CSE-571
Robotics

SLAM: Simultaneous Localization and Mapping

Many slides courtesy of Ryan Eustice, Cyrill Stachniss, John Leonard, Dieter Fox

The SLAM Problem

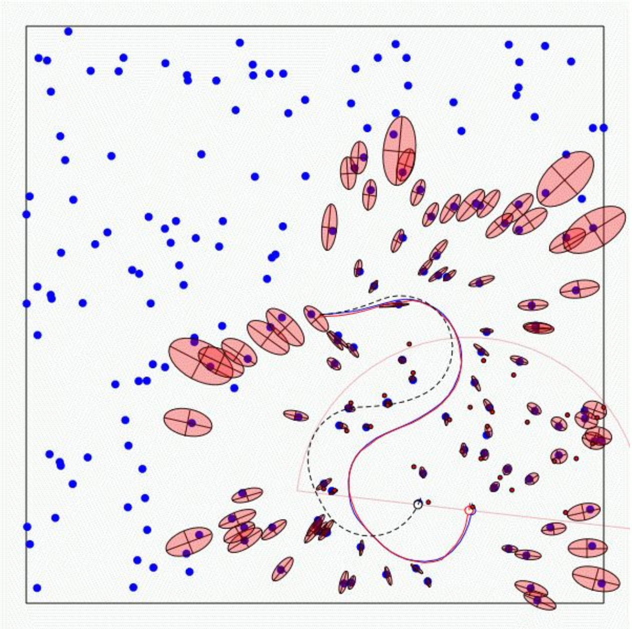
A robot is exploring an unknown, static environment.

Given:

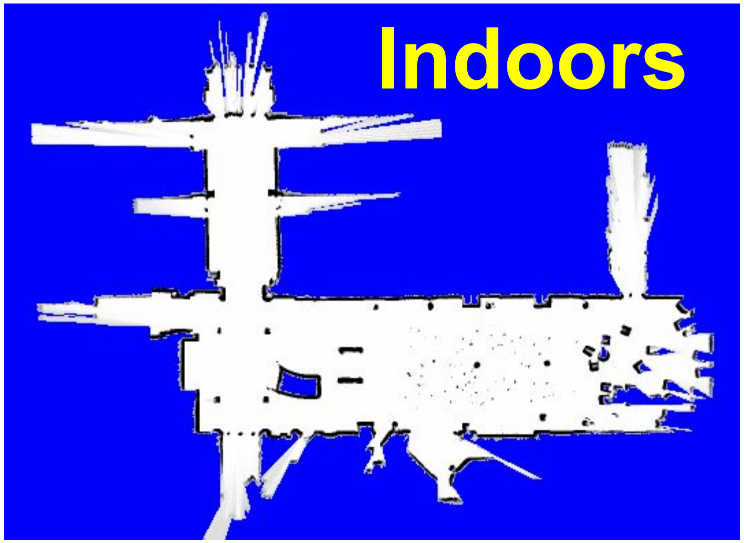
- The robot's controls
- Observations of nearby features

Estimate:

- Map of features
- Path of the robot



SLAM Applications



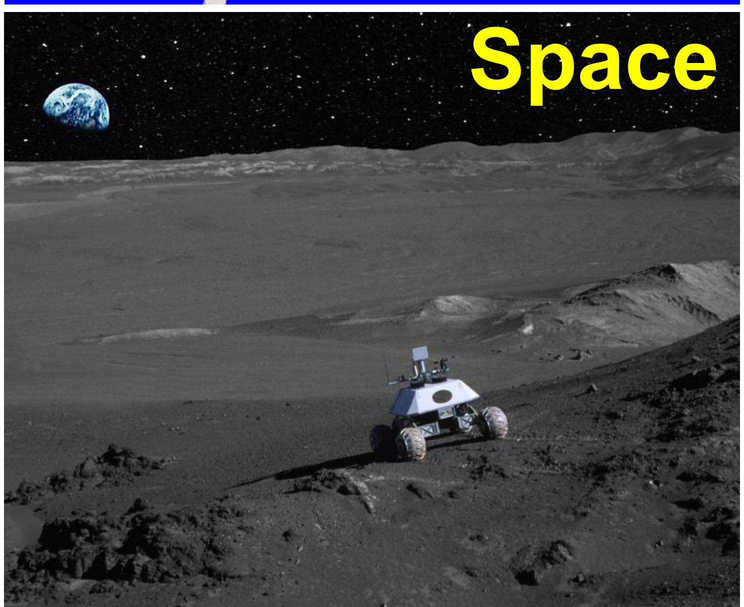
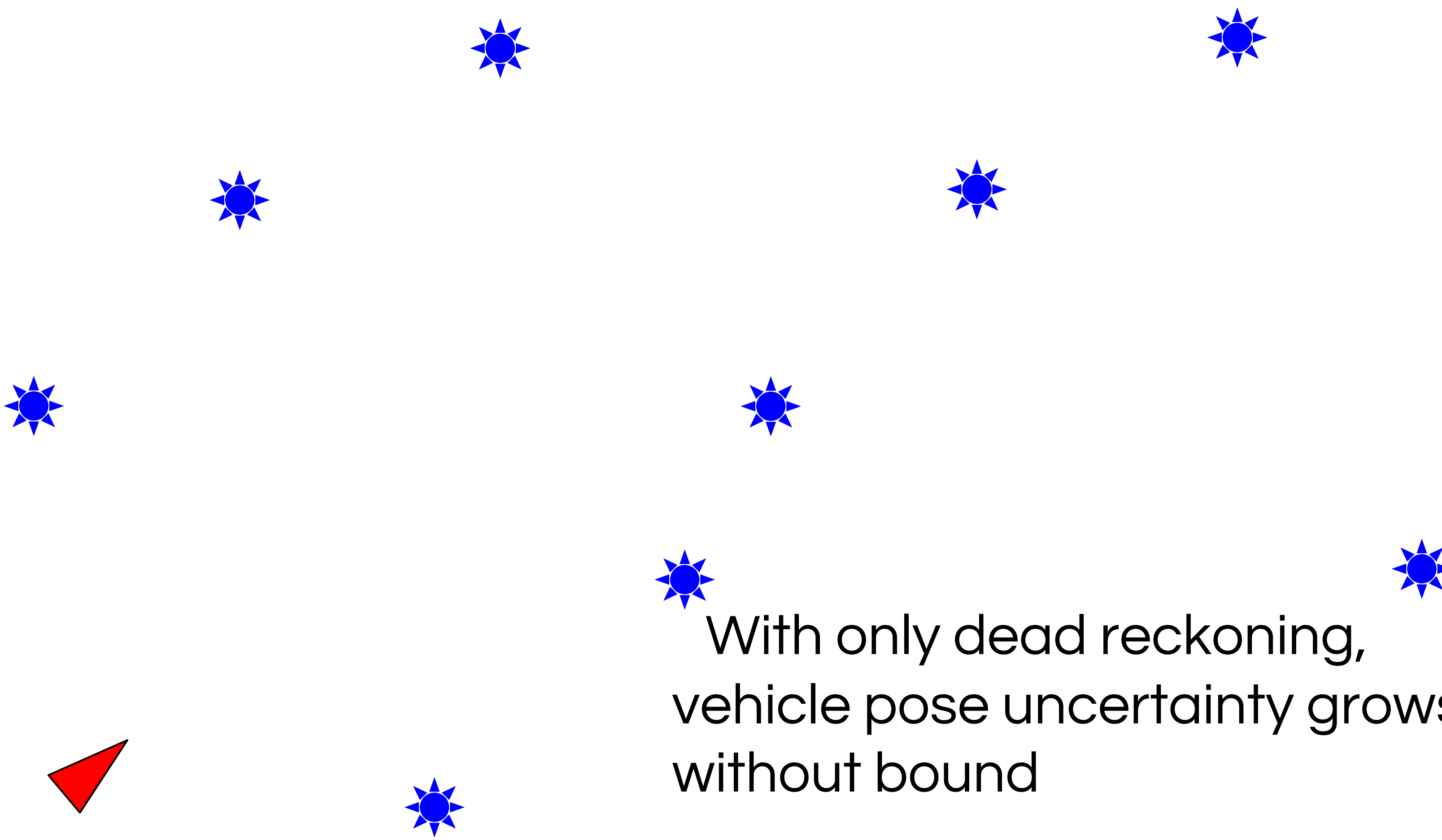
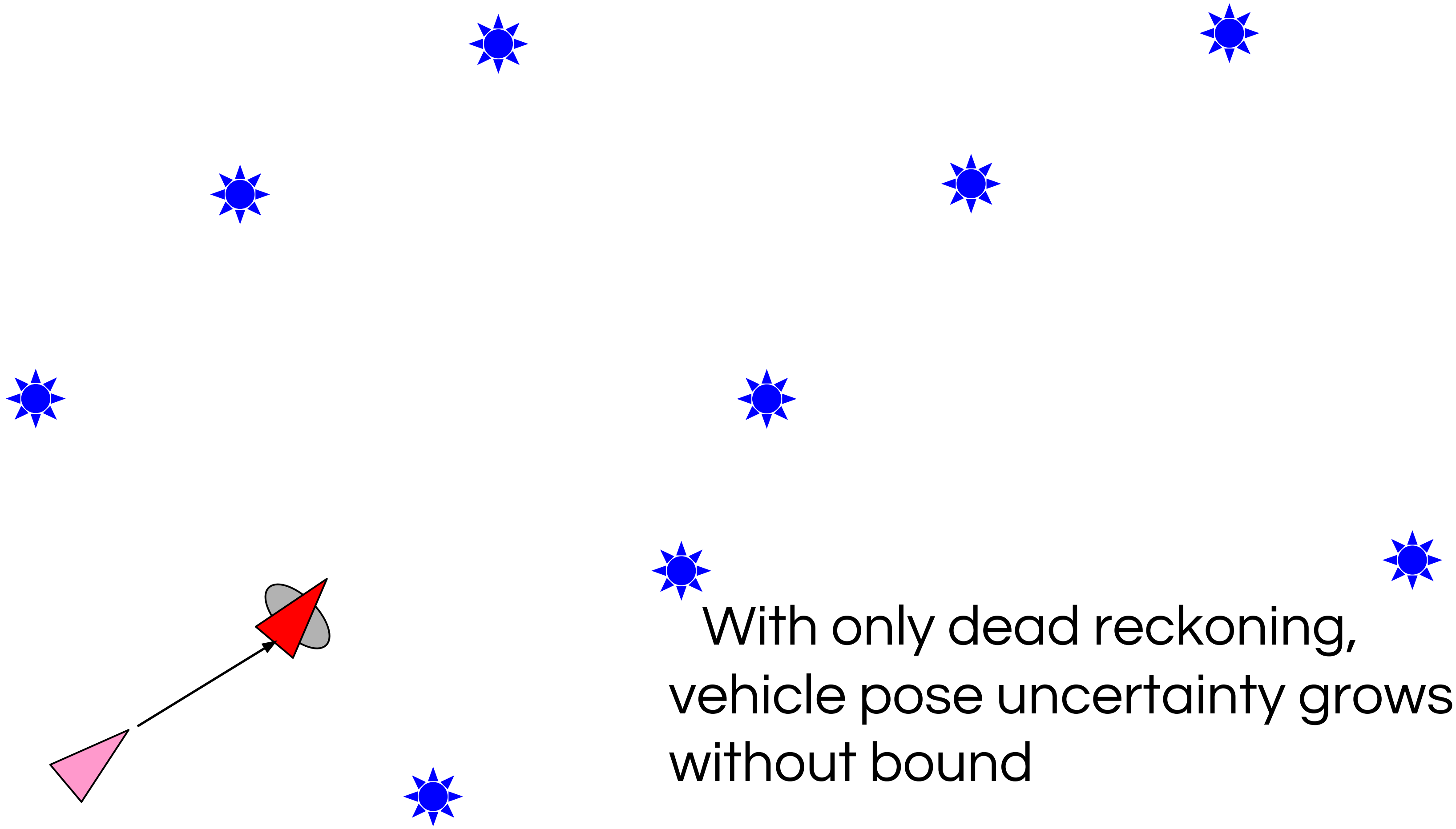


Illustration of SLAM without Landmarks



With only dead reckoning, vehicle pose uncertainty grows without bound

Illustration of SLAM without Landmarks



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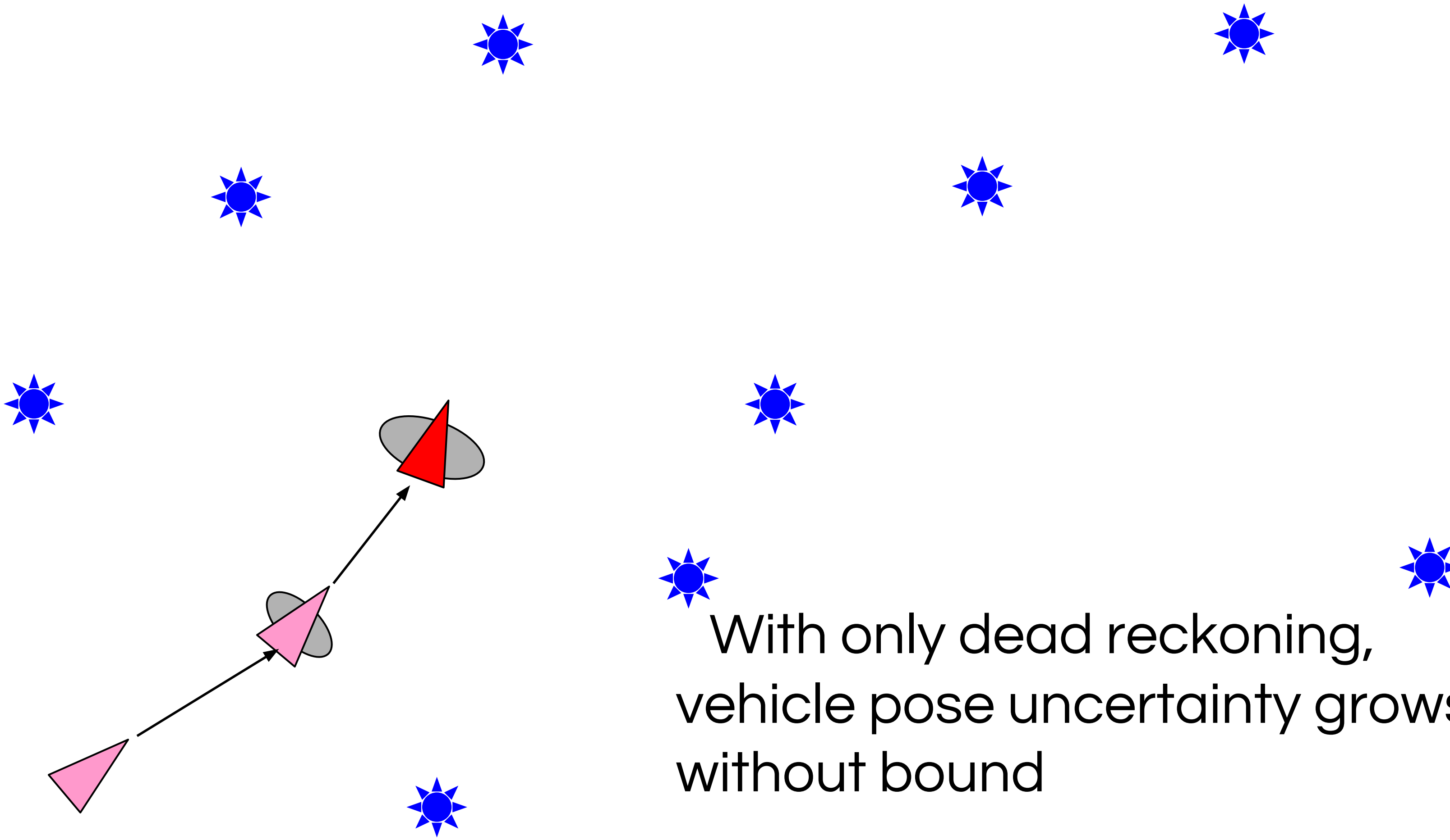

