Lecture 4: Mapping

Build a model (map) given the robot knows its location, from uncertain and noisy sensor data → Opposite of Localization

How? → Fiducials
→ GPS (outside) } location without map

Maps: Feature vs. volumetric
World-coordinate based vs. robot-coordinate based

Maps
- Metric (geometric properties)
  - Grids
    - Occupancy grid mapping (Elfes, Moravec – 85)
  - Shapes (Chatila, Laumond – 85)
- Topological (Connectivity properties)
  - ↳ Matarić (90)

Occupancy Grid Mapping: Represent a map of the environment as an evenly-spaced field of binary random variables

↳ solve as filtering problem.

↳ Calculate approx. posterior estimates of these random variables

Map m (grid) $= \{m_i\}$ collection of finitely many cells

Want to find: $p(m \mid z_{1:t}, x_{1:t})$ → Path is known, so, $u_{1:t}$ play no role!

↳ Collection of separate problems

So, $p(m \mid z_{1:t}, x_{1:t}) = \prod_i p(m_i \mid z_{1:t}, x_{1:t})$

↳ This assumption may not hold true always.

↳ More complicated: Use adjacency info (Joint probabilities)

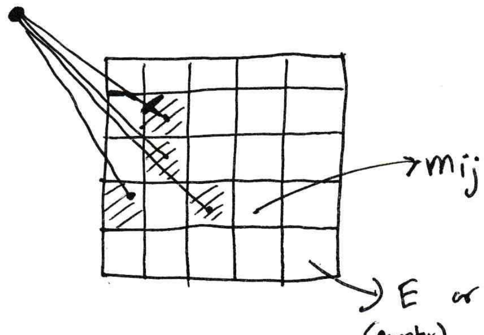
Two components:

State model / motion model: Map is static (Map is the state of the world)

↳ No explicit motion model

Sensor model / Measurement model: $p(z_t \mid m, x_t)$

Example :- 2D grid with LIDAR data



HITS :- When laser ~~detects~~ detects object

PASSTHROUGHS :- It doesn't detect or passes through.

<u>Binary</u> → Binary bayes filter to update each cell.

<u>Priors</u> :- $P(m_{ij}) = P_F(m_{ij})$ = Probability that m_{ij} is filled

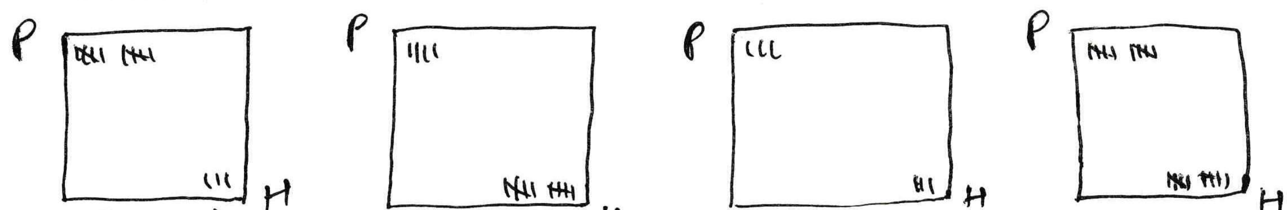
$= 1 - P_E(m_{ij})$

a. Uniform : 0.5

b. More complicated : Adjacency info. (Joint priors)

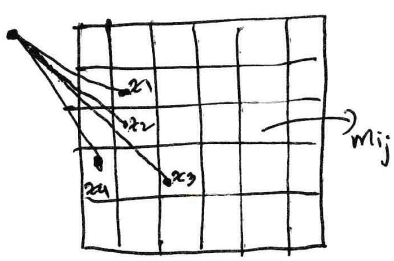
<u>Sensor model</u> :- $P(z_t \mid m_{ij}, x_t)$

<u>Examples of sensor data for 1 cell:</u>



(Empty) (Filled) (Not enough info) Door/glass (based on angle of incidence)

Unusual artifacts



m_{ij} location = E/F (labels)

$z_1, z_2, \ldots z_N$ → sensor measurements (One laser scan z)

P.T.O

③

$$P_F(m_{ij}|z_{1:t},x_{1:t}) = \frac{P(z_t|m_{ij},z_{1:t-1},\overset{Ⓐ}{x_{1:t}})\;P(m_{ij}|z_{1:t-1},x_{1:t})^{Ⓑ}}{P(z_t|z_{1:t-1},x_{1:t})}$$

Using Markov assumption,

Ⓐ $P(z_t|m_{ij},z_{1:t-1},x_{1:t}) = P(z_t|m_{ij},x_t) = \dfrac{P(m_{ij}|z_t,x_t)\,P(z_t|x_t)}{P(m_{ij}|x_t)}$

