CSE-571
Robotics

Bayes Filter Implementations

Particle filters

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Motivation

- So far, we discussed the
  - Kalman filter: Gaussian, linearization problems, multi-modal beliefs
- Particle filters are a way to **efficiently** represent **non-Gaussian distributions**
- Basic principle
  - Set of state hypotheses (“particles”)
  - Survival-of-the-fittest

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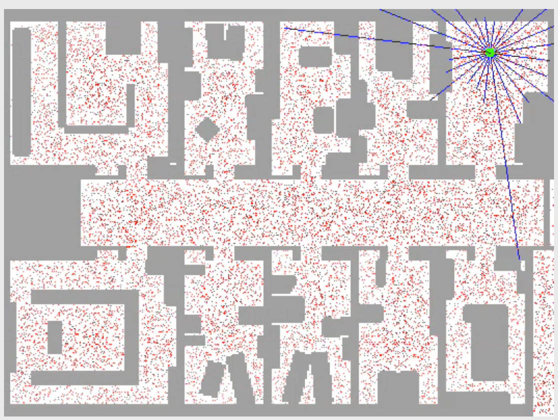
Sample-based Localization (sonar)



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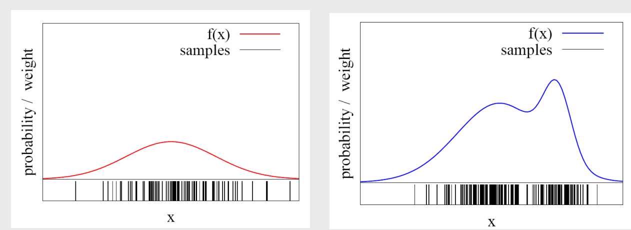
Density Approximation

- Particle sets can be used to approximate densities



- The more particles fall into an interval, the higher the probability of that interval
- How to draw samples form a function/distribution?

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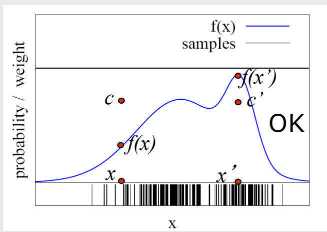
Rejection Sampling

- Let us assume that $f(x) <= 1$ for all x
- Sample x from a uniform distribution
- Sample c from [0,1]
- if $f(x) > c$ keep the sample
  otherwise reject the sampe



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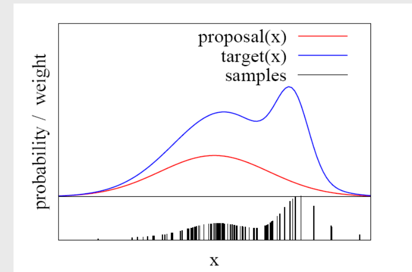
Importance Sampling Principle

- We can even use a different distribution g to generate samples from f
- By introducing an importance weight w, we can account for the “differences between g and f ”
- $w = f / g$
- f is often called target
- g is often called proposal



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Importance Sampling with Resampling: Landmark Detection Example

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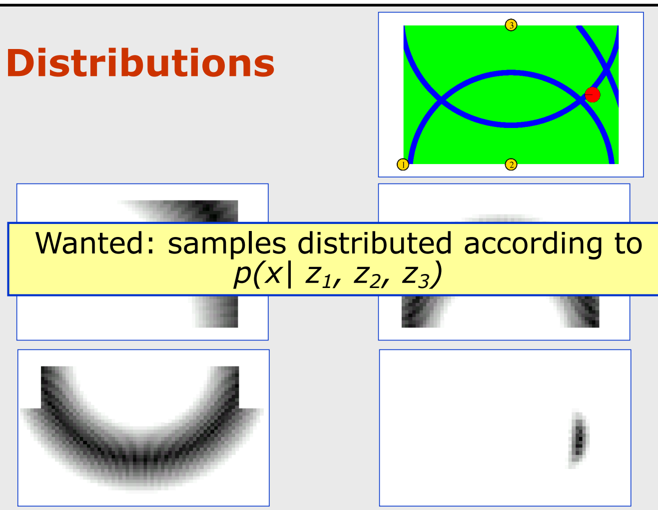
Distributions



Wanted: samples distributed according to $p(x| z_1, z_2, z_3)$

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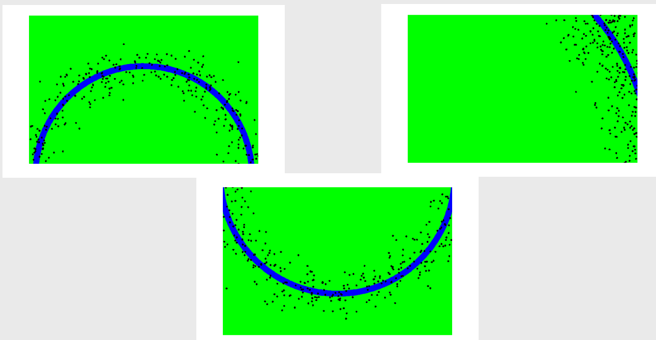
This is Easy!

We can draw samples from $p(x|z_l)$ by adding noise to the detection parameters.



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Importance Sampling with Resampling

$$\text{Target distribution f}: p(x \mid z_1, z_2, \ldots, z_n) = \frac{\prod_k p(z_k \mid x) \quad p(x)}{p(z_1, z_2, \ldots, z_n)}$$

$$\text{Sampling distribution g}: p(x \mid z_l) = \frac{p(z_l \mid x) p(x)}{p(z_l)}$$

$$\text{Importance weights w}: \frac{f}{g} = \frac{p(x \mid z_1, z_2, \ldots, z_n)}{p(x \mid z_l)} = \frac{p(z_l) \prod_{k \neq l} p(z_k \mid x)}{p(z_1, z_2, \ldots, z_n)}$$

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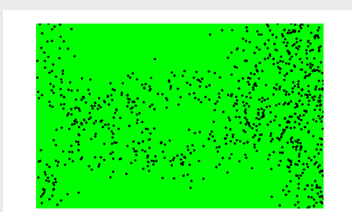
Importance Sampling with Resampling



Weighted samples

After resampling

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Resampling

- **Given**: Set S of weighted samples.
- **Wanted** : Random sample, where the probability of drawing x_i is given by w_i.
- Typically done n times with replacement to generate new sample set S'.