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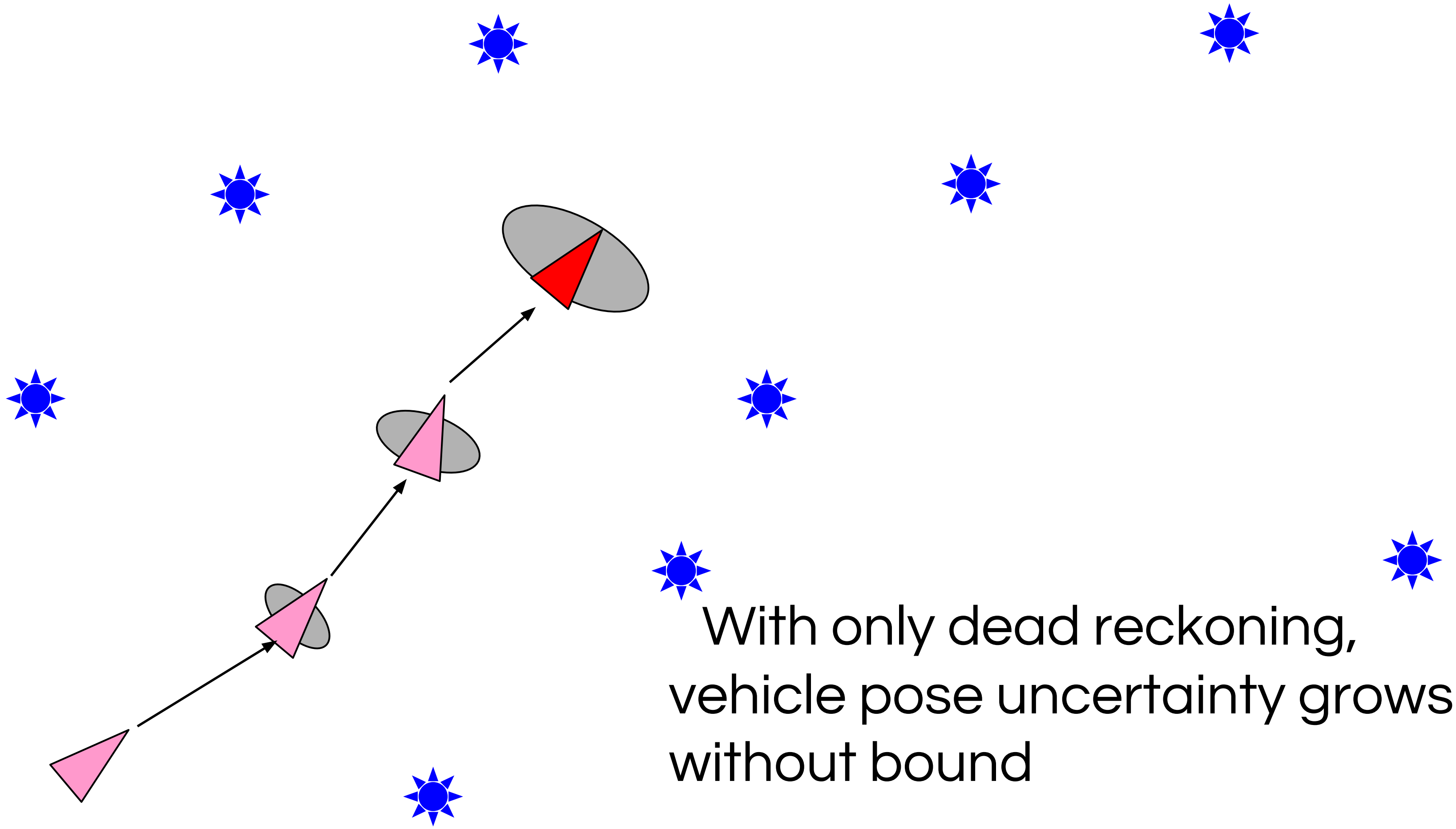
Illustration of SLAM without Landmarks



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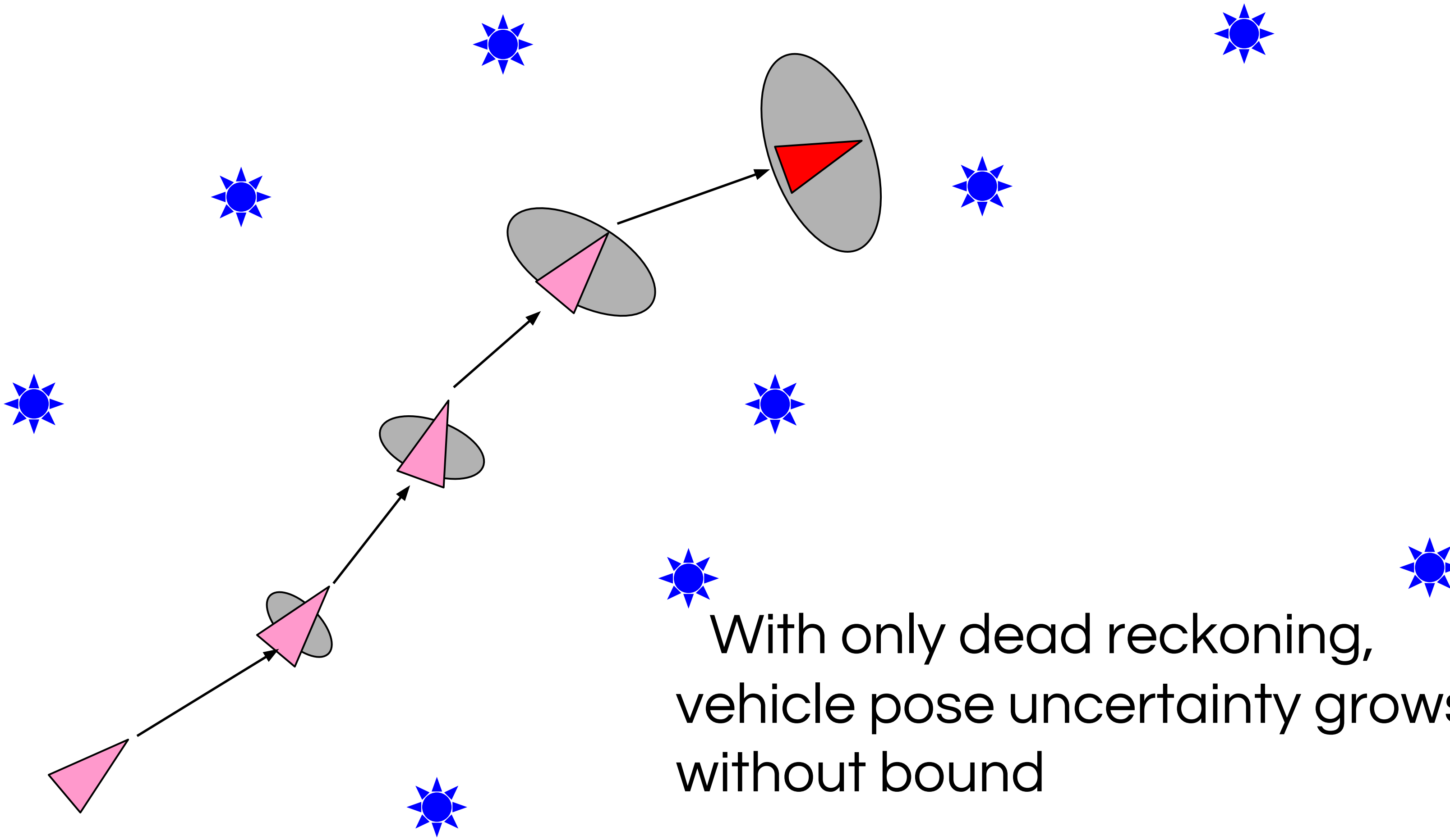

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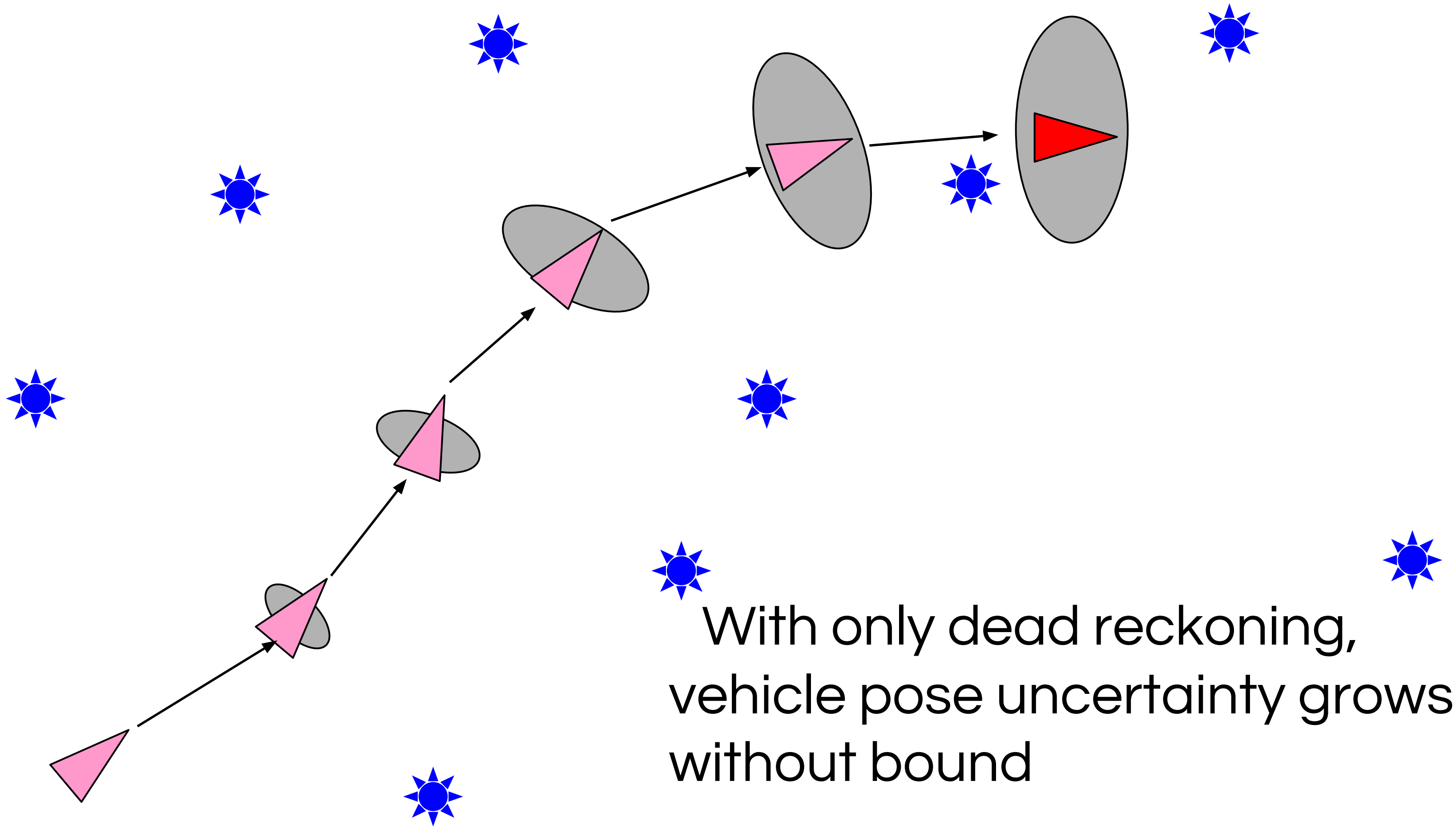
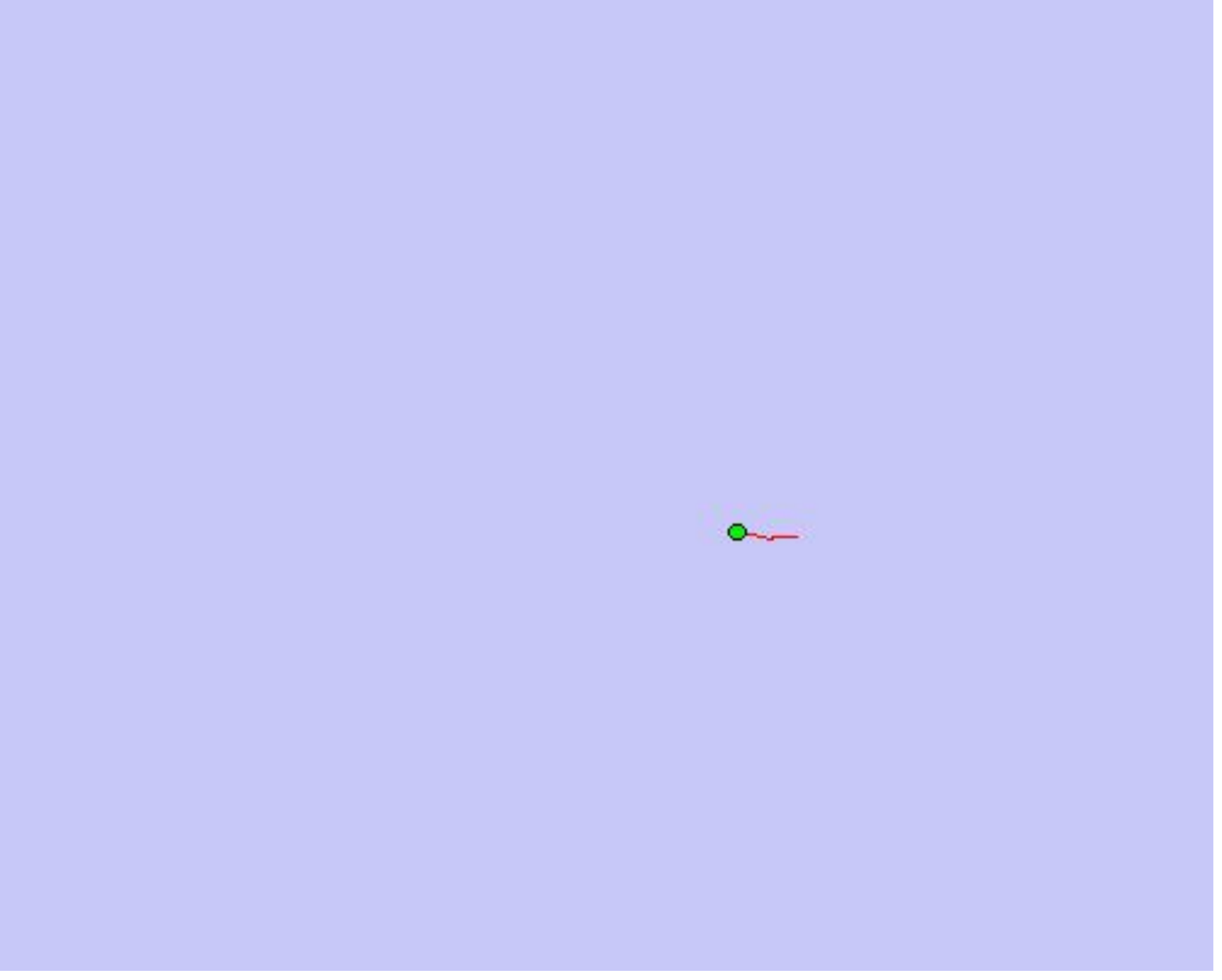
Illustration of SLAM without Landmarks