Ⓑ $P(m_{ij}|z_{1:t-1},x_{1:t}) = P(m_{ij}|z_{1:t-1},x_{1:t-1})$

So, $$P_F(m_{ij}|z_{1:t},x_{1:t}) = \frac{P(m_{ij}|z_t,x_t)P(z_t|x_t)P(m_{ij}|z_{1:t-1},x_{1:t-1})}{P(m_{ij}|x_t)\,P(z_t|z_{1:t-1},x_{1:t})}$$

Also, assuming probability that a map cell is filled does not depend on the location of the robot, -

$$P(m_{ij}|x_t) = P(m_{ij})$$

So, $$P_F(m_{ij}|z_{1:t},x_{1:t}) = \frac{P_F(m_{ij}|z_{1:t-1},x_{1:t-1})\cdot P_F(m_{ij}|z_t,x_t)\cdot P(z_t|x_t)}{P(z_t|z_{1:t-1},x_{1:t})\;P_F(m_{ij})}$$

So, $$P_E(m_{ij}|z_{1:t},x_{1:t}) = \frac{P_E(m_{ij}|z_{1:t-1},x_{1:t-1})\cdot P_E(m_{ij}|z_t,x_t)\cdot P(z_t|x_t)}{P(z_t|z_{1:t-1},x_{1:t})\;P_E(m_{ij})}$$

So, ODDS RATIO:

$$\frac{P_F(m_{ij}|z_{1:t},x_{1:t})}{P_E(m_{ij}|z_{1:t},x_{1:t})} = \frac{P_F(m_{ij}|z_{1:t-1},x_{1:t-1})}{P_E(m_{ij}|z_{1:t-1},x_{1:t-1})}\cdot\frac{P_F(m_{ij}|z_t,x_t)}{P_E(m_{ij}|z_t,x_t)}\cdot\frac{P_E(m_{ij})}{P_F(m_{ij})}$$

Due to underflow error, taking Log:

$$\underbrace{\log\left(\frac{P_F(m_{ij}|z_{1:t},x_{1:t})}{P_E(m_{ij}|z_{1:t},x_{1:t})}\right)}_{l_{t,ij}} = \underbrace{\log\left(\frac{P_F(m_{ij}|z_{1:t-1},x_{1:t-1})}{P_E(m_{ij}|z_{1:t-1},x_{1:t-1})}\right)}_{l_{t-1,ij}} + \log\left(\frac{P_F(m_{ij}|z_t,x_t)}{P_E(m_{ij}|z_t,x_t)}\right) - \log\left(\frac{P_F(m_{ij})}{P_E(m_{ij})}\right)$$

Given $P_F = P$

$P_E = 1 - P_F = 1 - P$

P.T.O

$$l_{t,ij} = \log\left(\frac{p(m_{ij} | z_{1:t}, x_{1:t})}{1 - p(m_{ij} | z_{1:t}, x_{1:t})}\right) = \log \underline{\text{odds ratio}}$$

$$l_{t-1,ij} = \log\left(\frac{p(m_{ij} | z_{1:t-1}, x_{1:t-1})}{1 - p(m_{ij} | z_{1:t-1}, x_{1:t-1})}\right)$$

$$l_0 = \log\left(\frac{p(m_{ij})}{1 - p(m_{ij})}\right)$$

$$\text{inverse sensor model} = \log\left(\frac{p(m_{ij} | z_t, x_t)}{1 - p(m_{ij} | z_t, x_t)}\right)$$

So, $l_{t,ij} = l_{t-1,ij} + \text{Inverse sensor model} - l_0$ → for 2D

for 1D : $l_{t,i} = l_{t-1,i} + \text{Inverse sensor model} - l_0$

<u>Algorithm</u> occupancy-grid-mapping $(\{l_{t-1,i}\}, x_t, z_t)$: <u>1D-example</u>

for each cell m_i do:
  if m_i in perceptual field of z_t then
    $l_{t,i} = l_{t-1,i}$ + inverse sensor model $(m_i, x_t, z_t) - l_0$
  else
    $l_{t,i} = l_{t-1,i}$
  endif
endfor
return $\{l_{t,i}\}$

<u>For laser data, inverse sensor model</u>

$$p(m_{ij} | z_t, x_t) = \eta\, p(z_t | m_{ij}, x_t)\, p(m_{ij} | x_t)$$

$$= \eta \prod_{k=1}^{N} p(z_{tk} | m_{ij}, x_t) \cdot p(m_{ij})$$

→ Map doesn't depend on pose!

→ independent laser beams assumption!

<u>Multi-Sensor fusion</u>

$$p(m_i) = 1 - \prod_k \left(1 - p(m_i^k)\right)$$ → independent maps by each sensor

or pessimistic,

$$p(m_i) = \max_k p(m_i^k).$$