Lecture 2: Motion and Sensor Model Example

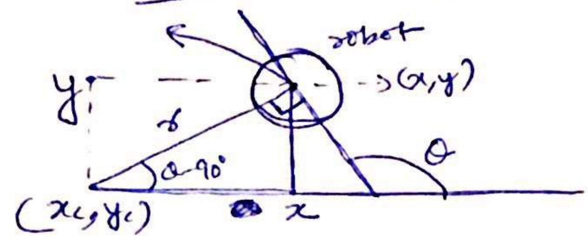


Figure 1.

Controls (v, ω)
State (x, y, θ)

$u = (v, \omega)^T$: control at an instant

Control is fixed for the time-interval ΔT

So, for a rigid body, the robot moves on a circle with radius $r = \left|\frac{V}{\omega}\right|$; in that time-interval.

$(V = \omega r)$ (Euler's theorem of rigid bodies: translation + rotation = pure rotation about some point (x_c, y_c).)

Pure translation : (x_c, y_c) is at infinity.

Let $x_{t-1} = (x, y, \theta)^T$ be pose at time $t-1$.

Let $x_t = (x', y', \theta')^T$ be pose at time t.

From Fig. 1;
$$x_c = x - \frac{V}{\omega}\sin\theta \qquad \text{— (A)}$$
$$y_c = y + \frac{V}{\omega}\cos\theta$$

$$\therefore \begin{pmatrix} x \\ y \\ \theta \end{pmatrix} = \begin{pmatrix} x_c + \frac{V}{\omega}\sin\theta \\ y_c - \frac{V}{\omega}\cos\theta \\ \theta \end{pmatrix}$$

After ΔT, $\theta' = \theta + \omega\Delta T$:
$$\therefore \begin{pmatrix} x' \\ y' \\ \theta' \end{pmatrix} = \begin{pmatrix} x_c + \frac{V}{\omega}\sin(\theta + \omega\Delta T) \\ y_c - \frac{V}{\omega}\cos(\theta + \omega\Delta T) \\ \theta + \omega\Delta T \end{pmatrix}$$

$$\therefore \begin{pmatrix} x' \\ y' \\ \theta' \end{pmatrix} = \begin{pmatrix} x_c + \frac{v}{w}\sin(\theta + w\Delta T) \\ y_c - \frac{v}{w}\cos(\theta + w\Delta T) \\ \theta + w\Delta T \end{pmatrix}$$

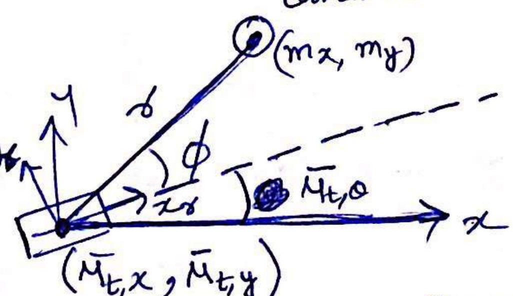
$$= \begin{pmatrix} x - \frac{v}{w}\sin\theta + \frac{v}{w}\sin(\theta + w\Delta T) \\ y + \frac{v}{w}\cos\theta - \frac{v}{w}\cos(\theta + w\Delta T) \\ \theta + w\Delta T \end{pmatrix} \text{ from (A)}$$

$$\therefore \boxed{\begin{pmatrix} x' \\ y' \\ \theta' \end{pmatrix} = \begin{pmatrix} x \\ y \\ \theta \end{pmatrix} + \begin{pmatrix} -\frac{v}{w}\sin\theta + \frac{v}{w}\sin(\theta + w\Delta T) \\ \frac{v}{w}\cos\theta - \frac{v}{w}\cos(\theta + w\Delta T) \\ \theta + w\Delta T \end{pmatrix}}$$

→ Motion model

Sensor model

Landmark in a map 'm'



Range and bearing sensor: Range (r) → distance from landmark

Bearing (ϕ) → Heading in robot's local frame (x_r, y_r)

$$\boxed{\begin{aligned} r &= \sqrt{(m_x - \bar{\mu}_{t,x})^2 + (m_y - \bar{\mu}_{t,y})^2} \\ \phi &= \operatorname{atan2}(m_y - \bar{\mu}_{t,y},\ m_x - \bar{\mu}_{t,x}) - \bar{\mu}_{t,\theta} \end{aligned}}$$

$$z = \begin{pmatrix} r \\ \phi \end{pmatrix}$$ → Sensor model