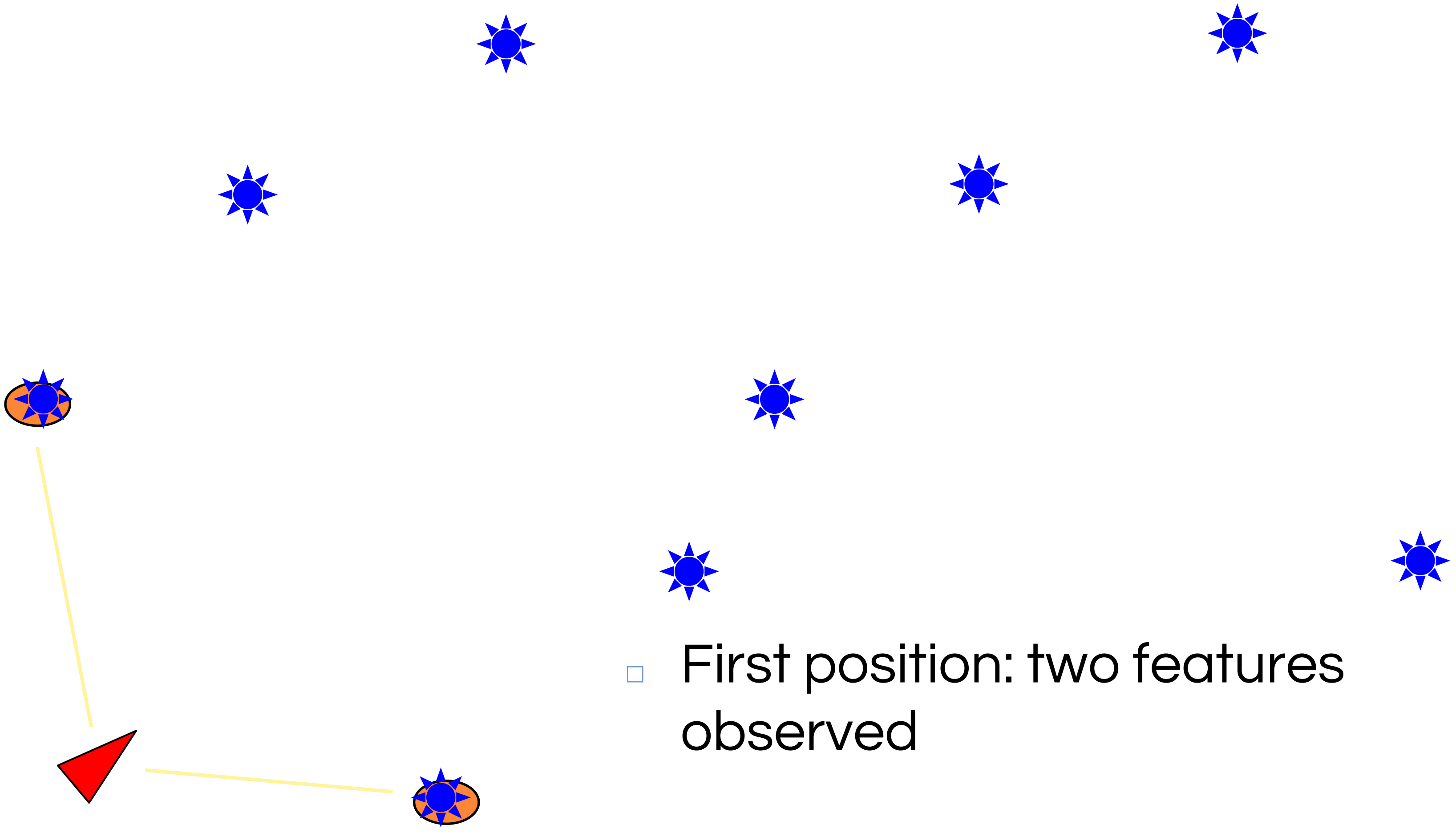


With only dead reckoning, vehicle pose uncertainty grows without bound

Mapping with Raw Odometry

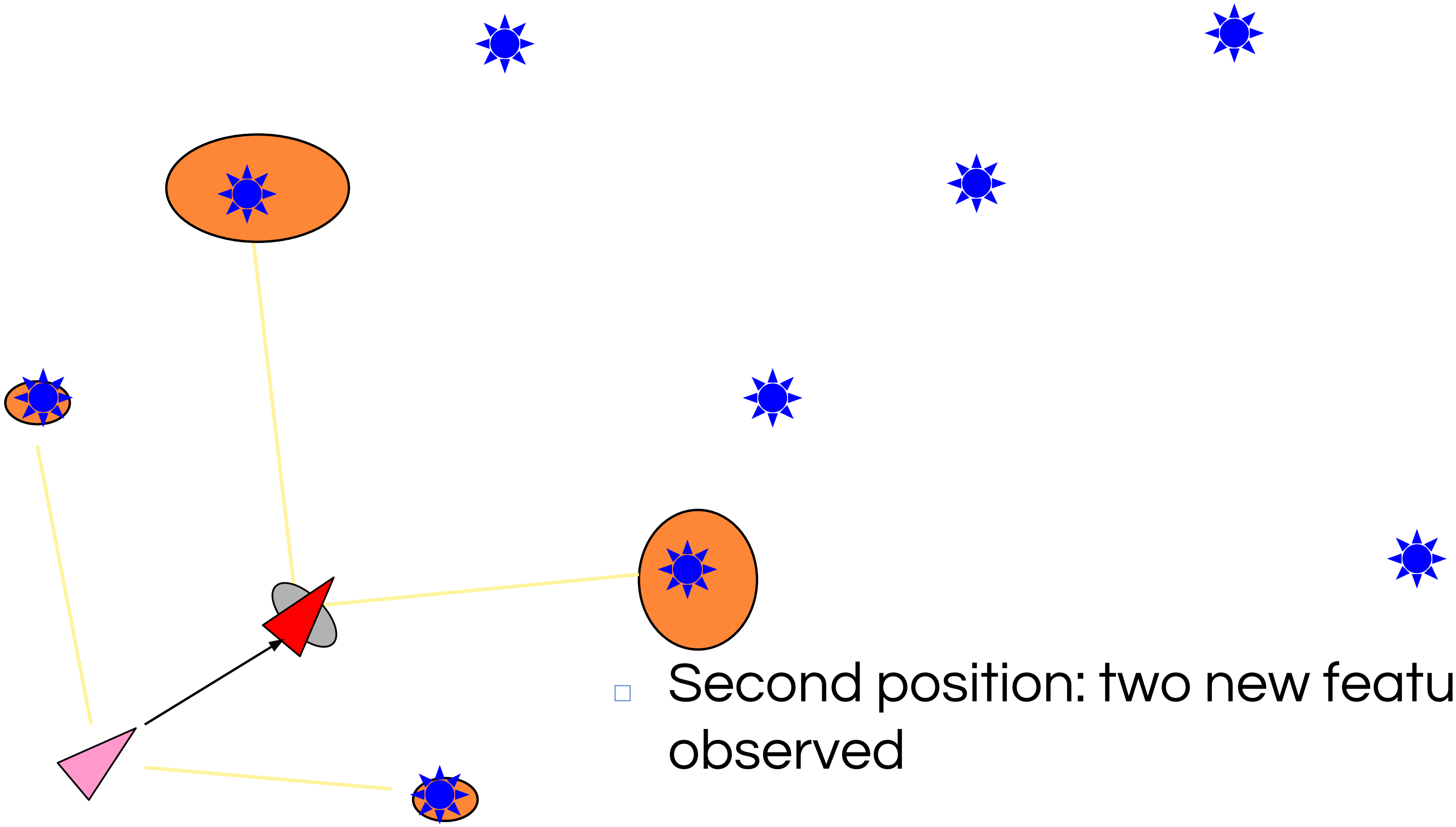


Repeat, with Measurements of Landmarks



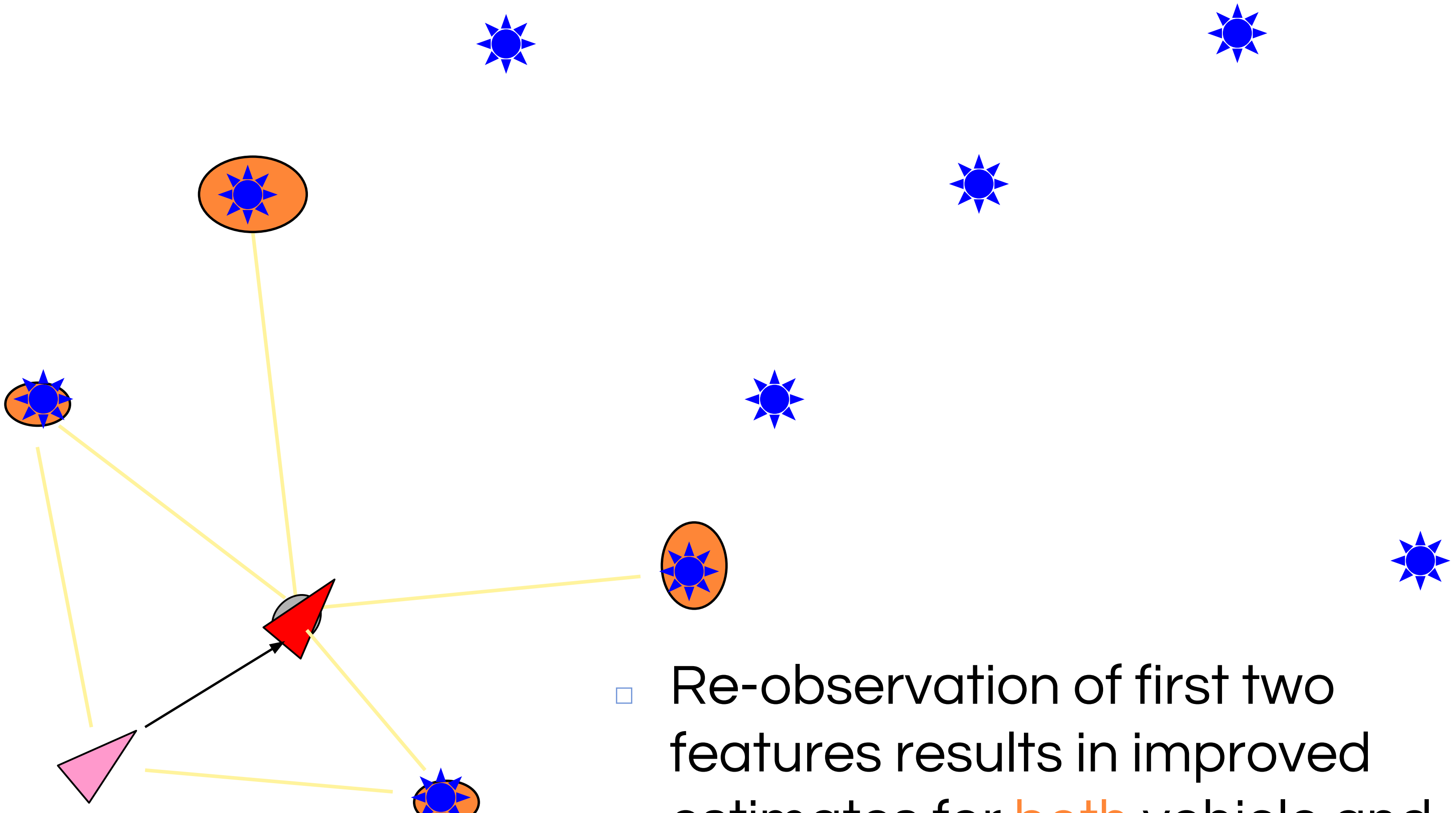
- First position: two features observed

Illustration of SLAM with Landmarks



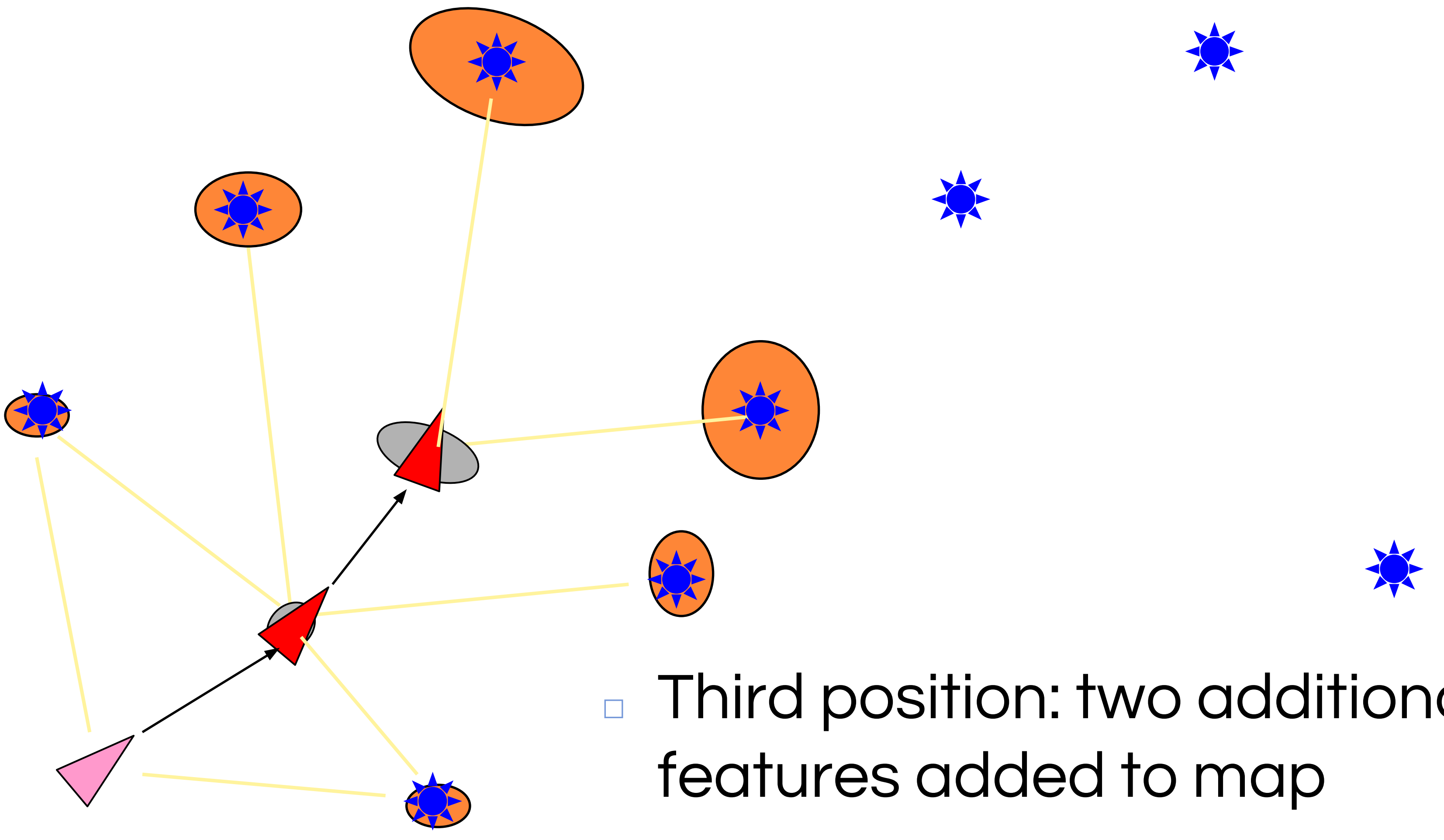
- Second position: two new features observed

Illustration of SLAM with Landmarks



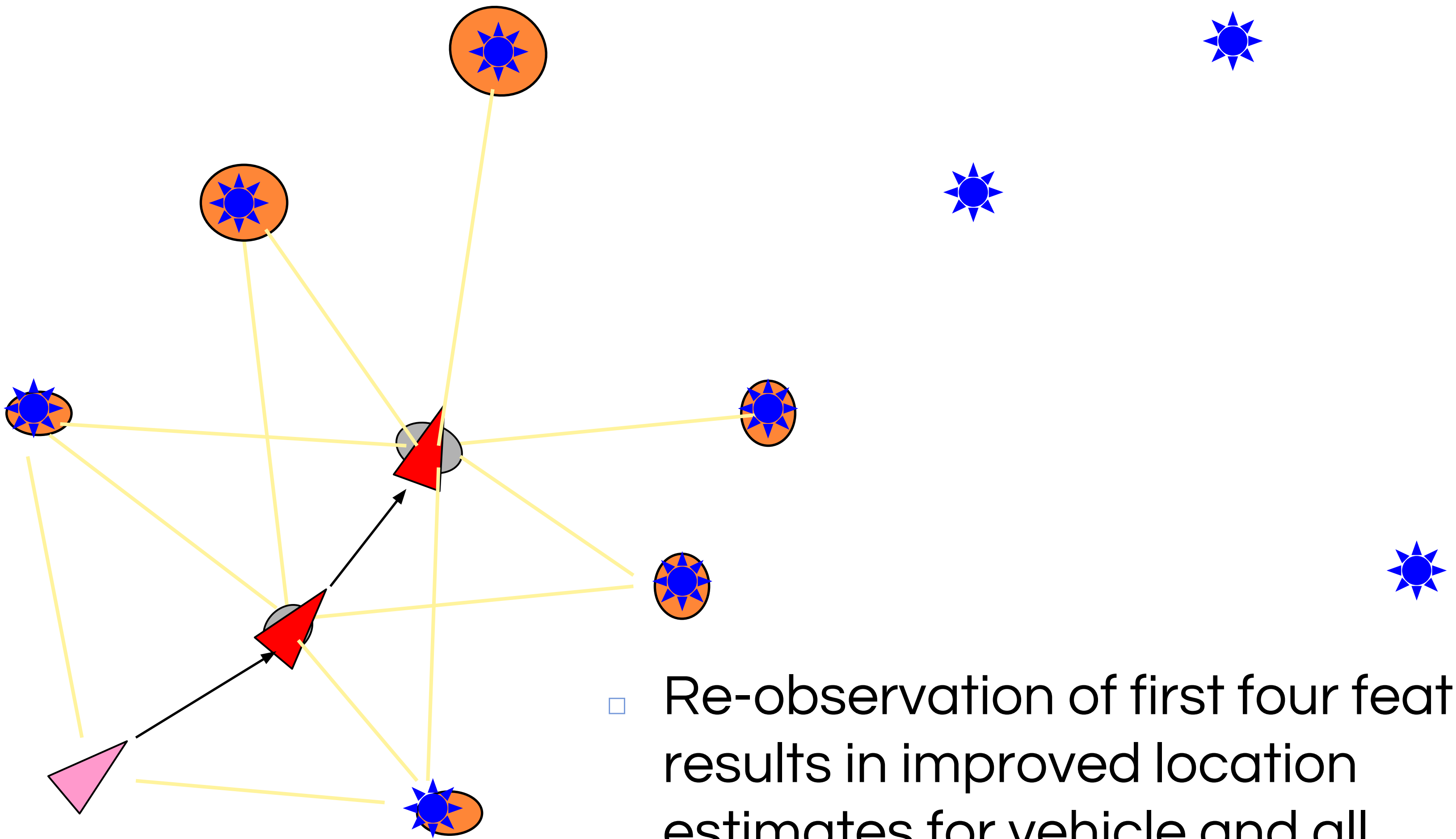
- Re-observation of first two features results in improved estimates for both vehicle <u>and</u> feature

