Lecture 1: Bayes Filter Derivation

$$Bel(x_t) = p(x_t \mid z_{1:t}, u_{1:t})$$

$$= p(x_t \mid z_t, z_{1:t-1}, u_{1:t})$$

$$= \frac{p(z_t \mid x_t, z_{1:t-1}, u_{1:t})\, p(x_t \mid z_{1:t-1}, u_{1:t})}{p(z_t \mid z_{1:t-1}, u_{1:t})}$$

let $p(z_t \mid z_{1:t-1}, u_{1:t}) = 1/\eta$

So, $Bel(x_t) = \eta\, \underbrace{p(z_t \mid x_t, z_{1:t-1}, u_{1:t})}_{A}\, \underbrace{p(x_t \mid z_{1:t-1}, u_{1:t})}_{B}$

(A) $p(z_t \mid x_t, z_{1:t-1}, u_{1:t}) \rightarrow p(z_t \mid x_t)$ → Using Markov Assumption

(B) $p(x_t \mid z_{1:t-1}, u_{1:t}) \rightarrow \int p(x_t, x_{t-1} \mid z_{1:t-1}, u_{1:t})\, dx_{t-1}$

(Unmarginalization using: $P(x) = \int p(x,y)\,dy$)

$$\therefore \int p(x_t, x_{t-1} \mid z_{1:t-1}, u_{1:t})\, dx_{t-1}$$

$$= \int p(x_t \mid x_{t-1}, z_{1:t-1}, u_{1:t})\, p(x_{t-1} \mid z_{1:t-1}, u_{1:t})\, dx_{t-1}$$

$[P(x,y) = P(x \mid y) P(y)]$

$$= \int p(x_t \mid x_{t-1}, u_t, z_{1:t-1}, u_{1:t-1})\, p(x_{t-1} \mid z_{1:t-1}, u_{1:t-1})\, dx_{t-1}$$

↓ Using Markov Assumption

[∵ u_t does not affect x_{t-1}]

$$= \int p(x_t \mid x_{t-1}, u_t)\, Bel(x_{t-1})\, dx_{t-1}$$

$$\therefore Bel(x_t) = \eta\, \underbrace{p(z_t \mid x_t)}_{\text{sensor model}} \int \underbrace{p(x_t \mid x_{t-1}, u_t)}_{\text{motion model}}\, Bel(x_{t-1})\, dx_{t-1}$$