}} & \sigma_{m_{1,x}m_{n,y}} \\ \sigma_{m_{1,y}x} & \sigma_{m_{1,y}y} & \sigma_{\theta} & \sigma_{m_{1,y}m_{1,x}} & \sigma_{m_{1,y}m_{1,y}} & \cdots & \sigma_{m_{1,y}m_{n,x}} & \sigma_{m_{1,y}m_{n,y}} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \sigma_{m_{n,x}x} & \sigma_{m_{n,x}y} & \sigma_{\theta} & \sigma_{m_{n,x}m_{1,x}} & \sigma_{m_{n,x}m_{1,y}} & \cdots & \sigma_{m_{n,x}m_{n,x}} & \sigma_{m_{n,x}m_{n,y}} \\ \sigma_{m_{n,y}x} & \sigma_{m_{n,y}y} & \sigma_{\theta} & \sigma_{m_{n,y}m_{1,x}} & \sigma_{m_{n,y}m_{1,y}} & \cdots & \sigma_{m_{n,y}m_{n,x}} & \sigma_{m_{n,y}m_{n,y}} \end{pmatrix}}_{\Sigma}$$

Courtesy: Cyrill Stachniss

28

EKF SLAM: State Representation

- More compactly

$$\underbrace{\begin{pmatrix} x_R \\ m_1 \\ \vdots \\ m_n \end{pmatrix}}_{\mu} \quad \underbrace{\begin{pmatrix} \Sigma_{x_R x_R} & \Sigma_{x_R m_1} & \dots & \Sigma_{x_R m_n} \\ \Sigma_{m_1 x_R} & \Sigma_{m_1 m_1} & \dots & \Sigma_{m_1 m_n} \\ \vdots & \vdots & \ddots & \vdots \\ \Sigma_{m_n x_R} & \Sigma_{m_n m_1} & \dots & \Sigma_{m_n m_n} \end{pmatrix}}_{\Sigma}$$

Courtesy: Cyrill Stachniss

29

EKF SLAM: State Representation

- Even more compactly (note: $x_R \rightarrow x$)

$$\underbrace{\begin{pmatrix} x \\ m \end{pmatrix}}_{\mu} \quad \underbrace{\begin{pmatrix} \Sigma_{xx} & \Sigma_{xm} \\ \Sigma_{mx} & \Sigma_{mm} \end{pmatrix}}_{\Sigma}$$

Courtesy: Cyrill Stachniss

30

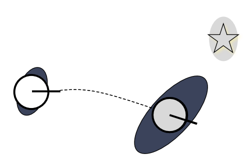
EKF SLAM: Filter Cycle

1. State prediction
2. Measurement prediction
3. Measurement
4. Data association
5. Update

Courtesy: Cyrill Stachniss

31

EKF SLAM: State Prediction

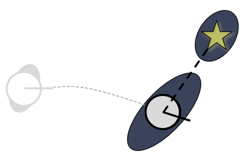


$$\underbrace{\begin{pmatrix} x_R \\ m_1 \\ \vdots \\ m_n \end{pmatrix}}_{\mu} \quad \underbrace{\begin{pmatrix} \Sigma_{x_R x_R} & \Sigma_{x_R m_1} & \dots & \Sigma_{x_R m_n} \\ \Sigma_{m_1 x_R} & \Sigma_{m_1 m_1} & \dots & \Sigma_{m_1 m_n} \\ \vdots & \vdots & \ddots & \vdots \\ \Sigma_{m_n x_R} & \Sigma_{m_n m_1} & \dots & \Sigma_{m_n m_n} \end{pmatrix}}_{\Sigma}$$

Courtesy: Cyrill Stachniss

32

EKF SLAM: Measurement Prediction

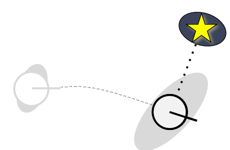


$$\underbrace{\begin{pmatrix} x_R \\ m_1 \\ \vdots \\ m_n \end{pmatrix}}_{\mu} \quad \underbrace{\begin{pmatrix} \Sigma_{x_R x_R} & \Sigma_{x_R m_1} & \dots & \Sigma_{x_R m_n} \\ \Sigma_{m_1 x_R} & \Sigma_{m_1 m_1} & \dots & \Sigma_{m_1 m_n} \\ \vdots & \vdots & \ddots & \vdots \\ \Sigma_{m_n x_R} & \Sigma_{m_n m_1} & \dots & \Sigma_{m_n m_n} \end{pmatrix}}_{\Sigma}$$

Courtesy: Cyrill Stachniss

33

EKF SLAM: Obtained Measurement

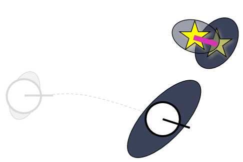


$$\underbrace{\begin{pmatrix} x_R \\ m_1 \\ \vdots \\ m_n \end{pmatrix}}_{\mu} \quad \underbrace{\begin{pmatrix} \Sigma_{x_R x_R} & \Sigma_{x_R m_1} & \dots & \Sigma_{x_R m_n} \\ \Sigma_{m_1 x_R} & \Sigma_{m_1 m_1} & \dots & \Sigma_{m_1 m_n} \\ \vdots & \vdots & \ddots & \vdots \\ \Sigma_{m_n x_R} & \Sigma_{m_n m_1} & \dots & \Sigma_{m_n m_n} \end{pmatrix}}_{\Sigma}$$