Illustration of SLAM with Landmarks



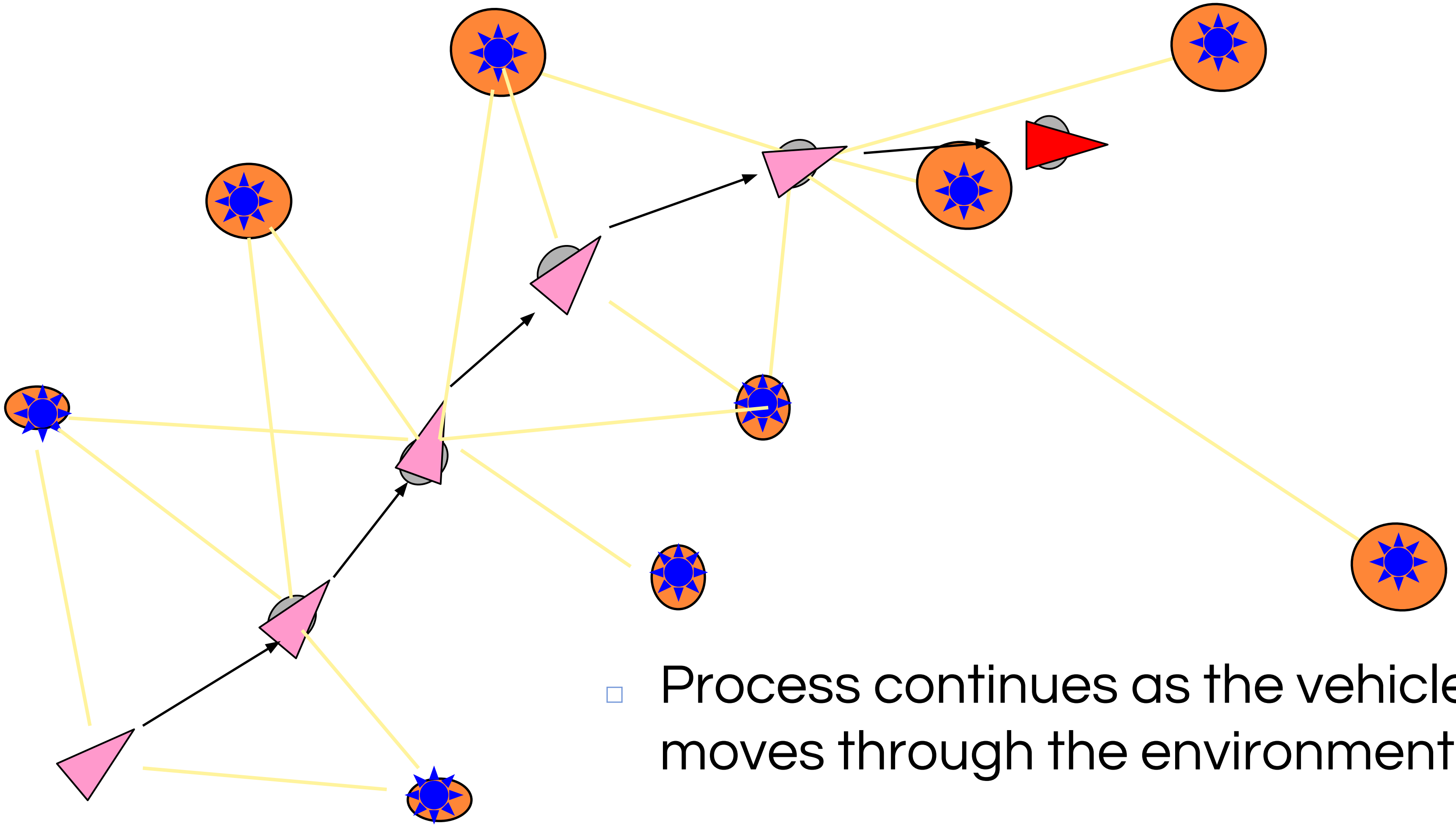
- Third position: two additional features added to map

Illustration of SLAM with Landmarks



- Re-observation of first four features results in improved location estimates for vehicle and all features

Illustration of SLAM with Landmarks



- Process continues as the vehicle moves through the environment

What is SLAM?

Localization: Estimate current pose, given map, controls, and observations

$$p(x_t | u_{1:t}, z_{1:t}, m)$$

Mapping: Build map given poses and observations

$$p(m | x_{1:t}, z_{1:t})$$

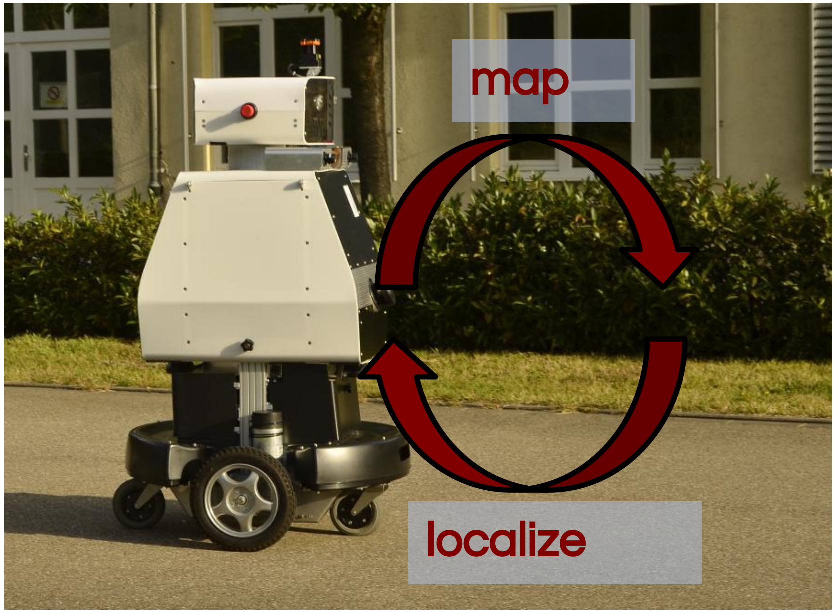
Simultaneous Mapping and Localization (SLAM):

Find poses and map given controls and observations

$$p(x_{1:t}, m | u_{1:t}, z_{1:t})$$

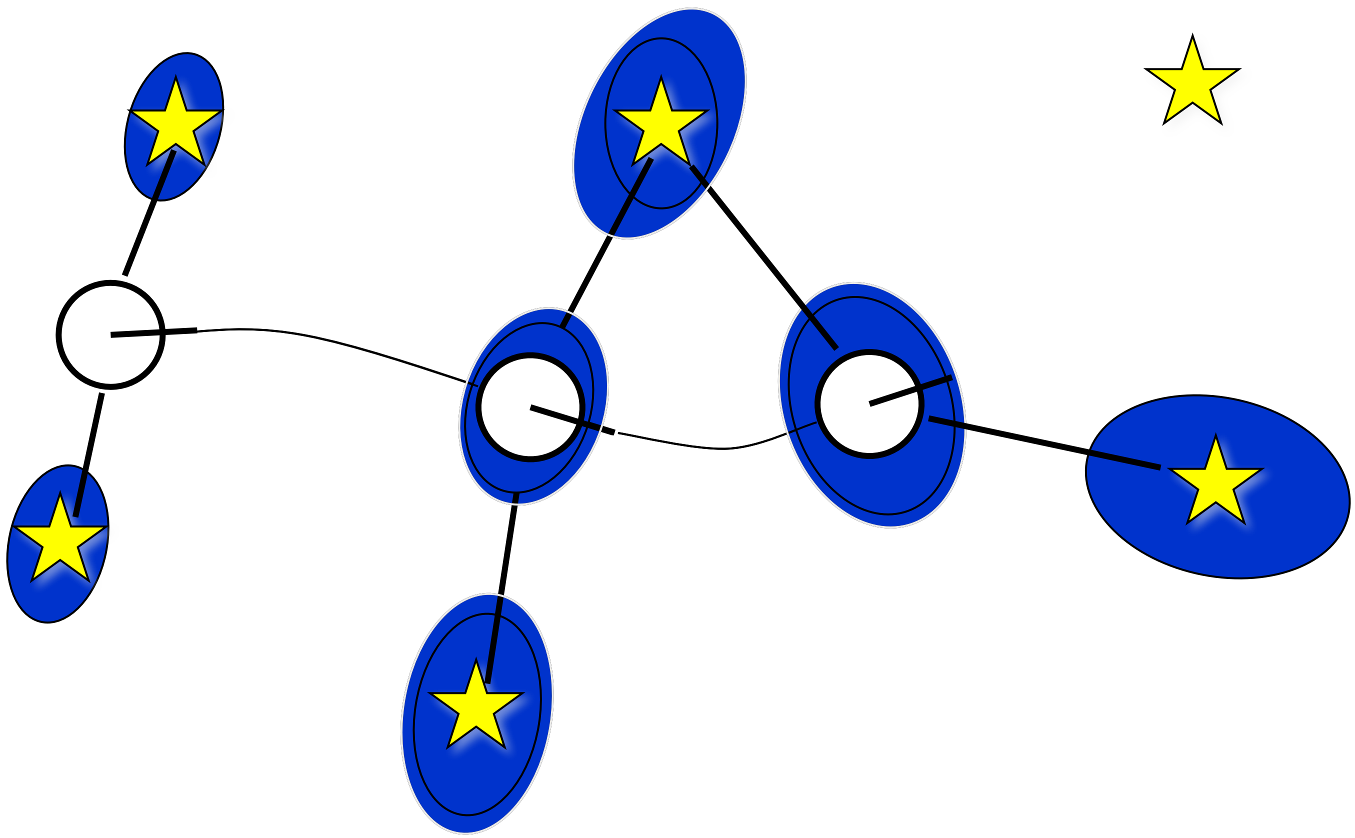
Simultaneous Localization and Mapping (SLAM)

- Building a map and locating the robot in the map at the same time
- Chicken-and-egg problem



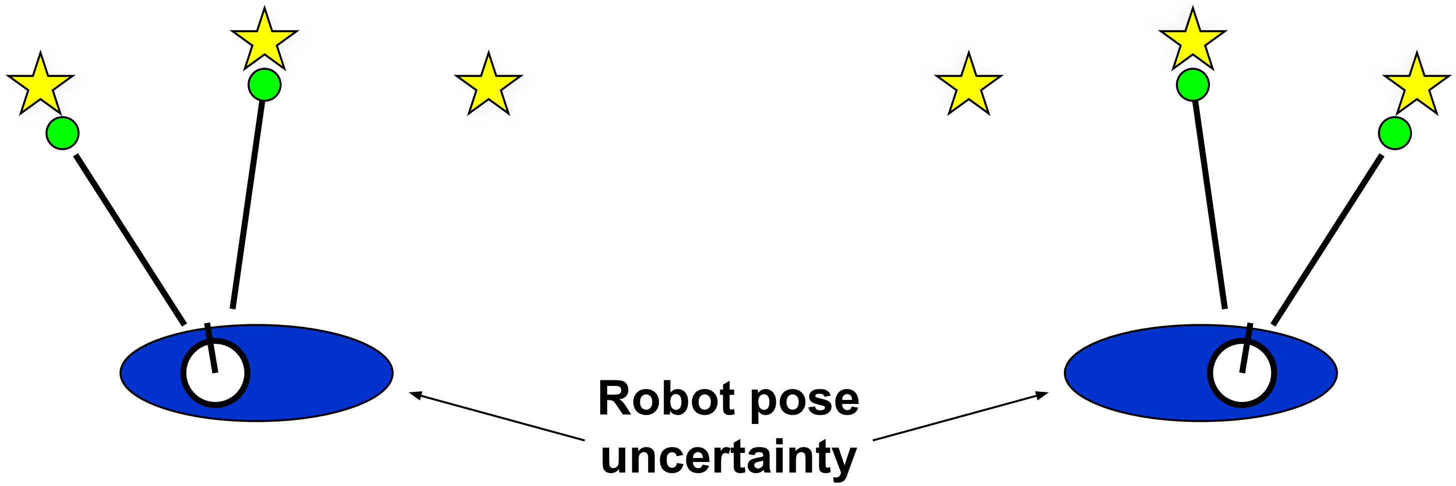
Why is SLAM a hard problem?

SLAM: robot path and map are both **unknown**



Robot path error correlates errors in the map

Why is SLAM a hard problem?



- In the real world, the mapping between observations and landmarks is unknown
- Picking wrong data associations can have catastrophic consequences
- Pose error correlates data associations

SLAM:

Simultaneous Localization and Mapping

- Full SLAM:

Estimates entire path and map!

$$p(x_{1:t}, m \mid z_{1:t}, u_{1:t})$$

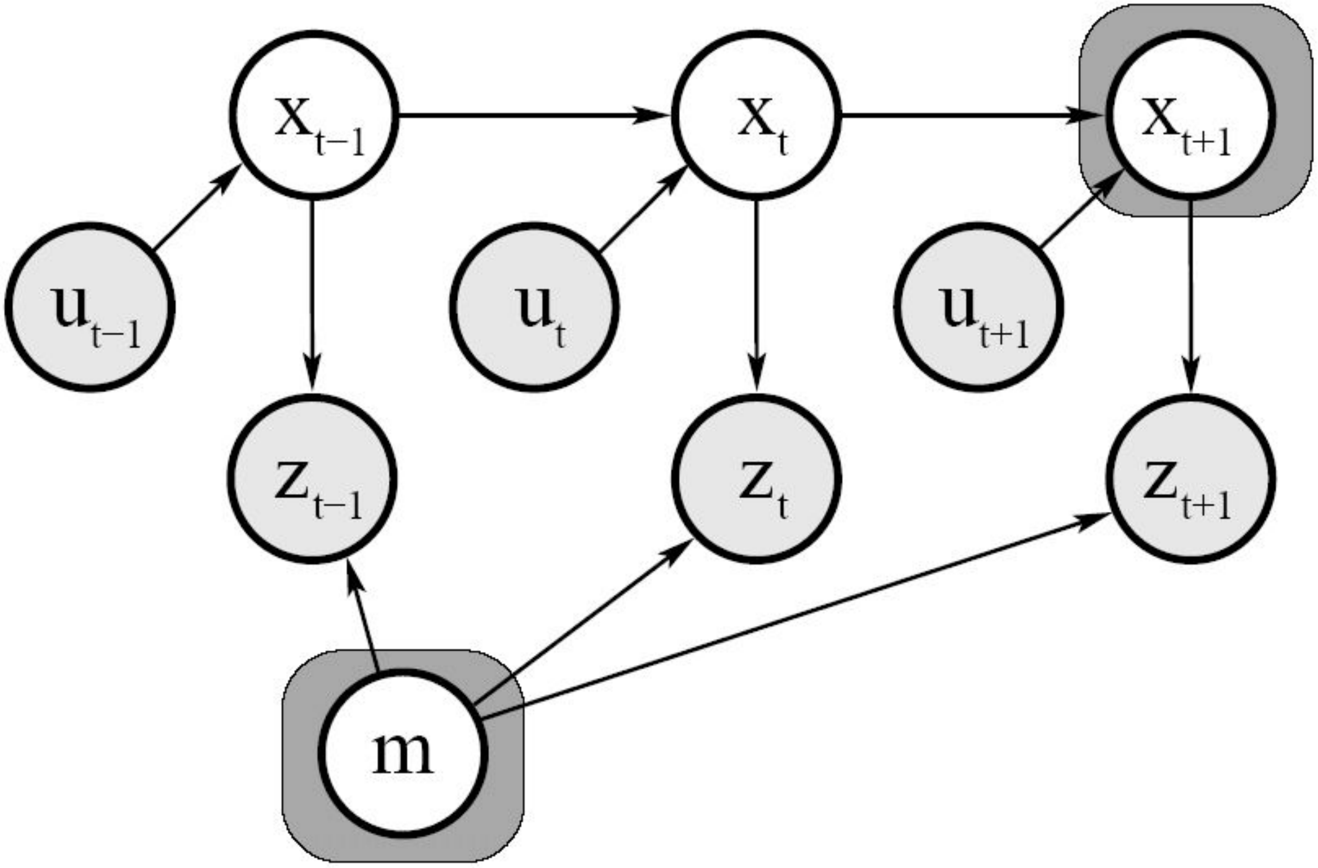
- Online SLAM:

$$p(x_t, m \mid z_{1:t}, u_{1:t}) = \int\int \cdots \int p(x_{1:t}, m \mid z_{1:t}, u_{1:t}) \, dx_1 dx_2 \ldots dx_{t-1}$$

