CSE-571
Probabilistic Robotics

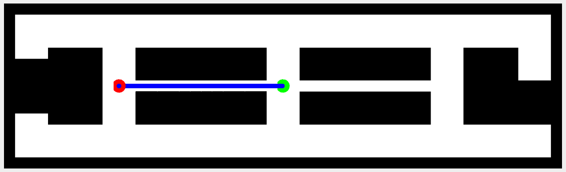
Planning and Control:

Markov Decision Processes

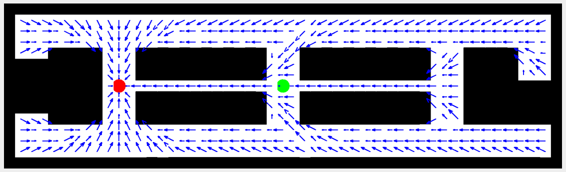
Problem Classes

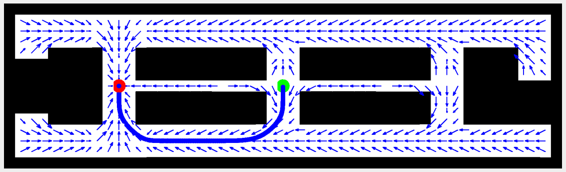
- Deterministic vs. stochastic actions
- Full vs. partial observability

Deterministic, fully observable

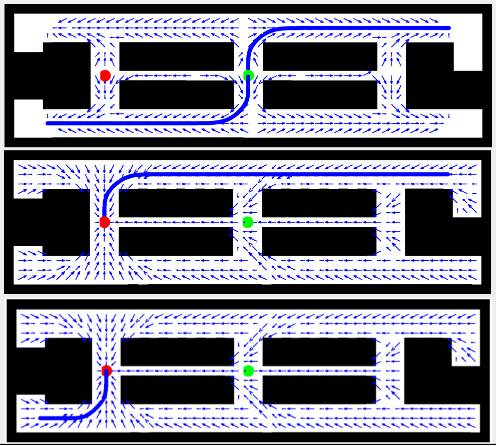


Stochastic, Fully Observable

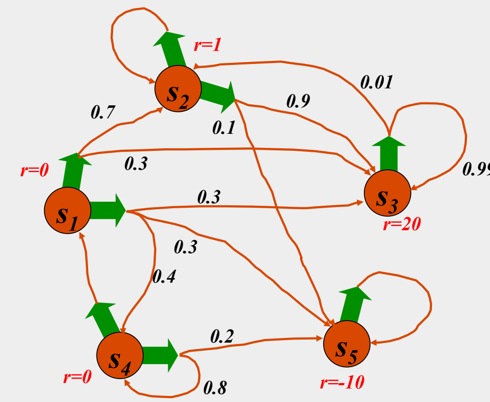




Stochastic, Partially Observable



Markov Decision Process (MDP)



Markov Decision Process (MDP)

- **Given:**
- States x
- Actions u
- Transition probabilities $p(x'|u,x)$
- Reward / payoff function $r(x,u)$

- **Wanted:**
- Policy $p(x)$ that maximizes the future expected reward

Rewards and Policies

- Policy (general case):

$$\pi : \quad z_{1:t-1}, u_{1:t-1} \rightarrow \quad u_t$$

- Policy (fully observable case):

$$\pi : \quad x_t \rightarrow \quad u_t$$

- Expected cumulative payoff:

$$R_T = E \left[\sum_{\tau=1}^{T} \gamma^\tau r_{t+\tau} \right]$$

  - T=1: greedy policy
  - T>1: finite horizon case, typically no discount
  - T=infty: infinite-horizon case, finite reward if discount < 1

Policies contd.