Courtesy: Cyrill Stachniss

34

EKF SLAM: Data Association and Difference Between h(x) and z

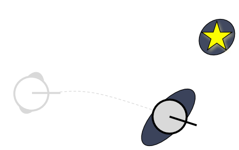


$$\underbrace{\begin{pmatrix} x_R \\ m_1 \\ \vdots \\ m_n \end{pmatrix}}_{\mu} \quad \underbrace{\begin{pmatrix} \Sigma_{x_R x_R} & \Sigma_{x_R m_1} & \dots & \Sigma_{x_R m_n} \\ \Sigma_{m_1 x_R} & \Sigma_{m_1 m_1} & \dots & \Sigma_{m_1 m_n} \\ \vdots & \vdots & \ddots & \vdots \\ \Sigma_{m_n x_R} & \Sigma_{m_n m_1} & \dots & \Sigma_{m_n m_n} \end{pmatrix}}_{\Sigma}$$

Courtesy: Cyrill Stachniss

35

EKF SLAM: Update Step



$$\underbrace{\begin{pmatrix} x_R \\ m_1 \\ \vdots \\ m_n \end{pmatrix}}_{\mu} \quad \underbrace{\begin{pmatrix} \Sigma_{x_R x_R} & \Sigma_{x_R m_1} & \dots & \Sigma_{x_R m_n} \\ \Sigma_{m_1 x_R} & \Sigma_{m_1 m_1} & \dots & \Sigma_{m_1 m_n} \\ \vdots & \vdots & \ddots & \vdots \\ \Sigma_{m_n x_R} & \Sigma_{m_n m_1} & \dots & \Sigma_{m_n m_n} \end{pmatrix}}_{\Sigma}$$

Courtesy: Cyrill Stachniss

36

EKF SLAM: Concrete Example

Setup

- Robot moves in the 2D plane
- Velocity-based motion model
- Robot observes point landmarks
- Range-bearing sensor
- Known data association
- Known number of landmarks

Courtesy: Cyrill Stachniss

37

Initialization

- Robot starts in its own reference frame (all landmarks unknown)
- 2N+3 dimensions

$$\mu_0 = \begin{pmatrix} 0 & 0 & 0 & \dots & 0 \end{pmatrix}^T$$

$$\Sigma_0 = \begin{pmatrix} 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \infty & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & \infty \end{pmatrix}$$

Courtesy: Cyrill Stachniss

38

Extended Kalman Filter Algorithm

1: $\textbf{Extended\_Kalman\_filter}(\mu_{t-1}, \Sigma_{t-1}, u_t, z_t)$**:**

2: $\bar{\mu}_t = g(u_t, \mu_{t-1})$

3: $\bar{\Sigma}_t = G_t\, \Sigma_{t-1}\, G_t^T + R_t$

4: $K_t = \bar{\Sigma}_t\, H_t^T (H_t\, \bar{\Sigma}_t\, H_t^T + Q_t)^{-1}$

5: $\mu_t = \bar{\mu}_t + K_t(z_t - h(\bar{\mu}_t))$

6: $\Sigma_t = (I - K_t\, H_t)\, \bar{\Sigma}_t$

7: $return\ \mu_t, \Sigma_t$

Courtesy: Cyrill Stachniss

39

Prediction Step (Motion)

- Goal: Update state space based on the robot's motion
- Robot motion in the plane

$$\begin{pmatrix} x' \\ y' \\ \theta' \end{pmatrix} = \underbrace{\begin{pmatrix} x \\ y \\ \theta \end{pmatrix} + \begin{pmatrix} -\frac{v_t}{\omega_t}\sin\theta + \frac{v_t}{\omega_t}\sin(\theta + \omega_t \Delta t) \\ \frac{v_t}{\omega_t}\cos\theta - \frac{v_t}{\omega_t}\cos(\theta + \omega_t \Delta t) \\ \omega_t\, \Delta t \end{pmatrix}}_{g_{x,y,\theta}(u_t,(x,y,\theta)^T)}$$

- How to map that to the 2N+3 dim space?