Integrations typically done one at a time

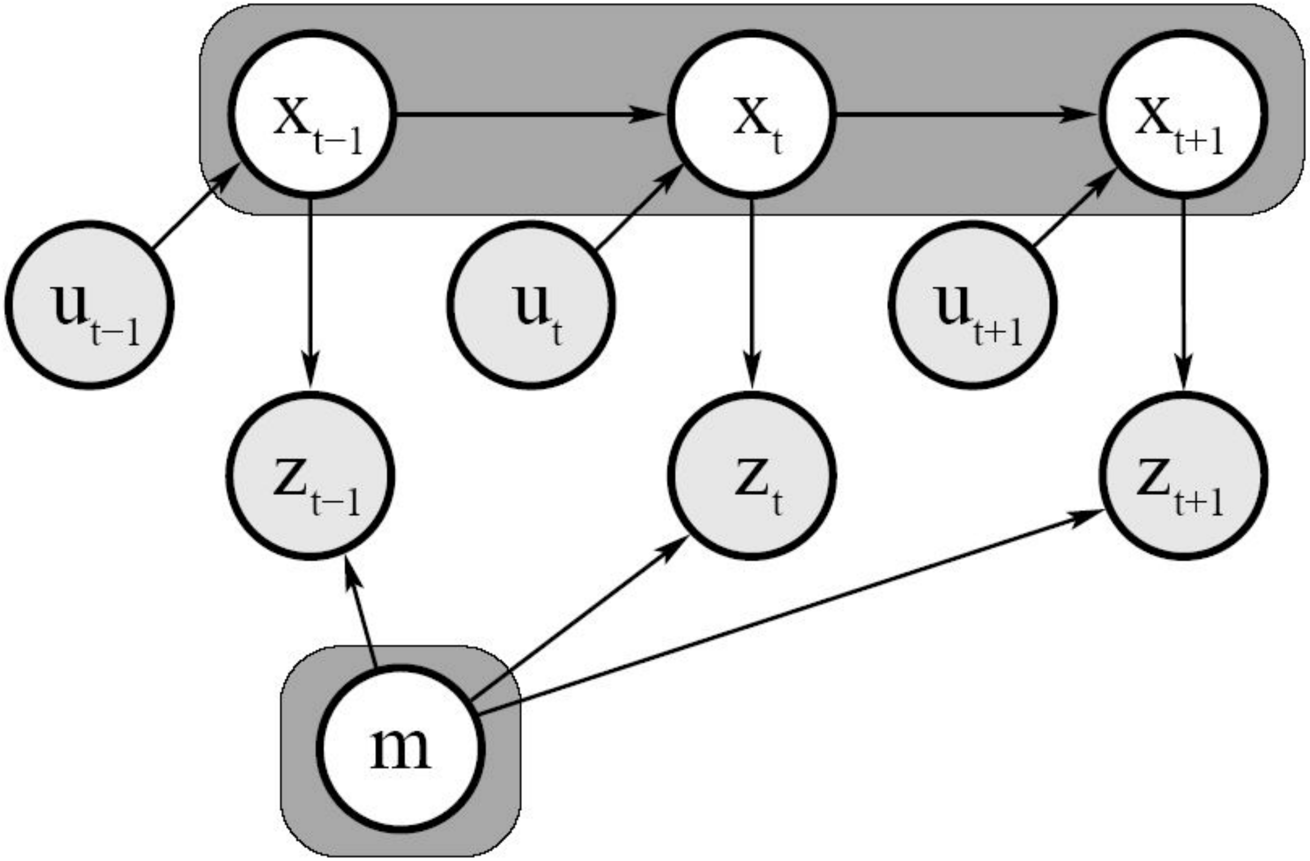
Estimates most recent pose and map!

Graphical Model of Online SLAM:



$$p(x_t, m \mid z_{1:t}, u_{1:t}) = \int\int \cdots \int p(x_{1:t}, m \mid z_{1:t}, u_{1:t})\, dx_1\, dx_2 \ldots dx_{t-1}$$

Graphical Model of Full SLAM:



$$p(x_{1:t}, m \mid z_{1:t}, u_{1:t})$$

Three Main Paradigms

Kalman filter

Particle filter

Graph-based

Bayes Filter

- Recursive filter with prediction and correction step

- Prediction

$$\overline{bel}(x_t) = \int p(x_t \mid u_t, x_{t-1}) \; bel(x_{t-1}) \, dx_{t-1}$$

- Correction

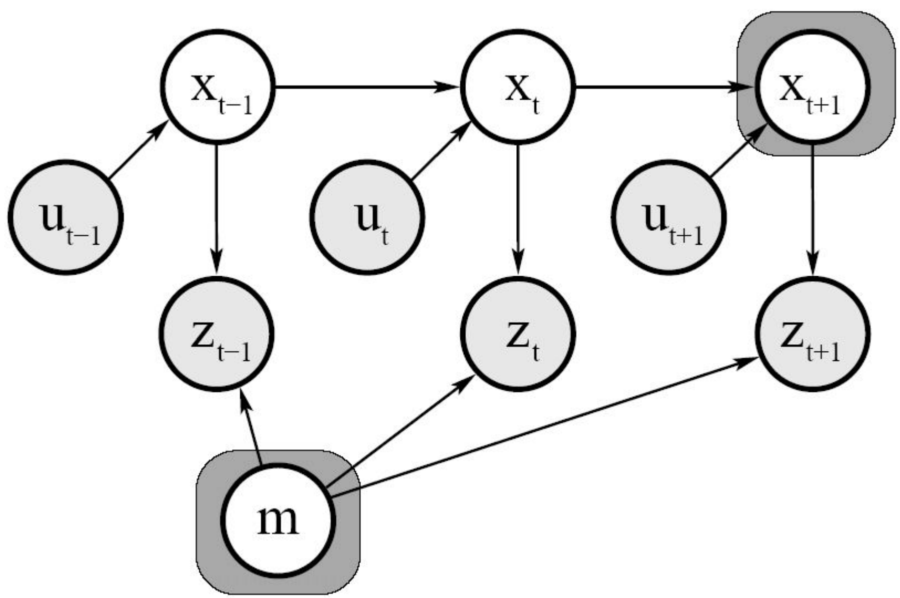
$$bel(x_t) = \eta \, p(z_t \mid x_t) \; \overline{bel}(x_t)$$

Courtesy: Cyrill

EKF for Online SLAM

- We consider here the Kalman filter as a solution to the online SLAM problem

$$p(x_t, m \mid z_{1:t}, u_{1:t})$$



Courtesy: Thrun, Burgard, Fox

Extended Kalman Filter Algorithm

1: **Extended_Kalman_filter**$(\mu_{t-1}, \Sigma_{t-1}, u_t, z_t)$**:**

2: $\bar{\mu}_t = g(u_t, \mu_{t-1})$

3: $\bar{\Sigma}_t = G_t \, \Sigma_{t-1} \, G_t^T + R_t$

4: $K_t = \bar{\Sigma}_t \, H_t^T (H_t \, \bar{\Sigma}_t \, H_t^T + Q_t)^{-1}$

5: $\mu_t = \bar{\mu}_t + K_t(z_t - h(\bar{\mu}_t))$

6: $\Sigma_t = (I - K_t \, H_t) \, \bar{\Sigma}_t$

7: $return \; \mu_t, \Sigma_t$

Courtesy: Cyrill

EKF SLAM

- Application of the EKF to SLAM
- Estimate robot's pose and locations of landmarks in the environment
- Assumption: known correspondences
- State space (for the 2D plane) is

$$x_t = (\underbrace{x, y, \theta}_{\text{robot's pose}}, \underbrace{m_{1,x}, m_{1,y}}_{\text{landmark 1}}, \ldots, \underbrace{m_{n,x}, m_{n,y}}_{\text{landmark n}})^T$$

Courtesy: Cyrill

Literature

EKF SLAM

- Thrun et al.: “Probabilistic Robotics”, Chapter 10
- Smith, Self, & Cheeseman: “Estimating Uncertain Spatial Relationships in Robotics”
- Dissanayake et al.: “A Solution to the Simultaneous Localization and Map Building (SLAM) Problem”
- Durrant-Whyte & Bailey: “SLAM Part 1” and “SLAM Part 2” tutorials

Three Main Paradigms

Kalman filter

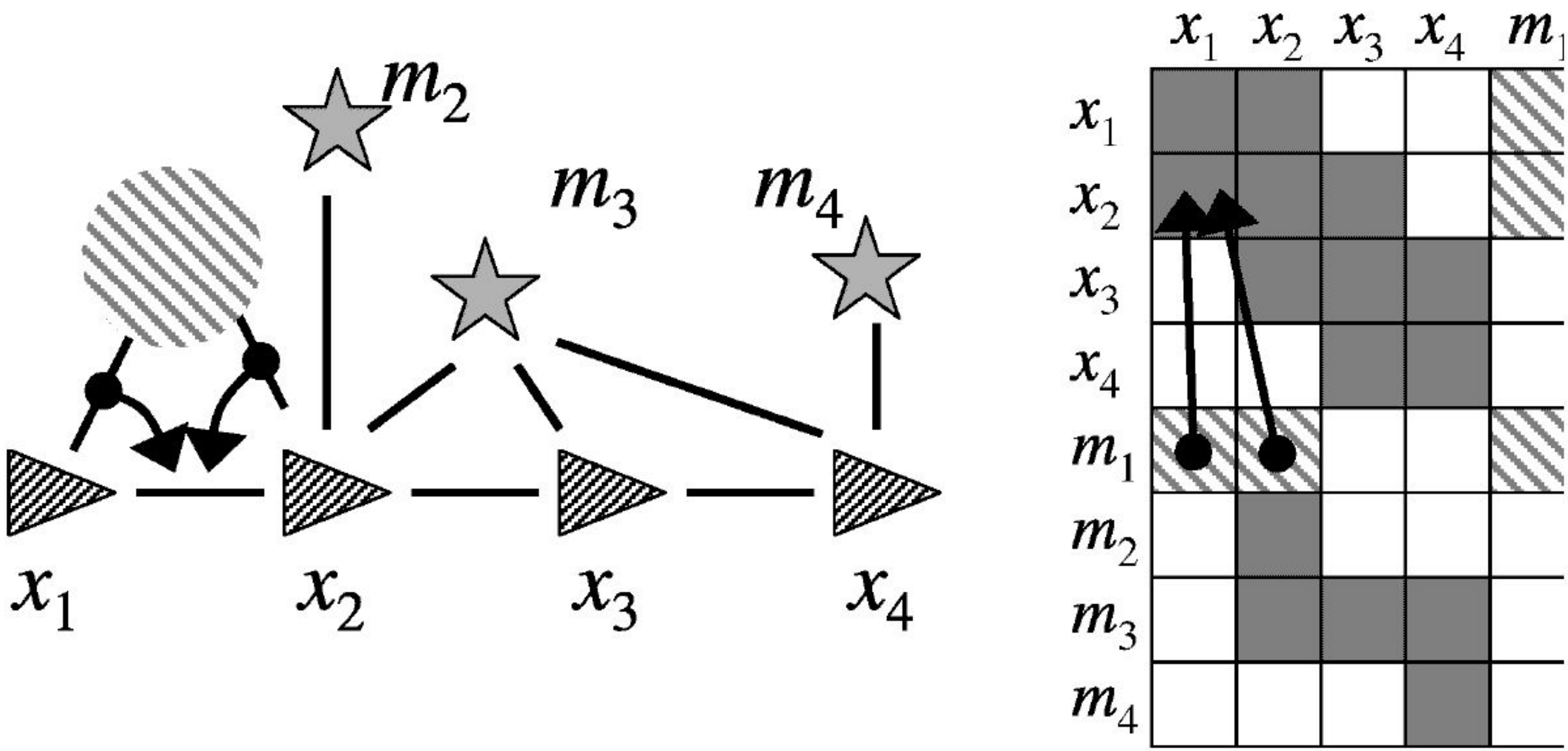
Particle filter

Graph-based

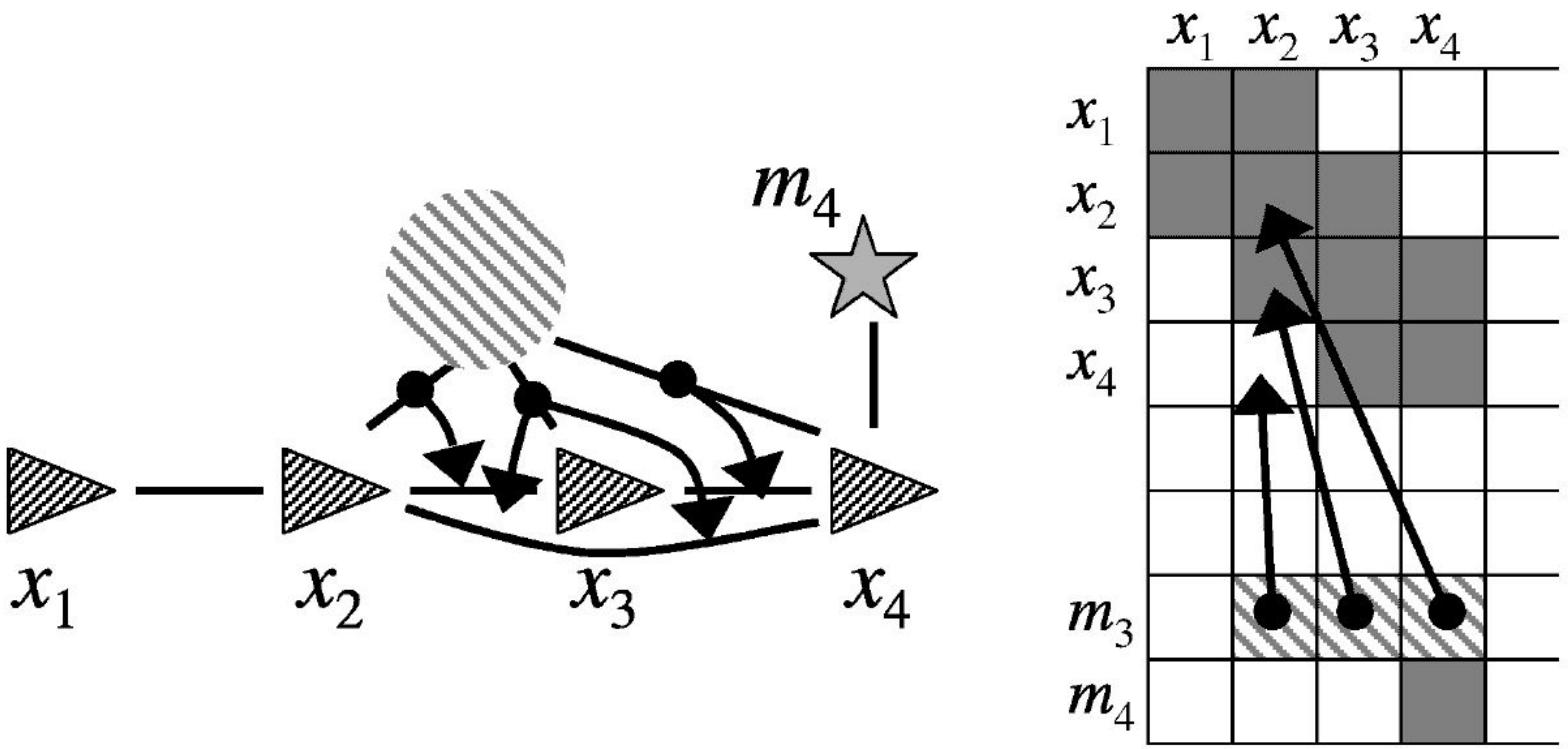
Graph-SLAM

- Full SLAM technique
- Generates probabilistic links
- Computes map only occasionally
- Based on Information Filter form

Graph-SLAM Inference (1)



Graph-SLAM Inference (2)



Graph-SLAM Summary

- Adresses full SLAM problem
- Constructs link graph between poses and poses/landmarks
- Graph is sparse: number of edges linear in number of nodes
- Inference performed by building information matrix and vector (linearized form)
- Map recovered by reduction to robot poses, followed by conversion to moment representation, followed by estimation of landmark positions
- ML estimate by minimization of $J_{GraphSLAM}$
- Data association by iterative greedy search

Three Main Paradigms

Kalman filter	Particle filter	Graph-based

Particle Filters

- Represent belief by random samples
- Sampling Importance Resampling (SIR) principle
  - Draw the new generation of particles
  - Assign an importance weight to each particle
  - Resampling
- Applications are localization, tracking, …

Particle Filter Algorithm

1. Sample the particles from the proposal distribution

$$x_t^{[j]} \sim \pi(x_t \mid \ldots)$$

2. Compute the importance weights

$$w_t^{[j]} = \frac{target(x_t^{[j]})}{proposal(x_t^{[j]})}$$

1. Resampling: Draw sample i with probability $w_t^{[i]}$ and repeat J times

Courtesy: C. Stachniss

Particle Filters for SLAM

- Localization: state space is $<x, y, \theta>$
- SLAM: state space is $<x, y, \theta, map>$
  - For grid maps: $<c_{11}, c_{12}, \ldots, c_{1n}, c_{21}, \ldots, c_{nm}>$
  - For feature maps: $<l_1, l_2, \ldots, l_m>$
- Problem: The number of particles needed to represent a posterior grows exponentially with the dimension of the state space!