- Expected cumulative payoff of policy:

$$R_T^{\pi}(x_t) = E\left[\sum_{\tau=1}^{T} \gamma^{\tau} r_{t+\tau} \mid u_{t+\tau} = \pi(z_{1:t+\tau-1} u_{1:t+\tau-1})\right]$$

- Optimal policy:

$$\pi^* = \underset{\pi}{\operatorname{argmax}} \quad R_T^{\pi}(x_t)$$

- 1-step optimal policy:

$$\pi_1(x) = \underset{u}{\operatorname{argmax}} \quad r(x,u)$$

- Value function of 1-step optimal policy:

$$V_1(x) = \gamma \max_u \; r(x,u)$$

2-step Policies

- Optimal policy:

$$\pi_2(x) = \underset{u}{\operatorname{argmax}} \quad \left[r(x,u) + \int V_1(x') p(x' \mid u,x) dx'\right]$$

- Value function:

$$V_2(x) = \gamma \max_u \quad \left[r(x,u) + \int V_1(x') p(x' \mid u,x) dx'\right]$$

T-step Policies

- Optimal policy:

$$\pi_T(x) = \underset{u}{\operatorname{argmax}} \quad \left[r(x,u) + \int V_{T-1}(x') p(x' \mid u,x) dx'\right]$$

- Value function:

$$V_T(x) = \gamma \max_u \quad \left[r(x,u) + \int V_{T-1}(x') p(x' \mid u,x) dx'\right]$$

Infinite Horizon

- Optimal policy:

$$V_\infty(x) = \gamma \max_u \quad \left[r(x,u) + \int V_\infty(x') p(x' \mid u,x) dx'\right]$$

- Bellman equation
- Fix point is optimal policy
- Necessary and sufficient condition

Value Iteration

- for all x do

$$\hat{V}(x) \leftarrow r_{\min}$$

- endfor

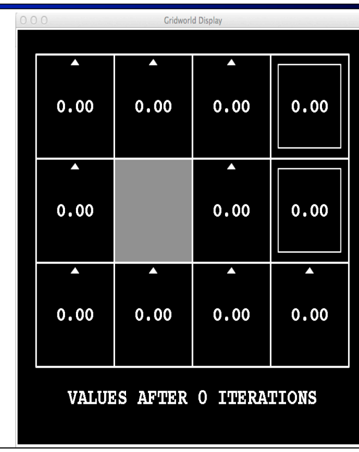
- repeat until convergence
  - for all x do

$$\hat{V}(x) \leftarrow \gamma \max_u \left[r(x,u) + \int \hat{V}(x') p(x' \mid u,x) dx' \right]$$

  - endfor
- endrepeat

$$\pi(x) = \operatorname*{argmax}_u \left[r(x,u) + \int \hat{V}(x') p(x' \mid u,x) dx' \right]$$

