CSE-571
Robotics

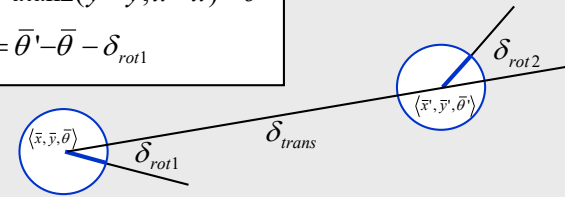**Probabilistic Motion Models**

Probabilistic Kinematics

- Robot moves from $\langle \bar{x}, \bar{y}, \bar{\theta} \rangle$ to $\langle \bar{x}', \bar{y}', \bar{\theta}' \rangle$.
- Odometry information $u = \langle \delta_{rot1}, \delta_{rot2}, \delta_{trans} \rangle$.

$$\delta_{trans} = \sqrt{(\bar{x}' - \bar{x})^2 + (\bar{y}' - \bar{y})^2}$$
$$\delta_{rot1} = \operatorname{atan2}(\bar{y}' - \bar{y}, \bar{x}' - \bar{x}) - \bar{\theta}$$
$$\delta_{rot2} = \bar{\theta}' - \bar{\theta} - \delta_{rot1}$$

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Noise Model for Motion

- The measured motion is given by the true motion corrupted with noise.

$$\hat{\delta}_{rot1} = \delta_{rot1} + \varepsilon_{\alpha_1 |\delta_{rot1}| + \alpha_2 |\delta_{trans}|}$$
$$\hat{\delta}_{trans} = \delta_{trans} + \varepsilon_{\alpha_3 |\delta_{trans}| + \alpha_4 |\delta_{rot1} + \delta_{rot2}|}$$
$$\hat{\delta}_{rot2} = \delta_{rot2} + \varepsilon_{\alpha_1 |\delta_{rot2}| + \alpha_2 |\delta_{trans}|}$$

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Sample Odometry Motion Model
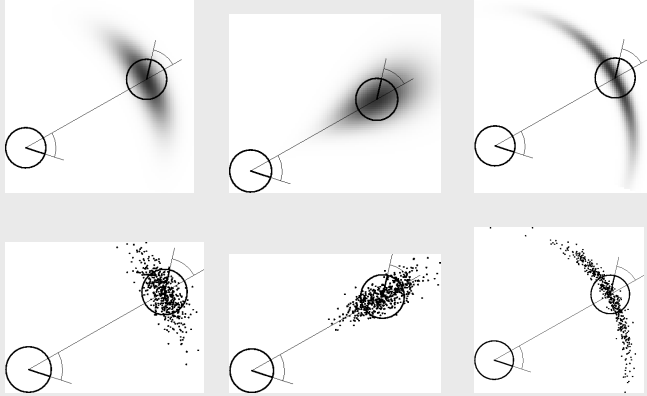
1. Algorithm **sample_motion_model**(u, x):
   $$u = \langle \delta_{rot1}, \delta_{rot2}, \delta_{trans} \rangle, x = \langle x, y, \theta \rangle$$
1. $\hat{\delta}_{rot1} = \delta_{rot1} + \operatorname{sample}(\alpha_1 |\delta_{rot1}| + \alpha_2\, \delta_{trans})$
2. $\hat{\delta}_{trans} = \delta_{trans} + \operatorname{sample}(\alpha_3\, \delta_{trans} + \alpha_4\, (|\delta_{rot1}| + |\delta_{rot2}|))$
3. $\hat{\delta}_{rot2} = \delta_{rot2} + \operatorname{sample}(\alpha_1 |\delta_{rot2}| + \alpha_2\, \delta_{trans})$

4. $x' = x + \hat{\delta}_{trans} \cos(\theta + \hat{\delta}_{rot1})$
5. $y' = y + \hat{\delta}_{trans} \sin(\theta + \hat{\delta}_{rot1})$
6. $\theta' = \theta + \hat{\delta}_{rot1} + \hat{\delta}_{rot2}$

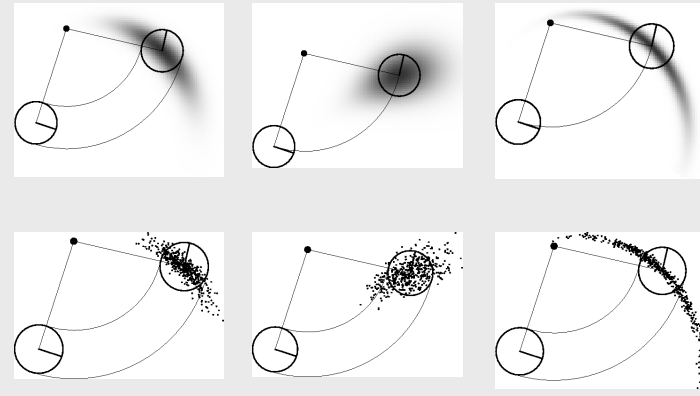7. Return $\langle x', y', \theta' \rangle$

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Examples (odometry based)

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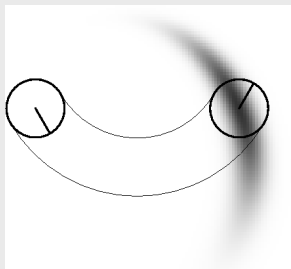## Examples (velocity based)

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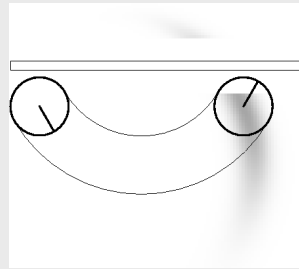## Motion Model with Map

$P(x \mid u, x')$

$P(x \mid u, x', m) \approx P(x \mid m) \, P(x \mid u, x')$

- When does this approximation fail?

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Sample-based Motion

Start

10 meters

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