Dependencies

- Is there a dependency between the dimensions of the state space?
- If so, can we use the dependency to solve the problem more efficiently?

- In the SLAM context
  - The map depends on the poses of the robot.
  - We know how to build a map given the position of the sensor is known.

Can We Exploit Dependencies Between the Different Dimensions of the State Space?

$$x_{1:t}\,, m$$

If We Know the Poses of the Robot, Mapping is Easy!

$$\underline{x_{1:t}}\,,\ \underline{m}$$

Key Idea

$$\underline{x_{1:t}}\ ,\ \underline{m}$$

If we use the particle set only to model the robot's path, each sample is a path hypothesis. For each particle, we can compute an individual map using it's path.

Rao-Blackwellization

- Factorization to exploit dependencies between variables:

$$p(a, b) \;=\; p(b \mid a)\, p(a)$$

- If $p(b \mid a)$ can be computed efficiently, represent only $p(a)$ with samples and compute $p(b \mid a)$ for every sample

Courtesy: C. Stachniss

Rao-Blackwellization for SLAM

- Factorization of the SLAM posterior

poses map observations & controls

$$p(x_{1:t}, m \mid z_{1:t}, u_{1:t}) =$$

First introduced for SLAM by Murphy in 1999

K. Murphy, Bayesian map learning in dynamic environments, In Proc. Advances in Neural Information Processing Systems, 1999

Courtesy: C. Stachniss

Rao-Blackwellization for SLAM

- Factorization of the SLAM posterior

poses map observations & controls

$$p(x_{1:t}, m \mid z_{1:t}, u_{1:t}) =$$
$$p(x_{1:t} \mid z_{1:t}, u_{1:t})\, p(m \mid x_{1:t}, z_{1:t})$$

path posterior map posterior

First introduced for SLAM by Murphy in 1999

K. Murphy, Bayesian map learning in dynamic environments, In Proc. Advances in Neural Information Processing Systems, 1999

Courtesy: C. Stachniss

Rao-Blackwellization for SLAM

- Factorization of the SLAM posterior

$$p(x_{1:t}, m \mid z_{1:t}, u_{1:t}) =$$
$$p(x_{1:t} \mid z_{1:t}, u_{1:t})\, \underline{p(m \mid x_{1:t}, z_{1:t})}$$

Grid cells are conditionally independent given the poses

First exploited in FastSLAM by Montemerlo et al., 2002

Courtesy: C. Stachniss

Rao-Blackwellization for SLAM

- Factorization of the SLAM posterior

$$p(x_{1:t}, m \mid z_{1:t}, u_{1:t}) =$$
$$p(x_{1:t} \mid z_{1:t}, u_{1:t})\, p(m \mid x_{1:t}, z_{1:t})$$
$$p(x_{1:t} \mid z_{1:t}, u_{1:t}) \underline{\prod_{i=1}^{N} p(m^i \mid x_{1:t}, z_{1:t})}$$

Grid cells are conditionally independent given the poses

First exploited in FastSLAM by Montemerlo et al., 2002

Courtesy: C. Stachniss

Rao-Blackwellization for SLAM

- Factorization of the SLAM posterior

$$
\begin{aligned}
p(x_{1:t}, m \mid z_{1:t}, u_{1:t}) = \\
p(x_{1:t} \mid z_{1:t}, u_{1:t})\, p(m \mid x_{1:t}, z_{1:t}) \\
\underbrace{p(x_{1:t} \mid z_{1:t}, u_{1:t})}_{\text{particle filter for localization}}\, \underbrace{\prod_{i=1}^{N} p\left(m^i \mid x_{1:t}, z_{1:t}\right)}_{\text{occupancy grid mapping}}
\end{aligned}
$$

First exploited in FastSLAM by Montemerlo et al., 2002

Courtesy: C. Stachniss

Modeling the Robot's Path

- Sample-based representation for
  $p(x_{1:t} \mid z_{1:t}, u_{1:t})$
- Each sample is a path hypothesis

$$x_1 \qquad x_2 \qquad x_3 \qquad \ldots$$

starting location,
typically (0,0,0)

pose hypothesis
at time *t*=2

- Past poses of a sample are not revised
- No need to maintain past poses in the sample set

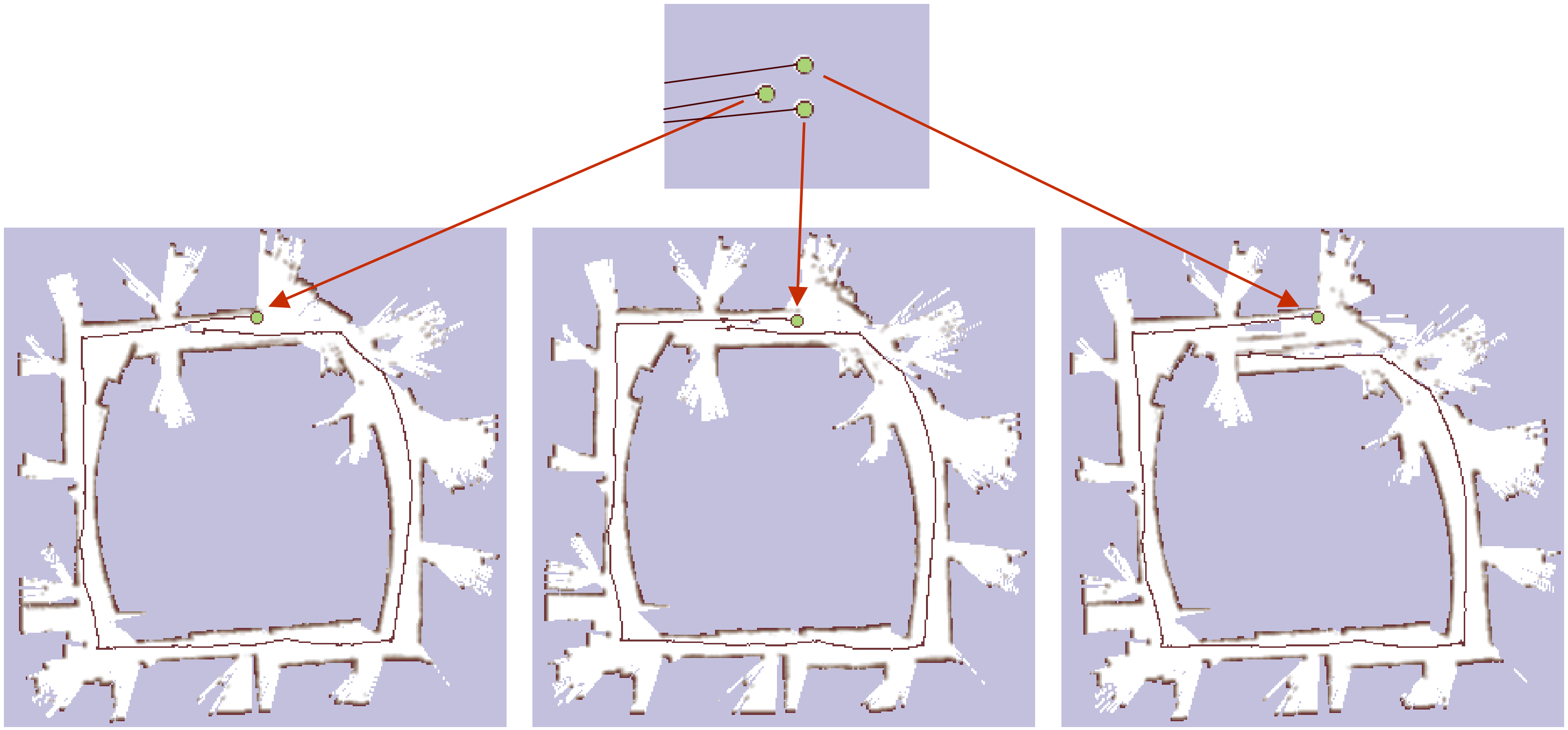
Courtesy: C. Stachniss

FastSLAM

- Proposed by Montemerlo et al. in 2002 (for landmark based SLAM)
- Each particle has a pose and a map

Particle 1	x, y, θ	**Occupancy grid map**
Particle 2	x, y, θ	**Occupancy grid map**
$\vdots$		
Particle *N*	x, y, θ	**Occupancy grid map**

FastSLAM – Particle representation



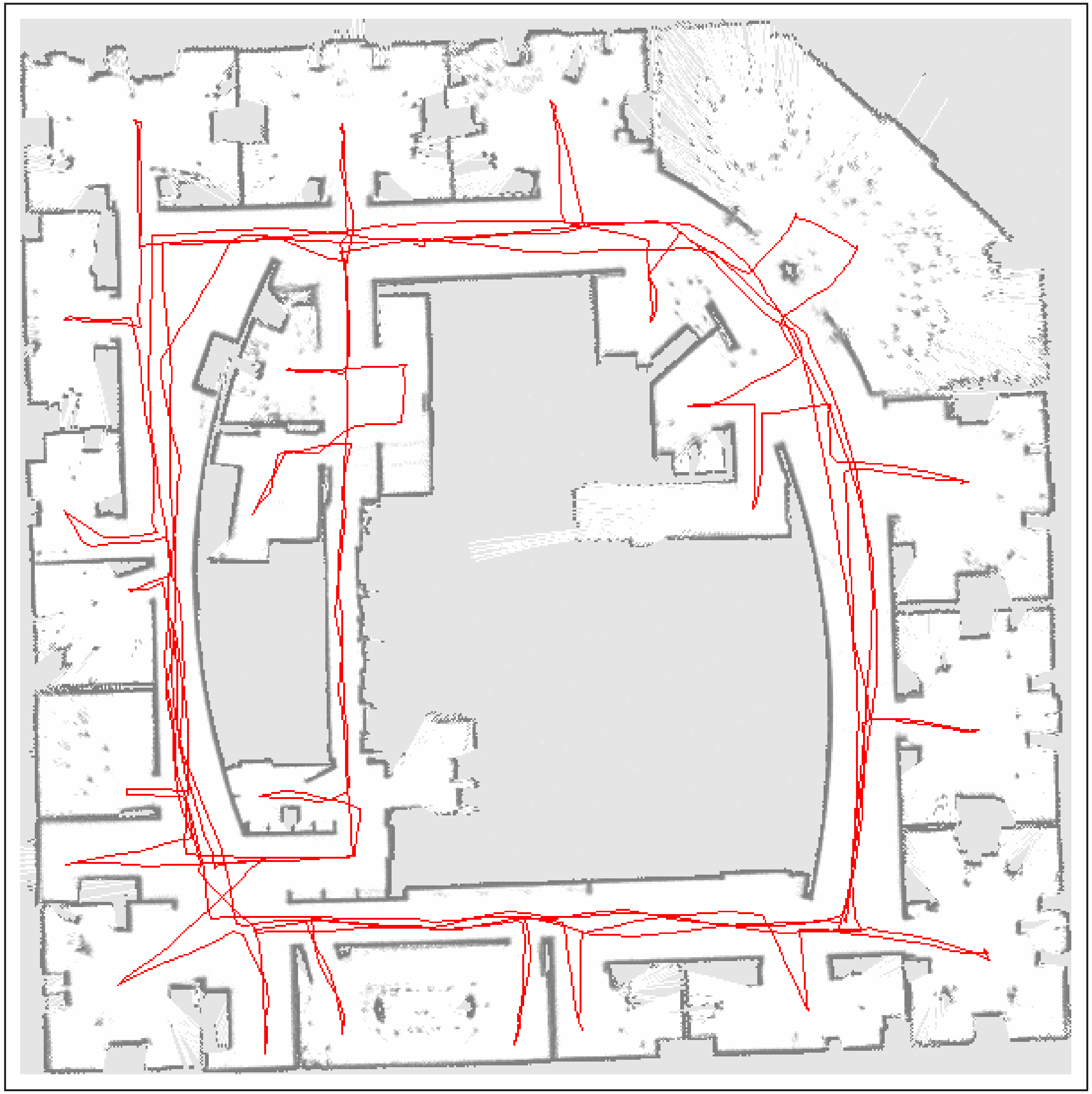
FastSLAM Algorithm

Algorithm FastSLAM_occupancy_grids($\mathcal{X}_{t-1}, u_t, z_t$**):**

$\bar{\mathcal{X}}_t = \mathcal{X}_t = \emptyset$
for $k = 1$ *to* M *do*
$x_t^{[k]} = \textbf{sample\_motion\_model}(u_t, x_{t-1}^{[k]})$
$w_t^{[k]} = \textbf{measurement\_model\_map}(z_t, x_t^{[k]}, m_{t-1}^{[k]})$
$m_t^{[k]} = \textbf{updated\_occupancy\_grid}(z_t, x_t^{[k]}, m_{t-1}^{[k]})$
$\bar{\mathcal{X}}_t = \bar{\mathcal{X}}_t + \langle x_t^{[k]}, m_t^{[k]}, w_t^{[k]} \rangle$
endfor
for $k = 1$ *to* M *do*
draw i *with probability* $\propto w_t^{[i]}$
add $\langle x_t^{[i]}, m_t^{[i]} \rangle$ *to* $\mathcal{X}_t$
endfor
return $\mathcal{X}_t$

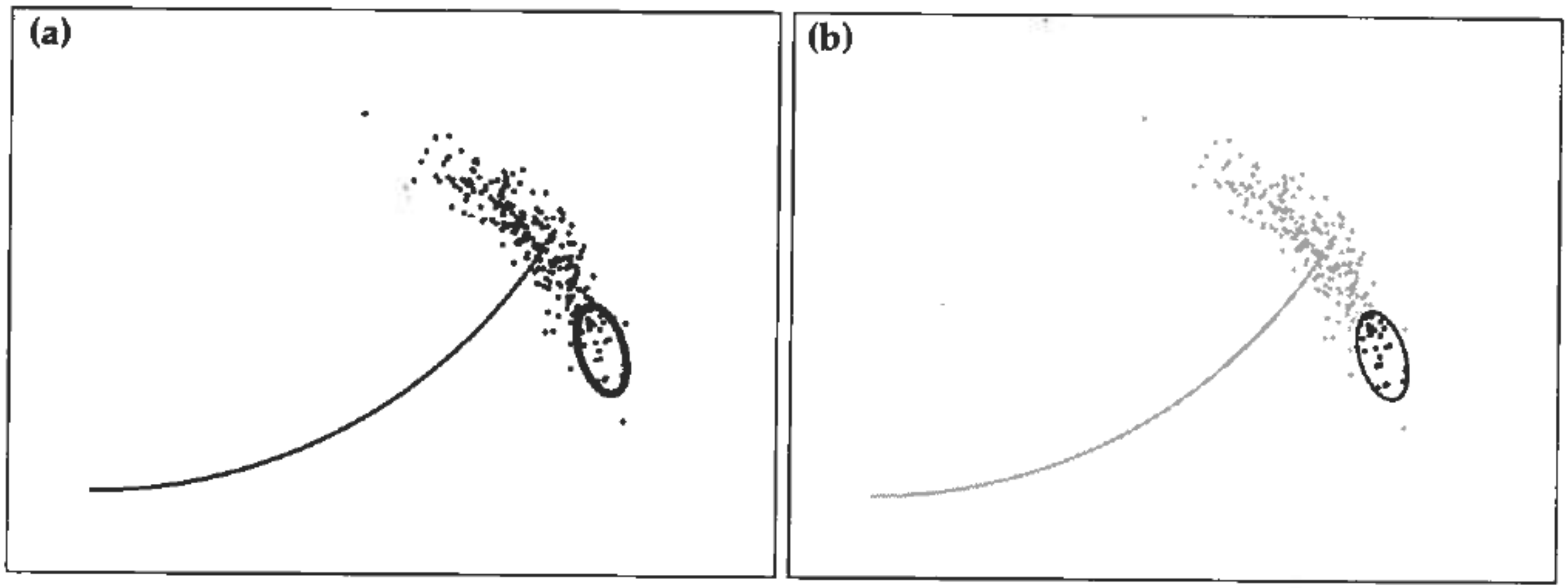
Pure odometry

FastSLAM – Best particle



Weakness of FastSLAM 1.0

- Proposal Distribution
- Importance weighting



FastSLAM 1.0 to FastSLAM 2.0

- FastSLAM 1.0 uses the motion model as the proposal distribution

$$x_t^{[k]} \sim p(x_t \mid x_{t-1}^{[k]}, u_t)$$

- FastSLAM 2.0 **considers also the measurements during sampling**
- Especially useful if an accurate sensor is used (compared to the motion noise)