k=0

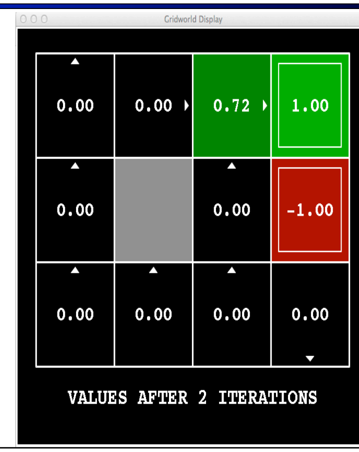


Noise = 0.2
Discount = 0.9
Living reward = 0

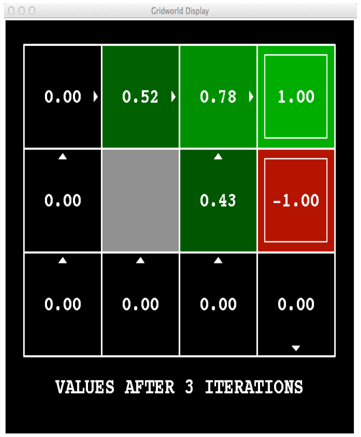
k=1

Noise = 0.2
Discount = 0.9
Living reward = 0

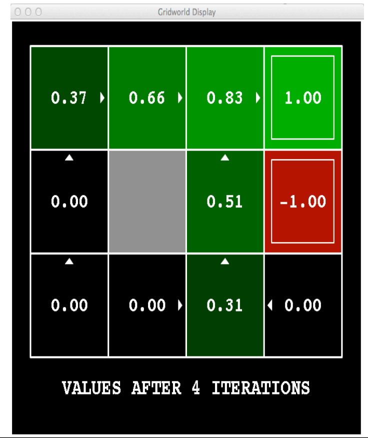
k=2



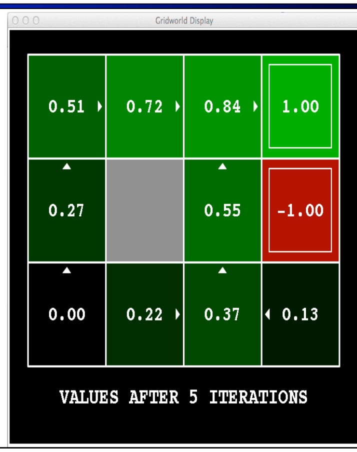
k=3


Gridworld Display
0.00
0.52
0.78
1.00
0.00
0.43
-1.00
0.00
0.00
0.00
0.00
VALUES AFTER 3 ITERATIONS

k=4


Gridworld Display
0.37
0.66
0.83
1.00
0.00
0.51
-1.00
0.00
0.00
0.31
0.00
VALUES AFTER 4 ITERATIONS

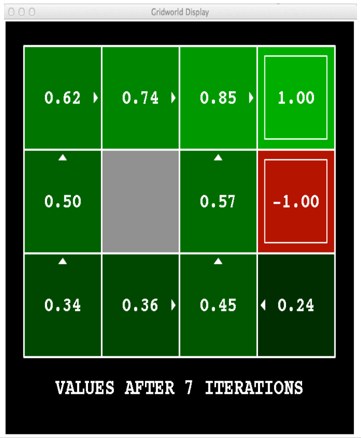
k=5


Gridworld Display
0.51
0.72
0.84
1.00
0.27
0.55
-1.00
0.00
0.22
0.37
0.13
VALUES AFTER 5 ITERATIONS

k=6


Gridworld Display
0.59
0.73
0.85
1.00
0.41
0.57
-1.00
0.21
0.31
0.43
0.19
VALUES AFTER 6 ITERATIONS