Courtesy: Cyrill Stachniss

40

Update the State Space

- From the motion in the plane

$$\begin{pmatrix} x' \\ y' \\ \theta' \end{pmatrix} = \begin{pmatrix} x \\ y \\ \theta \end{pmatrix} + \begin{pmatrix} -\frac{v_t}{\omega_t}\sin\theta + \frac{v_t}{\omega_t}\sin(\theta + \omega_t\Delta t) \\ \frac{v_t}{\omega_t}\cos\theta - \frac{v_t}{\omega_t}\cos(\theta + \omega_t\Delta t) \\ \omega_t\Delta t \end{pmatrix}$$

- to the 2N+3 dimensional space

$$\begin{pmatrix} x' \\ y' \\ \theta' \\ \vdots \end{pmatrix} = \underbrace{\begin{pmatrix} x \\ y \\ \theta \\ \vdots \end{pmatrix} + \underbrace{\begin{pmatrix} 1 & 0 & 0 & 0\ldots0 \\ 0 & 1 & 0 & 0\ldots0 \\ 0 & 0 & 1 & \underbrace{0\ldots0}_{2N cols} \end{pmatrix}^T}_{F_x^T} \begin{pmatrix} -\frac{v_t}{\omega_t}\sin\theta + \frac{v_t}{\omega_t}\sin(\theta + \omega_t\Delta t) \\ \frac{v_t}{\omega_t}\cos\theta - \frac{v_t}{\omega_t}\cos(\theta + \omega_t\Delta t) \\ \omega_t\Delta t \end{pmatrix}}_{g(u_t, x_t)}$$

Courtesy: Cyrill Stachniss

41

Extended Kalman Filter Algorithm

1: **Extended_Kalman_filter**$(\mu_{t-1}, \Sigma_{t-1}, u_t, z_t)$:

2: $\cancel{\bar{\mu}_t = g(u_t, \mu_{t-1})}$ DONE

3: $\bar{\Sigma}_t = G_t\ \Sigma_{t-1}\ G_t^T + R_t$

4: $K_t = \bar{\Sigma}_t\ H_t^T (H_t\ \bar{\Sigma}_t\ H_t^T + Q_t)^{-1}$

5: $\mu_t = \bar{\mu}_t + K_t(z_t - h(\bar{\mu}_t))$

6: $\Sigma_t = (I - K_t\ H_t)\ \bar{\Sigma}_t$

7: *return* μ_t, Σ_t

Courtesy: Cyrill Stachniss

42

Update Covariance

- The function g only affects the robot's motion and not the landmarks

Jacobian of the motion (3x3)

$$G_t = \begin{pmatrix} G_t^x & 0 \\ 0 & I \end{pmatrix}$$

Identity (2N x 2N)

Courtesy: Cyrill Stachniss

43

This Leads to the Time Propagation

1: **Extended_Kalman_filter**$(\mu_{t-1}, \Sigma_{t-1}, u_t, z_t)$:

2: $\cancel{\bar{\mu}_t = g(u_t, \mu_{t-1})}$ Apply & DONE

3: $\bar{\Sigma}_t = G_t\ \Sigma_{t-1}\ G_t^T + R_t$

$$\begin{aligned} \bar{\Sigma}_t &= G_t \Sigma_{t-1} G_t^T + R_t \\ &= \begin{pmatrix} G_t^x & 0 \\ 0 & I \end{pmatrix} \begin{pmatrix} \Sigma_{xx} & \Sigma_{xm} \\ \Sigma_{mx} & \Sigma_{mm} \end{pmatrix} \begin{pmatrix} (G_t^x)^T & 0 \\ 0 & I \end{pmatrix} + R_t \\ &= \begin{pmatrix} G_t^x \Sigma_{xx} (G_t^x)^T & G_t^x \Sigma_{xm} \\ (G_t^x \Sigma_{xm})^T & \Sigma_{mm} \end{pmatrix} + R_t \end{aligned}$$

Courtesy: Cyrill Stachniss

48

Extended Kalman Filter Algorithm

1: $\textbf{Extended\_Kalman\_filter}(\mu_{t-1}, \Sigma_{t-1}, u_t, z_t)$:

2: $\cancel{\bar{\mu}_t = g(u_t, \mu_{t-1})}$ DONE

3: $\cancel{\bar{\Sigma}_t = G_t\, \Sigma_{t-1}\, G_t^T + R_t}$ DONE

4: $K_t = \bar{\Sigma}_t\, H_t^T (H_t\, \bar{\Sigma}_t\, H_t^T + Q_t)^{-1}$

5: $\mu_t = \bar{\mu}_t + K_t(z_t - h(\bar{\mu}_t))$

6: $\Sigma_t = (I - K_t\, H_t)\, \bar{\Sigma}_t$

7: $return\ \mu_t, \Sigma_t$

Courtesy: Cyrill Stachniss

51

EKF SLAM: Correction Step

- Known data association
- $c_t^i = j$: i-th measurement at time t observes the landmark with index j
- Initialize landmark if unobserved
- Compute the expected observation
- Compute the Jacobian of h
- Proceed with computing the Kalman gain

Courtesy: Cyrill Stachniss

52

Range-Bearing Observation

- Range-Bearing observation $z_t^i = (r_t^i, \phi_t^i)^T$
- If landmark has not been observed

$$\begin{pmatrix} \bar{\mu}_{j,x} \\ \bar{\mu}_{j,y} \end{pmatrix} = \begin{pmatrix} \bar{\mu}_{t,x} \\ \bar{\mu}_{t,y} \end{pmatrix} + \begin{pmatrix} r_t^i\ \cos(\phi_t^i + \bar{\mu}_{t,\theta}) \\ r_t^i\ \sin(\phi_t^i + \bar{\mu}_{t,\theta}) \end{pmatrix}$$

observed location of landmark j — estimated robot's location — relative measurement