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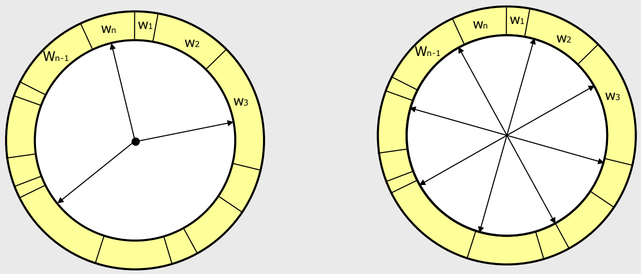
Resampling



- Roulette wheel
- Binary search, n log n

- Stochastic universal sampling
- Systematic resampling
- Linear time complexity
- Easy to implement, low variance

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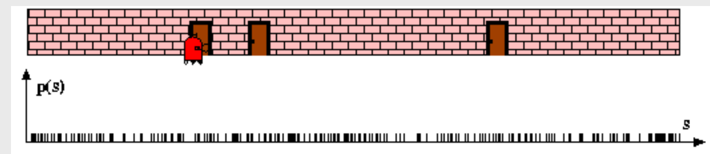
Particle Filters



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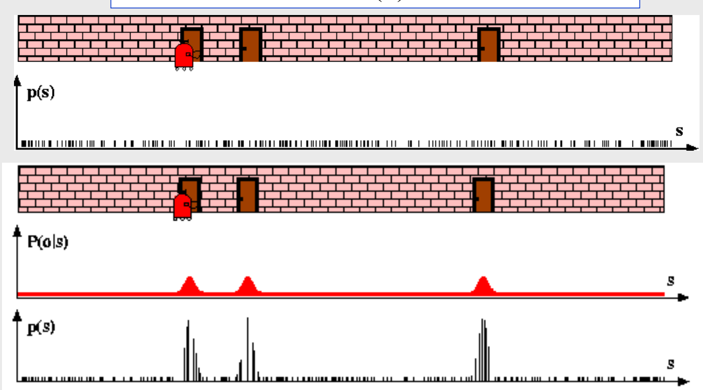
Sensor Information: Importance Sampling

$$Bel(x) \leftarrow \alpha\, p(z \mid x)\, Bel^{-}(x)$$

$$w \leftarrow \frac{\alpha\, p(z \mid x)\, Bel^{-}(x)}{Bel^{-}(x)} = \alpha\, p(z \mid x)$$



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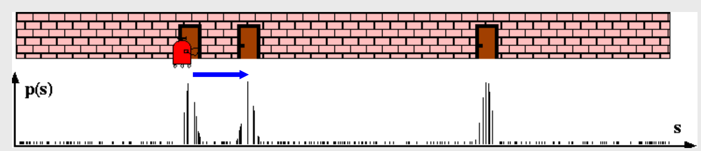
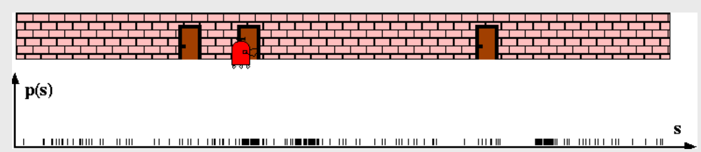
Robot Motion

$$Bel^{-}(x) \leftarrow \int p(x \mid u, x')\, Bel(x')\, \mathrm{d}x'$$





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Sensor Information: Importance Sampling

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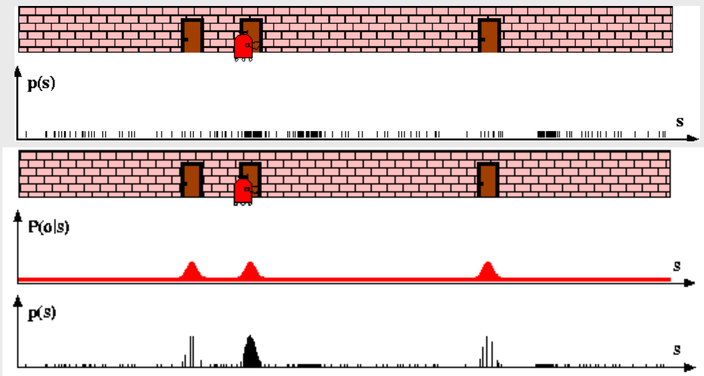

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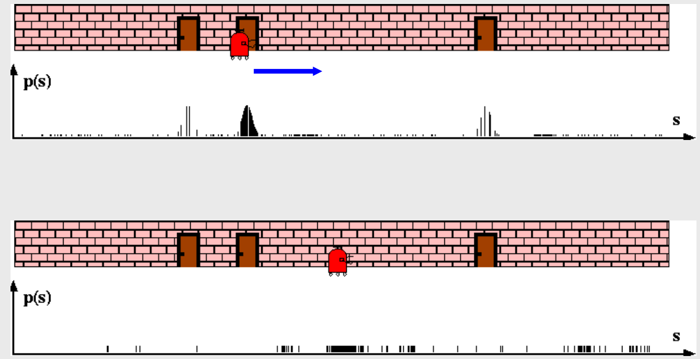
Robot Motion

$$Bel^-(x) \leftarrow \int p(x|u,x')\, Bel(x')\, \mathrm{d}x'$$



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Particle Filter Algorithm

1. Algorithm **particle_filter**(S_{t-1}, u_{t-1} z_t):
2. $S_t = \emptyset, \quad \eta = 0$
3. **For** $i = 1 \ldots n$ ***Generate new samples***
4. Sample index $j(i)$ from the discrete distribution given by w_{t-1}
5. Sample x_t^i from $p(x_t \mid x_{t-1}, u_{t-1})$ using $x_{t-1}^{j(i)}$ and u_{t-1}
6. $w_t^i = p(z_t \mid x_t^i)$ ***Compute importance weight***
7. $\eta = \eta + w_t^i$ ***Update normalization factor***
8. $S_t = S_t \cup \{\langle x_t^i, w_t^i \rangle\}$ ***Insert***
9. **For** $i = 1 \ldots n$
10. $w_t^i = w_t^i / \eta$ ***Normalize weights***

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Particle Filter Algorithm

$$Bel(x_t) = \eta\, p(z_t \mid x_t) \int p(x_t \mid x_{t-1}, u_{t-1})\, Bel(x_{t-1})\, dx_{t-1}$$

draw x^i_{t-1} from $Bel(x_{t-1})$

draw x^i_t from $p(x_t \mid x^i_{t-1}, u_{t-1})$

Importance factor for x^i_t:

$$w_t^i = \frac{\text{target distribution}}{\text{proposal distribution}} = \frac{\eta\, p(z_t \mid x_t)\, p(x_t \mid x_{t-1}, u_{t-1})\, Bel(x_{t-1})}{p(x_t \mid x_{t-1}, u_{t-1})\, Bel(x_{t-1})} \propto p(z_t \mid x_t)$$

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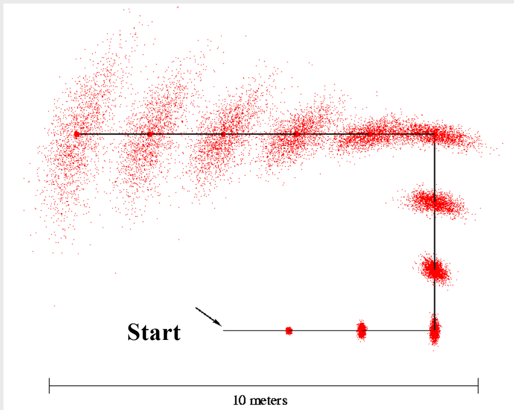
Motion Model Reminder