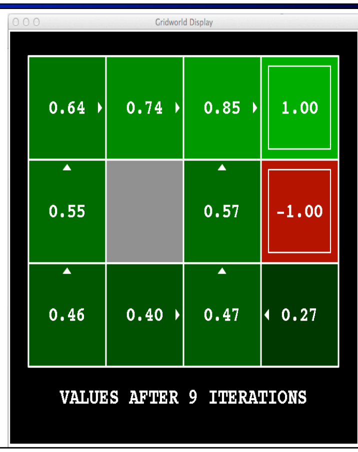
k=7


0.62
0.74
0.85
1.00
0.50
0.57
-1.00
0.34
0.36
0.45
0.24
VALUES AFTER 7 ITERATIONS

k=8


0.63
0.74
0.85
1.00
0.53
0.57
-1.00
0.42
0.39
0.46
0.26
VALUES AFTER 8 ITERATIONS

k=9


0.64
0.74
0.85
1.00
0.55
0.57
-1.00
0.46
0.40
0.47
0.27
VALUES AFTER 9 ITERATIONS

k=10


0.64
0.74
0.85
1.00
0.56
0.57
-1.00
0.48
0.41
0.47
0.27
VALUES AFTER 10 ITERATIONS

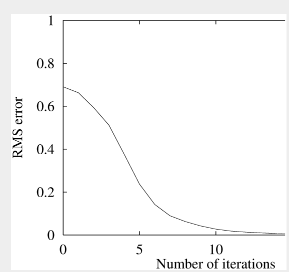
k=11

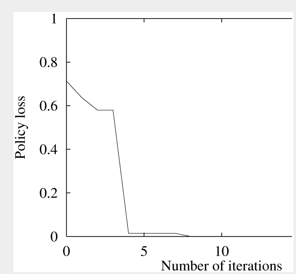
k=12

k=100

Value Function and Policy

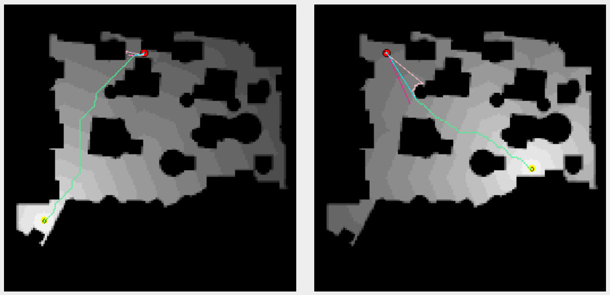
- Each step takes O(|A| |S| |S|) time.
- Number of iterations required is polynomial in |S|, |A|, 1/(1-gamma)





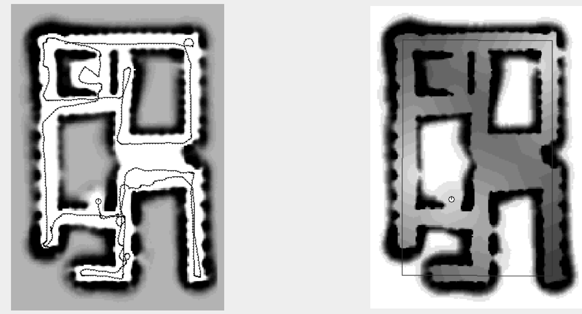
Value Iteration for Motion Planning

(assumes knowledge of robot' s location)

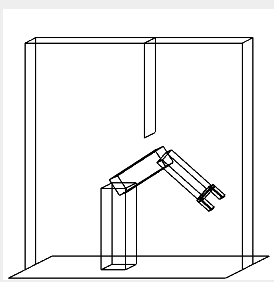


Frontier-based Exploration

- Every unknown location is a target point.



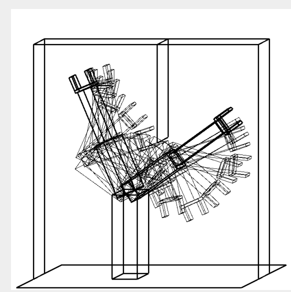
Manipulator Control



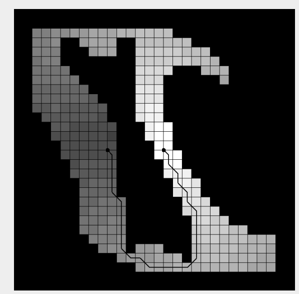
Arm with two joints

Configuration space

Manipulator Control Path

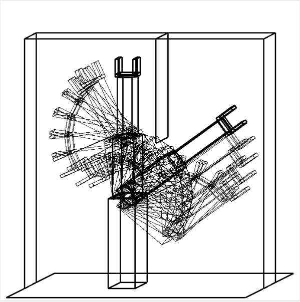


State space

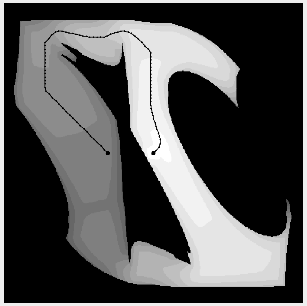


Configuration space

Manipulator Control Path



State space



Configuration space

POMDPs

- In POMDPs we apply the very same idea as in MDPs.
- **Since the state is not observable**, the agent has to **make its decisions based on** the belief state which is a **posterior distribution over states**.
- For finite horizon problems, the resulting value functions are piecewise linear and convex.
- In each iteration the **number of linear constraints grows exponentially**.
- Full fledged POMDPs have only been applied to very small state spaces with small numbers of possible observations and actions.
- **Approximate solutions are becoming more and more capable**.