Courtesy: Cyrill Stachniss

53

Jacobian for the Observation

- Based on

$$\begin{aligned} \delta &= \begin{pmatrix} \delta_x \\ \delta_y \end{pmatrix} = \begin{pmatrix} \bar{\mu}_{j,x} - \bar{\mu}_{t,x} \\ \bar{\mu}_{j,y} - \bar{\mu}_{t,y} \end{pmatrix} \\ q &= \delta^T \delta \\ \hat{z}_t^i &= \begin{pmatrix} \sqrt{q} \\ \operatorname{atan2}(\delta_y, \delta_x) - \bar{\mu}_{t,\theta} \end{pmatrix} \end{aligned}$$

- Compute the Jacobian

$$\begin{aligned} {}^{\text{low}}H_t^i &= \frac{\partial h(\bar{\mu}_t)}{\partial \bar{\mu}_t} \\ &= \frac{1}{q} \begin{pmatrix} -\sqrt{q}\delta_x & -\sqrt{q}\delta_y & 0 & +\sqrt{q}\delta_x & \sqrt{q}\delta_y \\ \delta_y & -\delta_x & -q & -\delta_y & \delta_x \end{pmatrix} \end{aligned}$$

Courtesy: Cyrill Stachniss

58

Jacobian for the Observation

- Use the computed Jacobian

$$
{}^{\text{low}}H_t^i = \frac{1}{q}\begin{pmatrix} -\sqrt{q}\delta_x & -\sqrt{q}\delta_y & 0 & +\sqrt{q}\delta_x & \sqrt{q}\delta_y \\ \delta_y & -\delta_x & -q & -\delta_y & \delta_x \end{pmatrix}
$$

- map it to the high dimensional space

$$
H_t^i = {}^{\text{low}}H_t^i \, F_{x,j}
$$

$$
F_{x,j} = \begin{pmatrix} 1 & 0 & 0 & 0\cdots0 & 0 & 0 & 0\cdots0 \\ 0 & 1 & 0 & 0\cdots0 & 0 & 0 & 0\cdots0 \\ 0 & 0 & 1 & 0\cdots0 & 0 & 0 & 0\cdots0 \\ 0 & 0 & 0 & 0\cdots0 & 1 & 0 & 0\cdots0 \\ 0 & 0 & 0 & \underbrace{0\cdots0}_{2j-2} & 0 & 1 & \underbrace{0\cdots0}_{2N-2j} \end{pmatrix}
$$

Courtesy: Cyrill Stachniss

59

Next Steps as Specified…

Extended_Kalman_filter$(\mu_{t-1}, \Sigma_{t-1}, u_t, z_t)$**:**

$\cancel{\bar{\mu}_t = g(u_t, \mu_{t-1})}$ DONE

$\cancel{\bar{\Sigma}_t = G_t\, \Sigma_{t-1}\, G_t^T + R_t}$ DONE

$K_t = \bar{\Sigma}_t\, H_t^T (H_t\, \bar{\Sigma}_t\, H_t^T + Q_t)^{-1}$

$\mu_t = \bar{\mu}_t + K_t(z_t - h(\bar{\mu}_t))$

$\Sigma_t = (I - K_t\, H_t)\, \bar{\Sigma}_t$

return μ_t, Σ_t

Courtesy: Cyrill Stachniss

61

Extended Kalman Filter Algorithm

Extended_Kalman_filter$(\mu_{t-1}, \Sigma_{t-1}, u_t, z_t)$**:**

$\cancel{\bar{\mu}_t = g(u_t, \mu_{t-1})}$ DONE

$\cancel{\bar{\Sigma}_t = G_t\, \Sigma_{t-1}\, G_t^T + R_t}$ DONE

$\cancel{K_t = \bar{\Sigma}_t\, H_t^T (H_t\, \bar{\Sigma}_t\, H_t^T + Q_t)^{-1}}$ Apply & DONE

$\cancel{\mu_t = \bar{\mu}_t + K_t(z_t - h(\bar{\mu}_t))}$ Apply & DONE

$\cancel{\Sigma_t = (I - K_t\, H_t)\, \bar{\Sigma}_t}$ Apply & DONE

return μ_t, Σ_t

Courtesy: Cyrill Stachniss

62

EKF SLAM – Correction (1/2)

EKF_SLAM_Correction

$Q_t = \begin{pmatrix} \sigma_r{}^2 & 0 \\ 0 & \sigma_\phi{}^2 \end{pmatrix}$

for all observed features $z_t^i = (r_t^i, \phi_t^i)^T$ *do*

  $j = c_t^i$

  if landmark j *never seen before*

    $\begin{pmatrix} \bar{\mu}_{j,x} \\ \bar{\mu}_{j,y} \end{pmatrix} = \begin{pmatrix} \bar{\mu}_{t,x} \\ \bar{\mu}_{t,y} \end{pmatrix} + \begin{pmatrix} r_t^i \cos(\phi_t^i + \bar{\mu}_{t,\theta}) \\ r_t^i \sin(\phi_t^i + \bar{\mu}_{t,\theta}) \end{pmatrix}$

  endif

  $\delta = \begin{pmatrix} \delta_x \\ \delta_y \end{pmatrix} = \begin{pmatrix} \bar{\mu}_{j,x} - \bar{\mu}_{t,x} \\ \bar{\mu}_{j,y} - \bar{\mu}_{t,y} \end{pmatrix}$