[Montemerlo et al., 2003]

Courtesy: C. Stachniss

FastSLAM 2.0 (Informally)

- FastSLAM 2.0 samples from

$$x_t^{[k]} \sim p(x_t \mid x_{1:t-1}^{[k]}, u_{1:t}, z_{1:t})$$

- Results in a more peaked proposal distribution
- Less particles are required
- More robust and accurate
- But more complex…

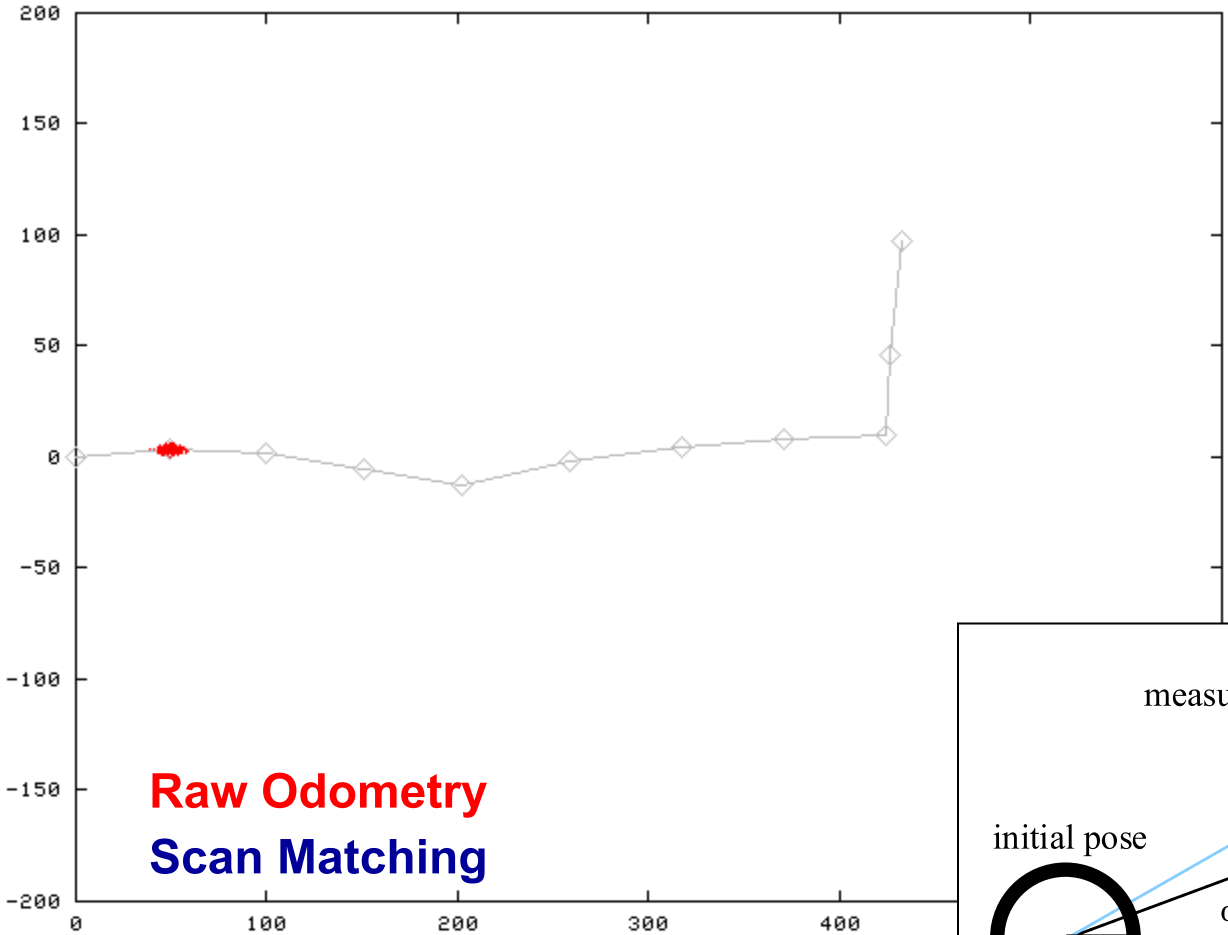
[Montemerlo et al., 2003]

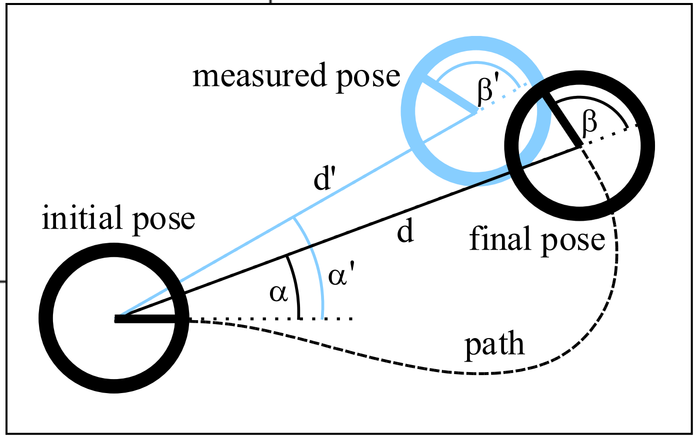
Courtesy: C. Stachniss

Generating better proposals

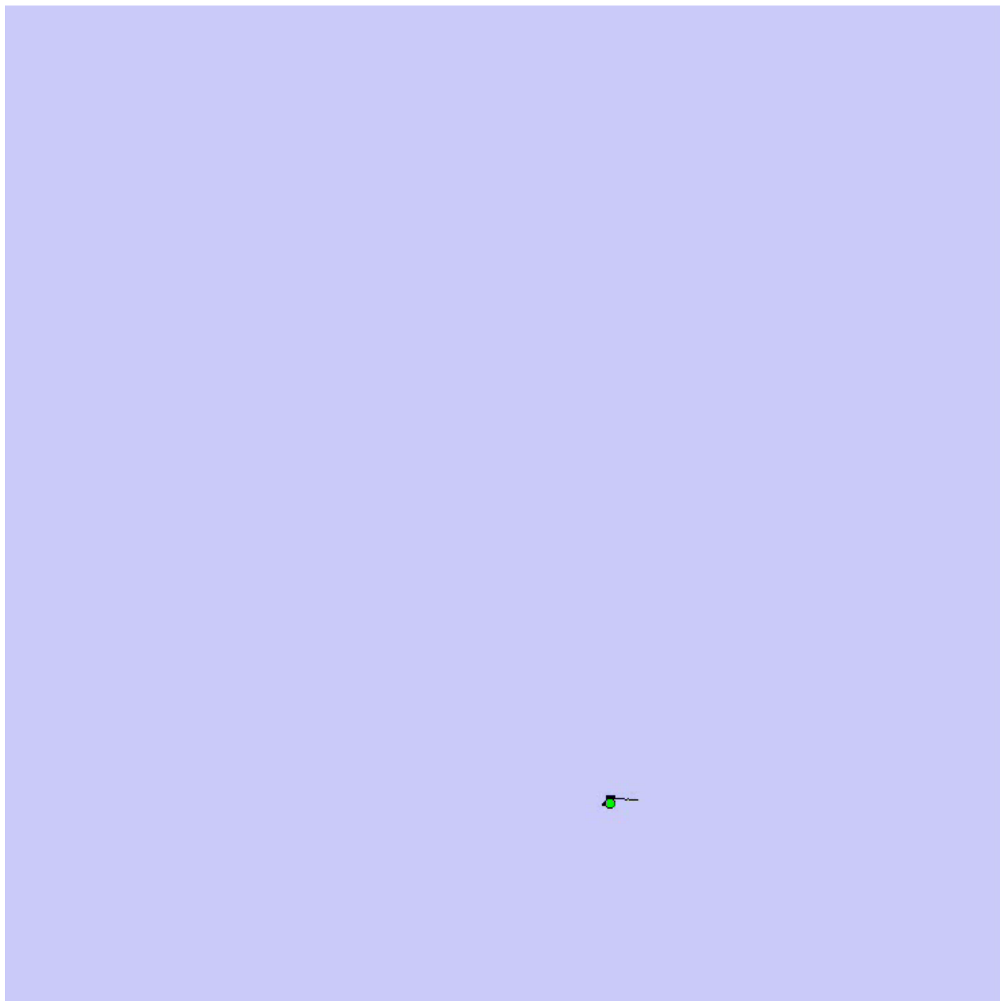
- Use scan-matching to compute highly accurate odometry measurements from consecutive range scans.
- Use the improved odometry in the prediction step to get highly accurate proposal distributions.

Motion Model for Scan Matching





Rao-Blackwellized Mapping with Scan-Matching



Map: Intel Research Lab Seattle

Loop Closure

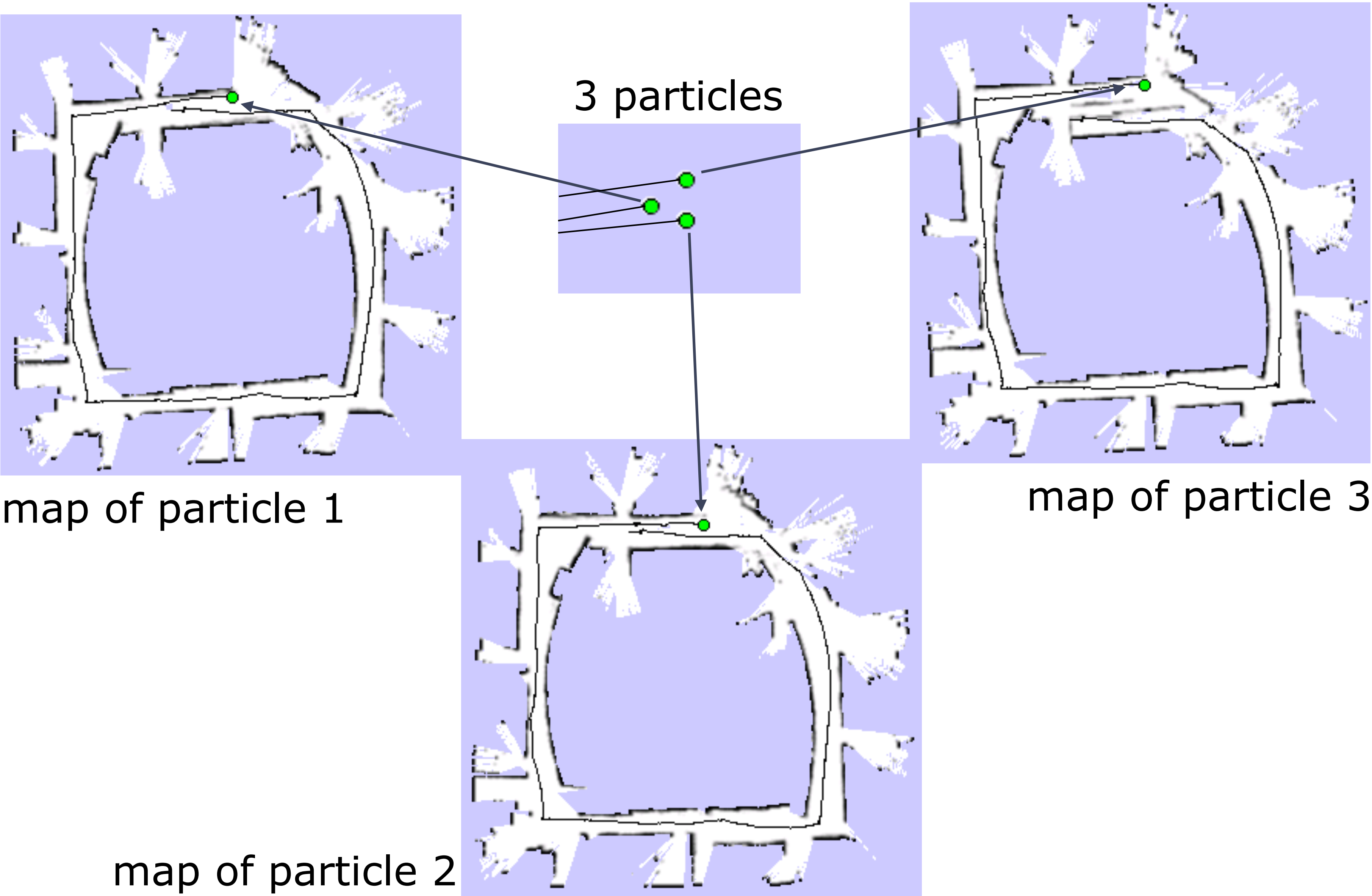
- Loop closure involves
  - Recognizing when the robot has returned to a previously mapped region
  - Using this information to reduce the uncertainty in the map estimate

- Without loop closure, the uncertainty can grow without bounds

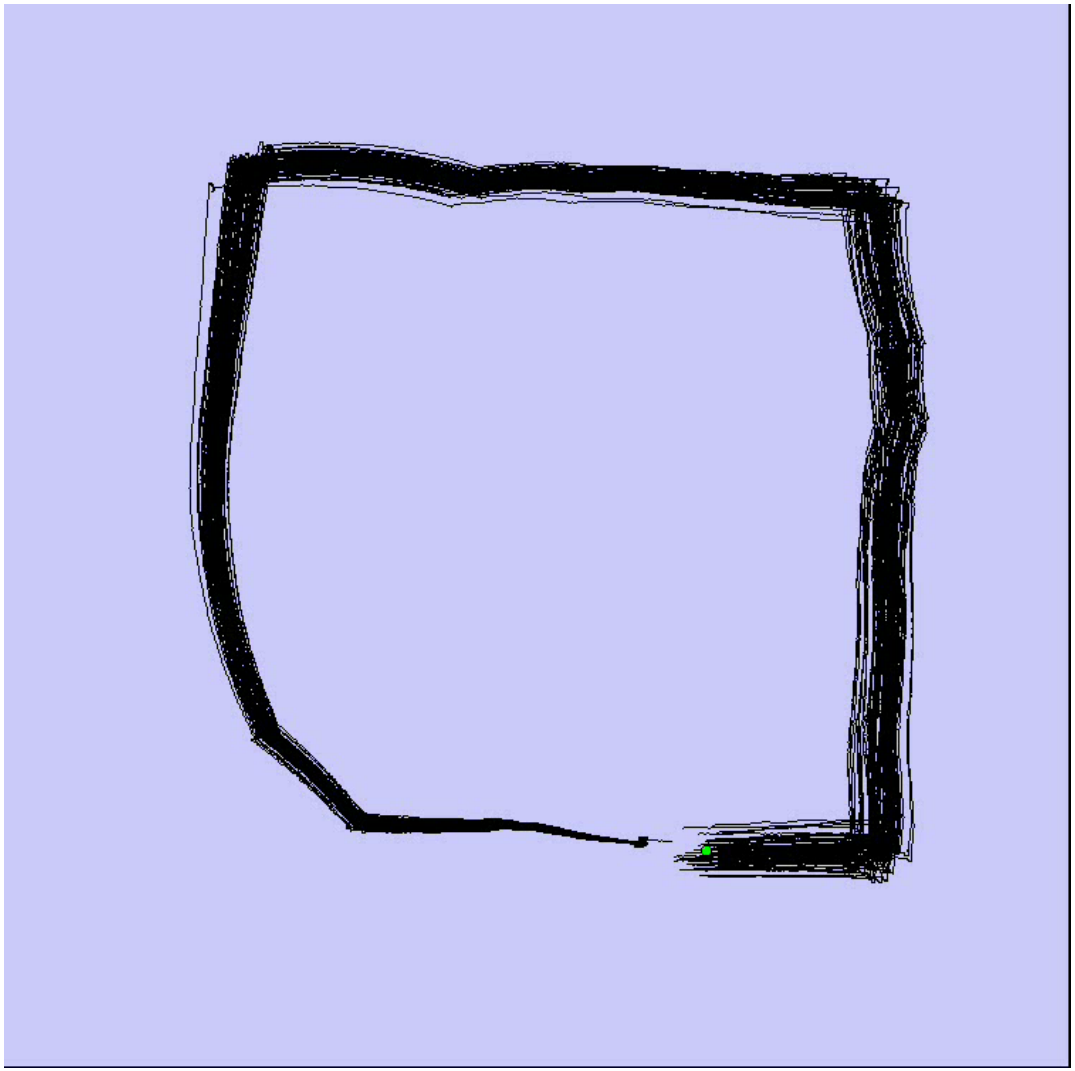
Loop Closure in FastSLAM

- Each particle has it’s own map
- Maps which agree to closing the loop are weighed higher than others
- These maps are more likely to be resampled
- **Key:** Need diversity of paths/particles/maps