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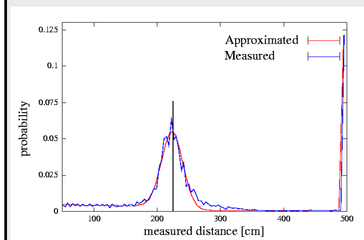
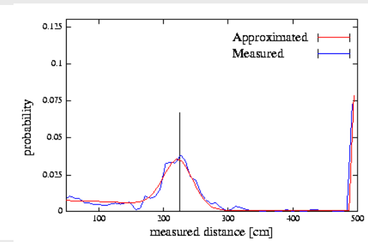
Proximity Sensor Model Reminder



Laser sensor



Sonar sensor

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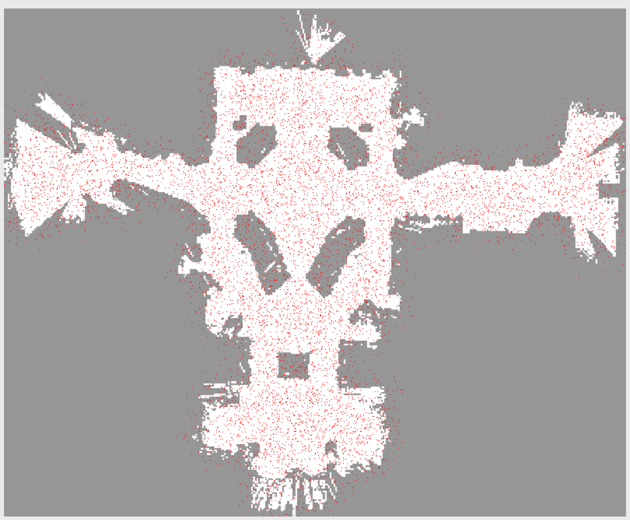

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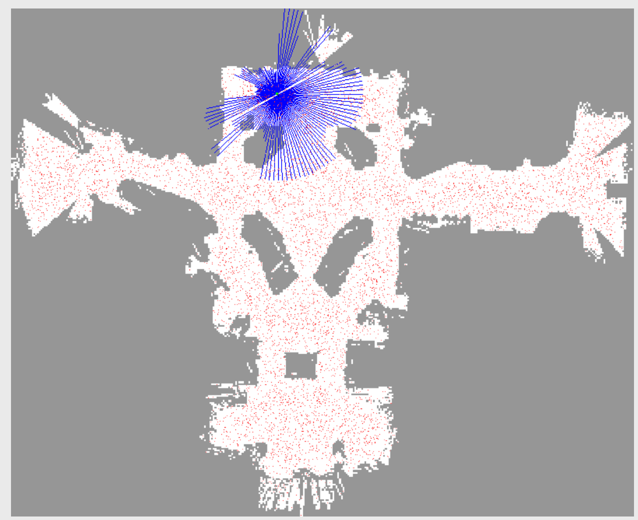

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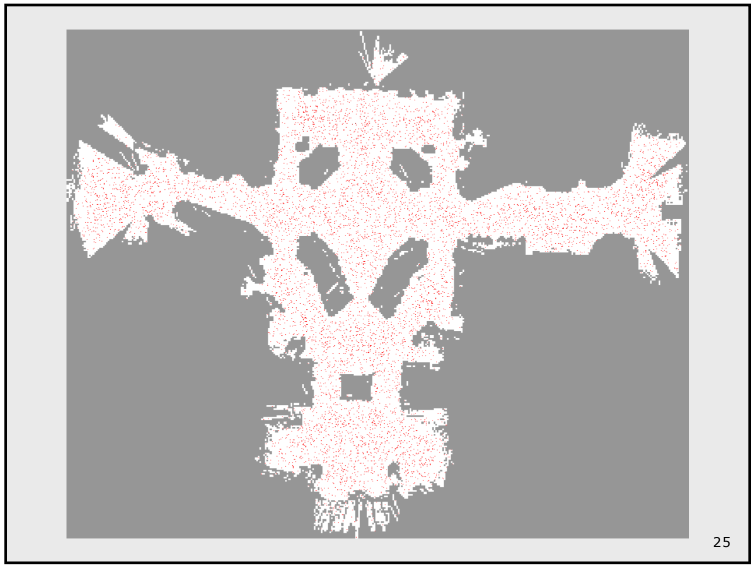

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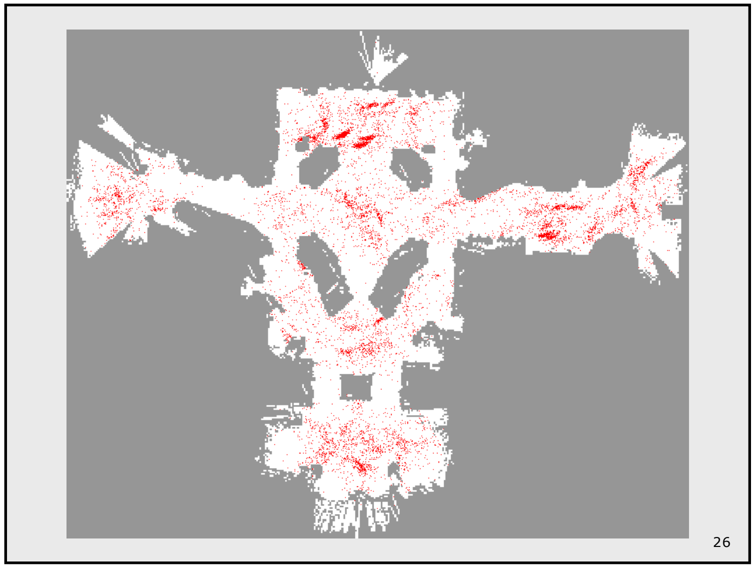

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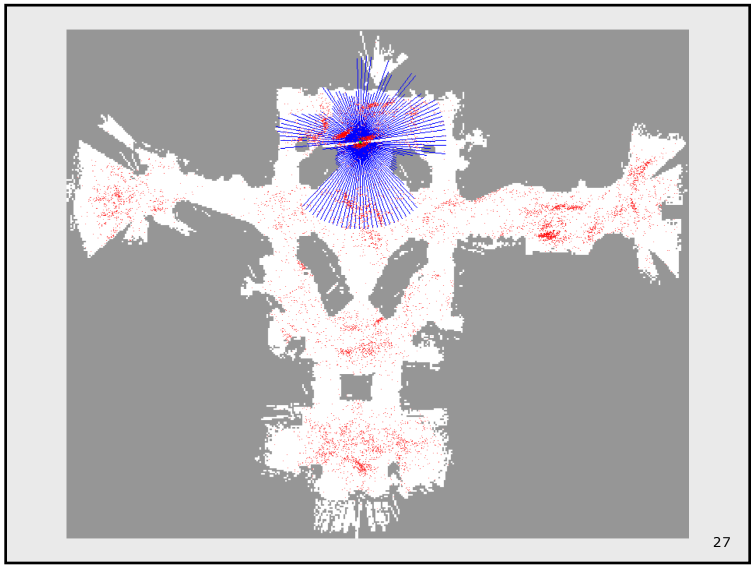