  $q = \delta^T \delta$

  $\hat{z}_t^i = \begin{pmatrix} \sqrt{q} \\ \operatorname{atan2}(\delta_y, \delta_x) - \bar{\mu}_{t,\theta} \end{pmatrix}$

Courtesy: Cyrill Stachniss

63

EKF SLAM – Correction (2/2)

$$F_{x,j} = \begin{pmatrix} 1 & 0 & 0 & 0\cdots0 & 0 & 0 & 0\cdots0 \\ 0 & 1 & 0 & 0\cdots0 & 0 & 0 & 0\cdots0 \\ 0 & 0 & 1 & 0\cdots0 & 0 & 0 & 0\cdots0 \\ 0 & 0 & 0 & 0\cdots0 & 1 & 0 & 0\cdots0 \\ 0 & 0 & 0 & \underbrace{0\cdots0}_{2j-2} & 0 & 1 & \underbrace{0\cdots0}_{2N-2j} \end{pmatrix}$$

$$H_t^i = \frac{1}{q}\begin{pmatrix} -\sqrt{q}\delta_x & -\sqrt{q}\delta_y & 0 & +\sqrt{q}\delta_x & \sqrt{q}\delta_y \\ \delta_y & -\delta_x & -q & -\delta_y & +\delta_x \end{pmatrix} F_{x,j}$$

$$K_t^i = \bar{\Sigma}_t\, H_t^{iT}(H_t^i\, \bar{\Sigma}_t\, H_t^{iT} + Q_t)^{-1}$$

$$\bar{\mu}_t = \bar{\mu}_t + K_t^i(z_t^i - \hat{z}_t^i)$$

$$\bar{\Sigma}_t = (I - K_t^i\, H_t^i)\, \bar{\Sigma}_t$$

endfor

$$\mu_t = \bar{\mu}_t$$

$$\Sigma_t = \bar{\Sigma}_t$$

return μ_t, Σ_t

Courtesy: Cyrill Stachniss

64

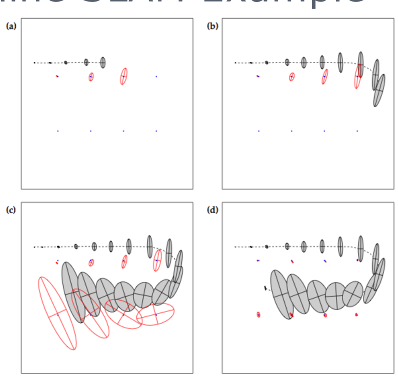
EKF SLAM Complexity

- Cubic complexity depends only on the measurement dimensionality
- Cost per step: dominated by the number of landmarks: $O(n^2)$
- Memory consumption: $O(n^2)$
- The EKF becomes computationally intractable for large maps!

Courtesy: Cyrill Stachniss

65

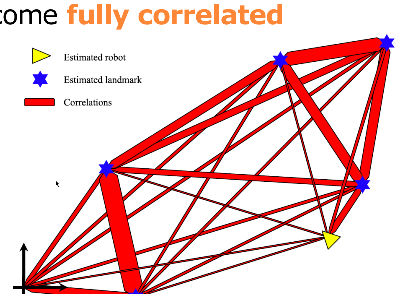
Online SLAM Example



69

EKF SLAM Correlations

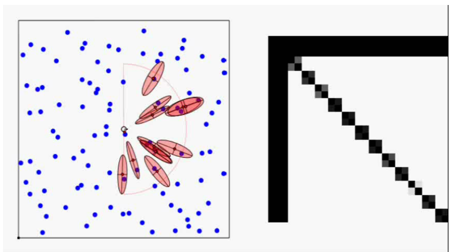
- In the limit, the landmark estimates become **fully correlated**



Courtesy: Dissanayake

70

EKF SLAM Correlations



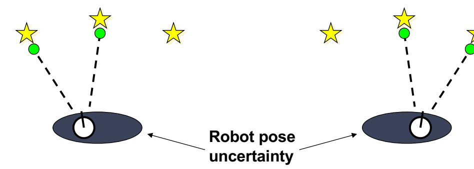
Blue path = true path **Red path** = estimated path **Black path** = odometry

- Approximate the SLAM posterior with a high-dimensional Gaussian [Smith & Cheesman, 1986] …
- Single hypothesis data association

Courtesy: M. Montemerlo

71

Data Association in SLAM



- In the real world, the mapping between observations and landmarks is unknown
- Picking wrong data associations can have catastrophic consequences
  - EKF SLAM is brittle in this regard
- Pose error correlates data associations

