lecture 1: Bayes Filter Derivation

\[ \text{Bel}(x_t) = \frac{p(x_t | z_{1:t}, u_{1:t})}{p(z_t | z_{1:t-1}, u_{1:t})} \]

\[ = \frac{p(x_t | z_{1:t}, z_{1:t-1}, u_{1:t})}{p(z_t | z_{1:t-1}, u_{1:t}) \cdot p(x_t | z_{1:t-1}, u_{1:t})} \]

\[ = \frac{\eta p(z_t | x_t, z_{1:t-1}, u_{1:t})}{p(z_t | z_{1:t-1}, u_{1:t})} \]

\[ \text{let } p(z_t | z_{1:t-1}, u_{1:t}) = \eta \]

\[ \text{so, } \text{Bel}(x_t) = \eta \frac{p(z_t | x_t, z_{1:t-1}, u_{1:t})}{p(z_t | z_{1:t-1}, u_{1:t})} \]

\[ \text{A) } p(z_t | x_t, z_{1:t-1}, u_{1:t}) \rightarrow p(z_t | x_t) \rightarrow \text{Using Markov Assumption} \]

\[ \text{B) } p(x_t | z_{1:t-1}, u_{1:t}) \rightarrow \int p(x_t | x_{t-1}, z_{1:t-1}, u_{1:t}) \text{d}x_{t-1} \]

\[ \text{Unmarginalization using } p(x) = \int p(x | y) p(y) \text{d}y \]

\[ = \int p(x_t | x_{t-1}, z_{1:t-1}, u_{1:t}) p(x_{t-1} | z_{1:t-1}, u_{1:t}) \text{d}x_{t-1} \]

\[ = \int p(x_t | x_{t-1}, u_t, z_{1:t-1}, u_{1:t}) p(x_{t-1} | z_{1:t-1}, u_{1:t}) \text{d}x_{t-1} \]

\[ \downarrow \text{Using Markov Assumption} \]

\[ = \int p(x_t | x_{t-1}, u_t) \text{Bel}(x_{t-1}) \text{d}x_{t-1} \]

\[ \therefore \text{Bel}(x_t) = \eta \frac{p(z_t | x_t) \int p(x_t | x_{t-1}, u_t) \text{Bel}(x_{t-1}) \text{d}x_{t-1}}{\text{sensor model motion model}} \]