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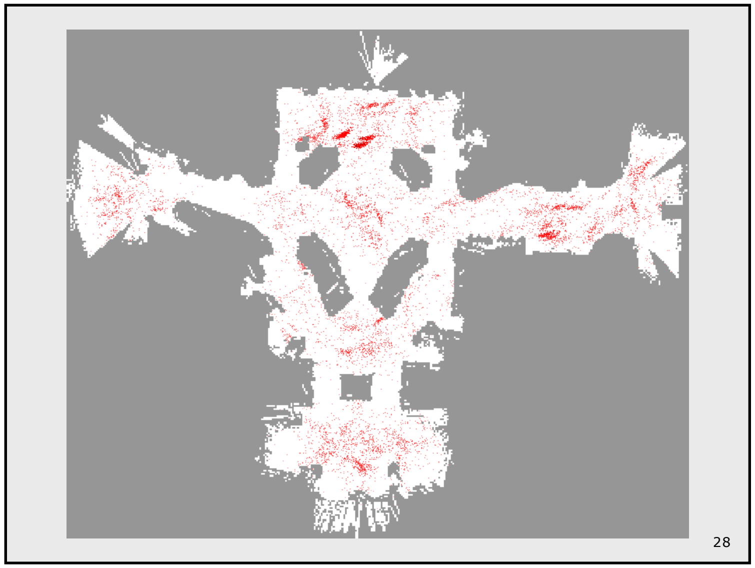

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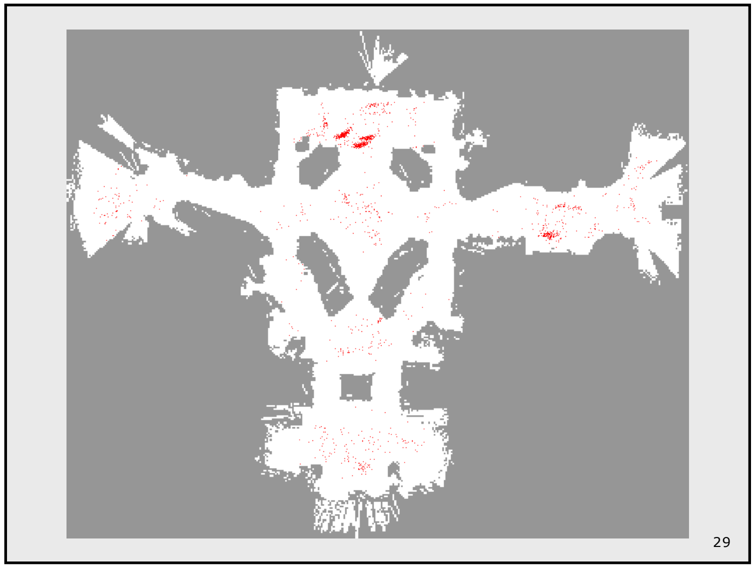

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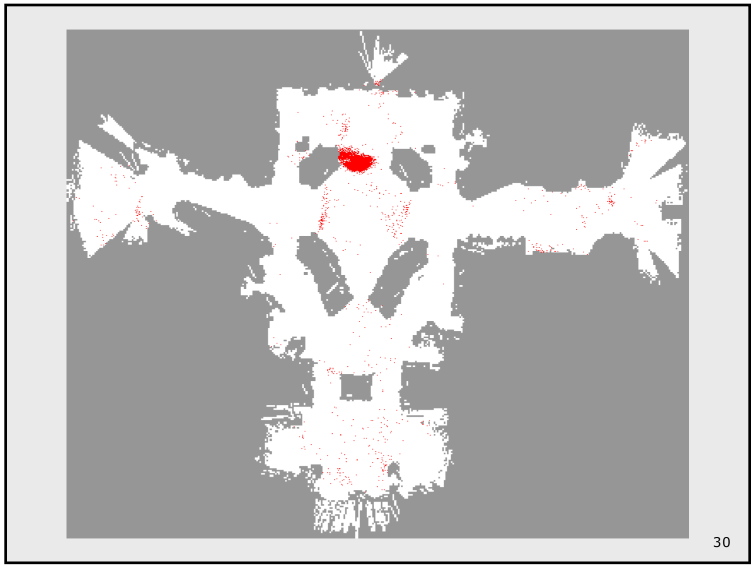

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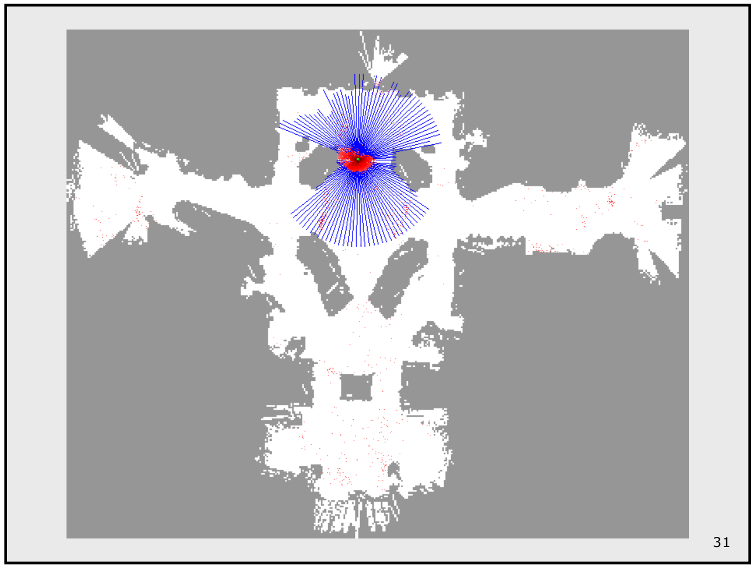