Loop Closure Example

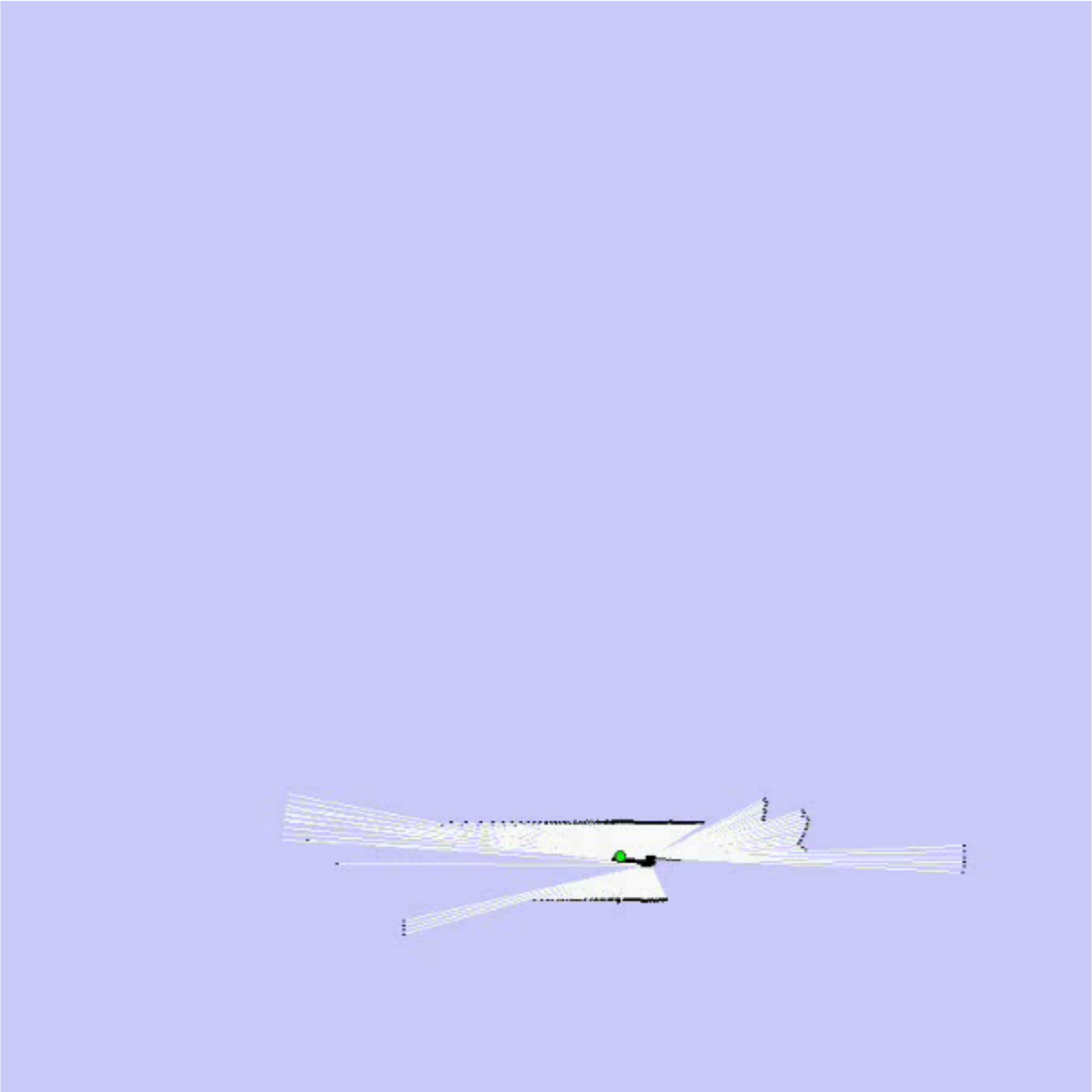


Rao-Blackwellized Mapping with Scan-Matching



Map: Intel Research Lab Seattle

Rao-Blackwellized Mapping with Scan-Matching



Map: Intel Research Lab Seattle

Example (Intel Lab)

- **15 particles**
- four times faster than real-time P4, 2.8GHz
- 5cm resolution during scan matching
- 1cm resolution in final map

Work by Grisetti et al.

Outdoor Campus Map

- **30 particles**
- 250x250m^2
- 1.088 miles (odometry)
- 20cm resolution during scan matching
- 30cm resolution in final map

Work by Grisetti et al.

FastSLAM Summary

- Particle filter-based SLAM
- Rao-Blackwellization: model the robot's path by sampling and compute the landmarks given the poses
- Allow for per-particle data association
- FastSLAM 1.0 and 2.0 differ in the proposal distribution
- Complexity $\mathcal{O}(N \log M)$

Courtesy: C. Stachniss

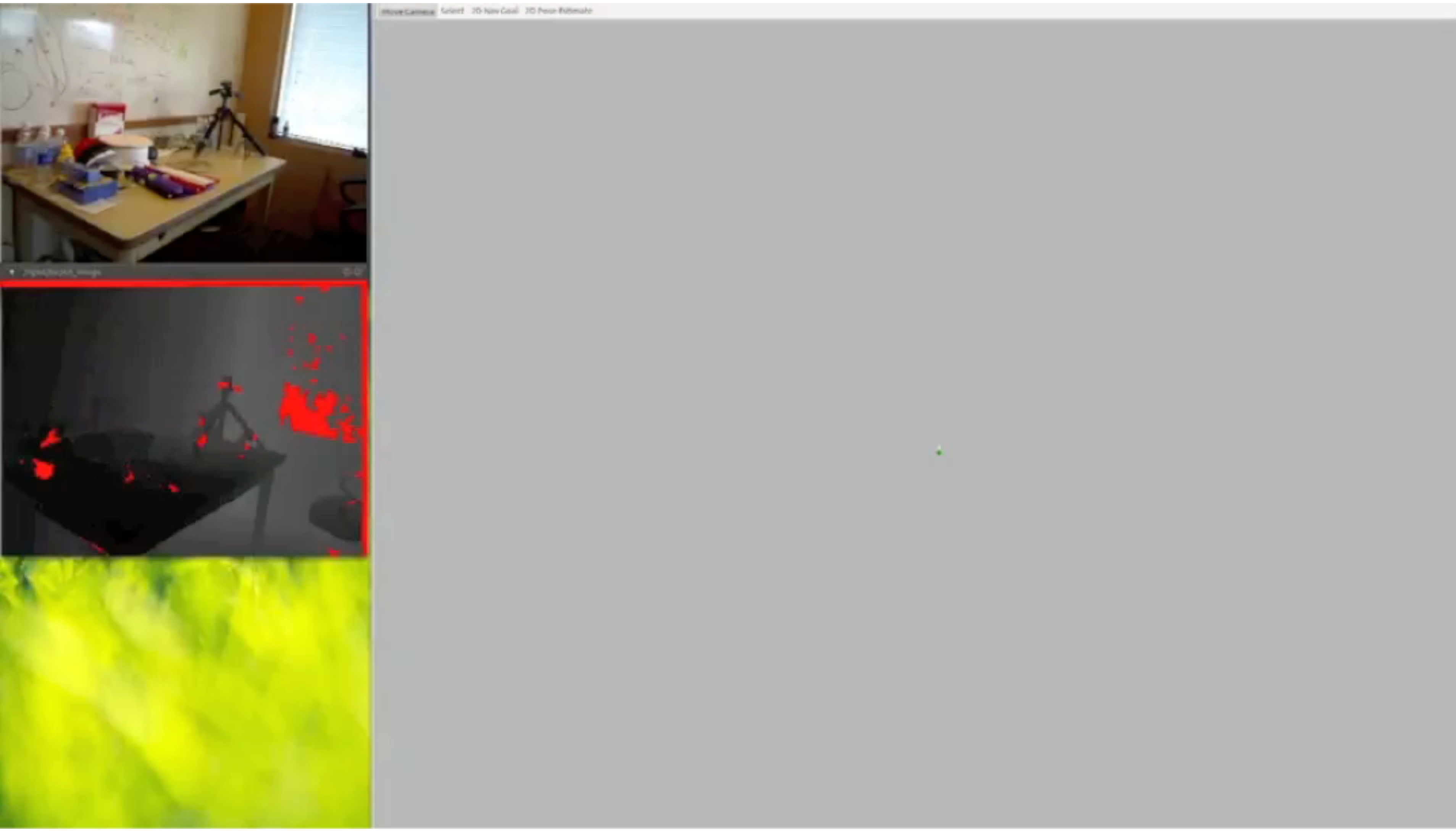
Literature

FastSLAM

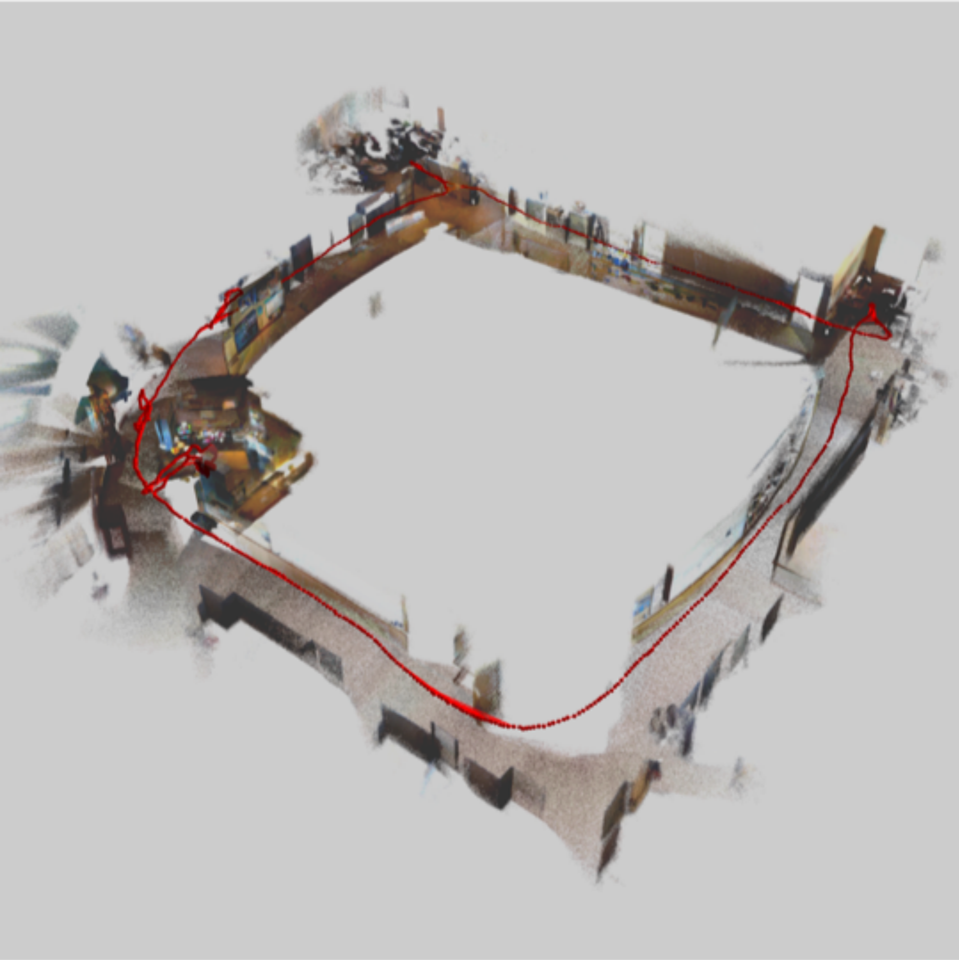
- Thrun et al.: “Probabilistic Robotics”, Chapter 13.1-13.3 + 13.8 (see errata!)
- Montemerlo, Thrun, Kollar, Wegbreit: FastSLAM: A Factored Solution to the Simultaneous Localization and Mapping Problem, 2002
- Montemerlo and Thrun: Simultaneous Localization and Mapping with Unknown Data Association Using FastSLAM, 2003

Courtesy: C. Stachniss

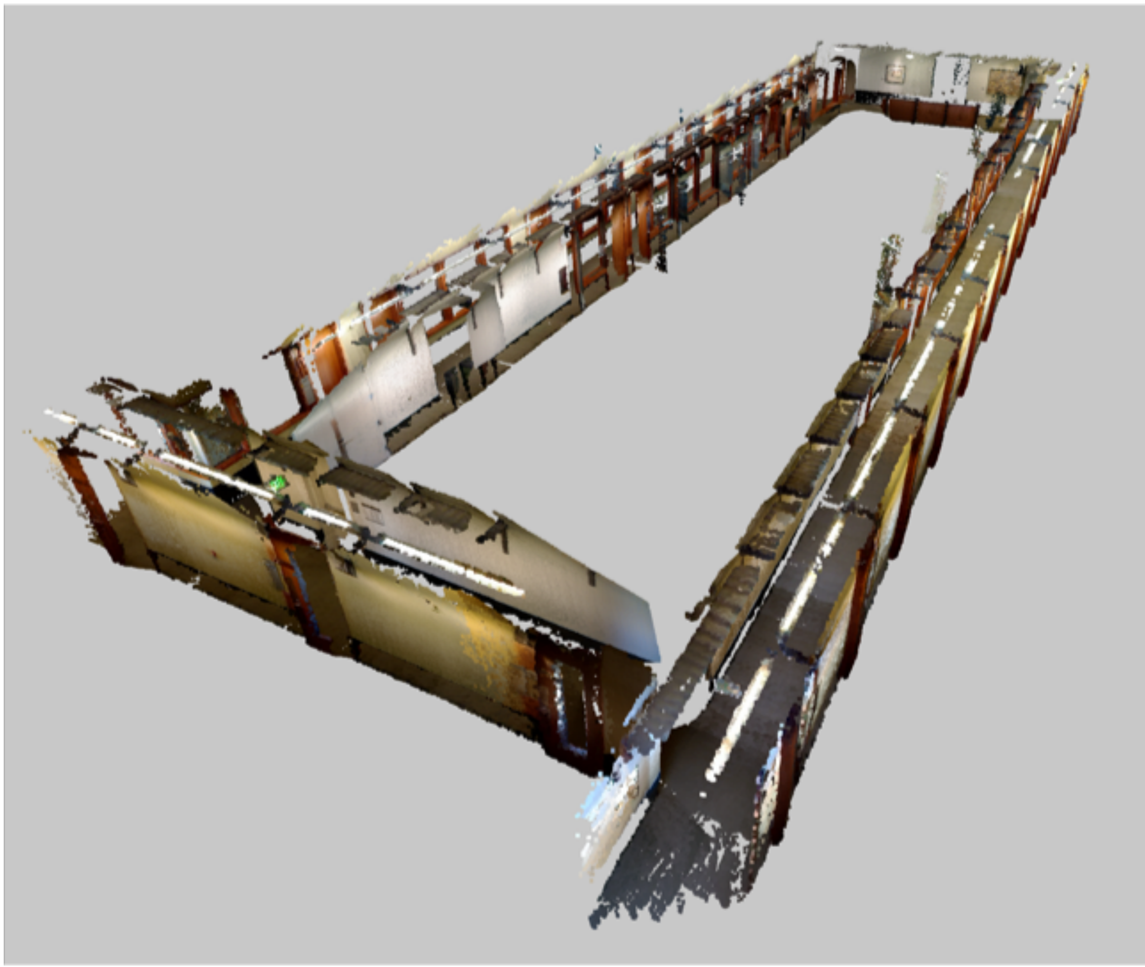
RGBD SLAM

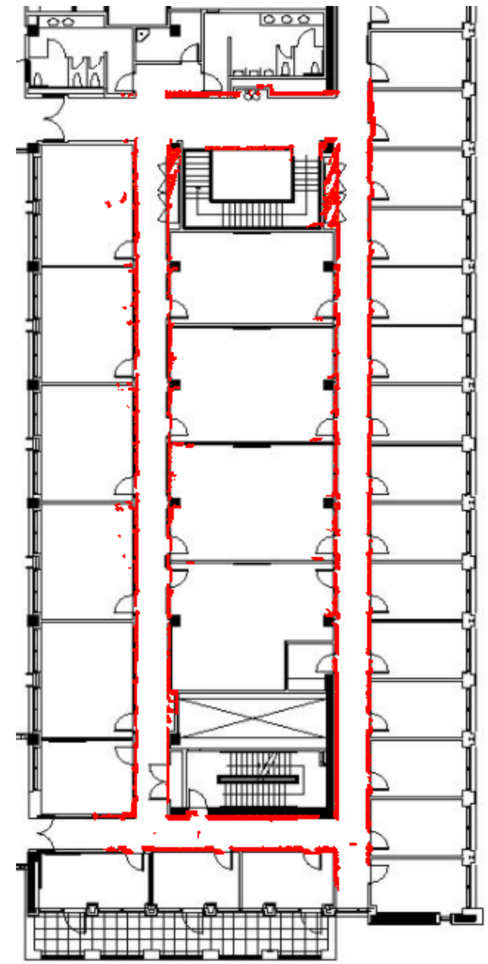


Resulting Map



Experiments: Overlay 1





Experiments: Overlay 2