CONFIGURATION SPACE

Configuration: \( q \in C \)

Everything we need to describe where the robot is now.

\[ A(q) : C \to W \to \text{world} = \mathbb{R}^2 \text{ or } \mathbb{R}^3 \]

Example:

\[ q = (x, y) \in \mathbb{R}^2 \quad A(q) = \bigcirc \]

\[ q = (x, y, \theta) \in SE(2) \quad A(q) = \]

\[ q = (q_1, q_2) \in S^2 \quad A(q) = \]

\[ q = (x, y, \theta, R) \in SE(3) \quad A(q) = \]

\( C \): configuration space

Any \( q \) in \( C \) is a point in \( C \)

Define obstacle region \( O \subset W \)

\[ \text{Cobs} = \{ q \in C \mid A(q) \cap O \neq \emptyset \} \]

\( \text{free} = C \setminus \text{Cobs} \)

Technical note: \( \text{Cobs} \) is a closed set

\( \text{Cobs} \) includes \( O \) and \( A(q) \) touching

\( \implies \) free is an open set

\( \implies \) No touching

\( \text{If we want to allow touching, then?} \)
Piano Movers' Problem

1. A world $W = \mathbb{R}^2$ or $\mathbb{R}^3$
2. An obstacle region $O \subseteq W$
3. A robot $A$ and its configuration space $C$
4. $C_{\text{free}} = C \setminus O$
5. $q_i \in C_{\text{free}}$ INITIAL CONFIG
6. $q_f \in C_{\text{free}}$ GOAL CONFIG
7. Compute a path $Z: [0, 1] \to C_{\text{free}}$ such that $q_i = Z(0)$, $q_f = Z(1)$
**GRAPH-BASED ALGORITHMS**

1. Create a graph \( G(V, E) \) with vertices and edges.

2. Plan a path in \( G \).

**IMPLICIT vs. EXPLICIT GRAPH**

- Graph only accessible via `succ` function
- `succ` returns successors

\[
succ(u) = \{ (u, v) \mid (u, v) \in E \}\]

**KEY CHALLENGE:** Collision checking > 90% of planning effort

**KEY IDEA:** Perform edge evaluations *LAZILY*

\[
c(e) = \text{dist}(u, v) \text{ if not in collision}\]

Cost of an edge \( = \infty \) if in collision.