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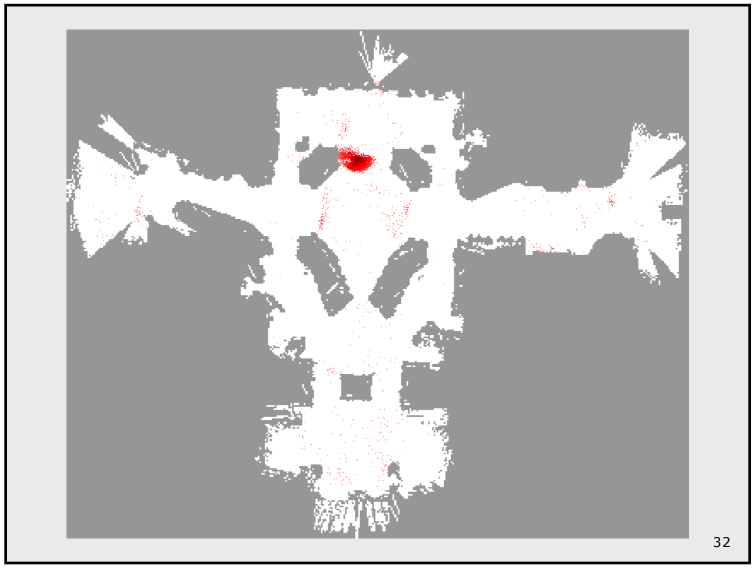

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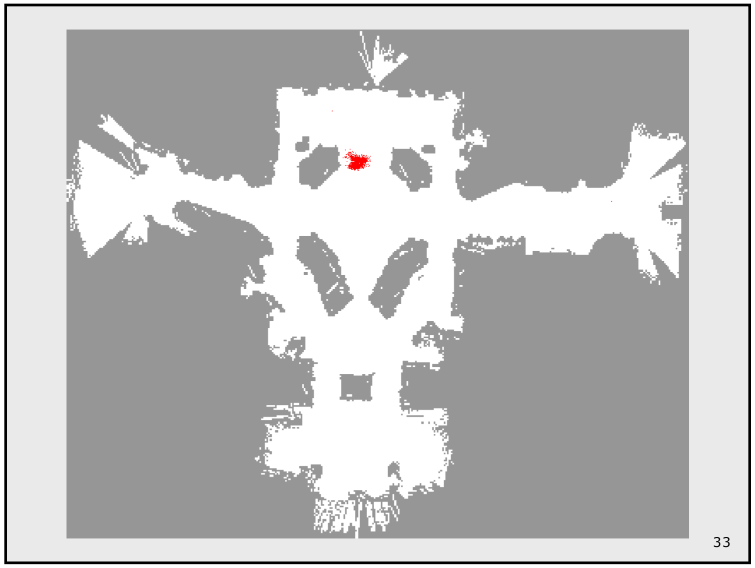

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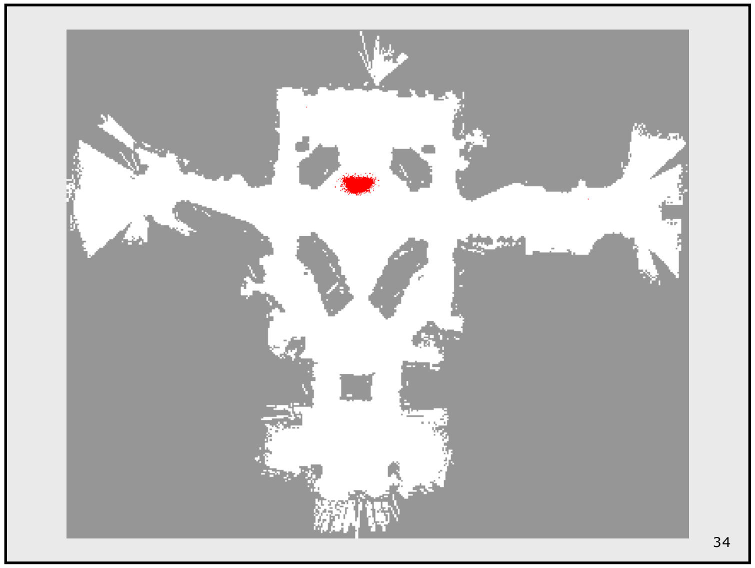

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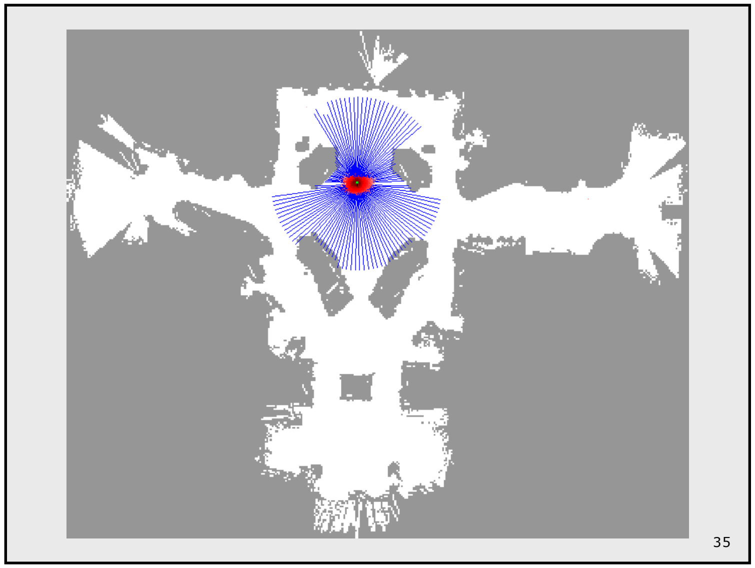

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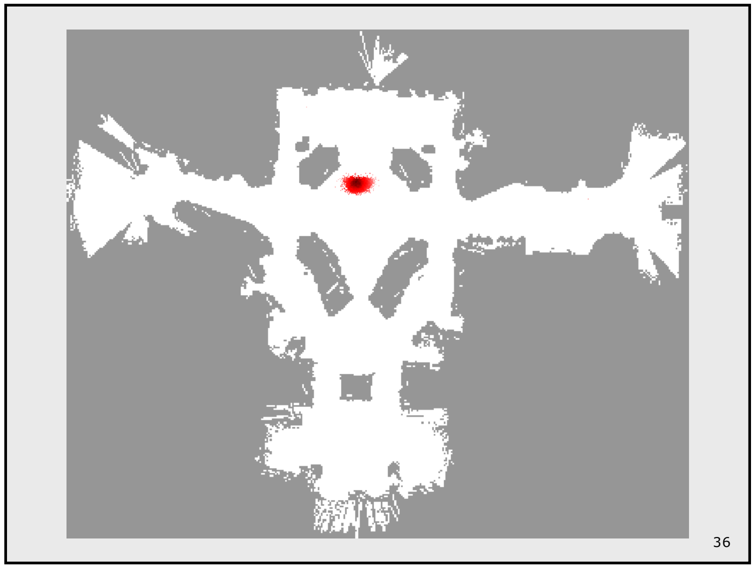