72

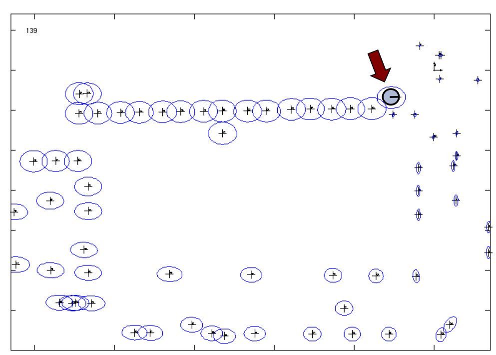
Loop-Closing

- Loop-closing means recognizing an already mapped area
- Data association under
  - high ambiguity
  - possible environment symmetries
- Uncertainties **collapse** after a loop-closure (whether the closure was correct or not)

Courtesy: Cyrill Stachniss

73

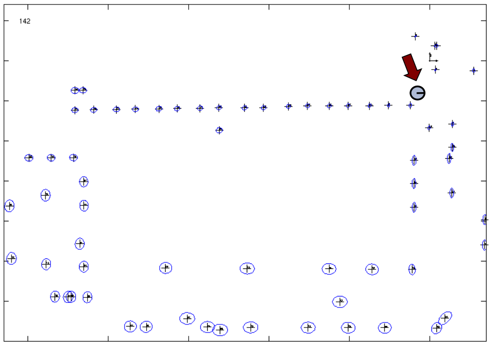
Before the Loop-Closure



Courtesy: K. Arras

74

After the Loop-Closure



Courtesy: K. Arras

75

Example: Victoria Park Dataset

Courtesy: E. Nebot

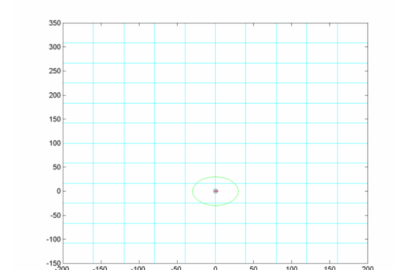
76

Victoria Park: Data Acquisition

Courtesy: E. Nebot

77

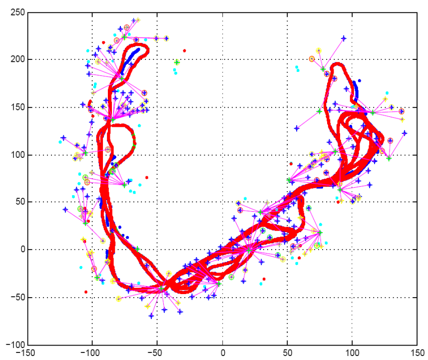
Victoria Park: EKF Estimate



Courtesy: E. Nebot

78

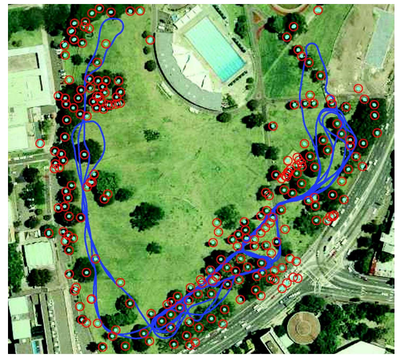
Victoria Park: EKF Estimate



Courtesy: E. Nebot

79

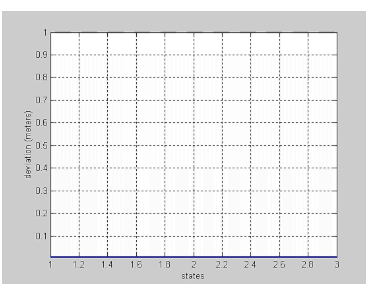
Victoria Park: Landmarks



Courtesy: E. Nebot

80

Victoria Park: Landmark Covariance



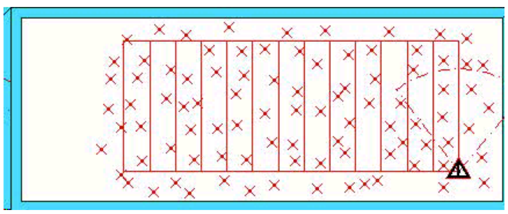
Courtesy: E. Nebot

81

Andrew Davison: MonoSLAM

82

Sub-maps for EKF SLAM



[Leonard et al 1998]

83

EKF SLAM Summary

- Quadratic in the number of landmarks: O(n^2)
- Convergence results for the linear case.
- Can diverge if nonlinearities are large!
- Have been applied successfully in large-scale environments.
- Approximations reduce the computational complexity.