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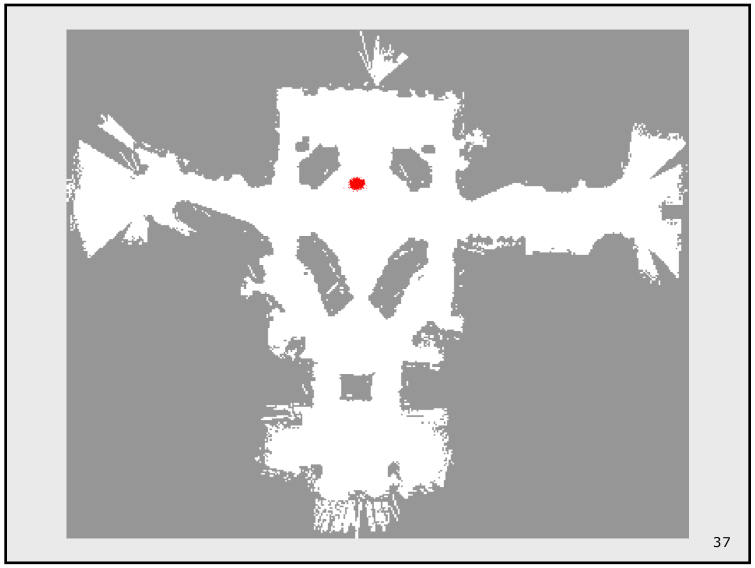

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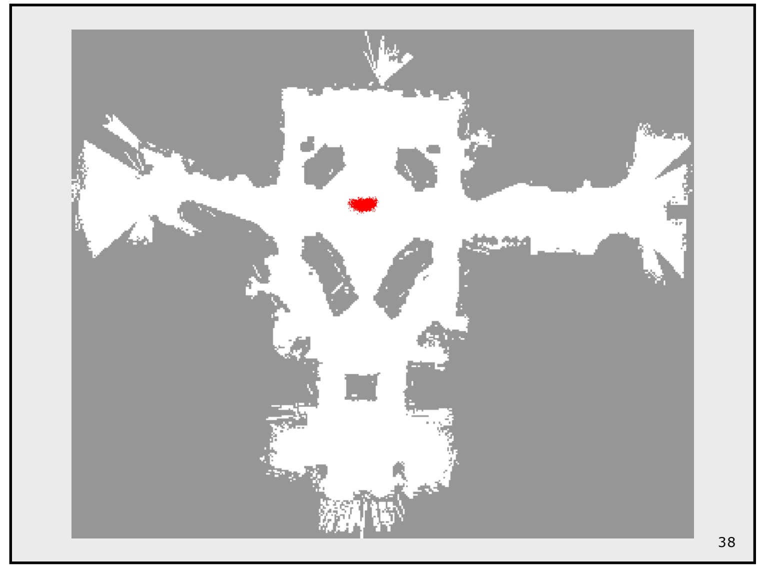

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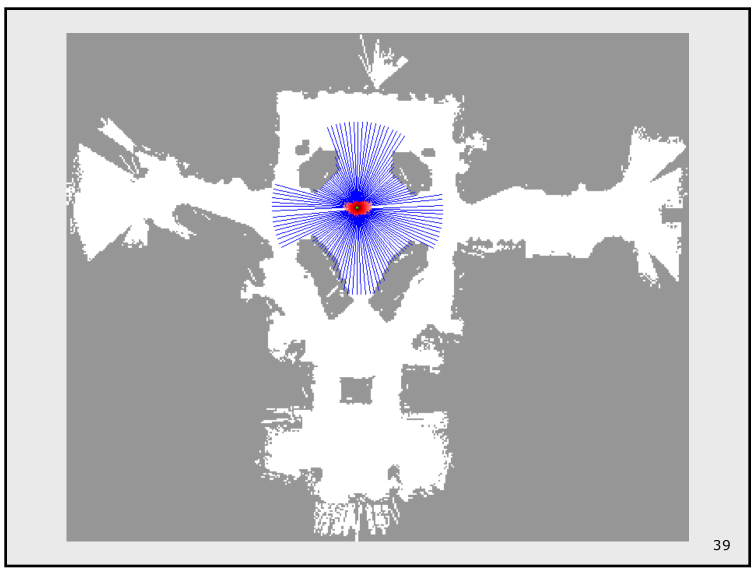

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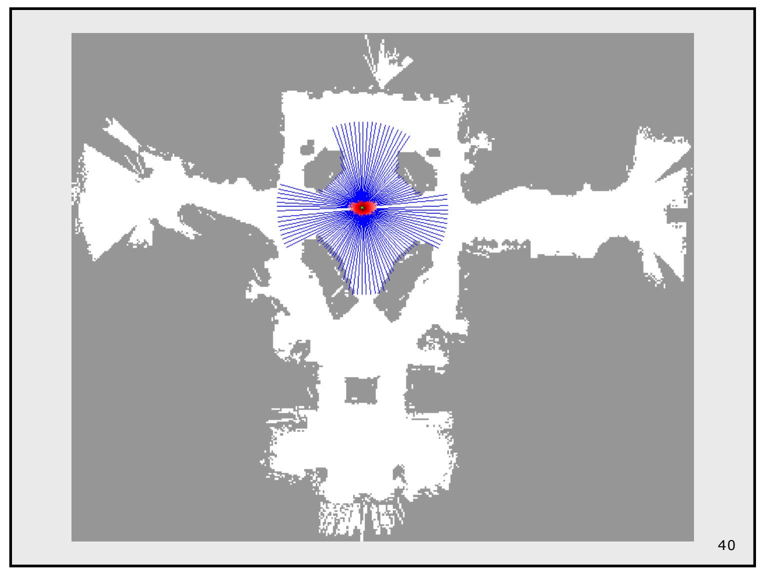

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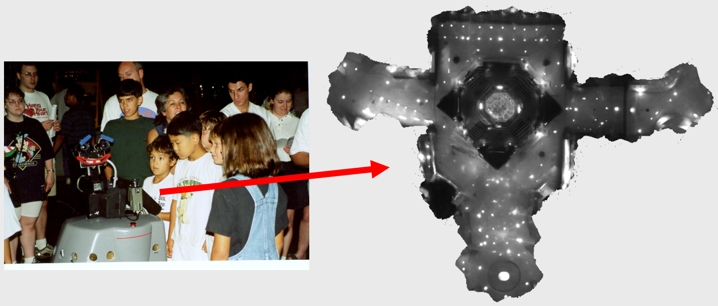
Using Ceiling Maps for Localization



[Dellaert et al. 99]

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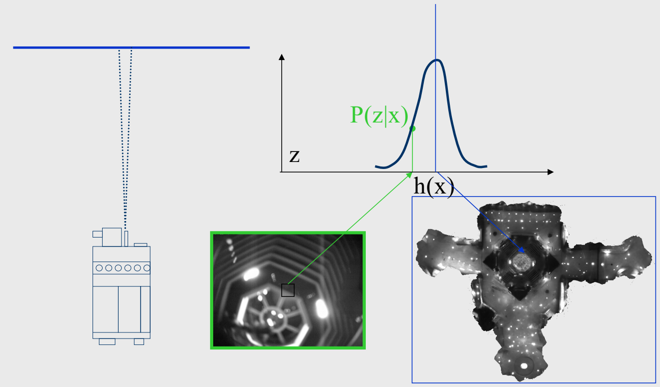
Vision-based Localization



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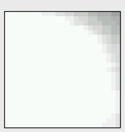
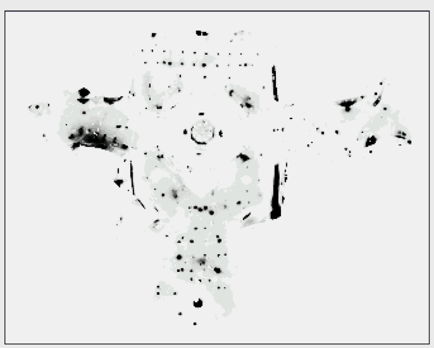
Under a Light

Measurement z:



***P(z|x)*:**



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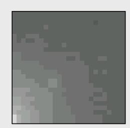
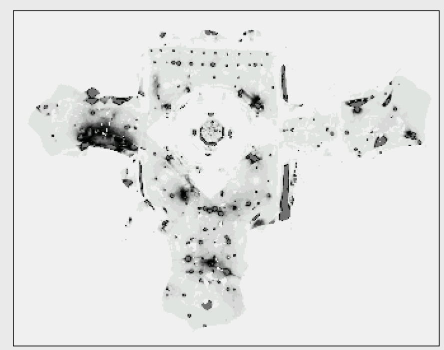
Next to a Light

Measurement z:



***P(z|x)*:**



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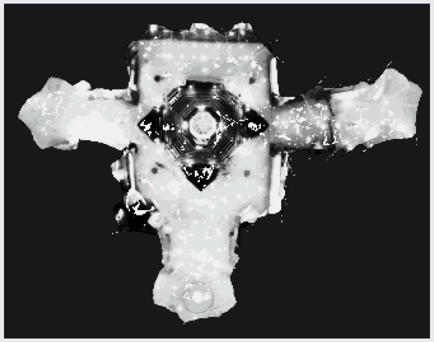
Elsewhere

Measurement z: 

***P(z|x)*:** 

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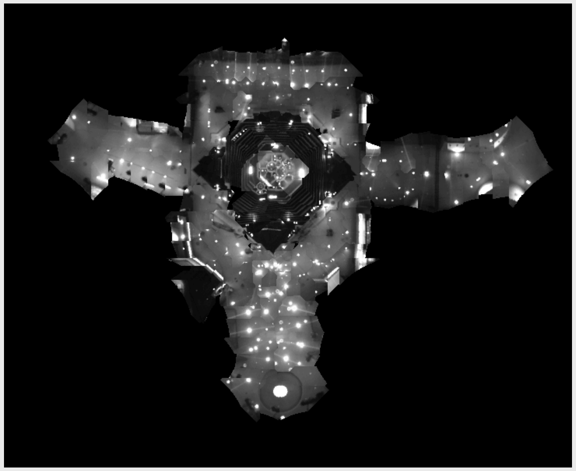
Global Localization Using Vision



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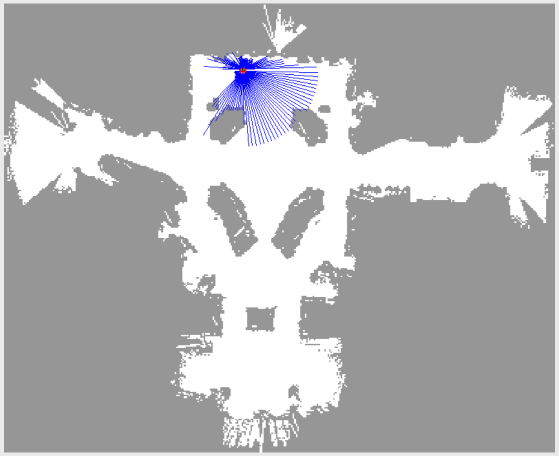
Recovery from Failure



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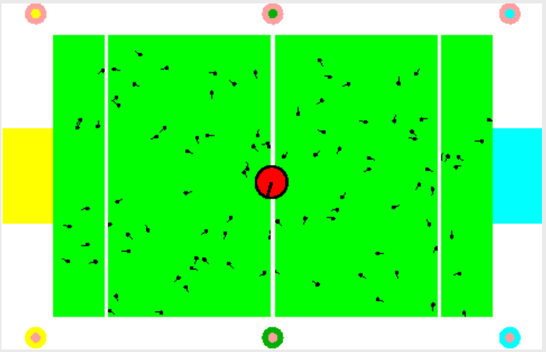
Localization for AIBO robots



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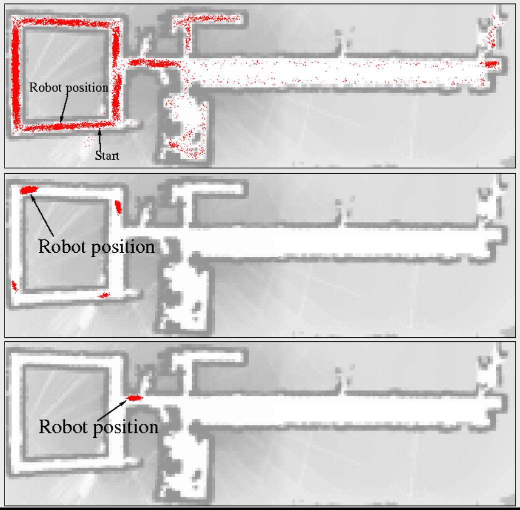
Adaptive Sampling



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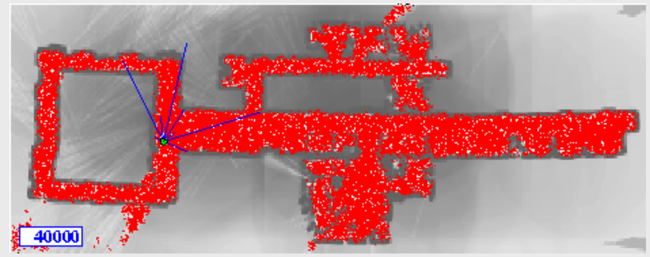
KLD-Sampling Sonar



Adapt number of particles on the fly based on statistical approximation measure

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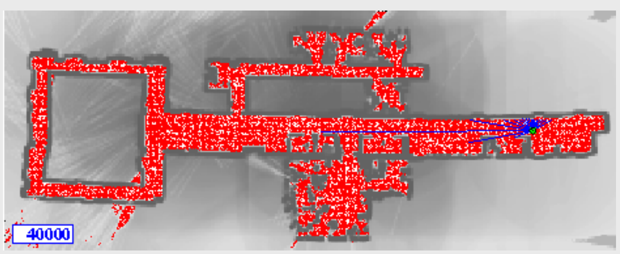
KLD-Sampling Laser