84

Literature

EKF SLAM

- "Probabilistic Robotics", Chapter 10
- Smith, Self, & Cheeseman: "Estimating Uncertain Spatial Relationships in Robotics"
- Dissanayake et al.: "A Solution to the Simultaneous Localization and Map Building (SLAM) Problem"
- Durrant-Whyte & Bailey: "SLAM Part 1" and "SLAM Part 2" tutorials

Courtesy: Cyrill Stachniss

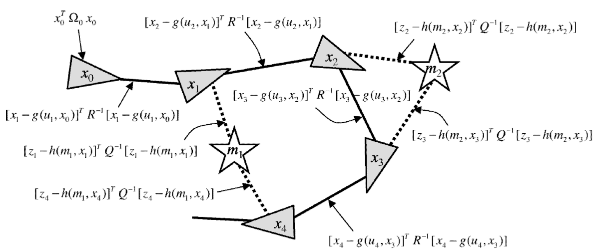
85

Graph-SLAM

- Full SLAM technique
- Generates probabilistic links
- Computes map only occasionally
- Based on Information Filter form

86

Graph-SLAM Idea



Sum of all constraints:

$$J_{\text{GraphSLAM}} = x_0^T \Omega_0 x_0 + \sum_t [x_t - g(u_t, x_{t-1})]^T R^{-1} [x_t - g(u_t, x_{t-1})] + \sum_t [z_t - h(m_{c_t}, x_t)]^T Q^{-1} [z_t - h(m_{c_t}, x_t)]$$

87

Information Form

- Represent posterior in canonical form

$\Omega = \Sigma^{-1}$ Information matrix

$\xi = \Sigma^{-1}\mu$ Information vector

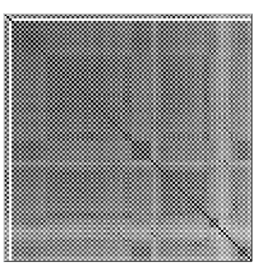
- One-to-one transform between canonical and moment representation

$$\Sigma = \Omega^{-1}$$

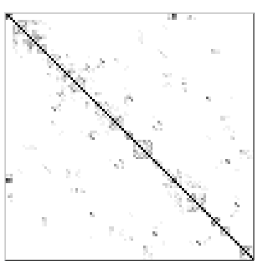
$$\mu = \Omega^{-1}\xi$$

88

Information vs. Moment Form



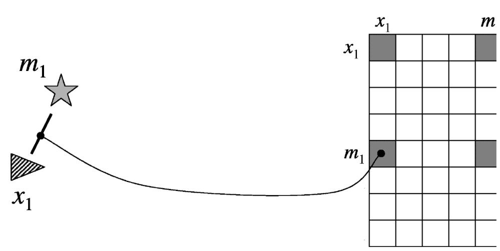
Correlation matrix



Information matrix

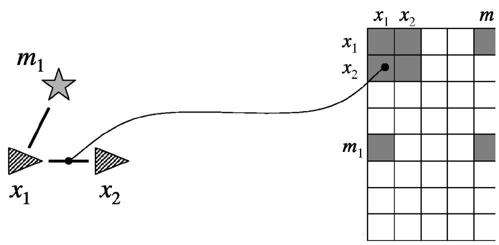
89

Graph-SLAM Idea (1)



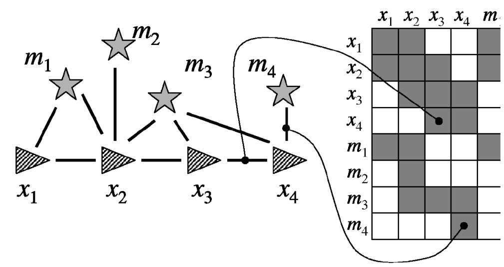
90

Graph-SLAM Idea (2)



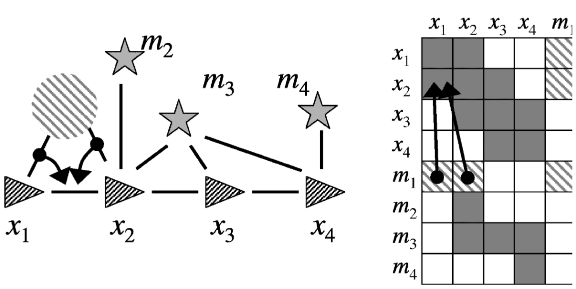
91

Graph-SLAM Idea (3)



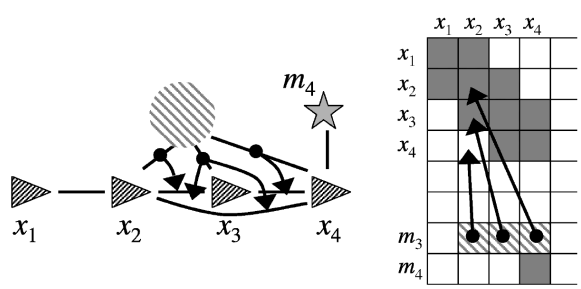
92

Graph-SLAM Inference (1)



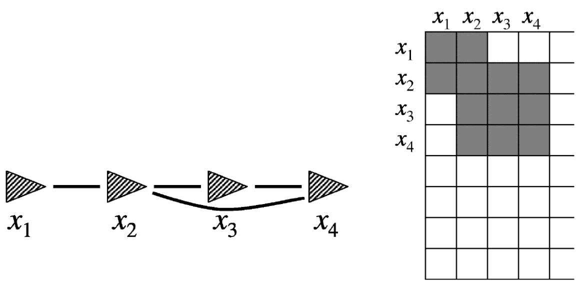
93

Graph-SLAM Inference (2)