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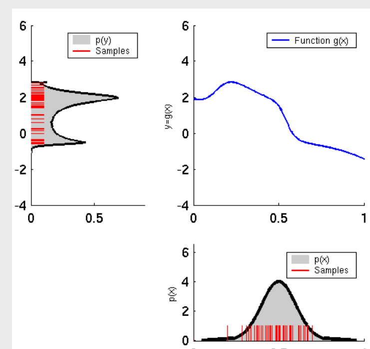
Particle Filter Projection



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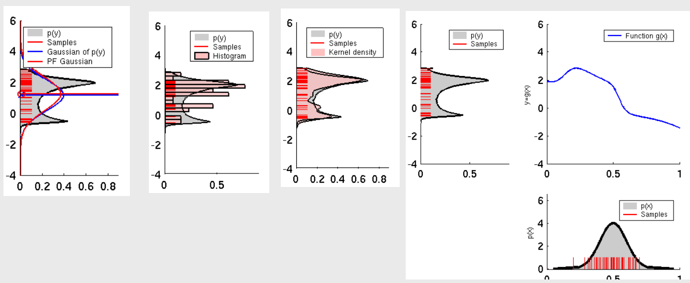
Density Extraction



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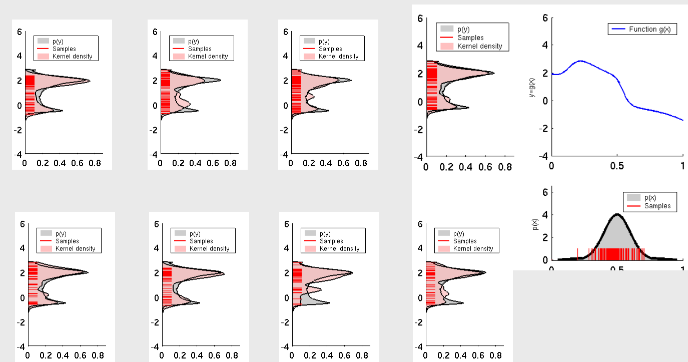
Sampling Variance



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CSE-571
Robotics

Bayes Filter Implementations

Discrete filters

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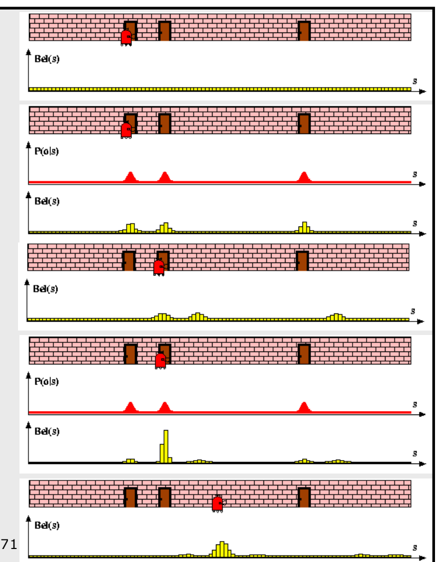
Piecewise Constant



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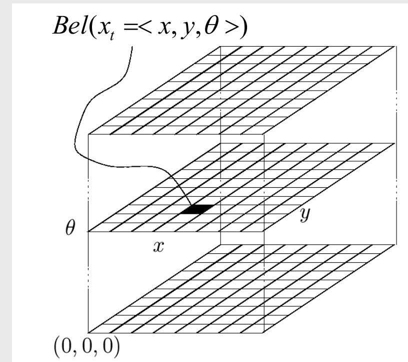
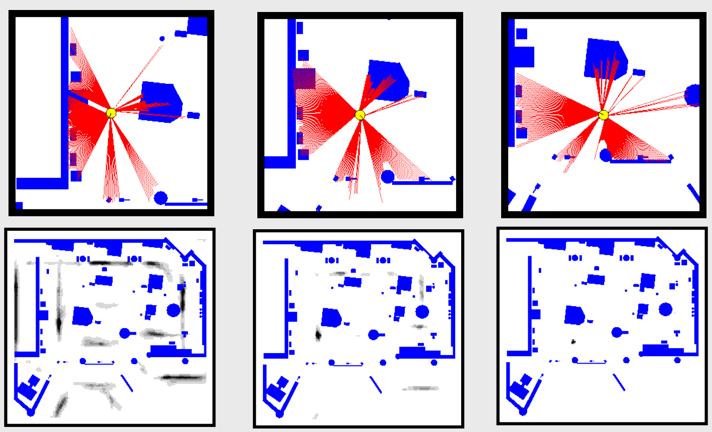
Discrete Bayes Filter Algorithm

Algorithm **Discrete_Bayes_filter**(*Bel(x),d*):
η=0
If *d* is a perceptual data item *z* then
    For all *x* do
        $Bel'(x) = P(z \mid x) Bel(x)$
        $\eta = \eta + Bel'(x)$
    For all *x* do
        $Bel'(x) = \eta^{-1} Bel'(x)$
Else if *d* is an action data item *u* then
    For all *x* do
        $Bel'(x) = \sum_{x'} P(x \mid u, x')\, Bel(x')$
Return *Bel'(x)*

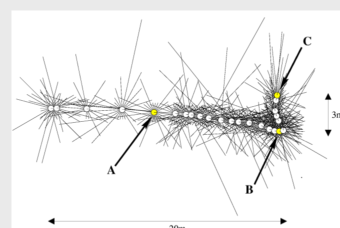
Piecewise Constant Representation



Grid-based Localization

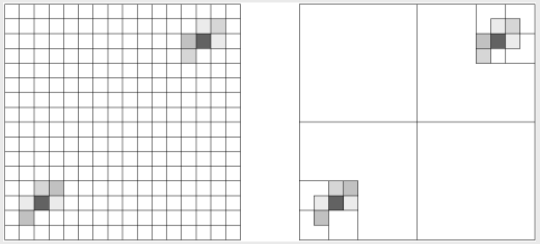


Sonars and Occupancy Grid Map



Tree-based Representation

Idea: Represent density using a variant of Octrees



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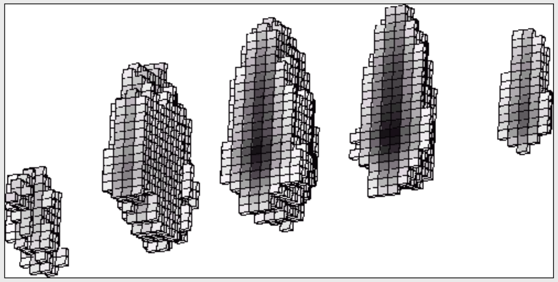
Tree-based Representations

- Efficient in space and time
- Multi-resolution



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