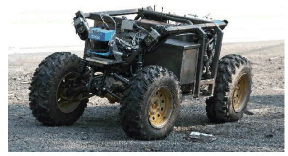
94

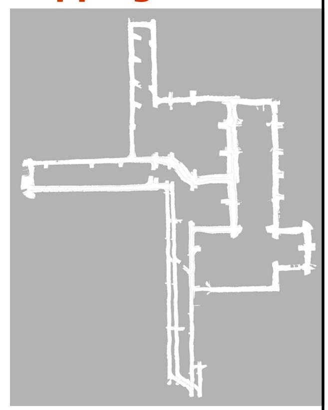
Graph-SLAM Inference (3)



95

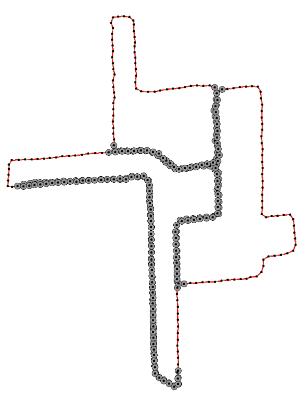
Mine Mapping





96

Mine Mapping: Data Associations



97

Efficient Map Recovery

- Information matrix inversion can be avoided if only best map estimate is required
- Minimize constraint function $J_{GraphSLAM}$ using standard optimization techniques (gradient descent, Levenberg Marquardt, conjugate gradient)

98

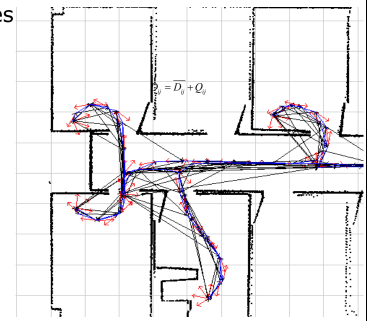
Robot Poses and Scans [Lu and Milios 1997]

- Successive robot poses connected by odometry
- Sensor readings yield constraints between poses
- Constraints represented by Gaussians

$$D_{ij} = \overline{D_{ij}} + Q_{ij}$$

- Globally optimal estimate

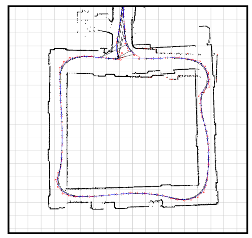
$$\arg\max_{X} \left[P(D_{ij} \mid \overline{D_{ij}}) \right]$$



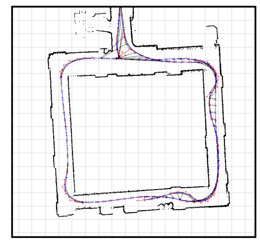
99

Loop Closure

- Use scan patches to detect loop closure
- Add new position constraints
- Deform the network based on covariances of matches



Before loop closure



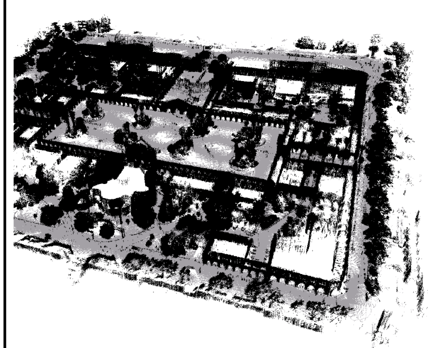
After loop closure

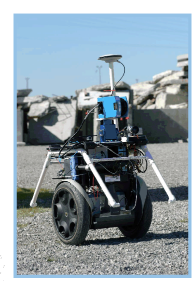
100

Mapping the Allen Center

101

3D Outdoor Mapping

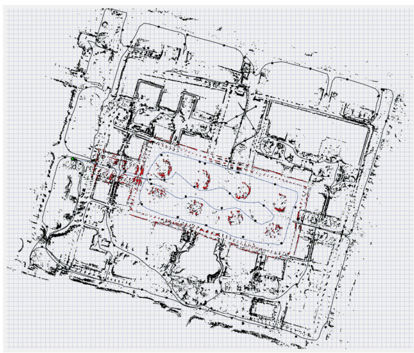




10^8 features, 10^5 poses, only few secs using cg.

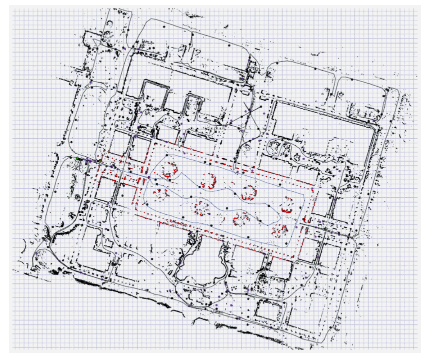
102

Map Before Optimization



103

Map After Optimization



104

Graph-SLAM Summary

- Adresses full SLAM problem
- Constructs link graph between poses and poses/landmarks
- Graph is sparse: number of edges linear in number of nodes
- Inference performed by building information matrix and vector (linearized form)
- Map recovered by reduction to robot poses, followed by conversion to moment representation, followed by estimation of landmark positions
- ML estimate by minimization of $J_{GraphSLAM}$
- Data association by iterative greedy search

105